Trading Tasks and Quality

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Abstract

I present a trade model featuring North-South differences in demand for quality and in quality of task supply. The model explains a number of stylised facts: Southern firms charge higher factory-gate prices for their products in rich than in poor, and in distant than in near markets. The model predicts that firms vary the quality of their products across markets by changing, between varieties, the fractions of low and high-quality tasks. This mechanism for quality differentiation introduces a new margin to trade: the extensive margin of intermediate imports. Extension of the model to general equilibrium with heterogeneous firms shows that even under low fixed and zero variable trade costs, only the more productive Southern firms export to the rich Northern market. Compared to their domestic market, they charge higher prices in the North, with the most productive ones earning higher revenues.

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1 Introduction

In 2004, Regal, a Turkish brand of white goods, launched a successful series of TV commercials. Each commercial advertised a particular product of Regal. The plot was the following. Inside an interrogation room, there is an interrogator and a customer. The interrogator offers the customer two brands of a product—a luxury brand and Regal. Then, he explains to the customer that the Regal product shares the same features as the luxury one, but cheaper. Finally, the interrogator asks her to choose between the two, and she invariably goes for the luxury one. The plot ends with the interrogator insulting the customer because of her *irrational* choice.

The commercials would have attracted even greater public interest, if the interrogator had also explained that the two products were manufactured using the same machinery and workers in the same product line in the same factory located in Manisa, Turkey. Regal is the low-end brand of Vestel Group which is one of the leading manufacturers of consumer durables in Turkey. The Group sells its products under different brands in the domestic, Middle-Eastern and European markets. In each product line, the Group’s factory produces multiple brands that differ slightly in their appearance but share the same features. The brands, however, differ significantly from each other in terms of price, with SEG being the lowest-priced one.

In this paper, I develop an international trade model that rationalises Vestel Group’s strategy—which, as I will argue below, is quite common for Southern (developing) country exporters. In the model, a firm adapts its product quality to meet local demand—known as quality-to-market. In doing so, it sources some of its inputs from multiple suppliers—known as multi-sourcing. In short, the model predicts that a firm adapts its product quality to meet local demand by changing the fractions of low and high-quality inputs across its product varieties. The varieties can share the same features and look almost the same, but inside they are different. Who knows, maybe the customer appearing in the commercials of Regal was making a rational choice!

Anecdotal evidence for the validity of multi-sourcing as a mechanism for within-firm quality differentiation comes from an interview that I conducted with an advisor to the Customs Administration of Turkey. Based on his extensive experience with Turkish exporters, he provided various examples of how Turkish manufacturers vary the quality of their inputs to produce different versions of the same product. Such strategies, he argued, are common among firms operating particularly in textiles, consumer durables and electronics. In one example, he described a manufacturer of electric cookers. The manufacturer produces two identical-looking electric cookers, but one is more expensive than the other and sold mostly in the European market. In the

\[1\] For instance, it sells Regal only in the domestic market, and exports its products under a number of high-quality world brands, such as Electrolux and Whirlpool, in the European market.
production of the more-expensive one, the manufacturer uses higher-quality coating and large-diameter electrical cables, both of which prolong the life of the oven.

It is not only Turkish exporters that engage in quality-to-market. A number of studies provide econometric evidence that supports its validity in other Southern countries as well (see Table 1). The studies find that firms charge higher factory-gate (fob) prices for their products in rich than in poor markets (F1), and also in distant than in near markets (F2). Both correlations are found to be stronger for differentiated products. Assuming that high quality is associated with high price, F1 suggests that a firm sells high-quality products to its customers located in rich markets.\footnote{Crozet et al. (2009) report a similar finding for French wine exporters. The paper is very interesting and insightful as the authors rely on objective criteria to distinguish between high-quality and low-quality wine. However, since I focus on export strategy of firms from Southern countries, I am not exploring the paper further.} With respect to distance, the finding suggests that a firm-level Alchian-Allen effect is at work: distance raises transport costs; a higher (per unit) transport cost is associated with a lower relative price of high-quality to low-quality products, and thus a higher relative demand for the high-quality one. In other words, the firm "ships the good apples out".\footnote{Hummels and Skiba (2004) build a simple model to show that a country changes the relative share of its high-quality exports owing to per unit transport costs; they also provide empirical support for the Alchian-Allen hypothesis.} These two observations, F1 and F2, point to the quality-to-market hypothesis as a plausible explanation.

There also exists evidence supporting multi-sourcing of inputs as a mechanism for within-firm quality differentiation. Manova and Zhang (2009) use transaction-level data from China, and find that firm-product-level variation in input prices is positively associated with firm-product-level variation in export prices (F3). They also report that firms that export to a large number of destinations import their inputs from a large number of origins. In other words, using data on Chinese exporters, they find evidence supporting firm-level multi-sourcing.

Imported inputs can provide Southern firms access to high-quality inputs. Kugler and Verhoogen (2009) use transaction-level data from a Southern country, Colombia, and present evidence pointing to the existence of quality differences between domestic and imported varieties of an input. Assuming that high quality commands a high price, quality difference between imported and domestic varieties should be reflected in a price difference between them. Indeed, this is what Kugler and Verhoogen find in the data (F4). After exploring other possible reasons for firm-product-level differences between domestic and imported varieties, they suggest that quality difference appears as the most plausible one.

I argue that firm-level quality-to-market and multi-sourcing of inputs together consistently explain the empirical findings F1-F4. The model predicts both quality-to-market and multi-sourcing at the firm-level by combining two types of heterogeneity.
task (input) quality and taste for quality. On the supply side, to produce a final good, a Southern firm combines a continuum of tasks that vary according to their skill requirements. The firm can source each task from domestic or foreign suppliers. Assuming that skills of foreign workers are of higher quality than those of domestic workers, the quality of tasks produced by foreign suppliers is higher. High quality comes at a high price; sourcing a task from foreign suppliers is more expensive than sourcing it from domestic suppliers. Facing a trade-off between quality and cost, the firm decides which inputs to source domestically and which ones to import. The model predicts that it imports more skill-intensive ones from North, and sources the rest domestically. Since, compared to Southern consumers, rich Northern consumers demand higher-quality products, the firm adapts its product to each market by changing the fraction of its imported tasks. Also, it uses a higher fraction of imported tasks to sell its product in a distant than in a near market since the relative demand for high quality products is higher in the distant market. Thus, in line with the empirical evidence, the firm engages in quality-to-market strategy by sourcing some tasks from multiple sources.

With its focus on firms’ outsourcing decision, this study is inspired by two seminal papers (Feenstra and Hanson (1996), and Grossman and Rossi-Hansberg (2008)). The present work differs mainly in two respects: first, it incorporates quality differences between suppliers into the firm’s decision of where to outsource; second, it allows for multi-sourcing of an input at the firm-level. To my knowledge, such extensions, which appear to be useful in understanding Southern firm’s importing strategy, have not been explored so far in the literature.

The model suggests a new firm-level trade margin: extensive margin of intermediate imports. Imagine a Southern country that has gone through a substantial trade liberalisation process. Through trade liberalisation, domestic firms gain access to Northern (i.e. OECD) markets. Compared to the domestic market, demand for quality is higher in rich Northern markets.\(^4\) A Southern firm that desires to sell its product in those markets chooses to add new varieties to its product line, which are of higher-quality than that of the existing ones. To produce the new varieties, the firm needs higher-quality intermediate inputs. If such high-quality intermediates are not available in the domestic inputs market, it imports them from Northern markets.\(^5\) So, at the firm-level, the extensive intermediate imports margin is positively associated with the extensive product margin. Goldberg et al. (2009, 2010) find that better access of Indian firms to high-quality OECD inputs in the aftermath of trade liberalisation led them to increase the number of their product varieties. The resulting increase in

\(^4\)See Bils and Klenow (2001) and Broda and Romalis (2009) for evidence that richer consumers consume higher-quality goods.

\(^5\)There is extensive empirical evidence suggesting a positive correlation between per capita income and quality of exported products (Hummels and Klenow (2005), Schott (2004)).
the extensive product margin contributed significantly to the growth in manufacturing output in that period.

After studying firm’s problem, I solve for the industry equilibrium. I show that, compared to low-productivity firms, high-productivity firms use higher fraction of imported tasks, produce higher quality products, and although they follow a more costly outsourcing strategy, they charge lower quality-adjusted prices. Also, the extension modifies the specifications for quality choice in other quality heterogeneous firms (QHF) models by incorporating consumer preferences into firm’s decision. This modification relaxes the restriction on fixed trade costs to obtain the prediction that only the more productive firms select into exporting (Melitz (2003)): even under low fixed and zero variable trade costs, only the more productive Southern firms can export to the Northern market where the intensity of consumer preferences for quality is higher than in the Southern market. I also show that, compared to their domestic market, such firms charge higher prices in the Northern market, with the most productive ones also earning higher revenues. We are more likely to observe a positive correlation between firm-product-level prices and revenues across destinations, the less dispersed the productivity distribution is. This result, which has not been studied theoretically so far, is consistent with the empirical finding of Manova and Zhang (2009) (see Table 1).

Finally, I look into the effects of a skill-upgrading in the South – Southern workers upgrade the quality of their skills. As a result of a skill-upgrading, the South moves towards higher-quality and more skill-intensive tasks in the global value chain, leading to the exit of low-productivity final good exporters from the Northern market.

As I mentioned above, this paper is related to the recently growing theoretical literature on quality and firm heterogeneity. Two features distinguish it from others. First, the paper proposes a new mechanism for quality differentiation between firms: changing extensive margin of intermediate imports. All QHF studies, including the current one, tell us that high-productivity firms produce high-quality products. They, however, differ in their explanation for quality-differentiation between firms. Many QHF studies argue that quality is a mere product of firm productivity (e.g. Baldwin and Harrigan (2011), Johnson (2011), Kneller and Yu (2008)). There are only a few studies that try to underline the possible mechanisms that make quality differentiation possible. Among them, Verhoogen (2008) and Kugler and Verhoogen (2011) are closest to this paper. Both studies put forward input quality as the source of quality differentiation between firms – it is labour and capital in Verhoogen (2008) and a single domestic intermediate input in Kugler and Verhoogen (2011).

The second feature that distinguishes the current study from other QHF studies is that its main focus is within-firm quality differentiation rather than between-firms quality differentiation. Although he does not explore it further, Verhoogen (2008) also highlights the possibility of quality-to-market at the firm-level; the model he presents
predicts that firms sell higher-quality varieties in rich than in poor markets.

The present study, in a way, combines two important elements of the two models – the role played by intermediate inputs in product quality (Kugler and Verhoogen (2011)), and firm’s quality-to-market strategy (Verhoogen (2008)). On the other hand, it differs from the other two in a number of respects, and it also extends their results. First, unlike their focus on between-firms quality differentiation, the main focus of this paper is within-firm differentiation. That is why, it explores the pattern of within-firm quality differentiation more deeply, and explains the observed positive correlation of firm-product-level price not only with per capita income of the destination country but also with its distance from the firm’s home country. Second, this paper replaces Kugler and Verhoogen’s single domestically produced intermediate input with a continuum of horizontally differentiated inputs, each of which has two varieties – a low-quality Southern and a high-quality Northern variety. So, unlike the other two paper’s focus on domestic input markets, this paper shows that foreign input markets also matter for the product quality of a domestic firm – it can import those varieties that are not available in the domestic input market and use them to produce different varieties of its product in a single product line. This mechanism, first, allows for multi-sourcing of inputs as a means of quality differentiation within a firm, and second it creates a new firm-level margin – extensive margin of intermediate imports– as a means of between-firms quality differentiation. Finally, I derive the industry-level implications of within-firm differentiation in an industry populated by heterogeneous firms. Verhoogen (2008), on the other hand, does not solve or discuss the industry-equilibrium. The simple structure of the model allows me to derive closed-form solutions for both firm-level and industry-level variables.

To sum up, motivated by some recently observed empirical regularities in Southern countries, I build a trade model that consistently explains a firm’s exporting and importing behaviour: quality-to-market strategy on the exporting side, and multi-sourcing of some tasks on the importing side. I combine the two into a single strategy and call it within-firm differentiation. Then, I solve the model in general equilibrium to see the consequences of within-firm differentiation. First, I derive a well-founded specification for firm’s quality choice; it combines firm productivity and overall task quality that in turn depends on firm productivity and the intensity of consumer taste for quality. Second, I show that, even when variable trade costs are zero and fixed trade costs are low, Melitz’s productivity sorting prediction arises from North-South differences in quality demand; only the more productive Southern firms can export to the Northern market where quality demand is higher than in the Southern market. These firms set higher prices in the Northern market than in the domestic market. The most productive ones also earn higher revenues in the Northern market where they charge higher prices. Finally, the model predicts that when the South moves – driven by
a skill-upgrading in the region – towards the production of more skill-intensive tasks, low-productivity final good exporters drop out of the Northern market.

2 Model

I assume two regions, North (N) and South (S). I denote a region by \( c \) where \( c = N, S \). Both regions are populated by people who inelastically supply their skills. Suppliers employ skills in the production of tasks, and firms use tasks in the production of final goods.

2.1 Consumers

A two-tier utility function represents consumer preferences in both regions. The upper-tier is a Cobb-Douglas function which determines the allocation of a consumer’s budget between a homogeneous good \((x_{c0})\) and a continuum of horizontally (and vertically) differentiated varieties indexed by \( \phi \). The lower-tier is a CES aggregate of differentiated goods, where quality \((q(\phi))\) augments quantity \((x(\phi))\). The specification of the utility function follows Hallak (2006):

\[
U_c = x_{c0}^{1-\mu} \left[ \int_{\phi \in \Omega_c} [q_c(\phi)^{\gamma_c} x_c(\phi)]^{\sigma-1} d\phi \right]^{\frac{\mu}{\sigma}} ; \quad \sigma > 1, \gamma_c > 1, 0 < \mu < 1; \tag{1}
\]

where \( \mu \) denotes budget share of differentiated goods, \( \sigma \) elasticity of substitution between varieties, and \( \gamma_c \) intensity of consumer preferences for quality in region \( c \). In (1), \( \Omega_c \) gives the set of available varieties of the differentiated good in region \( c \).

The specification in (1) incorporates heterogeneous consumer preferences for quality since the parameter \( \gamma \) is region-specific. Hallak (2006) finds that \( \gamma \) is increasing in consumer income: richer consumers have more intense preferences for quality. Hallak’s finding is also consistent with the demand side of the Linder hypothesis (1961). Linder argues that countries with higher per capita income demand relatively higher-quality goods.

2.2 Suppliers

A supplier is a task producer. Following the approach developed by Feenstra and Hanson (1996), I assume a continuum of intermediate goods industries. I depart from their approach by calling the product of an industry a Task rather than an Input.\(^6\) Grossman and Rossi-Hansberg (2008) suggest the term Task to describe the 21\(^{st}\) century trade in intermediate goods. Rapid improvements in information and communication technologies have eased the coordination of activities across different industries.

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\(^6\)I may use two terms interchangeably in the paper.
geographic areas, and have broken down production into smaller tasks. As a result, 
trading tasks that used to be non-tradable, such as accounting, can now be traded.

Production of a final good consists of a continuum of tasks indexed by \( j, j \in [0, 1] \). 
Northern and Southern suppliers produce vertically (quality) differentiated varieties of 
a task. Depending on its choice of product quality, a final good producer decides from 
which supplier to source each task. Using this tractable supply side model, we can 
study within-firm product differentiation by quality.

Tasks lie on the unit interval such that their skill requirements are increasing; let 
\( a(j) \) denote the skill requirement of task \( j \), then \( a'(j) > 0 \).\(^7\) Northern skill is equally 
productive as Southern skill in the physical production of tasks. But it is more 
productive than the Southern in the quality production: one unit of Northern skill 
produces one unit of quality, and one unit of Southern skill produces \( \lambda \) units of quality, 
\( \lambda < 1 \).\(^8\)

Suppliers operate in perfectly competitive industries in both regions. There is a 
large number of suppliers in each industry. So, a supplier charges a price that is equal 
to its marginal cost of production. It implies that a Northern supplier of task \( j \) charges 
a price equal to 
\[
p^N_j = a(j)r_N, \tag{2}
\]
and a Southern supplier charges a price equal to 
\[
p^S_j = a(j)r_S, \tag{3}
\]
where \( r_c \) denotes the price of skill in region \( c \). Since Northern skill is more productive, 
its price is higher: \( r_N > r_S \).\(^9\)

2.3 Firms

A firm is a final good producer. There is a large number of firms, and they operate 
in a monopolistically competitive industry in both regions. Throughout the paper, I 
focus on Southern firms.

A firm pays a fixed entry cost to learn its productivity, and to create a brand. Upon 
entry, it pays a fixed cost to run the production facilities. Then, the firm combines an 
equal amount of each task to produce a variety of the final good.\(^10\)\(^11\) The production

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\(^7\)Indeed, skill requirement refers to efficiency labour requirement.
\(^8\)See Clark (1987) for a discussion about the possible reasons of differences in labour quality between South and North.
\(^9\)It is in line with the empirical evidence reported, for instance, by Easterly (2004) who finds that skilled workers earn more in Northern countries than in Southern ones.
\(^10\)As in Bernard et al. (2010), varieties in a market are differentiated from each other by their brand. A firm is allowed to supply only one horizontal variety in a market, although it is allowed to produce different vertically-differentiated varieties in different markets.
\(^11\)The firm can use the same production facilities to manufacture different (vertically differentiated)
consists of two parts: physical units and quality. For both, I use similar specifications to those used by Kugler and Verhoogen (2011). The following function represents the production of physical units:

\[ F(n) = n\phi^n, \]

where \( n \) denotes number of each task, \( \phi > 0 \) firm productivity, and \( 0 < \alpha < 1 \) sensitivity of unit cost to firm productivity. Thus, a firm with productivity \( \phi \) requires \( \phi^{-\alpha} \) units of each task to produce one unit of final good. Its marginal cost of production is lower, the higher is the sensitivity of costs to firm productivity (higher \( \alpha \)).

In the production of quality, the *quality-complementarity hypothesis* holds; overall task quality and firm productivity complement each other in the production of final good quality. So, the quality of final good \( (q) \) depends on overall task quality and firm productivity in the following way:

\[ q(\phi, I) = [\phi^{-b} + \Psi(I)^{-b}]^{\frac{1}{b}}, \]

where \( \Psi(I) \) denotes overall task quality, and \( b > 0 \) degree of complementarity between overall task quality and firm productivity. As I mentioned earlier, task quality is proportional to the quality of skills embodied in that task. Thus, if a firm sources tasks in \( [0, I] \) from South and the rest from North, the overall quality of its tasks is equal to

\[ \Psi(I) = \lambda \int_0^I a(j) dj + \int_I^1 a(j) dj. \]

In words, the higher the share of tasks sourced from Northern suppliers the higher is the overall task quality.\(^{12}\)

To sum up, firms combine a continuum of tasks that they source from North or South, and produce differentiated goods. Then, they sell their products to consumers.\(^{13}\)

3 Partial Equilibrium

In this section, I focus on the behaviour of a single firm with productivity \( \phi \). The firm sources tasks in \( [0, I] \) from Southern suppliers and the rest from Northern suppliers. The resulting quality of its product is equal to

\[ q(\phi, I) = [\phi^{-b} + \Psi(I)^{-b}]^{\frac{1}{b}}, \]

\(^{12}\frac{\partial \Psi(I)}{\partial I} = (\lambda - 1)a(I) < 0.\)

\(^{13}\)Throughout the paper, \( \phi \) is an index for both firms and brands.
and by (4) its marginal cost of production is equal to

$$C(\phi, I) = \phi^{-a} \left[ r_S \int_0^1 a(j) dj + r_N \int_1^1 a(j) dj \right]. \quad (7)$$

The demand for its product in region $c$ is

$$x_c(\phi) = \mu Y_c (q(\phi, I)^\gamma_c)^{\gamma-1} \left( \frac{p(\phi, I) + t_c}{P_c} \right)^{-\sigma}, \quad (8)$$

where $p$ is the price charged by firm $\phi$; $P_c = \int_{\phi \in \Omega_c} \left( \frac{p_c(\phi) + t_c}{q_c(\phi)} \right)^{1-\sigma} \frac{1}{1-\sigma}$ denotes quality-adjusted price index, $Y_c$ aggregate income, and $t_c$ per unit cost of transporting the good to region $c$. Define the consumer price of variety $\phi$ as $p^{cf}(\phi, I) = p(\phi, I) + t_c$.

Given the demand for its product, the firm chooses the price ($p$) and the fraction of tasks to be sourced from Southern suppliers ($I$). Its choice of $I$, which is the fraction of domestically-sourced tasks, determines the marginal cost of production by (7), and the quality of the final good by (6). Here is the profit maximisation problem of the firm:\(^\text{14}\)

$$\max_{p(\phi, I), I \in [0,1]} \Pi(\phi, I) = x(\phi) \left[ p(\phi, I) - C(\phi, I) \right]$$

subject to $x(\phi) = \frac{\mu Y}{P_c} (q(\phi, I)^\gamma)^{\gamma-1} \left( \frac{p^{cf}(\phi, I)}{P_c} \right)^{-\sigma}$.

The firm charges the following price to maximise its profits:

$$p(\phi, I) = \frac{\sigma}{\sigma - 1} C(\phi, I) + \frac{1}{\sigma - 1} t. \quad (10)$$

The price of a variety equals to a constant markup over its marginal cost, plus a fraction of the cost of transporting the variety to the final consumer. So, the firm charges a higher price for the variety that it produces at a higher marginal cost, and that it sells to a more distant consumer.

To maximise its profits, the firm also chooses the fraction of its domestically-sourced tasks, and this fraction solves the following equation:

$$(\sigma - 1)\gamma (p(\phi, I) - C(\phi, I)) \frac{\partial q(\phi, I)}{\partial I} - q(\phi, I) \frac{\partial C(\phi, I)}{\partial I} = 0. \quad (11)$$

The solution of the equation (12) defines an implicit function:

$$I^* = I(\Gamma, X),$$

\(^\text{14}\)Here, I drop region subscripts. Also, in this section, I assume that the firm makes profits. In the next section, I discuss a firm’s decision of being active in the industry.
where $\Gamma$ is a vector of the following parameters $\gamma, \phi, \lambda, \mu, t$; and $X$ is a vector of aggregate variables and skill prices $Y, P, r_S, r_N$. Now, substitute $p(\phi, I)$ from (10) into (11) to obtain:\textsuperscript{15}

$$\gamma (C(\phi, I) + t) \frac{\partial q(\phi, I)}{\partial I} - q(\phi, I) \frac{\partial C(\phi, I)}{\partial I} = 0.$$ (12)

In the absence of quality considerations, minimising costs would be the only motive that determines the firm-level task trade. It is what we learn from the model developed by Feenstra and Hanson (1996). On the other hand, when deciding on the fraction of its domestically-sourced tasks, a firm in this model finds a balance between two opposing effects. A higher $I$ reduces the firm’s marginal cost:

$$\frac{\partial C(\phi, I)}{\partial I} = \phi^{-a} a(I) (r_S - r_N) < 0.$$ 

At the same time, a higher $I$ lowers the quality of its product:

$$\frac{\partial q(\phi, I)}{\partial I} = \frac{\partial q(\phi, I)}{\partial \Psi(I)} \frac{\partial \Psi(I)}{\partial I} < 0,$$

as well as the demand for its product:

$$\frac{\partial x(\phi, I)}{\partial q} > 0.$$

In short, the firm’s profit-maximising fraction of domestically-sourced tasks satisfies

$$\frac{\partial}{\partial I} \left( \frac{C(\phi, I) + t}{q(\phi, I)^\gamma} \right)_{I=I^*} = 0.$$

In the rest of this section, I characterise the comparative statics of $I^*$ – the fraction of domestically-sourced tasks– with respect to the intensity of preferences for quality ($\gamma$), firm productivity ($\phi$), per unit transport costs ($t$), and productivity gap between Northern and Southern skills ($\lambda$).

### 3.1 Effect of consumer preferences

What is the effect of the intensity of consumer preferences for quality, $\gamma$, on the firm’s outsourcing decision – the fraction of its domestically-sourced tasks ($I^*$)? To answer this question, totally differentiate the firm’s first-order condition in (12) with respect

\textsuperscript{15}Please see Appendix A.1 for the derivation of the corresponding second-order conditions.
to the taste parameter ($\gamma$) and $I$, and evaluate this derivative at $I^*$:

$$\left\{ (\sigma - 1) (p(\phi, I) - C(\phi, I)) \frac{\partial q(\phi, I)}{\partial I} \right\} \frac{dI}{d\gamma} + D_{II} dI \right\|_{I=I^*} = 0,$$

where $D_{II}$ denotes the derivative of the combined first-order condition (12) with respect to $I$, and it is negative as long as the second-order conditions for profit-maximisation hold. Then,

$$\left( \frac{dI}{d\gamma} \right)_{I=I^*} = - \left\{ \frac{(C(\phi, I) + t) \frac{\partial q(\phi, I)}{\partial I}}{D_{II}} \right\|_{I=I^*}. \tag{13}$$

Since $\frac{\partial q(\phi, I)}{\partial I} < 0$ and $D_{II} < 0$, both the numerator and denominator of (13) are negative. This implies $\left( \frac{dI}{d\gamma} \right)_{I=I^*} < 0$. When it faces more intense preferences for quality, the firm sources a higher fraction of tasks from Northern suppliers, and by doing so it improves the overall quality of its inputs. Therefore it produces a higher-quality variety for such consumers:

$$\left( \frac{dq(\phi, I)}{d\gamma} \right)_{I=I^*} = \left\{ \frac{\partial q(\phi, I) dI}{\partial I < 0} \frac{d\gamma}{d\gamma < 0} \right\} \|_{I=I^*} > 0,$$

and it charges a higher price for this variety:

$$\left( \frac{dp(\phi, I)}{d\gamma} \right)_{I=I^*} = \frac{\sigma}{\sigma - 1} \left\{ \frac{\partial C(\phi, I) dI}{\partial I < 0} \frac{d\gamma}{d\gamma < 0} \right\} \|_{I=I^*} > 0.$$

The following result summarises this finding.

**Result 1** A Southern firm facing different demands for quality in different markets differentiates the quality of its product: it sells a higher-quality variety in high-demand than in low-demand markets. Compared to the low-quality variety, when producing the high-quality one, the firm uses an overall higher-quality of tasks by importing more tasks from North.

The firm chooses endogenously to become a multi-product firm.\textsuperscript{16} This firm produces multiple varieties (versions) of a product in a single product line, and sells them in different markets. Across the varieties, it changes the mix of domestic (Southern) and imported (Northern) tasks. As it pays a higher price for the imported variety of a

\textsuperscript{16}I use the term here in a different sense from the existing trade literature that studies firms that produce horizontally-differentiated products (e.g. Bernard et al. (2010), Eckel and Neary (2010)), and from the industrial organisation literature that studies firms that produce vertically differentiated products for a single market (e.g. Johnson and Myatt (2003) and the studies cited in that paper). Here, I study firms that produce vertically differentiated products for segmented markets.
task, the firm’s production cost varies across varieties. As a result, the price it charges also varies across markets.

Result 1 is in line with the empirical facts F1 and F3 in Table 1: a Southern firm charges a higher factory-gate prices for its product in rich compared to poor markets; and the dispersion of its export prices is positively correlated with the dispersion of its input prices. When consumer demand for quality is high in a market, the firm meets the demand by upgrading the quality of its product. It does so by using higher-quality inputs. Since such inputs are more expensive than the low-quality ones, the firm charges a higher price in that market.

The result highlights a link between firm’s extensive intermediate imports margin and its product margin. As Southern firms gain access to new markets where quality demand is higher than in their domestic market, they add new product varieties to their product lines. Since they sell the new varieties in the rich markets, they should be of higher quality than the existing ones. To produce them, firms need high-quality input varieties that may not be available in the domestic market. They can source such inputs from the North as Northern workers are able to produce higher-quality inputs than their Southern counterparts. In other words, Southern firms gain access to high-productivity Northern workers through intermediate imports. Thus, an expansion on Southern firms’ extensive product margin towards higher-quality varieties is associated with an expansion on their extensive intermediate imports margin towards higher-quality inputs.

As I mentioned in the introduction, its focus distinguishes the current study from other studies. Here the focus is within-firm quality differentiation between markets rather than between-firms differentiation within a market. Although it is not their primary aim, there are a few theoretical studies that mention the possibility of within-firm quality differentiation. Eckel et al. (2010) show that a multi-product firm invests differently in quality between its horizontally differentiated products that it sells in a market, but not in the quality of a single product between markets. Verhoogen (2008), on the other hand, notes that varying demand for quality across markets leads to within-firm quality differentiation. He does not, however, elaborate on this argument since the firm-level dataset he uses does not allow for testing such prediction. So, he focuses on firm-level average prices, wages and quality in the rest of the paper. In another paper, Crinò and Epifani (2010) observe among Italian manufacturing firms that the correlation between firm productivity and export intensity is increasing in per capita income of the export market. They build a partial equilibrium QHF model to rationalise this observation. Their model correctly predicts that a firm sells a higher-quality product in rich compared to poor markets. However, since the price

\[17\] But their empirical results show that firms upgrade the quality of their core-competence products and sell them at higher prices on average in their export markets.
charged by the firm does not vary with its product quality, the model does not explain
the positive correlation between firm-product level prices and per capita income of
the destination markets. The primary focus of Crinò and Epifani is the relationship
between firm productivity and its export intensity, and their model successfully explains
the observed relationship between the two. The current paper, on the other hand,
explicitly focuses on within-firm quality and price differentiation. Furthermore, I also
suggest a mechanism for such differentiation, which is supported by empirical evidence.

In summary, Result 1 gives an explanation for the stylised facts F1 and F3 in Table
1. To my knowledge, this study is the only one that tries to consistently explain them
within a single model.

3.2 Effect of firm productivity

Another interesting relationship is the one between the firm’s productivity ($\phi$), and its
profit maximising fraction of domestically-sourced tasks ($I^*$). To find the nature of
the relationship, totally differentiate the first-order condition in (12) with respect to $\phi$
and $I$, and then evaluate it at $I^*$:

$$\left\{ \frac{\partial}{\partial \phi} \left[ \gamma (C(\phi, I) + t) \frac{\partial q(\phi, I)}{\partial I} - q(\phi, I) \frac{\partial C(\phi, I)}{\partial I} \right] d\phi + D_{II} dI \right\}_{I=I^*} = 0.$$  

Simplify the expression using (12):

$$\left( \frac{dI}{d\phi} \right)_{I=I^*} = -\left\{ \frac{\partial q(\phi, I) b C(\phi, I) + t q(\phi, I)}{\partial I} \frac{\partial q(\phi, I)}{\partial \phi} + \frac{a t}{\phi} \right\}_{I=I^*}.$$  

Given the signs above, we obtain

$$\left( \frac{dI}{d\phi} \right)_{I=I^*} < 0. \quad (14)$$

In words, a high-productivity firm, compared to a low-productivity one, sources a
higher fraction of tasks from Northern suppliers. The result directly follows from the
complementarity between firm productivity and overall task quality in the production
of final good quality. Using the same fraction of imported tasks as a low-productivity
firm, a high-productivity one is able to produce a higher-quality product ($\frac{\partial q(\phi, I)}{\partial \phi} > 0$) at a lower cost ($\frac{\partial C(\phi, I)}{\partial \phi} < 0$) – implying a higher demand for its product. This creates
an incentive for the firm to upgrade the quality of its product by using a higher fraction
of imported tasks from North. From a consumer’s point of view, the quality-adjusted
price charged by a more productive firm is lower:

\[
\left( \frac{d \left[ p^{\text{cif}}(\phi, I)/q(\phi, I)^{\gamma} \right]}{d\phi} \right)_{I=I^{*}} = \left\{ \left[ \frac{\partial \left[ p^{\text{cif}}(\phi, I)/q(\phi, I)^{\gamma} \right]}{\partial I} \right] \left( I=I^{*} \right) \right\} + \left\{ \left[ \frac{\partial \left[ p^{\text{cif}}(\phi, I)/q(\phi, I)^{\gamma} \right]}{\partial \phi} \right] \left( I=I^{*} \right) \right\} < 0,
\]

since \( \left[ \frac{\partial \left[ p^{\text{cif}}(\phi, I)/q(\phi, I)^{\gamma} \right]}{\partial I} \right] \left( I=I^{*} \right) = 0 \) by the Envelope Theorem. The following result summarises these findings.

**Result 2** Within a market, a high-productivity Southern firm, compared to a low-productivity one, uses a higher fraction of imported tasks from North, and produces a higher-quality variety. Also, it charges a lower quality-adjusted price, and thus earns larger revenues.

The result that more productive firms produce higher-quality varieties is not new. Indeed, there is a growing pile of studies in the international trade literature focusing on between-firm quality differentiation. They find that firm productivity is positively associated with product quality, and negatively with the product’s quality-adjusted price (Baldwin and Harrigan (2011), Johnson (2011), Kneller and Yu (2008), Kugler and Verhoogen (2011), and Verhoogen (2008)). What Result 2 adds to their findings is that it offers a new mechanism by which productivity differentials between firms generate quality differentials between their products: different firms use different mixes of domestic and imported tasks. In line with the empirical evidence (F4), for a Southern firm, the imported variety of a task is of higher-quality than the domestic variety of the same task. So, a more productive Southern firm produces a higher-quality product by using a higher fraction of imported tasks.

### 3.3 Effect of transport costs

Another stylised fact that I try to explain is that a firm’s factory (fob) price in a market is positively correlated with its distance from the market, and this correlation is stronger for differentiated products (Görg et al. (2010) and Manova and Zhang (2009)). This empirical finding, F2 in Table 1, is closely related to Alchian-Allen’s "shipping the good apples out" hypothesis (1964), which argues that adding a per unit transport cost lowers the relative price of high-quality products, and thus increases their demand relative to low-quality products. It is a demand-side explanation. We need to complement it with a supply-side mechanism of within-firm quality differentiation to fully explain F2. It is where our model comes in.
It is hard to find such mechanism in existing QHF studies. In a model with linear demand, a firm reduces its markup in a more distant market – implying a negative correlation between a firm’s prices in different markets and its distance from them. In a model with logit demand, as in Verhoogen (2008), a firm lowers both the price and quality of its product in more distant markets (Martin (2009)). So, models with linear and logit demand imply the opposite of what F2 suggests. In a model with CES preferences, a firm adds to its fob price a fraction of the cost it bears to transport its product to a market (see (10)). Assuming that it bears a higher cost to transport its product to a more distant market, the firm charges a higher price in such market. Although this explanation is consistent with the observed positive correlation between a firm’s fob prices in different markets and its distance from them, it does not explain the second part of F2, which says that the correlation is stronger for differentiated products.

To see that this model is able to fully explain F2, totally differentiate (12) with respect to $t$ and $I$, and evaluate the derivative at $I^*$:

$$
\left( \frac{dI}{dt} \right)_{I=I^*} = -\left\{ \frac{\gamma \frac{\partial q_t(I)}{\partial I}}{D_I} \right\}_{I=I^*} < 0.
$$

A Southern firm uses a higher fraction of imported tasks from North, and thus produces a higher-quality variety to sell in a distant, compared to a near, market. It also charges a higher fob price in such market since it bears higher production and transportation costs.

As suggested by Alchian and Allen, adding a per unit transport cost to prices reduces the relative price of high-quality products, and thus increases their relative demand. Knowing it, a firm produces a higher-quality variety to sell in a more distant market. Since producing a higher-quality variety is more costly, it charges a higher price for this variety. So, a Southern firm charges a higher (fob) price for its product in a distant than in a near market for two reasons: first, it adds a fraction of unit transport cost to the price; second, it uses a higher fraction of imported tasks, which are of higher-quality and more expensive, to produce a higher-quality variety for such market. Although the first reason is common in models with CES preferences, the second one is unique to the current model.

**Result 3** A Southern firm’s product quality is higher in distant than in near market. Compared to the low-quality variety, when producing the high-quality one, the firm uses a higher fraction of imported tasks from North. Since the imported variety of a task is more expensive than the domestic variety, the firm bears a higher production cost, and thus charges a higher price for the variety it sells in the distant market.
Result 3 is consistent with the stylised fact F2. To fully explain it, a model has to feature within-firm quality differentiation across markets. It is what this model does, and Result 3 summarises: a firm uses an overall higher-quality of tasks to sell a higher-quality variety in a distant than in a near, market, and it charges a higher price for this variety. This model explains F2 in a comprehensive way to include other elements that are also missing in existing QHF studies – F1, F2 and F3 in Table 1. So, the current model provides a useful tool to study comprehensively the recently unearthed stylised facts that existing QHF models do not accommodate.

3.4 Effect of productivity gap

Here, I discuss an implication of the model that is not yet tested: what is the effect of a change in the relative productivity of Southern skill ($\lambda$) on the firm’s profit-maximising fraction of domestically-sourced tasks? To be specific, what happens if Southern workers upgrade the quality of their skills – $\lambda$ rises. To find the answer, totally differentiate the first-order condition in (12) with respect to $\lambda$ and $I$, and evaluate it at $I^*$:

$$
\left\{ \gamma (C(\phi, I) + t) \frac{\partial^2 q(\phi, I)}{\partial I \partial \lambda} - \frac{\partial q(\phi, I)}{\partial I} \frac{\partial C(\phi, I)}{\partial I} \right\} d\lambda + D_{II} dI = 0.
$$

In Appendix A.2, I prove that the term in the square brackets in (15) is positive. Since $D_{II} < 0$, we obtain

$$
\left( \frac{dI}{d\lambda} \right)_{I=I^*} > 0.
$$

**Result 4** Assume that Southern workers upgrade their skills – $\lambda$ rises. At constant skill prices, it leads a Southern firm to increase the fraction of its domestically-sourced tasks. The resulting impact on its product quality is ambiguous.

This is an intuitive result. At constant skill prices, an improvement in the productivity of Southern skill reduces its productivity-adjusted price. This would lead firms to substitute away from Northern suppliers, and to increase the fraction of their domestically-sourced tasks. This reasoning, however, ignores another effect that is at

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18 There is an analogy between Result 3 and the response of a firm to voluntary export restraints (VERs). An exporting country sets VERs to limit the quantity of its exports to a specific country. We know from other studies that a VER may also create an incentive for exporters to upgrade the quality of their products. In 1981, the Japanese government imposed a restriction on the number of its car exports to the U.S. This led to an increase in the price of Japanese cars imported by the U.S. Feenstra (1984) empirically illustrates that the two-thirds of this price increase resulted from quality improvement by the Japanese car exporters. Although the underlying mechanisms are different in two cases, this is an interesting analogy to note. I would like to thank Richard Baldwin for bringing this analogy to my attention.
work here. As a result of a skill-upgrading in South, even keeping $I$ unchanged, the quality of a firm’s product improves. The quality gain arises from quality improvement in the firm’s initially domestically-sourced tasks. Thus the demand for its product increases, and so does its profits. So, the initial $I$ is not optimal at the new level of $\lambda$. The firm can now afford to source a higher fraction of tasks from North: $I$ rises. Here, substitution effect dominates the quality-upgrading effect. As a result, when there is a skill-upgrading in South, a Southern firm increases the fraction of its domestically-sourced tasks.

The resulting effect of an increase in $\lambda$ on a firm’s product quality is ambiguous. On the one hand, the quality of tasks that the firm used to source, and is still sourcing from Southern suppliers increases. On the other hand, the quality of tasks that the firm used to source from North, but is now sourcing from South reduces. Thus the overall effect of a higher $\lambda$ on the firm’s product quality remains ambiguous.

A skill-upgrading in South unambiguously reduces the firm’s marginal cost and price. The reason is that the firm switches from high-price Northern suppliers to low-price Southern suppliers for a range tasks. Consumers benefit from lower product prices. Without determining the resulting effect of a higher $\lambda$ on the firm’s product quality, we cannot tell the direction of change in the quality-adjusted price that consumers face in a market.

This simple exercise shows that a skill-upgrading in South can result in the region moving up in the global value chain towards higher skill-intensive tasks. The result in part hinges on its partial equilibrium nature: it ignores the indirect effect working through changes in relative skill prices. We should expect the indirect effect to weaken the substitution effect. It is, however, ambiguous whether the indirect effect will be strong enough to turn the result around. In Section 4, where I embed the model in a general equilibrium framework, I will examine the other extreme: what happens if relative skill prices adjust one-to-one to a change in relative productivities?

So far, I have set up the model, and derived some results from a single firm’s perspective, which are in line with the empirical facts in Table 1. In the next section, I embed the model in a Melitz-type heterogeneous-firm framework. Before jumping to the next section, I summarise the results derived up to here. First, a Southern firm engages in quality-to-market by selling a higher-quality variety in rich than in poor, as well as in distant than in near markets. Second, a high-productivity Southern firm, compared to a low-productivity one, produces a higher-quality variety since it uses a higher fraction of imported tasks. Also, it charges a lower quality-adjusted price, and earns larger revenues in a market. Finally, when Southern workers upgrade their skills, a Southern firm increases the fraction of its domestically-sourced tasks, resulting in an ambiguous change in its product quality.
I start with describing the setup of the general equilibrium extension. Consumer preferences in each region are given by $(1)$. Let $L_c$ denote the population of region $c$, then the relative size of the Southern market satisfies $L_S / L_N = \varepsilon > 1$. In both regions, an individual inelastically supplies one unit of skill.

I explain the setup of intermediate goods and the differentiated final good industries in Section 2. In the homogenous good industry, which is perfectly competitive, a large number of firms produce $x_0$. They produce the good under constant returns to scale technology. Northern skills are more productive than Southern ones in the production of $x_0$: one unit of Northern skill produces one unit, and one unit of Southern skill produces $\lambda < 1$ units of $x_0$. In both regions, the industry is large enough so that both have a strictly positive output of $x_0$. It requires that productivity-adjusted skill abundances in the two regions are almost identical. Also, they trade the good freely. Thus the homogenous good industry equalises the productivity-adjusted skill prices across regions: $r_S = \lambda r_N$.

In the differentiated goods industry, there is a continuum of firms, indexed by their productivity $\phi$. There is Melitz-type uncertainty in the industry: all firms are ex-ante identical; they have to pay a fixed entry cost ($f_e > 0$ in terms of Northern skills, and $f_e / \lambda$ in Southern skills) to learn their productivity. Firms draw their productivities from a Pareto distribution ($G(\phi)$) with shape parameter $\nu$:

$$G(\phi) = 1 - \phi^{-\nu}, \text{ where } \phi \in [1, \infty) \text{ and } \nu > \eta_N > \eta_S > 2,$$

where $\eta_e = (\alpha + \gamma_e - 1)(\sigma - 1)$, $c = S, N$. The density function corresponding to $G(\phi)$ is $g(\phi)$.

A firm enters the industry only if its expected profits in that market are higher than the cost of entry. Upon entry, a firm pays a fixed production cost ($f > 0$ in terms of Northern skills, and $f / \lambda$ in terms of Southern skills) to run the production facilities. Only those firms that can cover the fixed production cost can produce. Thus, as in Melitz (2003), some firms are not active in the industry.

The active ones decide where to locate in the next stage. Locating in North produces a lump-sum benefit which increases with firm productivity. For instance, assuming that infrastructure quality is higher in North, a firm can benefit from lower fixed costs if it locates in this region. Owing to the complementary nature of public infrastructure, a more productive firm benefits more from locating in North. The following function represents the lump-sum benefits from locating in North:

$$\beta(\phi) = \beta \phi.$$
Nevertheless, it is more costly to locate in North since, for instance, land prices are higher. The additional fixed cost of locating in North is $\ell > 0$. So, a firm locates in North if and only if its productivity is greater than $\phi_1 = \ell / \beta$. In the rest of the paper, I assume this value is extremely large – $\phi_1 \to \infty$. Two implications arise from such an approach: first, a firm’s location decision is independent of its output and exporting decisions; and second, only the most productive firms are able to locate in North. Others locate in South, and, as it has been so far, their behaviour is what I focus here.\footnote{I keep firm’s location problem as simple as possible since it is not the focus of this paper. Under these assumptions, the distribution of firms across regions does not affect the key results derived from the model. As $\phi_1 \to \infty$, expected profits of Northern firms have negligible effect on the entry decision of a potential entrant, and thus I ignore them. The question of firm location can also be studied within this model by allowing a firm’s output decision to depend on its location.}

After deciding where to locate, a firm decides which markets to sell its products. To export its product, it has to pay a fixed cost that I express in terms of fixed production cost as $f_X = \theta f$, $\theta > 0$.\footnote{A firm pays $f_x$ in terms of Northern skills, and $f_x / \lambda$ in Southern skills.} Also, the firm has to determine the varieties to be produced in its product line. From then on, it earns the same per period profit unless hit by a bad shock that induces a forced exit from the industry. The probability of being hit by a bad shock is $\delta$, which is exogenous, and identical across firms and time.

As I mentioned in Section 2.1, Hallak (2006) finds that the taste parameter $\gamma$ is increasing in consumer income. So, since $r_N > r_S$, we have $\gamma_N > \gamma_S$: Northern consumers have more intense preferences for quality than the Southern ones.

For the sake of mathematical tractability, I assume that the elasticity of complementarity between productivity and task quality is unity ($b = 1$), and that transport costs are zero ($t = 0$).

Next, using (12), I derive the key firm-level expressions: price, task quality, and product quality. Without imposing a functional form on the unit skill requirement function $a(j)$, identifying a firm’s profit maximising fraction of domestically-sourced tasks ($I^*$) is not possible. We can, however, express $I^*$ for a firm as a function of its productivity, and consumer taste parameter $\gamma_c$:

$$\Psi(I^*) = \tilde{\Psi}_c(\phi) = \phi(\gamma_c - 1).$$

(16)

Remember that $\Psi(I)$ is a decreasing function of $I$. So, this expression confirms previous findings in the paper: within a market, a high-productivity Southern firm, compared to a low-productivity one, uses a higher fraction of imported tasks from North (Result 2); a firm which sells in both markets uses a higher fraction of imported tasks to produce a higher-quality variety for the Northern market since $\gamma_N > \gamma_S$ (Result 1).

Another implication of (16) is that a firm’s absolute markup $(p(\phi) - C(\phi))$ varies across markets, with its markup being higher in the Northern market. This results from
the interaction between demand and supply-driven quality differentiation in the model. This feature will prove to be important to explain the fifth stylised fact in Table 1, about which I have remained silent so far: a firm’s prices and revenues correlate positively across markets (F5). According to Manova and Zhang (2009), existing heterogeneous firm trade models do not explain F5. The models with standard CES preferences imply zero correlation as a firm’s costs and prices do not vary across markets; those with quadratic preferences imply ambiguous results. A consistent explanation for F5 requires within-firm quality differentiation and varying markups across markets.

Now, set \( r_N = 1 \), and use (16) to derive the marginal cost, price, quality and quality-adjusted price of a firm’s variety sold in region \( c \):

\[
C_c(\phi) = (\gamma_c - 1)\phi^{1-\alpha}, \quad (17a)
\]
\[
p_c(\phi) = \frac{\sigma}{\sigma - 1} (\gamma_c - 1)\phi^{1-\alpha}, \quad (17b)
\]
\[
q_c(\phi) = \frac{\phi(\gamma_c - 1)}{\gamma_c}, \quad (17c)
\]
\[
\frac{p_c(\phi)}{q_c(\phi)\gamma_c} = \frac{\sigma}{\sigma - 1} (\gamma_c - 1)^{1-\gamma_c}\gamma_c^{\gamma_c}\phi^{1-\alpha-\gamma_c}. \quad (17d)
\]

The expression in (17c) is a reduced form of a firm’s quality choice in a market. With the extension to heterogeneous firms, the model predicts that a firm’s quality choice depends on its own productivity \( \phi \), and its overall task quality \( \Psi(T^*) \) that, in turn, depends on firm productivity and consumer taste for quality from (16), implying multisourcing of some tasks. Thus the study contributes to the QHF literature by suggesting a well-founded and simple specification for a firm’s optimal quality choice, which features quality-to-market strategy. As the imported variety of a task is more expensive than the domestic variety, the firm charges a higher price for the variety that it sells in the Northern market (17b). Within a market, compared to a low-productivity firm, a high-productivity firm charges a lower quality-adjusted price (17d).

A firm, first, decides whether to enter the industry. Second, it decides whether to be active in each market. Finally, it decides on its task composition (domestically-sourced and imported), product quality, and price for each variety it produces. I started from the firm’s final decision. So, the next step is to study the firm’s market participation decision: produce or remain inactive in a market. A firm with productivity \( \phi \) produces and sells a variety in a market if and only if it makes profits: the firm’s variable profits should, at least, cover its fixed costs. Otherwise, it remains inactive in the market. This condition determines a cutoff productivity: if a firm’s productivity is above the cutoff, it produces in the market; if below the cutoff, it remains inactive. Here is the condition:

\[
\Pi_c(\phi_S^*) = \Pi_S^c(\phi_S^*) - fr_N = 0 \iff x_S(\phi_S^*) [p_S(\phi_S^*) - C_S(\phi_S^*)] - (f/\lambda)r_S = 0, \quad (18)
\]
where $\phi^*_S$ denotes the cutoff productivity in South, and $\Pi^S_{var}(\phi^*_S)$ variable profits at $\phi^*_S$. Appendix A.3 shows, and Figure 1 illustrates that profits are increasing in productivity. Within a market, a high-productivity firm, compared to a low-productivity one, produces a higher-quality variety at a lower cost. So, it charges a lower quality-adjusted price, and makes larger profits.

**Figure 1: Cut-off Productivity Condition**

It is easy to see that if a Southern firm exports to the Northern market then it should also sell in the domestic market. The reason is the following: if a firm is profitable in the Northern market, then its variable profits in this market should cover the sum of fixed production cost $f$ and fixed trade cost $f_X$. Since it does not have to pay any additional fixed costs to sell its product in the domestic market, the firm should also be active there. But the reverse is not true: a Southern firm that is profitable in the domestic market may not be profitable in the Northern market as it has to pay an additional fixed cost $f_X$ to sell its product in there. In other words, only a fraction of active firms in the Southern market export to the Northern market. Appendix A.4 shows that the model predicts a sorting of Southern firms by productivity level: only the more productive Southern firms export to the rich Northern market. The prediction requires the following condition to hold:

$$\theta > \frac{\nu - \eta_N}{\nu - \eta_S}. \quad (19)$$

As $\eta_N > \eta_S$, we have $(\nu - \eta_N)/(\nu - \eta_S) < 1$. Under zero variable trade costs in Melitz’s original model, sorting of firms into exporting arises only if fixed export cost is larger than fixed production cost – $\theta > 1$. In this paper, on the other hand, sorting of Southern firms into exporting to the Northern market can arise even when fixed cost of exporting is smaller than the fixed production cost – $(\nu - \eta_N)/(\nu - \eta_S) < \theta < 1$. So, cross-market differences in quality demand relax the condition for partitioning of
firms by export status; the condition in (19) is less restrictive than the corresponding condition in Melitz (2003).

Appendix A.4 derives the cutoff productivity for exporting relative to that of selling in the domestic market:

\[ \frac{\phi^*_N}{\phi^*_S} = \left( \frac{\theta \varepsilon \lambda \nu - \eta_S}{\nu - \eta_N} \right)^{1/\nu}. \]

The difference between two cutoff productivities depends on two factors: aggregate income and consumer preferences. The ratio \(1/\varepsilon \lambda\) captures the Northern aggregate income relative to the Southern. If this ratio rises, then the threshold productivity for selling to the Northern market, relative to the Southern market, falls. This prediction is in line with the findings of Chaney (2008); it is easier to sell in a larger than in a smaller market. The second factor that changes the Northern threshold productivity relative to the Southern is the intensity of preferences for quality. The parameter \(\gamma\) captures this effect, and it directly depends on the consumer taste parameter \(\gamma\). Since \(r_N > r_S\), and in line with the empirical evidence that consumer preference for quality becomes more intense with higher income, we have \(\gamma_N > \gamma_S\). Therefore the threshold productivity for selling to the Northern market should be higher than for the Southern market. This prediction of the model is novel to the QHF literature.

**Result 5** The intensity of consumer preferences for quality differs in North and South: \(\gamma_N > \gamma_S\). It implies that the threshold productivity for selling to the Northern market is higher: \(\phi^*_N > \phi^*_S\).

Result 5 modifies the productivity-sorting prediction of Melitz, which says that the productivity threshold for exporting is higher than for selling in the domestic market. In Melitz’s paper (2003), the prediction arises from a combination of fixed and variable trade costs. Here, it arises from a combination of fixed trade costs and differences in consumer tastes for quality between markets. In this model, productivity differences between firms are reflected in differences in their costs and differences in their product quality. A firm’s product quality depends on its productivity and the overall task quality used in the production, which, in turn, depends positively on the firm’s productivity. So, as (17c) and (17d) show, compared to a low-productivity firm, a high-productivity firm produces a higher quality variety and charges a lower quality-adjusted price in a market. In other words, low-productivity firms have comparative disadvantage in producing high-quality goods. Such disadvantage is magnified by consumer’s greater taste for quality in the Northern market. Thus, even when the fixed export cost is acceptably low, only the more productive Southern firms export to the Northern market where the taste for quality is more intense than in the domestic market.

The prediction is open to testing. When testing it, one should find a proxy for consumer preferences for quality, and control for aggregate income. Controlling for
aggregate income is necessary since, as I argued above, the threshold productivity for selling to a higher-income market is lower. Here, the appropriate measure is gross domestic product (GDP). On the other hand, finding a proxy for consumer preferences for quality is difficult. In doing so, one can benefit from the demand side of the Linder hypothesis (1961). It argues that, in a market, per capita income is the most important determinant of consumer preferences. To be specific, richer consumers demand higher-quality products. So, when testing Result 5, one can use per capita GDP as a proxy for the intensity of consumer preferences for quality. According to this result, controlling for aggregate income, the productivity threshold for selling to a higher-per-capita-income market is higher.

In Appendix A.5, I solve the model in general equilibrium. To emphasise the role of consumer preferences in sorting of firms into exporting, the solution assumes two conditions: productivity-adjusted skill abundances in two regions are almost identical – \( \varepsilon \lambda \) does not deviate significantly from one; and fixed export cost satisfies \( \theta > (\nu - \eta_N)/\nu - \eta_S \).

Now, I will examine the general equilibrium implications of a skill-upgrading in South. In partial equilibrium, assuming that skill prices remain unchanged, a skill-upgrading in South leads all firms to substitute away from Northern suppliers. As a result, product prices, unadjusted for quality, fall. Here, I examine the other extreme: relative skill prices adjust to match a change in relative skill productivities.

From expression (16), we see that the profit-maximising overall task quality for a firm depends only on its productivity and the intensity of consumer preferences for quality, implying that its overall task quality will not change with \( \lambda \). For this result to hold, the firm should be switching from Northern suppliers to Southern ones for a range of tasks; \( I^* \) increases.

\[
\frac{dI^*}{d\lambda} = \frac{\int_0^{I^*} a(j) dj}{(1-\lambda)a(I^*)} > 0.
\]

But unlike the partial equilibrium case, here the increase in \( I^* \) does not arise from a substitution effect. Because after a skill-upgrading in South, relative skill price adjust such that quality-adjusted relative skill prices remain unchanged. Then, why would a firm move part of its outsourcing from North to South? At the initial composition of tasks \((I_0^* vs 1-I_0^*)\), a skill upgrading in South raises the firm’s overall task quality, and thus the quality of and demand for its product:

\[
\lambda \mapsto \Psi(I_0^*) \left( = \lambda \int_0^{I_0^*} a(j) dj + \int_{I_0^*}^{1} a(j) dj \right) \mapsto q(\phi, I_0^*) \mapsto x_c(\phi) \mapsto .
\]

And the associated increase in the relative skill prices in South raises the firm’s marginal
cost and price:

\[ r_S \uparrow \Rightarrow C(\phi, I_0^*) = \phi^{-\alpha} \left[ r_S \int_0^{I_0^*} a(j) dj + r_N \int_{I_0^*}^1 a(j) dj \right] \uparrow \Rightarrow p_c(\phi, I_0^*) \uparrow . \]

The resulting change in the firm’s profits is, however, negative. So, the firm offsets the negative change in its profits by reducing its costs; it lowers the fraction of its imported tasks: \( I_c^* \) increases. As a result, the firm’s marginal cost, price, and quality-adjusted price will remain unchanged.

On the extensive margin, we only observe the income-raising effect of higher \( \lambda \): aggregate income in South increases, cutoff productivity for selling in the Southern market falls, and more firms enter the market. The mass of firms selling in the Southern market increases. On the other hand, the mass of those selling in the Northern market remains unchanged. In other words, the Southern market can accommodate a larger mass of firms, but the same is not true for the Northern market. It implies that competition to sell in the Northern market becomes tougher; the upward pressure on skill prices drives low-productivity (and low-quality) firms out of the market – \( \phi_N^* \) rises.

**Result 6** Assume that Southern workers upgrade their skills and become more productive – \( \lambda \) rises. The presence of the homogeneous good causes a matching increase in the relative skill prices, \( r_S/r_N \). At the firm-level, a firm increases the fraction of its domestically-sourced tasks, keeping its product quality, marginal cost, and price unchanged. At the aggregate-level, income in South increases. So, the cutoff productivity for selling in the Southern market falls, and the mass of firms increases. In the Northern market, intense competition raises the cutoff productivity, leaving the mass of firms unchanged.

Now, let us go back to Table 1. Only one stylised fact, F5, remains unexplained. The extension of the model to heterogeneous firms allows us to explain it in a consistent way with the other stylised facts. F5 says that a firm earns higher revenues in a market where it charges a higher fob price. Manova and Zhang (2009) report this empirical finding using Chinese transaction-level data. They highlight that it is robust to controlling for a firm’s country-product specific market share. The QHF studies that use standard CES preferences do not explain F5 because a firm’s fob price does not vary across markets, implying zero correlation. The studies that use quadratic preferences generate ambiguous results.

21Firm’s profits are negatively related to its quality-adjusted price: \( \frac{p_c(\phi)}{q_c(\phi)} \). As a result of an equal increase in \( \lambda \) and \( r_S \), the resulting change in the firm’s quality-adjusted price is proportional to

\[ 1 - \phi^{-\alpha(\alpha+1)} > 0. \]

Thus the firm’s profits fall.
Can this model accommodate the empirical finding that a firm earns higher revenues in a market where it charges a higher fob price? First, let us check how a firm’s price varies across markets. From (17b), we see that a firm selling to both markets charges a higher price in the Northern market:

\[ \frac{p_N(\phi)}{p_S(\phi)} = \frac{\gamma_N - 1}{\gamma_S - 1} > 1. \]

What about its revenues? The corresponding ratio for the firm’s revenues is equal to:

\[ \frac{r_N(\phi)}{r_S(\phi)} = \left( \frac{\phi}{\phi_N^*} \right)^{\eta_N - \eta_S} \left( \frac{1}{\theta \alpha \nu - \eta_N} \right)^{\eta_S/\nu}. \]

So, we have \( r_N(\phi) > r_S(\phi) \) if and only if the following holds:

\[ \phi > \phi_N^* \left( \theta \alpha \nu - \eta_S \right)^{\eta_S/\nu} = \hat{\phi} > \phi_N^*. \]

Therefore, we can state the following result.\(^{22}\)

**Result 7** There exists a productivity threshold (\( \hat{\phi} \)) above which a firm’s fob prices and revenues are positively correlated across markets. Observing such correlation is more likely when firm productivities are less dispersed (\( \nu \) is higher).

Northern consumers demand higher-quality varieties than Southern consumers. Observing this, a Southern firm uses a higher fraction of imported tasks to produce a higher-quality variety for the Northern market. Since imported tasks are more expensive, producing such variety is more costly. Thus the firm charges a higher price for the variety it sells in the Northern market. If the firm’s productivity is high enough, between markets, its cost increases by less than the quality of its product does. Therefore the firm earns higher revenues in the Northern market where it charges a higher fob price.

This prediction does not, however, hold for any Southern firm selling to a Northern market. Whether we observe it for an average productivity firm depends on the productivity distribution. To be specific, if firm productivities concentrate around the mean –their dispersion is low–, the chance of observing a positive correlation between a firm’s (fob) prices and revenues across markets is high.

5 Conclusion

Recent transaction-level datasets from Southern countries have unearthed new findings that are either missing in, or conflicting with the predictions of existing international

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\(^{22}\)Please see Appendix A.6 for the proof of the second part of the following result.
trade models. First, I discuss that they all come together nicely into a consistent story about a firm’s integration into global markets, which can be titled \textit{within-firm differentiation} – a strategy that consists of two elements: quality-to-market and multi-sourcing of inputs. Then, I present a simple and tractable trade model to study the sources and consequences of within-firm differentiation.

The model predicts both quality-to-market and multi-sourcing at the firm-level. On the supply side, to produce a final good, a firm combines a continuum of tasks that vary according to their skill requirements. The firm can source each task from domestic or foreign suppliers. As skills of foreign workers are of higher quality than those of domestic workers, tasks produced by foreign suppliers are of higher quality. High quality commands high prices; sourcing a task from foreign suppliers is more expensive than sourcing it from domestic suppliers. Facing a trade-off between quality and cost, the firm decides which inputs to source domestically and which ones to import. The model predicts that it imports more skill-intensive ones from North, and sources the rest domestically. As consumers located in different regions value quality at varying degrees, the firm adapts its product to local market by changing the fraction of its imported tasks. Also, since the relative demand for high-quality products is higher in distant than in near market, the firm produces a higher-quality variety for the distant market by using a higher fraction of imported tasks. Thus, in line with the empirical evidence, the firm engages in quality-to-market strategy by sourcing some tasks from multiple sources.

Next, I extend the model to heterogeneous firms. The extension provides a simple specification for a firm’s quality choice, which depends on the firm’s productivity and its overall task quality which, in turn, depends positively on the firm’s productivity and the intensity of consumer taste for quality. Thus the extension contributes to the QHF literature by deriving a well-founded and simple expression for the quality choice of a firm that engages in quality-to-market strategy. I also solve the model in general equilibrium and show that taste differences between markets relax the restriction on fixed trade costs to obtain Melitz’s prediction that only the more productive firms select into exporting. Under zero variable trade costs, even when the fixed cost of exporting is acceptably low compared to Melitz’s threshold, only the more productive Southern firms can export to the Northern market where quality demand is higher than in the domestic market. These firms charge higher prices in the Northern market than in the domestic market. Also, among them, the most productive ones earn higher revenues in the Northern market – implying a positive relationship between firm-product-level prices and revenues across markets.

I propose another prediction that is open to testing. Regardless of what happens to skill prices, a skill upgrading in the South leads a Southern firm to increase the fraction of its domestically-sourced tasks. At one extreme, skill prices remain unchanged –
corresponding to partial equilibrium. In that case, the firm’s marginal cost and price fall, but the direction of change in its product quality and quality-adjusted price is ambiguous. At the other extreme, skill prices adjust one-to-one to the change in relative productivities. In such case, the firm’s product quality, marginal cost, and price remain unchanged. The general equilibrium effect is twofold. First, the South moves up towards more skill-intensive tasks in the global value chain. Second, the resulting increase in Southern aggregate income raises the mass of successful entrants into the domestic differentiated goods industry, and competition for exporting to the Northern market becomes tougher. So, the cutoff productivity for exporting increases. In short, an overall improvement in the quality of tasks produced in the South weeds out low-productivity final good exporters from the Northern market.

References


A Appendix

A.1 Second-order conditions for profit maximisation

To check the second-order conditions for a unique solution for $I \in (0, 1)$, define the corresponding Hessian matrix:

$$H = \begin{bmatrix} \frac{\partial^2 \Pi}{\partial p^2} & \frac{\partial^2 \Pi}{\partial p \partial I} \\ \frac{\partial^2 \Pi}{\partial I \partial p} & \frac{\partial^2 \Pi}{\partial I^2} \end{bmatrix}.$$
The second-order conditions for a maximum are:

\[
\frac{\partial^2 \Pi}{\partial p^2} < 0, \\
|H| > 0.
\]

The signs of the individual terms, evaluated at \((p^*, I^*)\), are\(^{23}\)

\[
\frac{\partial^2 \Pi}{\partial p^2} = \mu Y P^{\sigma - 1}(1 - \sigma)(p + t)^{-\sigma - 1}q^{\gamma(\sigma - 1)} < 0,
\]

\[
\frac{\partial^2 \Pi}{\partial I^2} = -\mu Y P^{\sigma - 1}(p + t)^{-\sigma}q^{\gamma(\sigma - 1)} - 1 \frac{\partial q}{\partial I} \frac{\partial \Psi}{\partial I} < 0,
\]

\[
+ \frac{\partial q}{\partial I} (\sigma - 1)\gamma(b + 1) (\nu - C) \frac{\partial \Psi}{\partial I} + 1 - \frac{q^b}{\Psi^b} < 0,
\]

\[
\frac{\partial^2 \Pi}{\partial I \partial p} = \frac{\partial^2 \Pi}{\partial p \partial I} = \mu Y P^{\sigma - 1}\sigma(p + t)^{-\sigma - 1}q^{\gamma(\sigma - 1)} \frac{\partial C}{\partial I} < 0.
\]

As \(\frac{\partial^2 \Pi}{\partial p^2} < 0\), the first condition is satisfied. To check the second one, we need to determine the sign of \(|H|\) at \((p^*, I^*)\):

\[
|H| = (\frac{\partial^2 \Pi}{\partial p^2})(\frac{\partial^2 \Pi}{\partial I^2}) - (\frac{\partial^2 \Pi}{\partial p \partial I})^2
\]

\[
= -\frac{\sigma}{(1 - \sigma)^2} \left( \frac{\partial \Pi}{\partial p} \right)^2 \frac{\partial C}{\partial I} \left[ (1 - 1/\gamma) \frac{\partial C}{\partial I} - (\sigma - 1)(b + 1) \frac{p - C}{\Psi} \frac{\partial \Psi}{\partial I} \left( 1 - \frac{q^b}{\Psi^b} \right) \right]
\]

\[
= \frac{\sigma}{1 - \sigma} \left( \frac{\partial \Pi}{\partial p} \right)^2 \frac{\partial C}{\partial I} \left[ \frac{p - C}{\psi} \frac{\partial q}{\partial I} (\gamma - 1 - \Psi^b \phi^{-b}) - \frac{p - C}{\Psi} \frac{\partial \Psi}{\partial I} \left( 1 - \frac{q^b}{\Psi^b} \right) \right]
\]

The second term in the square brackets is positive. \(|H|\) is positive as long as the second term dominates the first. A sufficient condition for \(|H| > 0\) is

\[
\gamma < 1 + \Psi^b \phi^{-b}.
\]

In words, the intensity of consumer preference for quality should not exceed unity significantly. Throughout the paper, I focus on the values of \(\gamma\) that satisfy this condition. So, \(I^*\) satisfies \(0 < I^* < 1\).

### A.2 Proof of Result 4

To determine the sign of \((dI/d\lambda)_{I=I^*}\), we need to know the sign of the following partial derivative:

\(^{23}\)I abuse the notation by dropping the arguments.
\[ \Upsilon = \left\{ \gamma \left( C(\phi, I) + t \right) \frac{\partial^2 q(\phi, I)}{\partial I \partial \lambda} - \frac{\partial q(\phi, I)}{\partial \lambda} \frac{\partial C(\phi, I)}{\partial I} \right\}_{I=I^*}. \] 

First, let us consider the terms in the expression individually:

\[
\frac{\partial^2 q(\phi, I)}{\partial I \partial \lambda} = \frac{\partial^2 q(\phi, I)}{\partial \Psi \partial \lambda} \frac{\partial \Psi}{\partial I} + \frac{\partial q(\phi, I)}{\partial \Psi} \frac{\partial^2 \Psi(I)}{\partial I \partial \lambda},
\]

\[
\frac{\partial q(\phi, I)}{\partial \lambda} = \frac{\partial q(\phi, I)}{\partial \Psi} \frac{\partial \Psi(I)}{\partial \lambda},
\]

where

\[
\frac{\partial q(\phi, I)}{\partial \Psi} = q(\phi, I)^{1+b} \Psi(I)^{-1-b},
\]

\[
\frac{\partial^2 q(\phi, I)}{\partial \Psi \partial \lambda} = (1 + b) \left( \frac{q(\phi, I)}{\Psi(I)} \right)^{1+b} \frac{\partial \Psi(I)}{\partial \lambda} \left[ \frac{1}{q(\phi, I)} \frac{\partial q(\phi, I)}{\partial \Psi} - \frac{1}{\Psi(I)} \right].
\]

We can use these to expand (20). Below, for simplicity, I drop the arguments of the functions:

\[
\gamma (C + t) \frac{\partial q}{\partial \Psi} \left[ \frac{1 + b}{q} \frac{\partial \Psi}{\partial \Psi} \frac{\partial \Psi}{\partial I} \frac{\partial \Psi}{\partial \lambda} - \frac{1 + b}{\Psi} \frac{\partial \Psi}{\partial I} \frac{\partial \Psi}{\partial \lambda} + a(I) \right] - \frac{\partial q}{\partial \Psi} \frac{\partial C}{\partial \lambda} \frac{\partial I}{\partial I}.
\]

Remember that we are calculating this partial derivative at \( I = I^* \). So, use the combined first-order condition in (12) to simplify the expression above, and obtain:

\[ \Upsilon = \left\{ \gamma (C + t) \frac{\partial q}{\partial \Psi} \left[ \frac{1 + b}{q} \frac{\partial \Psi}{\partial \Psi} \frac{\partial \Psi}{\partial I} \frac{\partial \Psi}{\partial \lambda} - \frac{1 + b}{\Psi} \frac{\partial \Psi}{\partial I} \frac{\partial \Psi}{\partial \lambda} + a(I) \right] \right\}_{I=I^*} \]

> 0.

This completes the proof.

A.3 Proof of the sign of \( \frac{d \Pi(\phi)}{d \phi} \)

To prove that a more productive firm makes higher profits, differentiate the variable profits of a firm with respect to its productivity \( \phi \):

\[
\frac{d \Pi(\phi)}{d \phi} = \frac{d}{d \phi} \left[ \pi(\phi) (p(\phi, I) - C(\phi, I)) \right]
\]

\[ = \frac{\partial \Pi(\phi)}{\partial \pi(\phi)} \left( \frac{\partial x(\phi)}{\partial \phi} + \frac{\partial x(\phi)}{\partial \phi} \frac{d I}{d \phi} \right) + \frac{\partial \Pi(\phi)}{\partial p(\phi, I)} \left( \frac{\partial p(\phi, I)}{\partial \phi} + \frac{\partial p(\phi, I)}{\partial \phi} \frac{d I}{d \phi} \right)
\]

\[ + \frac{\partial \Pi(\phi)}{\partial C(\phi, I)} \left( \frac{\partial C(\phi, I)}{\partial \phi} \frac{d I}{d \phi} \right). \]
It follows from the Envelope Theorem that $\frac{\partial \Pi(\phi)}{\partial p(\phi, I)} = 0$. Also, by the same theorem, 
\[ \frac{d}{d\phi} \left( \frac{\partial \Pi(\phi)}{\partial x(\phi)} \frac{\partial x(\phi)}{\partial \phi} + \frac{\partial \Pi(\phi)}{\partial C(\phi, I)} \frac{\partial C(\phi, I)}{\partial \phi} \right) = 0. \]
So, we obtain:
\[ \frac{d\Pi(\phi)}{d\phi} = \frac{\partial \Pi(\phi)}{\partial x(\phi)} \frac{\partial x(\phi)}{\partial \phi} + \frac{\partial \Pi(\phi)}{\partial C(\phi, I)} \frac{\partial C(\phi, I)}{\partial \phi} > 0. \]
This proves that profits are increasing in productivity.

### A.4 Proof of Result 5

First, remember the following firm-level expressions:

\[ C_c(\phi) = (\gamma_c - 1)\phi^{1-\alpha}, \quad (21a) \]
\[ p_c(\phi) = \frac{\sigma}{\sigma - 1}(\gamma_c - 1)\phi^{1-\alpha}, \quad (21b) \]
\[ q_c(\phi) = \frac{\phi(\gamma_c - 1)}{\gamma_c}, \quad (21c) \]
\[ \frac{p_c(\phi)}{q_c(\phi)^{\gamma_c}} = \Lambda_c\phi^{1-\alpha-\gamma_c}, \quad (21d) \]

where $\Lambda_c = \frac{\sigma}{\sigma - 1}(\gamma_c - 1)^{1-\gamma_c}\gamma_c^{\gamma_c}$.

If two firms $\phi$ and $\phi'$ both sell their products to region $c$, the ratio between their quality-adjusted prices is equal to

\[ \frac{\frac{p_c(\phi)}{q_c(\phi)^{\gamma_c}}}{\frac{p_c(\phi')}{q_c(\phi')^{\gamma_c}}} = \left( \frac{\phi}{\phi'} \right)^{1-\alpha-\gamma_c}. \]

Use this expression to re-write the aggregate price index $P_c$ in region $c$:

\[ P_c = M_c^{\frac{1}{\sigma}} \frac{p_c(\bar{\phi}_c)}{q_c(\bar{\phi}_c)^{\gamma_c}}, \quad (22) \]

where $\bar{\phi}_c$ is the aggregate (or average) productivity of the firms selling to region $c$:

\[ \bar{\phi}_c = \left[ \int_{\phi_c}^{\bar{\phi}_c} (\phi^{1-\alpha-\gamma_c})^{1-\sigma} \mu_c(\phi) \right]^{\frac{1}{1-\sigma(1-\alpha-\gamma_c)}}. \quad (23) \]

In Melitz’s model, the aggregate productivity depends only on elasticity of substitution between varieties ($\sigma$). Here, it also depends on the technology parameter $\alpha$, and the intensity of consumer preferences for quality $\gamma_c$. As in the original model, we can derive the other aggregate variables in terms of aggregate productivity.

Take $r_N$ as the numéraire. The following zero-profit condition pins down the cutoff
productivity for selling in the Southern market:

$$\mu Y_S \Lambda_S^{1-\sigma} P_S^{\sigma-1} \phi_S^{(1-\alpha-\gamma_S)(1-\sigma)} = \sigma f.$$  

Use (21d) and (22) to re-write this condition:

$$\sigma f M_S [k(\phi_S^*) + 1] = \mu Y_S,$$  \hspace{1cm} (24)

where

$$k(\phi_c^*) = \left( \frac{\sim \phi_c}{\phi_c} \right)^{(1-\alpha-\gamma_c)(1-\sigma)} - 1; \ c = S, N.$$  \hspace{1cm} (25)

So, a firm with productivity $\phi < \phi_S^*$ remains inactive in the domestic market. The distribution of the active firms in that region becomes:

$$\mu_S(\phi) = \frac{g(\phi)}{1 - G(\phi_S^*)}.$$  

Assume that the distribution of firm productivities is Pareto with a shape parameter $\nu > \eta_N > \eta_S > 2$. So, we can write the cutoff productivity condition as:

$$\sigma f M_S \frac{\nu}{\nu - \eta_S} = \mu Y_S.$$  

Similarly, we can write the cutoff for exporting to the Northern market by setting $f_X = \theta f, \ \theta > 0$

$$\sigma \theta f M_N \frac{\nu}{\nu - \eta_N} = \mu Y_N.$$  

Use the Pareto distribution assumption, and combine the two cutoff productivity conditions to obtain

$$\frac{Y_S}{Y_N} = \frac{1}{\theta} \frac{\nu - \eta_N}{\nu - \eta_S} \frac{M_S}{M_N}.$$  \hspace{1cm} (26)

Based on conditional factor price equalisation, two regions have almost the same aggregate income. Since I am interested in the effect of the difference in consumer tastes on the cutoff productivities, I ignore the minor differences between the aggregate incomes and, thus, equate the left-hand side of (26) to unity. On the right-hand side, as $\eta_N > \eta_S$, $(\nu - \eta_N)/(\nu - \eta_S)$ is less than unity. Now, consider the following case. First, let $\theta < (\nu - \eta_N)/(\nu - \eta_S)$, fixed cost of exporting is very small. For the conditional factor price equalisation to hold, we must then have $M_S < M_N$: there are some Southern firms whose variable profits in the Northern market cover both the fixed cost of production ($f$) and the cost of exporting ($f_X$), and they are not profitable in their domestic market. But, under monopolistic competition and CES preferences, such a case cannot arise because there are no additional fixed costs that those firms have to incur to sell their products in the domestic market. So, they should be profitable and
thus active in their domestic market as well, implying $M_S > M_N$. For this inequality to hold, we must have

$$\theta > \frac{\nu - \eta_N}{\nu - \eta_S}. \quad (27)$$

Thus the model applies when the fixed cost of exporting is not too small. Under this assumption, the cutoff productivity for exporting satisfies

$$\Rightarrow \phi^*_N = \phi^*_S \left( \theta \varepsilon \lambda \frac{\nu - \eta_S}{\nu - \eta_N} \right)^{1/\nu}.$$

Given (27), we have $\phi^*_N > \phi^*_S$. This completes the proof.

A.5 Characterisation of the heterogeneous-firm extension of the model

Appendix A.4 derives the cutoff productivities in North and South. There is also a free-entry condition: a firm enters the market if and only if it expects to cover the fixed entry cost. A firm’s expected value is equal to

$$v(\phi) = \max \left\{ 0, \frac{1}{\delta} E[\pi(\phi)] \right\}.$$

Owing to the free-entry condition, its expected per period profits are equal to

$$E[\pi(\phi)] = \delta f_e \quad (28)$$

$$\Rightarrow \Pi(\phi^*_S)(1 - G(\phi^*_S)) + \Pi(\phi^*_N)(1 - G(\phi^*_N)) = \delta f_e. \quad (29)$$

One of its implications is that aggregate profits do not contribute to aggregate income. Because, by (28), aggregate profits net of entry costs is equal to zero.$^{24}$ Thus, income of a region is simply equal to the total payments to workers – value of the stock of skills – in the region:

$$Y_S = \varepsilon \lambda L_N,$$

$$Y_N = L_N.$$

$^{24}$The aggregate profits of firms that are located in a region may not equal to zero. I propose two different mechanisms so that the argument in the text holds. First, as Chaney (2008) proposes, profits of all firms are collected by a global fund, which then re-distributes them. Second, one can solve for the integrated world equilibrium to pin down the world income. Then, distribute the world income to each region in proportion to their stock of skills. Under both mechanisms, since aggregate profits net of entry costs are zero, they do not contribute to regional income.
Use the Pareto assumption, and the equality \( \frac{r_c(\phi)}{r_c(\phi')} = \left( \frac{\phi}{\phi'} \right)^{\eta_c} \) to express the free-entry condition as;

\[
\frac{\eta_S}{\nu - \eta_S} \phi_S^{\star - \nu} + \theta \frac{\eta_N}{\nu - \eta_N} \phi_N^{\star - \nu} = \frac{\delta f_e}{f}.
\]

(30)

I am interested in stable equilibria: the mass of firms entering the market should be equal to the mass of firms exiting so that the distribution of firms in the industry remains unchanged. Let \( M_e \) denote the mass of firms entering the market, then we can write the stability condition as;

\[
(1 - G(\phi_S^*)) M_e = \delta M,
\]

(31)

where \( M = M_S \). Also, the mass of firms selling to the Northern market satisfies

\[
M_N = M \frac{1 - G(\phi_N^*)}{1 - G(\phi_S^*)}.
\]

(32)

As a result, we have five equations: two cut-off productivity conditions (24), one free-entry condition (30), one stability condition (31), and the equation that pins down the mass of firms selling to the Northern market (32). We can solve them to determine the five endogenous variables: \( \phi_S^*, \phi_N^*, M, M_e, M_N \). Here is the solution:

\[
\phi_S^* = \left( \frac{f}{\delta f_e} \frac{\eta_S + \eta_N/(\varepsilon \lambda)}{\nu - \eta_S} \right)^{1/\nu};
\]

\[
\phi_N^* = \left( \frac{\theta \varepsilon \lambda}{\delta f_e} \frac{\eta_S + \eta_N/(\varepsilon \lambda)}{\nu - \eta_N} \right)^{1/\nu};
\]

\[
M = \frac{\mu (\varepsilon \lambda L) (\nu - \eta_S)}{\sigma f_e \nu} = M_S;
\]

\[
M_N = \frac{\mu L (\nu - \eta_N)}{\theta \sigma f_e \nu};
\]

\[
M_e = \frac{\mu L (\varepsilon \lambda \eta_S + \eta_N)}{f_e \nu},
\]

where \( L = L_N \). Any surplus of skills is absorbed by the homogenous good industry in both regions.

**A.6 Proof of Result 7**

Here, I prove that observing a positive correlation between a firm’s fob prices and its revenues across markets is more likely, the closer \( \bar{\phi} \) is to the cutoff productivity in the Northern market \( (\phi_N^*) \). Differentiate the expression for \( \bar{\phi} \) with respect to \( \nu \):

\[
\frac{d(\bar{\phi}/\phi_N^*)}{d\nu} = \frac{\bar{\phi}}{\phi_N^*} \frac{\eta_N}{\nu} \left[ - \frac{1}{\nu (\eta_N - \eta_S)} \ln \left( \frac{\theta \varepsilon \lambda \nu - \eta_S}{\nu - \eta_N} \right) - \frac{1}{(\nu - \eta_N) (\nu - \eta_S)} \right].
\]
If $\phi^*_N > \phi^*_S$, we must have $(2\frac{\nu - \eta_S}{1 - \eta_N}) > 0$. So, we obtain $\frac{d(\phi^*/\phi^*_N)}{d\phi^*_N} < 0$. This completes the proof.

Table 1: Some Unexplained Observations in Recent Empirical Studies

<table>
<thead>
<tr>
<th>Stylised Fact</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>F2: A firm charges a high price for a product in distant markets. The correlation is stronger for differentiated products.</td>
<td>Görg et al. (2010), Manova and Zhang (2009)</td>
</tr>
<tr>
<td>F3: A firm that pays a wide range of input prices also offers a wide range of export prices across markets.</td>
<td>Manova and Zhang (2009)</td>
</tr>
<tr>
<td>F4: A firm pays a higher price for imported input than for domestic input in the same product category.</td>
<td>Kugler and Verhoogen (2009)</td>
</tr>
<tr>
<td>F5: A firm earns higher revenues in a market where it charges a higher fob price.</td>
<td>Manova and Zhang (2009)</td>
</tr>
</tbody>
</table>