Informative Advertising, Consumer Search and Transparency Policy (Job Market Paper)

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Abstract

Information about a new or non-frequently purchased product is often produced by both sides of the market. We construct a monopoly pricing model consisting of both seller's information disclosure and consumer's information acquisition. The presence of consumer search, which lowers the probability of making sales, creates incentive for the monopolist to deter search. In contrast with most previous literature, we show that, partial information disclosure arises in equilibrium when the search cost is low. As the search cost increases to medium level, the monopolist hides information but lowers the price to prevent consumers from searching. When the search cost is very high, the monopolist charges high price and hides information. The equilibrium price is thus non-monotonic in search cost. Information disclosure and consumer search co-exist only when the search cost is low, and thus complement each other. We show that transparency policies on advertising cannot improve social welfare. Nevertheless, they benefit consumers in a wide range of values of the search costs by improving matching quality and reducing the expense of searching. But for some medium levels of search costs, transparency policies hurt consumers due to the induced high price in equilibrium.

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1 Introduction

When a new product or service becomes available in the market, both buyers and sellers are usually uncertain about how much buyers value it. Consequently, access to information about the matching between product attributes and buyers' tastes is important for all market participants to make a decision. In many markets, this information is generated by both sides of the market. While sellers can choose how much information to convey to the market, buyers are not completely passive, and can obtain information through their own costly endeavors. For example, automobile buyers have different tastes in cars' appearance, horse-power, inner decoration, and fuel saving. Automobile manufacturers and retailers may reveal information about those product attributes completely or partially. Meanwhile, consumers can collect that information by themselves, but collecting information is costly. The associated search cost is for gathering more information about the matching value. For instance, consumers can collect information from the internet, attend promotion campaigns, seek advice from experienced users and so on.

In this paper, we explore a monopolist's information disclosure choice and pricing strategies in the presence of consumer search. Our primary goal is to answer the questions that who produces information in equilibrium when both sides of the market are able to do so and how much information is released. If there is no strategic interaction between sellers and buyers, sellers seem to be in a better position as information providers, because they have better knowledge about products and because it is less costly for them to convey this information. However, information disclosure is often strategically used by sellers as a tool to extract consumer surplus. Often, it is not of sellers' interest to efficiently release information to the market. Nevertheless, if information can be produced by the demand side, sellers' choice of revealing information is constrained by consumers' search option. If the information in ads is unsatisfactory to the buyers, consumers can collect more precise information by themselves anyway. However, to what extent consumer search will be conducted is determined by the magnitude of search costs. Thus, it is interesting to explore, when search cost changes, how the advertising precision and search intensity change accordingly. In particular, we aim to address the question that whether advertising precision and consumer search interact as substitutes or complements in the process of information provision.

The aim of this paper is not purely theory-oriented, but also motivated by the gap between the advertising theory and the real world advertising. In most theoretical works regarding sellers' choice of revealing information (see Lewis and Sappington, 1994, Johnson and Myatt, 2006, Ivanov, 2010), a surprisingly uniform answer is that sellers always prefer extreme choice, either perfect information disclosure or no information disclosure. However, empirical evidences suggest a different story. Over 4 studies of US television advertising, Abernethy and Franke (1996) showed that the mean number of "information cues"(e.g., price, quality, performance, availability, nutrition, and warranties) was 1.06, with only 27.7 percent having two or more cues, and 37.5 percent having no cues. Over 7 studies of US magazines, the mean number of cues was slightly higher at 1.59, with only 25.4 percent having three or more cues, and 15.6 percent having no cues. That is, in real world, a large portion of daily advertising is featured by partial information disclosure. Our model offers an explanation for why the sellers may prefer partial information disclosure to extreme choices. Finally, we also use our model to examine the welfare implications of transparency policies that force sellers to reveal information.

In our model, a monopolist and consumers sequentially produce information. The monopolist first chooses the price and how much information to reveal to the market. Any information disclosure will lead to dispersion in the distribution of consumers' valuations. Upon observing the monopolist's choice and the advertising content, all consumers update their estimation of their true valuation. If they are satisfied with the information provided by the monopolist, they can choose to make an immediate purchase with no further cost, as in the case of online payments. If they feel more information is needed, they can engage in search by incurring a fixed search cost. After search, they have perfect knowledge about their true valuation and their purchase decision is based on that. In this setting, it is noticeable that consumers can use search as a counter-strategy to the monopolist's first period choice; the monopolist needs to take this into account in choosing advertising precision and price.

In this paper, we show that when the search cost is high, the monopolist hides all information and charges a high price. When the search cost is medium, the monopolist still hides all information but charges a lower price to deter consumer search. When the search cost is low, the monopolist reveals partial information to the market but charges a high price. Consumers actively search for information only when the search cost is low. Therefore, information disclosure and consumer search co-exist when the search cost is low and, neither occurs when the search cost is higher than a certain threshold. They behave as complements in the process of producing information. Equilibrium price is non-monotonic in search cost.

For any given price, a monopolist always prefers consumers to make immediate purchase rather than search, because the probability of make sales is reduced after consumers knowing their true valuations. Thus the monopolist have intentions to, fully or partially, deter consumer search. Consumer search is not very effective when the search cost is high. Foreseeing that consumers are not likely to search even when the advertising is uninformative and the price is high, the monopolist's optimal strategy is to charge a high price but hide all information. The ex-ante consumer surplus is fully extracted. Any leak of information discourages immediate purchase and is thus suboptimal, because the consumers hearing "bad news" (receiving low signal realizations) lower their willingness to pay.

When the search cost deceases, consumers choose to search if the price is too high. There are two ways to deter search: reducing price and revealing information. Reducing price is better than information disclosure when the search cost is not too low. Information disclosure creates heterogeneity among consumers so that some immediate purchase are lost unless the price is very low. Instead, by reducing price but hiding all information so that all consumers still prefer immediate purchase, the monopolist can retain the full base of immediate purchase. When search cost is very low, reducing the price to prevent search is no longer optimal. Some (never full) information is revealed to convince consumers who hear "good news" (receive high signal realizations) to make immediate purchase at a high price. The consumers who receive "bad news" will search. The implication of this argument is two-fold. First, why use information disclosure rather than reducing price to prevent search? Notice that the strategy of reducing price itself destroys profits. When the search cost is low, the price would have to be reduced drastically to prevent search so that the profit drops off very quickly. Especially, when search cost tends to zero, the price needs to be close to zero to prevent search. On the contrary, information disclosure itself is costless and it always induces consumers who hear "good news" to make immediate purchase at high price. The profit decrease slowly as search cost decreases. Second, why information disclosure should be partial? If information disclosure is perfect, consumers receiving "bad news" will quit the market immediately without search. If there is no information disclosure, no immediate purchase will be made unless the price is set at a very low level.

Our model also provides a new insight for the desirability of transparency policies. Transparency policies are implemented in many markets such as food, pharmaceutical, and financial markets. It is widely believed that greater product transparency can enhance consumer surplus and/or social welfare. Building on our model, we show that transparency policies have positive effects on consumer surplus by improving matching quality and reducing expense on searching. However, transparency policy hurts consumers when search cost is medium. When the transparency policy is absent, to prevent search, the monopolist would like to reduce price when the search cost is medium. If transparency policy is imposed, the monopolist always charges a high price since deterring search is no longer a concern. For some medium level of search cost, the negative effect from the induced high price dominates the positive effect from the increased transparency. We also show that transparency policies cannot improve social welfare as monopolist's profit decreases drastically under transparency policy. These policy implications provide theoretical justification for using transparency policies if consumer surplus is under consideration, but remind us that effectiveness crucially depends on regulators' target and on the magnitude of the search cost.

The rest of the paper is organized as follows. Section 2 discusses the related literature. Section 3 presents the model and the information structure. Section 4 explores the equilibrium with partial information disclosure under a general setting. In Section 5, we completely characterize the equilibrium in a setting with uniform valuation distribution and truth-or-noise advertising. Transparency policy and welfare issues are discussed in Section 6. Section 7 concludes.

2 Related Literature

There is a growing literature modeling sellers' strategic control of match-specific information. Lewis and Sappington (1994) is the pioneer work which emphasizes the fact that in many markets sellers can affect consumers' access to product information and use it to better discriminate between consumers. Johnson and Myatt (2006) and, Ivanov (2010) extend the model of Lewis and Sappington (1994) to general information disclosure technology and a competitive environment. On the other side of the market, consumers may actively search for information. Wolinsky (1986), Anderson and Renault (1999) introduce a sequential search model where consumers search for their best matches in the market. In Bergemann and Valimaki (2002), Shi (2009), the consumers can acquire information before an auction or other mechanisms. Both sets of works considers only one side of the market in the process of producing information. But, in many markets, both sides produce information. Having this fact in mind, our model consists of both information disclosure by the seller side and information acquisition by the consumer. In our model, after consumers observe ads by the seller, they can choose to collect more information if they are unsatisfied with the information provided.

The information disclosure technology we use is based on the idea of Lewis and Sappington (1994) that more informative advertising leads to a more dispersed valuation distribution. This method of modeling informativeness is also used by Johnson and Myatt (2006) and is generalized by Ganuza and Penalva (2010). In most information disclosure literature with monopolist pricing, the monopolist usually prefers either perfect information disclosure or no information disclosure. The reason is that the demand structure always induces a trade off between segmenting the market and endowing consumer information rent. By hiding all information or revealing all information, the monopolist can choose to serve either a niche market or serve a mass market. In a competitive environment, Ivanov (2010) shows that when the number of firms reaches a certain value, fully revealing information is the unique symmetric Nash equilibrium. Our paper differs from previous works in the way in which consumers can actively engage in search. This provides incentive to the seller to deter search so that partial information disclosure can occur in equilibrium. This complements results with extreme levels of information disclosure.

Our paper is most closely related to Anderson and Renault (2006). In their model, the monopolist is able to provide match-specific information and price-specific information prior to consumer search. Consumers have to incur a search cost to make purchase. This means purchase is always made under perfect information. Then, consumers only buyer when their true valuation is higher than price. When the price is not advertised, the potential hold-up problem prevents some valuable trades to be realized. The monopolist thus may provide information to alleviate this problem. The main result is that the monopolist chooses threshold value advertising where consumers are only informed about whether their true valuations are higher than a threshold value. We depart from Anderson and Renault (2006) in several aspects. First, we interpret search cost differently. Anderson and Renault assume that consumers need to incur a search cost even when they are perfectly informed. We separate the cost of collecting information from the transportation cost. Consumers have an option to make immediate purchase without incurring search cost. Search cost is incurred purely for collecting information and transportation cost is normalized to zero. Second, in Anderson and Reanult's model, to mitigate hold-up problem, the monopolist provide information to induce consumers to visit the store. Thus, information disclosure is used to "pull consumers over". In contrast, in our model, to deter search, the monopolist reveals information to guarantee the product quality. That is, information disclosure is used to "push consumers away". Finally, our model yields different policy implications. In Anderson and Renault (2006), since only efficient trades occur, implementing transparency policy reduces the volumes of trades and thus hurt both sides. In our model, as immediate purchase is available, it is possible for consumers to buy product with value lower than the price. Greater market transparency can correct this inefficiency, so consumers may get better off. Bar-Isaac, Caruana and Cunat (2011) focus on the product design choice (which has similar consequence as information disclosure) in a perfect competitive market with consumer search. However, the search cost is still interpreted as a mixture of information-collecting cost and transportation cost.

Another closely related work is Bar-Isaac, Caruana and Cunat (2010). They study the relation between consumers' information gathering and the monopolist's marketing strategy. The monopolist first chooses the marketing strategy, which determines how hard it is for consumers to know their true valuations. In the second stage, consumers have exactly the same options as in our model: they can make an immediate purchase, search, or quit the market. When consumers are ex-ante heterogeneous, they find that the monopolist might choose an intermediate marketing strategy, by setting a positive but finite search cost. In our model, the monopolist cannot manipulate the search cost, but takes it as given. Though market obfuscation and information disclosure are two sides of the same coin, we find it is of its own interest to explore the market consequence of information disclosure. Because search cost is exogenous in our model, we are able to characterize how the precision of advertising and search intensity change as search cost changes. Most importantly, different from Bar-Isaac, Caruana and Cunat (2010), we show that even when consumers are ex-ante homogeneous, the monopolist may prefers partial information disclosure.

Finally, our paper is also related to homogeneous product models where consumers search for the lowest price in the market but sellers also advertise prices. Stegeman (1986) extends Butters Model to include consumer search. Robert and Stahl (1993) investigate firms' advertising strategies in a sequential search model with homogeneous consumers. They show that advertising and consumer search are complements in the sense that for parameter values for which firms do not advertise, consumers do not search either. Janssen and Non (2008, 2009) study the sequential search model with ex-ante heterogeneous consumers and show that advertising and consumer search can be substitutes. Our model differs from those models in that we assume the price is always advertised and advertising content is only about match-information. As in Robert and Stahl (1993), we show that information disclosure and consumer search are complements.

3 Model Setting

3.1 Preliminaries

Consider a monopolist that sells a new or non-frequently purchased product to a continuum of ex-ante homogeneous consumers. Each consumer has unit demand. Denote a consumer's true valuation of the product as $v \in [0, 1]$. We assume that this matching value is *a priori* unknown to both sides. Consumers have undelying tastes that are heterogeneous, so v could be different across consumers. Consumers are risk-neutral. If they purchase the product at price p, they get realized surplus $v - p$. We assume the marginal production cost constant and normalize it to zero. The monopolist is also risk-neutral and obtains expected profit $pD(p)$ if it sets price p, where $D(p)$ is the expected demand.

Without additional information, the true valuation can be seen as a random variable

V. It is common knowledge that V has distribution function $G(v)$ with support $[0, 1]$. We assume that the density function $g(v)$ is strictly positive and differentiable. Furthermore, we focus on the case where there is no correlation between consumers' valuations so that the distribution of true valuation is i.i.d across consumers. We suppose that the "hazard rate" $g/(1 - G)$ is strictly increasing. One example is the standard uniform distribution with $g(v) = 1$ for $v \in [0, 1]$. This yields a standard linear expected demand curve with unit intercepts.

Besides setting the price, the monopolist is free to supply product information to consumers to maximize expected profit. We call this process informative advertising. Throughout this paper, we assume that advertising is costless. This is a common assumption in the literature where advertising is about revealing product attributes (see Lewis and Sappington, 1994 and Johnson and Myatt, 2006) rather than informing sellers' existence in the spirit of Butters' model.[1]

Since our focus is not on how advertising alleviates hold-up problem, price is always advertised and the monopolist commits to the advertised price. This is different from Anderson and Renault (2006). Upon observing the advertising content including price, consumers update their beliefs about true valuation v in a Bayesian manner. If consumers are satisfied with the estimated valuation, they are free to make an immediate purchase without incurring any further cost except the price. However, consumers may need more information to make a purchase decision. In that case, they can conduct search by incurring search cost $s \in [0, 1]$. For simplicity, we assume that consumers are perfectly informed about their true valuations v after search. Once a consumer knows his true valuation, he buys if and only if $v \geq p$. Upon receiving the ads, a consumer maybe very pessimistic about his true valuation and quit the market immediately without purchase or search.

The timing of the model is summarized as follows: 1) the monopolist simultaneously chooses the price and how much information to convey when advertising; 2) consumers observe advertising content, update their valuation estimates and independently choose to make an immediate purchase, search or quit; 3) if a consumer searches, he makes a purchase decision after knowing his true valuation; 4) payoffs realize after any purchase. Furthermore, if a consumer is indifferent between immediate purchasing and searching, we assume he buys immediately; if he is indifferent between searching and quitting, we assume he searches.

[1] The following two reasons also justify this assumption. First, the advertising modeled here can be understood as disclosing information about attributes, which is different from hyper advertising. There is no significant cost difference between two advertisements, where one is a detailed illustration of the product and the other is an uninformative poster. Second, we emphasize that the partial information disclosure results stems from the monopolist's strategic consideration of differentiating buyers rather than simply saving on advertising costs.

Thus, we have constructed a model such in which both sides of the market can yield information to consumers. We will use the model as a workhorse to analyze the interplay in generating information between the monopolist and consumers.

3.2 Information Structure

Following the literature on informative advertising, the monopolist can not lie in its advertising. But it is free to choose the precision of advertising. Let $\mu = \int_0^1 v dG(v)$ be the prior mean of the true valuation. If there is no further information, this prior mean is a sufficient statistic for a consumer to make decision. If additional information is supplied by the monopolist, intuitively, the posterior valuations will move away from μ. Formally, we assume that the monopolist can choose a parameter $\eta \in [\underline{\eta}, \overline{\eta}]$, which measures the precision of its advertising. A higher η corresponds to a more informative advertising.

The choice of η is public knowledge. After the monopolist's choice of price and precision, each consumer independently and privately observes a realized signal $x \in [0, 1]$. Different consumers may observe different x. This does not mean they see different ads. Rather, they interpret the same ad in different ways due to the underlying heterogeneous tastes. As the signals reflect consumers' true valuations, let the correlation between the two be determined by a joint distribution $F(x, v \mid \eta) = \Pr[X \leq x, V \leq v \mid \eta]$, where X is the random variable representing the signal. Without loss of generality, the marginal distribution of X is assumed to be uniform.[2]

Having observed his signal, a consumer updates his belief about his true valuation. The posterior distribution of V is therefore $G(v \mid x, \eta) = \Pr[V \leq v \mid x, \eta]$. As is standard in the literature, a higher signal realization implies a higher likelihood of having a high true valuation. Formally, for any $x' > x$, $G(v \mid x', \eta) \leq G(v \mid x, \eta)$ with the strict inequality holds for some $v \in (0, 1)$. In other words, higher realizations of X is more likely to be associated with high true valuations in the sense of *First Order Stochastic Dominance* (FOSD).

The way we model information disclosure is based on the idea that more information creates larger dispersion of posterior valuations. This is similar to Johnson and Myatt (2006) and Ganuza and Penalva (2010). For intuition on this method, consider the following discrete example. A monopolist sells a product with two attributes: size and color. There are two types of consumers, type L and type R, with equal size in the population. The preference of

[2]Suppose the original signal is $\tilde{X}_\eta$ with continuous and strictly increasing marginal distribution $\{F(\tilde{x}|\eta)\}$. Define $X_\eta = F(\tilde{X} \mid \eta)$ for each η. According to the *probability integral transformation theorem*, X_η is uniformly distributed on $[0, 1]$. This simplifies the analysis when we compare realizations of different signals. Further details can be found in Ganuza and Penalva (2010).

consumers are heterogeneous. The type L consumers prefers blue color and large size while the type R consumers prefers the other way around. The preference is rationalized by the following payoff matrix

	L	R
red/large	2+4=6	4+2=6
blue/large	4+4=8	2+2=4
red/small	2=2=4	4+4=8
blue/small	4+2=6	2+4=6

The prior information is that each possible combination of attributes is of probability 1/2. Now if there is no information disclosure, each type of consumers hold the valuation 6. If the monopolist only reveals information about color, say blue, the type L consumers upgrade their valuations to 7 while the type R consumers downgrade their valuations to 5. Suppose the monopolist fully reveals information and the product is of color blue and size large. The type L consumers end up with valuation 8 while the type R consumers only value the product by 4. Clearly, the distribution of valuations becomes more dispersed along the way more information is provides. A more informative advertising thus leads to a more dispersed distribution of valuations.

Formally, a consumer who observes signal x and the monopolist's choice η forms posterior estimate about V as $w(x,\eta) = \int_0^1 vdG(v \mid x,\eta)$. Obviously, not only the signal realization x but also the precision η affects consumers' estimation. A more precise advertising raises (lowers) the prospect of a consumer who receives relatively good (bad) signals. We assume, for any $\eta \in [\underline{\eta}, \overline{\eta}]$, there exists a rotation point x_η^R such that, for any $v \in [0,1]$ and $\eta' > \eta$

$$G(v|x,\eta) \begin{cases} \geq G(v|x,\eta') & \text{when} \quad x > x_\eta^R \\ = G(v) & \text{when} \quad x = x_\eta^R \\ \leq G(v|x,\eta') & \text{when} \quad x < x_\eta^R \end{cases} \tag{1}$$

This condition says that for any precision level, there exists a signal realization x_η^R which does not change a consumer's expectation. However, for any signal better than x_η^R, the incremental precision leads to a higher expectation in the sense of FOSD and vice versa for any worse signal. Let $H(w,\eta)$ be the distribution of posterior valuation W when the precision is η. The way we model informative advertising as a posterior valuation dispersion leads to a *Rotation Order* ($H(w,\eta)$ rotates clock-wisely when η increases) introduced by

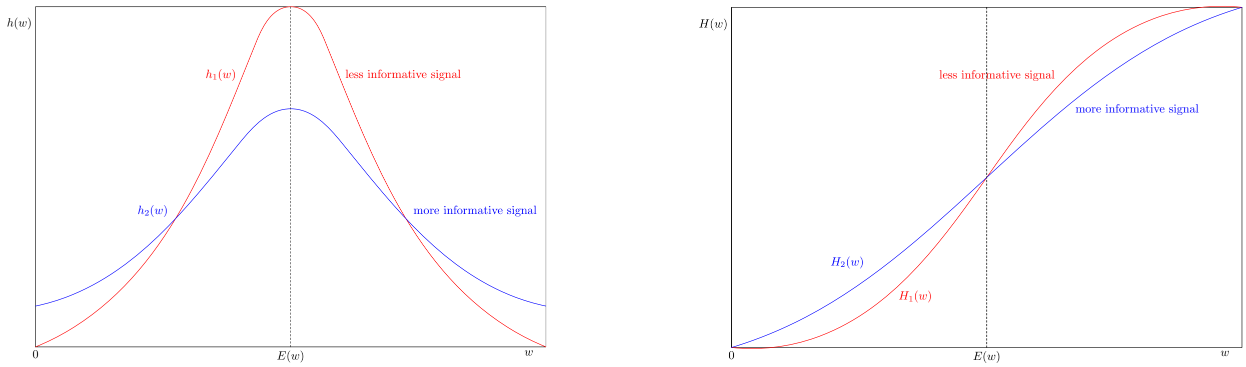

Figure 1: signal X_2 is more informative than signal X_1

Johnson and Myatt (2006).[3] Figure 1 shows an example of rotation-ordered distributions and their density functions.

3.3 An Example: Truth-or-Noise Advertising

There are many advertising technologies in the literature which satisfy our assumption, for example, Gaussian learning. Here, we use the truth-or-noise advertising technology, introduced by Lewis and Sappington(1994) to show how valuations become more dispersed as η increases. Consumers' valuations v are independently drawn from a distribution G, and G has an increasing hazard rate. Each consumer observes a signal x. With probability $\eta \in [0, 1]$, the signal x perfectly matches his true valuation v, and with probability $1 - \eta$, the signal x is a pure noise independently drawn from G. A Consumer who observes x with precision η will calculate his posterior estimate as

$$w(x, \eta) \equiv \mathbb{E}(v|x, \eta) = \eta x + (1 - \eta)\mu.$$

where μ is the prior expectation of v. The distribution of posterior valuations is

$$H(w, \eta) = G\left(\frac{w - (1 - \eta)\mu}{\eta}\right),$$

and we have

$$\frac{\partial H(w, \eta)}{\partial \eta} = -\frac{w - \mu}{\eta^2} h(w, \eta).$$

Thus, $H(w, \eta)$ satisfies rotation-ordered criteria with $w^+ = \mu$ for all η. In Section 5, we focus on truth-or-noise technology to explicitly characterize the monopolist's optimal strategy.

[3] The family of distribution $\{H(w, \eta)\}$ is rotation-ordered if, for each η, there exists a rotation point $w^\dagger_\eta$, such that $w \gtreqless w^\dagger_\eta \Leftrightarrow \frac{\partial H(w,\eta)}{\partial \eta} \lesseqgtr 0$.

4 General Analysis

Based on our model, this section analyzes the monopolist's optimal pricing and advertising strategies under general demand structure and advertising technology. We first identify consumers' best response to any pair of $(p, \eta) \in [0,1] \times [\underline{\eta}, \overline{\eta}]$. In the next stage, the monopolist makes its optimal choice. We show necessary conditions for the existence of partial information disclosure in equilibrium.

Suppose that a consumer receives signal x, and knows the monopolist's choice on p and η. Given his posterior belief $G(v \mid x, \eta)$, the expected benefit from making immediate purchase is given by

$$u_B(x, p; \eta) = w(x, \eta) - p = \int_0^1 (v - p) dG(v \mid x, \eta). \tag{2}$$

If he chooses to search, the expected payoff is

$$u_S(x, p, s; \eta) = \int_p^1 (v - p) dG(v \mid x, \eta) - s, \tag{3}$$

because he only purchases when $v \geq p$. We define three threshold values to classify consumers' choices into different regimes. Let $\overline{p}_{x|\eta}, \underline{p}_{x|\eta}, \hat{p}_{x|\eta} \in [0,1]$ defined by equations

$$\int_{\overline{p}_{x|\eta}}^1 (1 - G(v|x,\eta))dv = \int_0^{\underline{p}_{x|\eta}} G(v|x,\eta)dv = s \tag{4}$$

$$\int_0^{\hat{p}_{x|\eta}} G(v|x,\eta)dv = \int_{\hat{p}_{x|\eta}}^1 (1 - G(v|x,\eta))dv \tag{5}$$

if they exist.

If the consumer chooses to quit the market, his payoff is zero. We use the following lemma to fully characterize consumers' response.

Lemma 1 *When a buyer receives signal x with informativeness η, he goes to search if and only if (i) $\overline{p}_{x|\eta}$ and $\underline{p}_{x|\eta}$ exist, (ii)$\overline{p}_{x|\eta} \geq \underline{p}_{x|\eta}$, (iii)$p \in \left[\underline{p}_{x|\eta}, \overline{p}_{x|\eta}\right)$; makes immediate purchase if and only if $p \leq \min\{\underline{p}_{x|\eta}, \hat{p}_{x|\eta}\}$; quits the market otherwise.*

All the proofs are in the appendix. The next lemma shows that we can rank consumers' choice according to the signals that they receive.

Lemma 2 *If a consumer who observes signal $x \in [0,1]$ chooses immediate purchase, then all consumers receiving $y > x$ choose immediate purchase. If a consumer who observes $x \in [0,1]$ chooses to quit the market, then all consumers receiving $z \leq x$ quit the market.*

We introduce some notation defined in the signal space. Let $\mathcal{X}_B$, $\mathcal{X}_S$ and $\mathcal{X}_Q$ be the sets of the signals for which consumers choose to make an immediate purchase, search and quit respectively. We use $|\mathcal{X}_B|$, $|\mathcal{X}_S|$ and $|\mathcal{X}_Q|$ to denote the corresponding measures. Lemma 2 indicates these sets are connected and ordered. Suppose $\mathcal{X}_B$, $\mathcal{X}_S$ and $\mathcal{X}_Q$ all exist. Then, pick $x \in \mathcal{X}_B$, $y \in \mathcal{X}_S$ and $z \in \mathcal{X}_Q$, we must have $1 \geq x > y > z \geq 0$. Lemma 2 allows us to restrict our attention to those threshold signal realizations where some consumer is indifferent between two options. We use x_{BS}, x_{SQ} and x_{BQ} to represent critical types who are indifferent between making an immediate purchase and searching, searching and quitting, and making an immediate purchase and quitting respectively. The next lemma shows the impact of changing the advertising precision on critical types .

Lemma 3 *Let x_η^R be the rotation point, given $\eta \in [\underline{\eta}, \overline{\eta}]$. For $i = BS, BQ, SQ$, x_i^p increases as η increases when $x_i^p < x_\eta^R$ and x_i^p decreases as η increases when $x_i^p > x_\eta^R$.*

The monopolist's expected profit comes from two sources: immediate purchase and purchase after search. The expected profit then can be written as

$$\pi(p, \eta) = p\left(\int_{\mathcal{X}_B} dx + \int_{\mathcal{X}_S} \int_p^1 dG(v|x, \eta)dx\right) = p\left(\int_{\mathcal{X}_B \cup \mathcal{X}_S} dx - \int_{\mathcal{X}_S} G(p|x, \eta)dx\right), \quad (6)$$

where the greatest lower bound of $\mathcal{X}_B \cup \mathcal{X}_S$ is the lowest x such that $\max\{u_B(x, p; \eta), u_S(x, p, s; \eta)\} \geq 0$. It is clear that if the monopolist fully reveals information, no consumer will search. Also, there is no need to discuss outcomes involving no immediate purchase. If that is the case, all profits come from purchasing after search. The monopolist can instead choose perfect information and thereby secures no lower profit. Proposition 1 further shows that an equilibrium where all consumers make an immediate purchase must be associated with no information disclosure.

Proposition 1 *If all consumers make an immediate purchase in equilibrium, that is, if $\mathcal{X}_B = [0, 1]$, there is no information disclosure.*

From the discussion above, it is possible to provide some characterizations of an equilibrium. First, there must be a positive measure of consumers who make an immediate purchase; that is $|\mathcal{X}_B| > 0$. Second, from Proposition 1, if the information disclosure is partial, not every consumer makes an immediate purchase. There must exist strictly positive measures for consumers who engage in search and/or for consumers who quit the market; that is, $\max\{|\mathcal{X}_S|, |\mathcal{X}_Q|\} > 0$. These insights leave only three possible scenarios associated with a partial information disclosure equilibrium: i)$|\mathcal{X}_B| > 0$, $|\mathcal{X}_S| > 0$ and $|\mathcal{X}_Q| > 0$. ii)$|\mathcal{X}_B| > 0$, $|\mathcal{X}_S| > 0$ and $|\mathcal{X}_Q| = 0$; iii)$|\mathcal{X}_B| > 0$, $|\mathcal{X}_S| = 0$ and $|\mathcal{X}_Q| > 0$.

Let v_m be the solution to $\max_v \; v[1-G(v)]$. This is the optimal price a monopolist charges under perfect information. We consider the scenario where the monopolist hides all information. Suppose that the monopolist wants to induce all consumers to make immediate purchases. When a consumer prefers immediate purchase to searching, the maximum price $p_b(s)$ the monopolist can charge is defined by

$$s = \int_0^{p_b(s)} G(v)dv. \tag{7}$$

Let $p(s) = \min\{p_b(s), \mu\}$. This is the highest price at which all consumers make immediate purchases. Given the monopolist charging $p_b(s)$, the profit is also $p_b(s)$. Define a threshold search cost $\bar{s}$ by

$$p_b(\bar{s}) = v_m[1-G(v_m)]. \tag{8}$$

That is, at the search cost $\bar{s}$, the monopolist is indifferent between the two extreme choices on information disclosure. Also, the hazard rate of prior distribution is given by $r(v) = g(v)/[1-G(v)]$. The following proposition gives a sufficient condition for the existence of an equilibrium with partial information disclosure.

Proposition 2 *Suppose that the reciprocal hazard rate $1/r(v)$ is convex and the search cost $s \leq \bar{s}$. If there exists an advertising precision $\hat{\eta} \in (\underline{\eta}, \overline{\eta})$ such that*

$$v_m \leq \min\left\{\underline{p}_{G(v_m)|\hat{\eta}}, \hat{p}_{G(v_m)|\hat{\eta}}\right\}, \tag{9}$$

there exists a partial information disclosure level which is optimal.

5 The Uniform and Truth-or-Noise Case

The difficulty in our analysis lies in the fact that a consumer is concerned not only with his posterior valuation $w(x, \eta)$, but also with the whole posterior distribution $G(v \mid x, \eta)$. To derive some analytic results regarding the monopolist's optimal strategy, we focus on the case where the prior of true valuation is uniform (which generates the linear demand) and the monopolist uses the truth-or-noise advertising technology (the rotation point does not shift as η changes) as introduced in the previous example. In this section, we first establish some benchmark results where either the seller cannot communicate with consumers or the consumers have perfect information.

5.1 Benchmark: Perfect Information versus No Information

We first describe what happens if the monopolist has to fully reveal information. This can be imagined as a regulator enforcing a transparent policy. When information is perfect, every consumer knows his true valuation v exactly. With perfect knowledge about true valuation, no consumer needs to search, they make immediate purchase whenever $v \geq p$. In a setting with a continuum of consumers, the distribution of true valuation coincides with the prior distribution of valuation. The monopolist faces a standard downward sloping demand curve with unit intercept. The optimal price that the monopolist will charge is thus $p = 1/2$ and the profit it gains is $\pi = 1/4$.

The case with no information disclosure is less straightforward. Imagine that because of some technology barrier, the monopolist is unable to inform consumers of their true valuations. Thus, $\eta = 0$ and consumers only observe pure noise in advertising. Without additional information supplied by the monopolist, consumers are homogeneous in information and estimate v by using the prior $G(v)$. The expected payoff from an immediate purchase is

$$u_B(x, p; 0) = \mu - p = 1/2 - p. \tag{10}$$

The expected gain from search is

$$u_S(x, p, s; 0) = \int_p^1 (v - p) dv = (1 - p)^2/2 - s. \tag{11}$$

This comes from the fact that a consumer purchases after search if and only if he finds $v \geq p$. All consumers make immediate purchase when

$$p \leq \min\left\{\sqrt{2s}, 1/2\right\}, \tag{12}$$

and choose to search when

$$\sqrt{2s} \leq p < 1 - \sqrt{2s}. \tag{13}$$

The monopolist can expect two possible results: either all consumers make an immediate purchase or all consumers go to search. When the search cost is high, the monopolist can maintain a high price and all consumers make an immediate purchase. This is simply because the high search cost prevents consumers from finding out their true valuations. However, when the search cost is not high enough to deter consumers from search, the monopolist can still induce all consumers to make an immediate purchase by lowering the price. The next proposition characterizes the monopolist's optimal pricing strategy when no information disclosure via advertising is allowed.

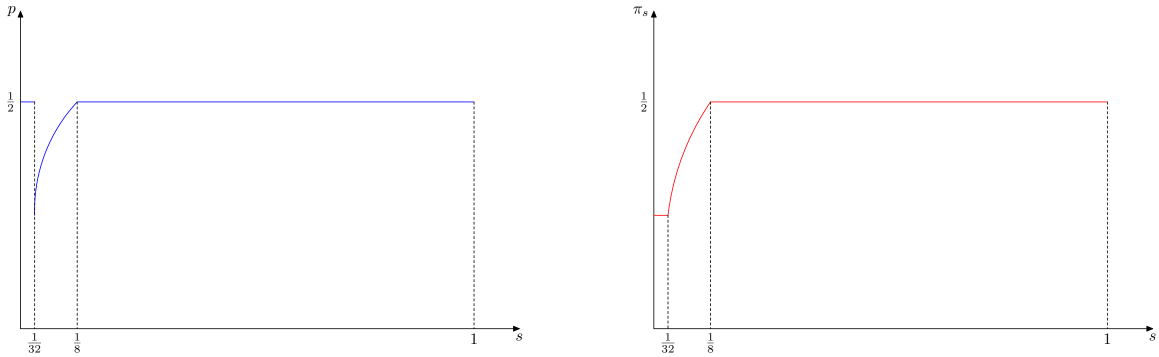

Figure 2: Seller's pricing strategy and expected profit when $v \sim U[0,1]$

Proposition 3 *Without information disclosure, the monopolist's optimal pricing strategy is as follows:*

$$p(s) = \begin{cases} \sqrt{2s} & \textit{when} \quad s \in (1/32, 1/8] \\ 1/2 & \textit{when} \quad s \in [0, 1/32] \bigcup (1/8, 1] \end{cases} \tag{14}$$

Its expected profit under this pricing strategy is

$$\Pi(s) = \begin{cases} 1/2 & \textit{when} \quad s \in (1/8, 1] \\ \sqrt{2s} & \textit{when} \quad s \in (1/32, 1/8] \\ 1/4 & \textit{when} \quad s \in [0, 1/32] \end{cases} \tag{15}$$

Proposition 3 says that, when there is no information disclosure, the monopolist's pricing strategy is non-monotonic in search cost; the expected profit is increasing in search cost; and consumer search only occurs when the search cost is low. The highest expected profit for a monopolist is 1/2. When the search cost is higher than 1/8, the monopolist can achieve this profit by charging $p = 1/2$ and all consumers will make an immediate purchase. When the search cost decreases to below 1/8, $p = 1/2$ does not induce all consumers to make an immediate purchase, and they would rather search. As the search cost does not fall far below 1/8, the monopolist can still induce all consumers to make an immediate purchase by lowering the price just a little bit. By doing this, the monopolist still occupies the whole market and does not forgo too much profit. Here, we show a new insight that the monopolist may want to deter consumers from search. In the benchmark case, the only way to deter search is to lower the price. However, as the search cost falls till it reaches 1/32, continuing to reduce the price is no longer the monopolist's optimal choice. Deterring search is too costly as it is now easy for consumers to find their true valuations. Rather than reducing the price, the monopolist returns to the high price of 1/2 to secure profit 1/4 in expectation. Then, as long as the search cost is lower than 1/32, consumers will search.

The underlying idea of Proposition 3 is that the monopolist faces a trade-off between deterring search and maintaining a relatively high price. Several authors, including Lewis and Sappington (1994) and Johnson and Myatt (2006) have shown that when information disclosure is allowed, a monopolist's choice is between serving a mass market and a niche market. For a monopolist who intends to serve a niche market, revealing information increases the marginal buyer's willingness to pay, so fully information disclosure is optimal. The opposite applies to a monopolist who wishes to serve a mass market. In our benchmark case where information disclosure is absent, although the demand structure is fixed to be linear, the monopolist still needs to make a choice between a mass market and a niche market. If all consumers make an immediate purchase, a mass market is served. When all consumers go to search, the monopolist serves a niche market. For any given price, only some consumers would find valuations higher than price, and hence make a purchase. The monopolist cannot differentiate consumers ex-ante as information disclosure is not feasible, but the search option of consumers may lead to heterogeneity ex-post. Unlike in previous work, here even information disclosure is not feasible, the monopolist faces the choice between serving a mass market and a niche market.

By comparing profits for the two extreme cases, we can see that under the linear demand the monopolist prefers no information disclosure. A natural question arises: can the monopolist earn a strictly higher profit when partial information disclosure is available?

5.2 Partial Information Disclosure

In both benchmark cases, the monopolist earns an expected profit 1/4 when the search cost is lower than 1/32. The monopolist cannot do better when information disclosure is infeasible simply because it is unable to differentiate between consumers. When the monopolist can transmit information to consumers, consumers who hear "good news" become more optimistic about their matching value and are more likely to make an immediate purchase; the opposite holds true for consumers hearing "bad news". Hence, even when the search cost is low, it is possible for the monopolist to convince some consumers to make an immediate purchase by supplying information. At the same time, the monopolist faces the problem of keeping consumers with bad news in the market. These consumers would quit the market if the information they received were very precise.

Under the uniform prior and truth-or-noise advertising, a buyer observing x has posterior valuation

$$w(x,\eta) = \eta x + \frac{1}{2}(1-\eta). \tag{16}$$

Notice that $w(x, \eta)$ increases as the advertising becomes more precise only when $x > 1/2$, otherwise it decreases. The consumer's expected payoff from immediate purchase is then

$$u_B(x, p; \eta) = w(x, \eta) - p = \eta x + \frac{1}{2}(1 - \eta) - p. \tag{17}$$

A consumer's expected payoff from search can be separated into two parts: one part comes from new information and the other part is from the prior. If the signal realization is relatively bad, $x < p$, the former part is always 0 as the consumer believes that with probability η he will not purchase after search. However, with probability $1 - \eta$ the signal is just pure noise drawn from a uniform distribution. Thus, the expected payoffs from search for consumers with $x < p$ only come from the latter part and are defined by

$$\underline{u}_S(x, p, s; \eta) = \frac{1}{2}(1 - \eta)(1 - p)^2 - s. \tag{18}$$

Upon observing a signal realization that is relatively good, i.e. $x \geq p$, search yields consumer expected payoff from both parts

$$u_S(x, p; \eta) = \eta(x - p) + (1 - \eta) \int_p^1 (v - p) dv - s = \eta(x - p) + \frac{1}{2}(1 - \eta)(1 - p)^2 - s. \tag{19}$$

The payoff function for search is then truncated at $x = p$. All payoff functions u_B, u_S and $\underline{u}_S$ are strictly decreasing in price while both u_S and $\underline{u}_S$ are strictly decreasing in search cost.

From Lemma 2, we know that consumers' choices are monotonic in the signal realizations that they receive. If there exists a signal realization with which a consumer makes an immediate purchase, then all consumers observing better signals make immediate purchases. Building on this observation, we can show that: for consumers who observe $x \geq p$, the choice between immediate purchase and search does not depend on the specific value of x. Suppose that a consumer observes x while price and advertising precision are given, and $x \geq p$. He prefers immediate purchase to search if $u_B(x, p; \eta) \geq u_S(x, p, s; \eta)$. This is equivalent to

$$p \leq \sqrt{\frac{2s}{1 - \eta}} \quad \forall \eta < 1. \tag{20}$$

It is clear that the choice not depend on the specific value of x. The next lemma shows that it has to be the case that in equilibrium some consumers make an immediate purchase. In other words, $p \leq \sqrt{2s/(1 - \eta)}$ has to hold in equilibrium.

Lemma 4 *In equilibrium, buyers observing $x \geq p$ must choose immediate purchase, that is, $p \leq \sqrt{2s/(1 - \eta)}$ holds in equilibrium.*

Lemma 4 says that at least the consumers who receive good news make immediate purchases in equilibrium. In the benchmark case, we show that the monopolist has an incentive to deter search and to induce consumers to make immediate purchases whenever it is possible. With the freedom of choosing any degree of information disclosure, the equilibrium outcome should be associated with some immediate purchase at least. Bearing this in mind, we can further rule out some cases from the possible equilibrium outcomes. Now assume that in equilibrium, the consumers who make an immediate purchase, who search, and who quit the market are all of positive measure. Lemma 2 implies $\underline{u}_S(x, p, s; \eta) < 0$. That is, consumers with $x < p$ do not search. But, if the consumers with $x \geq p$ search in equilibrium, we must have $p > \sqrt{2s/(1-\eta)}$. Then, no buyers make an immediate purchase. This contradicts our assumption. Therefore, we should have the following result: an equilibrium where consumers who choose immediate purchase, who search and who quit are all of positive measure does not exist. Therefore, there are three possible equilibrium outcomes: some buyers purchase immediately while all other buyers search (BI-SB); some buyers purchase immediately while all other buyers quit the market (BI-QM); all buyers make immediate purchase (BI).

For each possible equilibrium outcome, we can express the sufficient and necessary conditions. In a type (BI) equilibrium, even the consumer who receives the worst signal $x = 0$ makes an immediate purchase; equivalently, $u_B(0, p; \eta) \geq 0$. He also needs to prefer immediate purchase to search; that is $u_B(0, p; \eta) \geq \underline{u}_S(0, p, s; \eta)$. These two conditions are sufficient and necessary for a (BI) equilibrium, because by Lemma 2 all consumers with better signals will make an immediate purchase. The two conditions are summarized by

$$p \leq \min \left\{ \frac{-\eta + \sqrt{\eta^2 + 2s(1-\eta)}}{1-\eta}, \frac{1}{2}(1-\eta) \right\}. \tag{21}$$

For a type BI-SB equilibrium, consumers with $x = 0$ must search so that $\underline{u}_S(0, p, s; \eta) > u_B(0, p; \eta)$ and $\underline{u}_S(0, p, s; \eta) \geq 0$. Furthermore, as consumers with $x \geq p$ must make an immediate purchase, we have $p \leq \sqrt{2s/(1-\eta)}$. These conditions are summarized by

$$\frac{-\eta + \sqrt{\eta^2 + 2s(1-\eta)}}{1-\eta} < p \leq \min \left\{ \sqrt{\frac{2s}{1-\eta}}, 1 - \sqrt{\frac{2s}{1-\eta}} \right\}. \tag{22}$$

Similarly, the type BI-SB equilibrium requires that both $u_B(0, p; \eta)$ and $\underline{u}_S$ be strictly negative. To ensure that at least some consumers to make an immediate purchase, we need $p \leq \sqrt{2s/(1-\eta)}$ and $u_B(1, p; \eta) > 0$. These conditions are equivalent to

$$\max \left\{ \frac{1}{2}(1-\eta), 1 - \sqrt{\frac{2s}{1-\eta}} \right\} < p < \min \left\{ \frac{1}{2}(1+\eta), \sqrt{\frac{2s}{1-\eta}} \right\}. \tag{23}$$

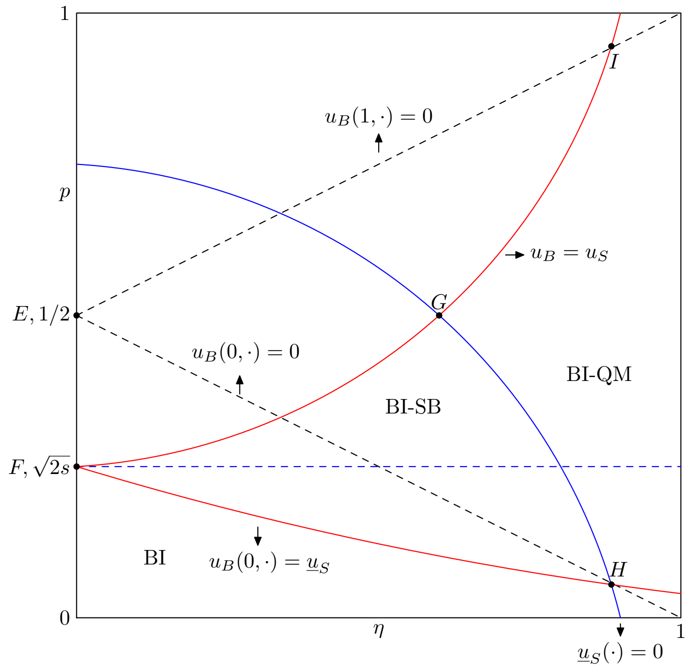

Figure 3: Buyers' response to (p, η).

Anticipating the response of consumers, the monopolist chooses the optimal combination of price and advertising precision. This choice depends on search cost. When $s>1/8$, the monopolist can make a profit $1/2$ by choosing $p = 1/2$ and $\eta = 0$ as shown in the extreme cases. Since the whole market is covered and the consumer base can not be expanded further by lowering price, the only way to make a higher profit is to charge $p > 1/2$. Truth-or-noise advertising causes the distribution of posterior valuation to rotate around $w = 1/2$. This implies that for any $p > 1/2$ and $\eta > 0$, there will be less than $1/2$ consumers who make an immediate purchase. Furthermore, for consumers with $w < 1/2$(i.e. $x < p$), their gain from search is $\underline{u}_S(x, p, \eta, s) = (1-\eta)(1-p)^2/2 < (1-\eta)/8 - s < 0$. Hence, the expected profit is all from immediate purchases, and it is strictly less than $1/2$. Consequently, when the search cost is higher than $1/8$, the monopolist's optimal strategy is to choose $p = 1/2$ and $\eta = 0$.

Our main focus is where the search cost is relatively low, that is, $s \leq 1/8$. When search cost is relatively low, consumer search becomes effective and we show that the monopolist prefers using information disclosure to deter search. In Figure 3, we depict the areas associated with the three possible equilibrium outcomes in precision-price space when $s \leq 1/8$. The area confined by $(0, 0), F, H$ and $(0, 1)$, including the boundary F, H and $(1, 1)$, corresponds to the BI regime. The area circled by F, G and H, including the boundary F, G and H, corresponds to the BI-SB regime. Finally, the area confined by $(1, 0), H, G, I$ and $(1, 1)$, including the boundary G, I, corresponds to the BI-QM regime.[4] The last lemma outlines the monopolist's exact maximization problem when $s \le 1/8$.

Lemma 5 *When the search cost s is less than $1/8$, the monopolist's optimization problem is*

$$\max\left\{\Pi(s), \sqrt{2s}\right\},$$

where $\Pi(s)$ is the maximum value of the following programming

$$\begin{aligned} \max_{p,\eta}\ & p\left\{\frac{(1+\eta)/2-p}{\eta} + (1-\eta)(1-p)\left[\frac{p-(1-\eta)/2}{\eta}\right]\right\} \\ s.t.\quad & 1/2(1-\eta)(1-p)^2 - s = 0, \\ & p \in [\sqrt{2s}, 1/2] \quad and \quad s \in (0, 1/8). \end{aligned} \tag{*}$$

The next proposition summarizes with the full characterization of the optimal pricing and advertising strategy, which is also the main result of this paper.

Proposition 4 *When the prior distribution of true valuation is uniform and the truth-or-noise advertising technology is adopted, the monopolist's optimal strategy of pricing and information disclosure is given by*

$$(p^*(s), \eta^*(s)) = \begin{cases} (1/2, 1-8s) & when \quad s \in [0, (3-2\sqrt{2})/4] \\ (\sqrt{2s}, 0) & when \quad s \in ((3-2\sqrt{2})/4, 1/8] \\ (1/2, 0) & when \quad s \in (1/8, 1] \end{cases} \tag{24}$$

The maximum profit is

$$\Pi^*(s) = \begin{cases} 1/4 + s & when \quad s \in [0, (3-2\sqrt{2})/4] \\ \sqrt{2s} & when \quad s \in ((3-2\sqrt{2})/4, 1/8] \\ 1/2 & when \quad s \in (1/8, 1] \end{cases} \tag{25}$$

Proposition 4 indicates that the monopolist charges high price, $p = 1/2$, both when the search cost is relatively high and low, and low price, $p = \sqrt{2s}$, when search cost is medium. Information disclosure is used only when the search cost is low, $s \le (3 - 2\sqrt{2s})/4$.

[4] Truth-or-noise advertising simplifies our analysis of monopolist's problem. In an equilibrium involving consumer search, truth-or-noise advertising implies that searchers with signal $x < p$ have the same probability of making a purchase, $(1-\eta)(1-p)$, after search. This is because there exists a positive probability mass η concentrating at some $x < p$; the remaining probability is still uniformly distributed over $[0, 1]$. Thus, the probability of eventual purchase does not depend on x.

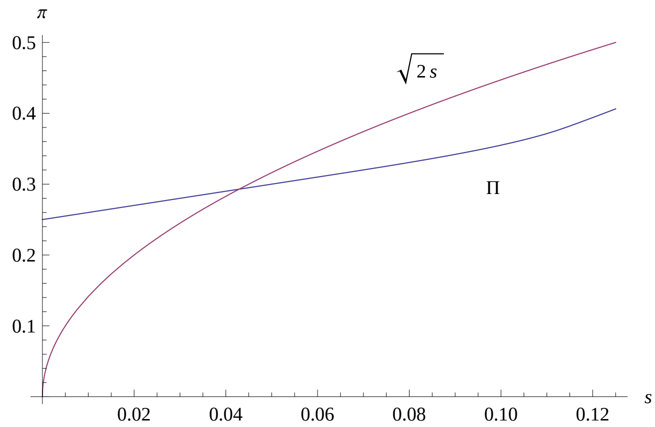

Figure 4: Comparison of Π and $\sqrt{2s}$.

The information disclosure is always partial provided the search cost is strictly positive. Within the range where information disclosure appears, the precision of advertising is strictly decreasing in search cost. Although the equilibrium price is non-monotonic in search cost, the expected profit is strictly increasing in search cost. We plot $\Pi(s)$ and $\sqrt{2s}$ for $s \in (0, 1/8)$ in Figure 4. The equilibrium profit is characterized by the upper envelope curve. Compared to the benchmark case where information disclosure is not allowed, the parameter range where the monopolist charges a high price expands because $1/32 \approx 0.03125 < (3 - 2\sqrt{2})/4 \approx 0.0428932$.

The monopolist's main trade-off is between maintaining a high price and deterring consumers from searching. Whenever consumer search is effective, a high price is likely to induce consumers to search and therefore reduces the purchase probability. The monopolist then may have intention to prevent consumers from searching. The optimal price and advertising precision is tailored to balance the price and the number of consumers who make an immediate purchase. When the search cost is relatively high, $s > 1/8$, the same logic in the benchmark case still applies. Consumer search, as a counter-strategy is not very effective and hence consumers are not likely to search. Anticipating that deterring search is not an important issue for high search cost, the monopolist charges a high price $p = 1/2$ and extracts the full ex-ante surplus. No information needs to be released, because the created heterogeneity makes $p = 1/2$ not maintainable if the monopolist wants to induce all consumers to make an immediate purchase.

When the search cost drops below $s = 1/8$, hiding all information and charging $p = 1/2$ is no longer sufficient to induce all consumers to make an immediate purchase. Reduction in search cost makes buyers favor search under high price. Deterring search becomes a serious issue for the monopolist. There are two ways to deter search: reducing price and revealing information. If the search cost is not too low, $(3 - 2\sqrt{2s})/4 < s \leq 1/8$, the monopolist chooses to reduce price rather than to reveal information. When search cost is not too low, small price reduction without information disclosure can convince all consumers to make an immediate purchase. Thus, the monopolist still have the whole base of immediate purchase and the profit does not fall too much. If additional information is released, consumers who hear "bad news" will choose to search unless the price is very low. In particular, if the price is set at $1/2$, only half consumers make immediate purchases. Therefore, price reduction is preferred for deterring search.

However, as the search cost continues to fall and drops below $(3 - 2\sqrt{2})/4$, to continue reducing the price is very costly. The price needs to be reduced a lot to prevent all consumers from searching. Especially, when s is very close to 0, the price needs to be set at almost 0 to deter search. This is not worthwhile for the monopolist. Instead, the monopolist switches to using information disclosure to deter search. The price goes back to the high level, $p = 1/2$. Only consumers who hear "good news" make an immediate purchase, while the others with "bad news" choose to search. That is, information disclosure only "partially" deters search. Also, the degree of information disclosure is partial, i.e. $\eta \in (0, 1)$, for any $s \neq 0$ within this range. Why information disclosure is always partial? If the information disclosure is perfect, consumers hearing "bad news" will quit the market immediately without search. If there is no information disclosure, no buyers will make immediate purchase under $p = 1/2$. Moreover, the lower the search cost is, the more information is revealed by the monopolist. As the search cost decreases, the monopolist needs to provide more precise information to induce consumers hearing "good news" to prefer immediate purchase to search. For consumers hearing "bad news", although the more precise adverting decreases their expected valuations, it is still able to grant them non-negative payoffs from searching because the search cost is now lower.

Finally, there are always half consumers searching when the search cost is below $(3 - 2\sqrt{2})/4$. Consumer search and information disclosure hence co-exist as complements in the sense that consumer only search at the parameter values where the monopolist reveals information. Robert and Stahl (1993) obtained a similar result in a sequential search model with homogeneous goods and price advertising. Our result can be seen as a complementary contribution to the literature with heterogeneous consumer tastes and informative advertising.

6 Transparent Policy and Welfare

Transparency policies that force producers to reveal product information are designed to improve social efficiency and/or protect consumers. Transparency policies regarding information disclosure of various products are employed across many industries. For example, in the food industry, the Nutrition Labeling and Education Act of 1990 (NLEA) requires food processors to label products with amounts of key nutrients as a public health measure. In the financial product market, the Securities and Exchange Acts of 1933 and 1934 require that publicly traded companies disclose information about their finances in standardized form in quarterly and annual reports. It is widely believed that consumer surplus can be enhanced as transparency policies improve matching quality between consumers' tastes and product attributes. However, this reasoning may not be valid when there is search frictions. When purchases have to be made after paying a search cost, as shown by Anderson and Renault (2006), both consumer surplus and social welfare can not be improved. Forcing a monopolist to use full-match advertising triggers a price increase which lowers the volumes of trades in the market. In their model, mismatch is not an issue as consumers only make purchase decisions after discovering the true valuation. In our setting, where the search cost is interpreted as an information-collecting cost, mismatch is a welfare-relevant problem because consumers can buy the product without incurring a search cost, and thus potentially realize a valuation lower than price they pay. Buiding on our model with linear demand, we evaluate the impact of transparency policies on consumer surplus and social welfare.

6.1 On Consumer Surplus

Consumer surplus is computed by taking the expectation over all possible signal realizations. When transparency policies are absent, the monopolist chooses price and advertising precision as in the equilibrium. In general, consumer surplus can be written as follows.

$$CS^*(s) = \underbrace{\int_{\mathcal{X}_B} \left[\eta(x-p) + (1-\eta)\int_0^1 (v-p)dv\right] dx}_{\text{surplus from immediate purchase}} + \underbrace{\int_{\mathcal{X}_S} \left[(1-\eta)\int_p^1 (v-p)dv - s\right] dx}_{\text{surplus from search}}, \tag{26}$$

where every consumer making an immediate purchase expects a surplus $\eta(x-p) + (1-\eta)\int_0^1 (v-p)dv$, while consumers engaging in search expects a surplus $(1-\eta)\int_p^1 (v-p)dv - s$. It is clear from the expression of consumer surplus from immediate purchasing, that there is always a chance that a buyer could buy a product with true valuation $v < p$. All consumers engaging in search have to incur a search cost. These two inefficiencies stem from the possibility that the monopolist does not fully reveal product information. We compute consumer surplus by using the equilibrium price and advertising precision derived in (24). When the search cost is low, $s \in [0, (3-2\sqrt{2}))/4]$, the equilibrium strategy $p = 1/2$ and $\eta = 1 - 8s$. Half the consumers (who receive $x \geq 1/2$) make an immediate purchase and the other half (who receive $x < 1/2$) choose to search. Consumer surplus is as follows:

$$CS^*(s) = 1/8 - s. \tag{27}$$

When search cost $s \in [(3-2\sqrt{2})/4, 1/8]$, the monopolist lowers the price to $\sqrt{2s}$ and hides all information. All consumers make immediate purchase and the consumer surplus is

$$CS^*(s) = 1/2 - \sqrt{2s}. \tag{28}$$

When the search cost $s \in (1/8, 1]$, the monopolist charges the highest price $p = 1/2$ and still hides all information. The consumer surplus is

$$CS^*(s) = 0. \tag{29}$$

Now suppose a transparency policy is implemented so that the monopolist has to fully reveal information to consumers, that is, choose $\eta = 1$. Perfect information disclosure corrects the two inefficiencies mentioned above. Mismatches are wiped out as all buyers make purchases only when the price is lower than than the true valuation. The expenditure on searching is saved as information is fully disclosed and no consumers need to search. Obviously, these are improvements on consumer surplus. However, when the information is perfect, the monopolist who faces a standard linear demand with unit intercept charges $p = 1/2$. The consumer surplus is

$$CS^\dagger = \int_p^1 (v - p)dv = \int_{1/2}^1 (v - 1/2)dv = 1/8. \tag{30}$$

We compare consumer surplus with and without transparency policies. When the search cost is either extremely high or extremely low (i.e. $s \in [0, (3-2\sqrt{2}))/4] \cup (1/8, 1]$), transparency policies improve consumer surplus. When $s \in [0, (3-2\sqrt{2}))/4]$, the price is the same but transparency policies save the expenditure on searching and improve matching quality. When $s \in (1/8, 1]$, the price is still the same and consumers do not search in either case; therefore the improvement in consumer surplus comes solely from better matching. When the search cost is at the medium level, $s \in [(3-2\sqrt{2})/4, 1/8]$, the gain to consumers from improved matching quality is diluted by the policy-induced high price. Recall that when the monopolist is free to choose η, it tends to reduce the price in order to prevent consumers from search in this range. A simple calculation gives that when $s \in [(3-2\sqrt{2})/4, 9/128)$, (30) is less than (28). So we have shown that transparency policies enhance consumer surplus for a wide range of search costs except for some medium level.

6.2 On Social Welfare

The effect of transparency policies also depends on the target of the regulator. Sometimes, a transparency policy is issued to improve social welfare, which takes the firms' profits into account. Assume the regulator weight consumer surplus and profits equally when it tries to enhance social welfare. So we aggregate them to compute social welfare. When the transparency policy is employed, consumer surplus is 1/8 while the monopolist gain expected profit 1/4 so the total welfare is $SW^{\dagger} = 3/8$. If the transparency policy is not issued and the monopolist is free to choose η, social welfare is given by

$$SW^*(s) = \begin{cases} 3/8 & s \in [0, (3-2\sqrt{2})/4) \\ 1/2 & s \in [(3-2\sqrt{2})/4, 1] \end{cases} \tag{31}$$

The comparison shows that, transparency policies cannot improve social welfare for the full range of search costs. This happens simply because the monopolist's profit drops drastically under the transparency policy, although consumers are better off for a wide range of search costs. The next proposition summarizes the welfare implication of transparency policies.

Proposition 5 *Assume $G(v)$ is uniform and the advertising technology is truth-or-noise. When the search cost $s \in [(3-2\sqrt{2})/4, 9/128)$, transparency policies reduces consumer surplus. In the remaining range of search cost, transparency policies always benefit consumers. Transparency policies cannot enhance social welfare, and strictly harms social welfare when the search cost $s \in [(3-2\sqrt{2})/4, 1]$.*

Our model yields a policy implication for social welfare similar to that of Anderson and Renault (2006), although the interpretations of search cost are different. However, in their model, transparency policy always hurts consumers. Some valuable trades are not realized because a high price is induced by transparency policies. In our model, two inefficiencies—incurring a search cost and making the wrong match—are ruled out by transparency policies. That is why for some range of search costs, our model shows that consumers can be better off under transparency policies. Our model provides two insights on the effectiveness of transparency policies: i) they can potentially enhance consumer surplus for extremely high or extreme low search costs; ii) their effectiveness depends on the target (social welfare or consumer surplus) of the regulator.

7 Conclusion

In most markets, information disclosure from supply side and information acquisition from demand side co-exist. It is important to understand how these two fundamental market forces interact in the process of producing information. Whether information disclosure and consumer search are substitutes or complements? How does advertising precision change according to the change of search cost? In particular, both empirical studies and daily observations suggest that information disclosure is usually partial, which contradicts the previous theoretical findings. This paper present a model integrating information disclosure and consumer search that is useful to answer these questions.

We show that, when the search cost is high, the monopolist charges a high price and hides all information; when the search cost is medium, the monopolist lowers the price to deter search but still hide all information; when the search cost is low, the monopolist partially reveals information but charges a high price. Partial information disclosure thus occurs only when the search cost is low. The equilibrium price is non-monotonic in the search cost. Information disclosure and consumer search co-exist when the search cost is low, and thus are complements to each other.

In line with Anderson and Renault (2006), our model predicts that transparency policies cannot improve social welfare. But importantly, transparency policies favor consumer surplus for a wide range of search costs by improving matching quality and reducing the expenditure on searching. Only for some medium level of search cost, do transparency policies harm consumers because the benefits mentioned above are diluted by the induced high price. Thus, our model provides a theoretical justification for the widely observed practice of transparency policy, but also reminds us that the effectiveness depends heavily on regulators' target and the magnitude of search cost.

An interesting extension is to incorporate sellers' competition into our model. The results will depend heavily on the detailed setting. In this paper, we assume that consumer search is one off since there is only one seller. However, when there are several sellers who produce differentiated products, the process of gathering information can be either simultaneous or sequential. In a simultaneous setting, a buyer knows his true valuation for each product after incurring a fixed cost once. In a sequential setting, a buyer makes sampling decision sequentially. As shown in Ivanov (2010), with a simultaneous setting without consumer search, tougher competition gives sellers more incentive to reveal information. But it is still ambiguous of the impact on advertising precision of competition when consumer search is introduced. Since competition is not the focus of this paper, we leave it to the future research.

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8 Appendix

Proof of Lemma 1.

Suppose that a consumer receives signal x. He chooses immediate purchase if and only if $u_B(x,p;\eta) \geq \max\{u_S(x,p,s;\eta),0\}$, or equivalently

$$s \geq \int_0^p G(v|x,\eta)dv \qquad \text{and} \qquad \int_p^1 (1-G(v|x,\eta))dv \geq \int_0^p G(v|x,\eta)dv. \tag{32}$$

Notice that $\int_p^1(1-G(v|x,\eta))dv - \int_0^p G(v|x,\eta)dv$ is non-increasing in p, and positive when $p=0$ and negative when $p=1$. Thus, $\hat{p}_{x|\eta}$ always exists when $G(v|x,\eta)$ is continuous in v. It is not necessary $\hat{p}_{x|\eta}$ to exist, because nonexistence can only happen when

$\int_0^1 G(v \mid x,\eta)dv < s$. Then the first inequality in (32) is satisfied automatically. Suppose that $p > \min\left\{\underline{p}_{x|\eta}, \hat{p}_{x|\eta}\right\}$. Then, $s < \int_0^p G(v|x,\eta)dv$ when $\min\left\{\underline{p}_{x|\eta}, \hat{p}_{x|\eta}\right\} = \underline{p}_{x|\eta}$, and $\int_p^1 1 - G(v|x,\eta)dv < \int_0^p G(v|x,\eta)dv$ when $\min\left\{\underline{p}_{x|\eta}, \hat{p}_{x|\eta}\right\} = \hat{p}_{x|\eta}$. It contradicts (32). Therefore, the consumer makes an immediate purchase if and only if $p \leq \min\left\{\hat{p}_{x|\eta}, \underline{p}_{x|\eta}\right\}$.

Suppose this consumer chooses to search. It is equivalent to

$$\int_p^1 (v-p)dG(v|x,\eta) - s > \int_0^1 (v-p)dG(v|x,\eta) \quad \text{and} \quad \int_p^1 (v-p)dG(v|x,\eta) - s \geq 0. \tag{33}$$

Simplifying using integration by parts, we have

$$s < \int_0^p G(v|x,\eta)dv \quad \text{and} \quad s \leq \int_p^1 (1 - G(v|x,\eta))dv. \tag{34}$$

First, if either $\bar{p}_{x|\eta}$ or $\underline{p}_{x|\eta}$ does not exist, condition (34) cannot hold. Second, if $\bar{p}_{x|\eta} \leq \underline{p}_{x|\eta}$, we can not find such a price p to make both inequalities in (34) hold. Therefore, consumer search requires $\bar{p}_{x|\eta} > \underline{p}_{x|\eta}$ and $p \in \left[\underline{p}_{x|\eta}, \overline{p}_{x|\eta}\right)$. ■

Proof of Lemma 2.

If a consumer observing x makes an immediate purchase, we must have $u_B(x,p;\eta) \geq \max\{u_S(x,p,s;\eta), 0\}$. Consider $y > x$. Since $G(v \mid y,\eta)$ FOSDs $G(v \mid x,\eta)$, we have $u_B(y,p;\eta) \geq \max\{u_S(y,p,s;\eta), 0\}$. Therefore, all buyers with signal realizations higher than x make immediate purchases. Now consider that a consumer quits the market after observing x. This implies $\max\{u_B(x,p;\eta), u_S(x,p,s;\eta)\} < 0$. Since both u_B and u_S are non-decreasing in signal realization, we must have $\max\{u_B(z,p;\eta), u_S(z,p,s;\eta)\} < 0$ if $z < x$. Therefore, a consumer receiving z should quit the market. ■

Proof of Lemma 3.

By definition, we have

$$\int_0^p G(v \mid x_{BS}^p, \eta)dv = s.$$

Let $\eta' > \eta$. By assumption, we have $G(v \mid x_{BS}^p, \eta) \geq G(v \mid x_{BS}^p, \eta')$ when $x_{BS}^p > x_\eta^R$. Then, we have

$$\int_0^p G(v \mid x_{BS}^p, \eta')dv \leq s.$$

Therefore, if there exists a $x_{BS}^{p\prime}$ so that a consumer is indifferent between immediate purchase and search under η', $x^{p\prime}_{BS}$ should satisfy

$$\int_0^p G(v \mid x^{p\prime}_{BS}, \eta')dv = s.$$

This can only happen when $x^{p\prime}_{BS} < x^p_{BS}$ by FOSD. When $x^p_{BS} < x^R_\eta$, we must have that x^p_{BS} increases as η increases. We can extend similar arguments to x^p_{SQ} and x^p_{BQ}. ■

Proof of Proposition 1.

Suppose there exists a pair (p, η) with $p > 0$ and $\eta > \underline{\eta}$ such that all consumers make an immediate purchase. This implies that even the consumer who receives the worst signal $x = 0$ makes an immediate purchase. Then we must have

$$u_B(0, p; \eta) = \int_0^1 (v - p)dG(v \mid 0, \eta) \geq 0,$$

and

$$u_B(0, p; \eta) \geq u_S(0, p, s; \eta) = \int_p^1 (v - p)dG(v \mid 0, \eta) \quad \Leftrightarrow \quad s \geq \int_0^p G(v \mid 0, \eta)dv.$$

Since $x = 0$ is the worst news, we surely have $0 \leq x^R_\eta$. The monopolist can decrease precision to $\eta' < \eta$ while keeping the price unchanged such that

$$\int_0^1 (v - p)dG(v \mid 0, \eta') > 0,$$

and

$$s > \int_0^p G(v \mid 0, \eta')dv.$$

These inequalities comes from the fact that $G(v \mid 0, \eta')$ FOSDs $G(v \mid 0, \eta)$ when $0 \leq x^R_\eta$. Now, the monopolist can increase price to $p' > p$ so that the above two inequalities still weakly hold. Therefore, all consumers still make immediate purchase under (p', η'). This process can be repeated till $\eta = \underline{\eta}$. This is a profitable deviation from (p, η), Proposition 1 thus follows. ■

Proof of Proposition 2

Step 1: Suppose the consumer search is absent. When the family of distribution $\{H(w, \eta)\}_{\eta \in [\underline{\eta}, \overline{\eta}]}$ is ordered by a sequence of rotation and the rotation point $w^\dagger_\eta$ is decreasing, the monopoly profits are quasi-convex in η, and hence maximized at extremes $\eta \in \{\underline{\eta}, \overline{\eta}\}$.

Proof of Step 1.

A general proof can be found in Proposition 1 in Johnson and Myatt (2006), so we omit it here. Under our setting, $\underline{\eta}$ corresponds to no information disclosure. Then we have

$$H(w, \underline{\eta}) = \begin{cases} 1 & \text{when } w \geq \mu, \\ 0 & \text{when } w < \mu. \end{cases}$$

The upper bound $\overline{\eta}$ corresponds to the perfect information disclosure, and $H(w, \eta)$ equals to the prior $G(v)$. ■

Step 2: If $1 - r(v)v$ is strictly pseudo-monotone for $v > 0$,[5] then $\mu \geq v_m$ if and only if $r(\mu)\mu \geq 1$.

Proof of Step 2.

We first show that $r(\mu)\mu \geq 1$ implies $\mu \geq v_m$. The first-order condition is $r(v_m)v_m = 1$. By assumption, $r(\mu)\mu \geq 1$. Since $1 - r(v)v$ is strictly pseudo-monotone for $v > 0$, we have $r(v)v \geq 1$ for all $v > \mu$. Therefore, $v_m < \mu$.

Then, we show that $r(\mu)\mu \leq 1$ implies $\mu \leq v_m$. We still have $r(v_m)v_m = 1$. The strict pseudo-monotonicity of $1 - r(v)v, v \geq 0$ implies $r(v)v > 1$ for all $v > v_m$. Then $\mu < v_m$. ■

Step 3: When the reciprocal hazard rate $1/r(v)$ is convex, $v_m < \mu$.

Proof of Step 3.

We have

$$\begin{aligned} \mu &= -\int_0^1 v d(1 - G(v)) = -v(1 - G(v))|_0^1 + \int_0^1 (1 - G(v))dv = \int_0^1 (1 - G(v))dv \\ &= \int_0^1 \left(\frac{1 - G(v)}{g(v)}\right) g(v)dv = \int_0^1 \frac{1}{r(v)} g(v)dv = \mathbb{E}\left[\frac{(1}{r(v)}\right]. \end{aligned}$$

Let $1/r(v)$ be convex. We claim that $1/r(v)$ is strictly decreasing in v. By contradiction, let $1/r(y) \geq 1/r(x)$ for some $y > x$. Note that $1/r(1) = 0$ so that $y = 1$ results in contradiction because $1/r(x) > 0$ for $x \neq 1$. For any $y \in (x, 1)$, we can express it as $y = \alpha x + (1 - \alpha)$ for some $\alpha \in (0, 1)$. Then, the convexity of $1/r(v)$ results in $1/r(y) \leq \alpha/r(x) + (1 - \alpha)/r(1) = \alpha/r(x) < 1/r(x)$. This contradicts with $1/r(y) \geq 1/r(x)$. Then, $r(v)$ is strictly increasing and $1 - r(v)v$ is strictly decreasing. From claim 2, $1 - r(v)v$ is strictly pseudo-monotone and we have $v_m \leq \mu$. ■

Step 4: The monopolist prefers $\underline{\eta}$ to $\overline{\eta}$ when consumer search is absent.

[5] A function $\varphi(v)$ is pseudo-monotone if for every v and $v' \neq v$, $\varphi(v)(v' - v) < 0$ implies $\varphi(v')(v' - v) < 0$ (Hadjisavvas, 2005). Equivalently, $\varphi(v) < 0$ implies $\varphi(v') < 0$ for all $v' > v$.

Proof of Step 4.

Step 1 shows that, for seller's maximization without the concern of consumer search, we only need to compare the profits under $\underline{\eta}$ and $\overline{\eta}$. When $\eta = \underline{\eta}$ and $p = \mu$, all consumers make immediate purchase and the profit is just μ. When $\eta = \overline{\eta}$, the standard monopoly price is charged and we denote it as v_m. The profit is $v_m[1 - G(v_m)]$. Notice that no matter $\underline{\eta}$ or $\overline{\eta}$ is chosen, the mean of W always equals to μ under rotation order. This can be easily seen by using Iterated Expectation Formula

$$\mathbb{E}[w \mid \eta] = \mathbb{E}_x \left[\mathbb{E}_v[v \mid x, \eta]\right] = \mathbb{E}[v] = \mu.$$

Then we must have $\mu > v_m[1 - G(v_m)]$. The monopolist will prefer choosing $\underline{\eta}$. $\blacksquare$

Step 5: when consumer search is present, The monopolist prefers choosing $\overline{\eta}$ to choosing $\underline{\eta}$ if the search cost $s < \bar{s}$.

Proof of Step 5.

We re-introduce consumer search into the model. In Step 4, the search cost s can be imagined as being infinity. However, when the search cost $s < \bar{s}$, choosing $\underline{\eta}$ is no longer optimal for the monopolist. This is immediate from the definition of $\bar{s}$. In particular, when $s \to 0$, to keep the full base of immediate purchase, $p \to 0$ and the profit tends to 0. If the monopolist does not reduce the price, all consumer search and it yields the same expected profit as choosing $\overline{\eta}$. This implies, when search cost is low enough, choosing $\overline{\eta}$ yields higher expected profit than choosing $\underline{\eta}$. $\blacksquare$

Step 6: If there exists a $\hat{\eta}$ such that $v_m \leq \min\left\{\underline{p}_{G(v_m)|\hat{\eta}}, \hat{p}_{G(v_m)|\hat{\eta}}\right\}$, partial information disclosure is optimal.

Proof of Step 6.

Set the price at $p = v_m$. Choose precision parameter $\hat{\eta} \in (\underline{\eta}, \overline{\eta})$ such that the consumer who receives signal $x = G(v_m) \in (0, 1)$ makes immediate purchase. From Lemma 1, it requires

$$v_m \leq \min\left\{\underline{p}_{G(v_m)|\hat{\eta}}, \hat{p}_{G(v_m)|\hat{\eta}}\right\}.$$

From Lemma 2, all consumers receiving $x > G(v_m)$ make immediate purchases. Thus, the monopolist can earn $v_m[1 - G(v_m)]$ from immediate purchase. This choice is no worse than choosing $\overline{\eta}$ as it is still possible to have purchase after search. Therefore, partial information disclosure is an optimal choice. $\blacksquare$

Proof of Proposition 3.

When no information is released, all buyers must behave exactly the same and only according to the prior distribution of true valuation. Buyers make an immediate purchase if and only if $u_{BI} \geq u_{SB}$, or equivalently, $p \leq \min\{\sqrt{2s}, 1/2\}$. When all consumers search, the monopolist's expected profit is $\Pi = p[1-G(p)] = p(1-p)$. The profit-maximizing solution is apparently $p = 1/2$ and $\Pi = 1/4$. When all consumers make immediate purchase, the highest price that the monopolist can charge is buyers' prior valuation, $\mu = 1/2$. Thus, the highest profit the monopolist can expect is $\Pi = 1/2$. When the search cost is high, $s \in (1/8, 1]$, such that all consumers make immediate purchase under $p = 1/2$, the monopolist will simply set $p = 1/2$.

When $p \leq 1/8$, if the monopolist tries to sell to all buyers immediately, it can charge $p = \sqrt{2s}$ at most. Preventing buyers from search yields immediate profit $\Pi = \sqrt{2s}$. Compare $\sqrt{2s}$ with the profit where all consumer search, $\Pi = 1/4$, the monopolist prefers deterring search if and only $s \in (1/32, 1/8]$. Therefore, the monopolist charges $p = \sqrt{2s}$ when $s \in (1/32, 1/8]$ and $p = 1/2$ when $s \in [0, 1/32]$. ■

Proof of Lemma 4.

We use a proof by contrapositive to show the results. Suppose that $p > \sqrt{2s/1-\eta}$ holds in equilibrium. Therefore, consumers observing $x \geq p$ do not choose to make an immediate purchase, or equivalently, $u_B(p,p;\eta) < u_S(p,p,s;\eta)$. Since $\underline{u}_S(x,p,s;\eta) = u_S(p,p,s;\eta)$ for all $x \leq p$ and $u_B(x,p;\eta)$ is decreasing in x, we conclude that $\underline{u}_S(x,p,s;\eta) > u_B(x,p;\eta)$ for all $x < p$. There are two possible cases for consumers with $x < p$:

i)they quit the market, that is, $u_S(p,p,s;\eta) < 0$, or

ii)they search the market, that is, $u_S(p,p,s;\eta) \geq 0$.

Now consider a potential deviation by the monopolist: fix p and increase η till $p = \sqrt{2s/1-\eta'}$. Since $p > \sqrt{2s/1-\eta}$ so that $\eta \neq 1$, this is always feasible. Consider consumers with $x \geq p$; now they switch to immediate purchase. For consumers with $x < p$, we have

$$\underline{u}_S(x,p,s;\eta') = u_S(p,p,s;\eta').$$

Case (i) is trivial, because no matter what those consumers choose, the monopolist cannot be worse off by increasing η. For case (ii), since we have

$$\underline{u}_S(x,p,s;\eta') = u_B(p,p;\eta') \geq 0$$

for all $x < p$, whenever $p \leq 1/2$. We will show later that $p > 1/2$ is never optimal. Then, all those consumers with $x < p$ choose to search. This is always a profitable deviation. ■

Proof of Lemma 5.

We show Lemma 5 by going through 4 steps. When $s \leq 1/8$, first, we show how the critical type's response changes with η. Second, we identify the upper bounds of the monopolist's profits in regimes BI and BI-QM. Third, we show that any choice made by the monopolist in regime BI-QM is strictly dominated by the choice it can make in other regimes. Thus, the monopolist's optimal strategy comes from comparison with the highest profits it can make in regimes BI and BI-SB. Finally, we show that the monopolist's optimal choice in BI-SB has to satisfy constraints $(1-\eta)(1-p)^2/2 - s = 0$ and $p \in [\sqrt{2s}, 1/2]$.

Step 1: Let $\hat{x}$ be the critical type between immediate purchase and search, $\tilde{x}$ be the critical type between search and quitting. $\partial\tilde{x}/\partial\eta \geq 0$ when $p \leq \sqrt{2s}$ and $\partial\tilde{x}/\partial\eta < 0$ when $p > \sqrt{2s}$. $\partial\hat{x}/\partial\eta \geq 0$ when $p \leq 1/2$ and $\partial\hat{x}/\partial\eta < 0$ when $p > 1/2$.

Proof of Step 1: Let $\tilde{x}, \hat{x} \in [0,1]$ such that $u_B(\tilde{x}, p; \eta) = \underline{u}_S$ and $u_B(\hat{x}, p; \eta) = 0$. Since $\eta\tilde{x} + (1-\eta)/2 - p = (1-\eta)(1-p)^2/2 - s$, the Implicit Function Theorem yields $d\tilde{x}/d\eta = [1-(1-p)^2 - 2\tilde{x}]/2\eta$. By substituting $\tilde{x}$ into it, we have $d\tilde{x}/d\eta = s - p^2/2\eta^2$. Therefore, $\partial\tilde{x}/\partial\eta \geq 0$ when $p \leq \sqrt{2s}$ and $\partial\tilde{x}/\partial\eta < 0$ when $p > \sqrt{2s}$. Similarly, $\partial\hat{x}/\partial\eta \geq 0$ when $p \leq 1/2$ and $\partial\hat{x}/\partial\eta < 0$ when $p > 1/2$. $\square$

Step 2: In regime BI, the highest expected profit that the monopolist can get is $\sqrt{2s}$. In regime BI-QM, the seller cannot gain more than 1/4 by charging $p > 1/2$.

Proof of Step 2: We analyze condition (21) which corresponds to the type BI equilibrium. Notice that both $\left[-\eta + \sqrt{\eta^2 + 2s(1-\eta)}\right]/(1-\eta)$ and $(1-\eta)/2$ are decreasing in η and cross only once. When η is small, $(1-\eta)/2$ is strictly higher and $\left[-\eta + \sqrt{\eta^2 + 2s(1-\eta)}\right]/(1-\eta)$ exceeds $(1-\eta)/2$ as η becomes larger. Since for any (p, η) in regime BI, all consumers make immediate purchase. The monopolist just needs to choose the highest possible price. This price is determined by function $p = \left[-\eta + \sqrt{\eta^2 + 2s(1-\eta)}\right]/(1-\eta)$ when $\eta = 0$, which is $p = \sqrt{2s}$.

Step 1 states that, for any $p \geq 1/2$ in regime BI-QM, the monopolist can attract more immediate purchase by increasing η. Therefore, seller will choose $\eta = 1$ if she chooses $p \geq 1/2$ in regime BI. The monopolist's maximization problem in regime BI-QM is $\max\limits_{p \geq 1/2, p \in BI-QM} p(1-p)$, constrained by $p \geq 1/2$. The maximum expected profit is 1/4. Any $P > 1/2$ yields profit strictly less than 1/4. $\square$

Step 3: For search cost $s < 1/8$, the optimal (p, η) is not in regime BI-QM.

Proof of Step 3: Suppose that the monopolist chooses $p < 1/2$ in BI-QM. From Step 1, decreasing η attracts more immediate purchase when p is fixed. The easiest way to understand our argument is to see from Figure 3. By decreasing η, the monopolist will eventually break the condition $\max\left\{\frac{1}{2}(1-\eta), 1 - \sqrt{\frac{2s}{1-\eta}}\right\} < p$ and reach the boundary line

J, H and $(1,0)$ in Figure 3. These boundaries belong to regime BI and BI-SB. This implies that any (p,η) in regime BI-QM is dominated by some (p,η') with $\eta' < \eta$. Combining this with Step 2, we know that the monopolist will not choose any price and precision combination in regime BI-QM. $\square$

Step 4: The monopolist's optimal choice in regime BI-SB has to satisfy constraints $1/2(1-\eta)(1-p)^2 - s = 0$ and $p \in [\sqrt{2s}, 1/2]$.

Proof of Step 4: If $p < 1/2$, the monopolist can fix p and decrease η and eventually goes into regime BI which can be seen clearly from Figure 3. Since all consumers make immediate purchases in that regime, the monopolist can be better off. Therefore, any (p,η) in regime BI-SB with $p < 1/2$ is dominated by some (p,η') with $\eta' < \eta$ in regime BI. For $p > 1/2$, the optimal (η, p) has to lie on the section joining G and J, which can be described by $1/2(1-\eta)(1-p)^2 - s = 0$ and $p \in [\sqrt{2s}, 1/2]$. $\square$

Combining the four claims, we have Lemma 5. $\blacksquare$

Proof of Proposition 4.
The expected demand is

$$D(p,\eta) = \frac{(1+\eta)/2 - p}{\eta} + (1-\eta)(1-p)\left[\frac{p-(1-\eta)/2}{\eta}\right].$$

Set the Lagrangian function for the constrained maximization problem $(*)$ as

$$\begin{aligned}\mathcal{L} =& p\left[\frac{(1+\eta)/2 - p}{\eta} + (1-\eta)(1-p)\frac{p-(1-\eta)/2}{\eta}\right] \\ &+ \lambda_1\left[s - 1/2(1-\eta)(1-p)^2\right] + \lambda_2(1/2-p) + \lambda_3(p-\sqrt{2s}).\end{aligned}$$

The first-order conditions are given by

$$\begin{aligned}
&\frac{\partial\mathcal{L}}{\partial\eta} = \frac{p\eta^2(p-1) + p^2(2p-1)}{\eta^2} + \lambda_1(1-p)^2 = 0 && \text{(w.r.t } \eta) \\
&\frac{\partial\mathcal{L}}{\partial p} = \frac{\eta(3-\eta)(1/2-p) + p(1-\eta)(1-3p)}{\eta} + \lambda_1(1-\eta)(1-p) - \lambda_2 + \lambda_3 = 0 && \text{(w.r.t } p) \\
&\frac{\partial\mathcal{L}}{\partial\lambda_1} = 1/2(1-\eta)(1-p)^2 - s = 0 && \text{(w.r.t } \lambda_1) \\
&\lambda_2(1/2-p) = 0 \qquad \text{and} \qquad \lambda_2 \geq 0 && \text{(w.r.t } \lambda_2) \\
&\lambda_3(p-\sqrt{2s}) = 0 \qquad \text{and} \qquad \lambda_3 \geq 0 && \text{(w.r.t } \lambda_3)
\end{aligned}$$

First, when $s \in (0, 1/16)$, $(p,\eta) = (1/2, 1-8s)$ is the solution to the above program. This can be verified by checking the Lagrangian multipliers. Substituting $(p, \eta) = (1/2, 1 - 8s)$ into the first-order conditions, we have

$$\lambda_1 = 1, \qquad \lambda_2 = 4s - \frac{2s}{1-8s}, \qquad \lambda_3 = 0,$$

where λ_2 is strictly positive when $s < 1/16$.

Second, $(p^*, \eta^*) = (\sqrt{2s}, 1 - 2s/(1-\sqrt{2s})^2)$ is the solution to the above maximization problem when $s \in (0.117324, 1/8)$. If $(p^*, \eta^*) = (\sqrt{2s}, 1 - 2s/(1-\sqrt{2s})^2)$ is the solution, we must have $\lambda_2 = 0$. From the first-order condition w.r.t η, we have

$$\lambda_1 = \frac{p(1-p)\eta^2 + p^2(1-2p)}{(1-p)^2\eta^2}.$$

Substituting λ_1 into the first-order condition w.r.t p, we have

$$\lambda_3 = \frac{p(1-\eta)[(p-1)\eta^2 + p(2p-1)] - \eta(1-p)[\eta(3-\eta)(1/2-p) + p(1-\eta)(1-3p)]}{\eta^2(1-p)}.$$

Substituting $p = \sqrt{2s}$ and $\eta = 1 - [2s/(1-\sqrt{2s})^2]$ into the expression of λ_3 and simplifying, we have

$$\lambda_3 = \frac{-1 + 8\sqrt{2s} - 51s + 78\sqrt{2}s^{3/2} - 96s^2 - 16\sqrt{2}s^{5/2} + 64s^3 + 8\sqrt{2}s^{7/2} - 32s^4}{(1 - 3\sqrt{2s} + 4s)^2}$$

The Lagrangian multiplier $\lambda_3 > 0$ when $s \in (\bar{s}, 1/8)$, where $\bar{s} \approx 0.117324$. Then, $\lambda_1 > 0$, $\lambda_2 = 0$ and $\lambda_3 > 0$ guarantees that $(p^*, \eta^*) = \left(\sqrt{2s}, 1 - 2s/(1-\sqrt{2s})^2\right)$ is the solution to $(*)$ when $s \in (\bar{s}, 1/8)$.

Finally, if $s \in [1/16, \bar{s}]$, the solution to program $(*)$ must be interior. Then, we have $\lambda_2 = \lambda_3 = 0$. It is hard to explicitly solve for interior solutions. Nevertheless, we can show that the value function $\Pi(s)$ over $[1/16, \bar{s}]$ is strictly lower than $\sqrt{2s}$. The value function $\Pi(s)$ is continuous over $[1/16, \bar{s}]$. Applying the Envelope Theorem, we have

$$\frac{d\Pi(s)}{ds} = \left.\frac{\partial \mathcal{L}}{\partial s}\right|_{p^*,\eta^*} = \lambda_1 \qquad \text{and} \qquad \left.\frac{d\Pi^2(s)}{ds^2}\right|_{p^*,\eta^*} = 0.$$

Thus, $\Pi^*(s)$ is increasing and linear over $[1/16, \bar{s}]$. The values at the end points are given by $\Pi(1/16) = 5/16$ and $\Pi(\bar{s}) = 0.387513$. Substituting $s = 1/16$ and $\bar{s}$ into $\sqrt{2s}$, we get $\sqrt{2}/4$ and 0.484405 respectively. Since both $\Pi(s)$ and $\sqrt{2s}$ are continuous and strictly increasing, $\sqrt{2s}$ is larger at both end points, we conclude that $\Pi(s) < \sqrt{2s}$ over $[1/16, \bar{s}]$.

Now we compare $\Pi(s)$ and $\sqrt{2s}$ on the full range of s. For $s < \frac{1}{4}(3 - 2\sqrt{2})$, $\sqrt{2s} < \Pi(s)$

when $s < \left(3-2\sqrt{2}\right)/4$ and vice versa for $s \geq \left(3-2\sqrt{2}\right)/4$. Since $\left(3-2\sqrt{2}\right)/4 < 1/16$ and $\sqrt{2s} > \Pi(s)$ when $s \in [1/16, \bar{s}]$, we conclude that $\sqrt{2s} \geq \Pi(s)$ when $s \in [\left(3-2\sqrt{2}\right)/4, \bar{s}]$. For $s \in (\bar{s}, 1/8]$, we have $\Pi(s) = \sqrt{2s}\ D(p,\eta)$, which is less than $\sqrt{2s}$ as $D(p,\eta) < 1$. Then, $\sqrt{2s} \geq \Pi(s)$ when $s \in [\left(3-2\sqrt{2}\right)/4, 1/8]$. Proposition 4 thus follows. ■