Long-term care policy, myopia and redistribution

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Abstract

This paper examines whether myopia (misperception of the LTC risk) and private insurance market loading costs can justify social LTC insurance and/or the subsidization of private insurance. We use a two-period model wherein individuals differ in three unobservable characteristics: level of productivity, survival probability and degree of ignorance concerning the risk of LTC (the former two being perfectly positively correlated). The decentralization of a first-best allocation requires that LTC insurance premiums of the myopic agents are subsidized (at a “Pigouvian” rate) and/or that there is public provision of the appropriate level of LTC. The support for the considered LTC policy instruments is less strong in a second-best setting. When social LTC provision is restricted to zero, a myopic agent’s tax on private LTC insurance premiums involves a tradeoff between paternalistic and redistributive (incentive) considerations and we may have a tax as well as a subsidy on private LTC insurance. Interestingly, savings (which goes untaxed in the first-best but plays the role of self-insurance in the second-best) is also subject to (positive or negative) taxation. Social LTC provision is never second-best optimal when private insurance markets are fair (irrespective of the degree of the proportion of myopic individuals and their degree of misperception). At the other extreme, when the loading factor in the private sector is sufficiently high, private coverage is completely crowded out by public provision. For intermediate levels of the loading factors, the solution relies on both types of insurance.

Keywords: Long-term care, myopia, optimal taxation

JEL-Classification: D91, H21, I13
1 Introduction

The demand for professional long-term care (LTC) services is likely to increase dramatically during the decades to come. This is due to two main trends. The first is the forecasted aging of societies, and the increase in of the proportion of individuals aged 80+, who represent the main group at risk for dependency. The second trend is related to ongoing changes in society and family values, which imply that the current main provider of LTC, namely the family, is likely to play a less significant role in the future. Consequently, dependency represents a major financial risk in old age. The probability that a 65-year-old will use a nursing home is quite significant, with estimates ranging from 35% to 49% (Brown and Finkelstein, 2009). Care provided in a nursing home may be expensive, e.g., a single bedroom in a nursing home can cost up to $75,000 per year (Genworth, 2010). One might expect that, faced with such a hazard, rational risk-averse individuals would buy LTC insurance in order to smooth their consumption over the different states of nature. However, in most countries only a small fraction of the population effectively buys dependency insurance protection. The economic literature has identified a number of factors that can explain the low level of LTC insurance demand. These include significant loading factors due to high administrative costs, adverse selection in the demand for insurance (Sloan and Norton, 1997; Finkelstein and McGarry, 2006; Brown and Finkelstein, 2007 and 2009) and the existence of cheaper substitutes like family care or public assistance (Pauly, 1990; Zweifel and Strüwe, 1998; Brown and Finkelstein, 2008). Last but not least, it is widely acknowledged that underestimation or ignorance of the dependency risk is a major cause of the deficient individual insurance protection (Brown and Finkelstein, 2009; Pestieau and Ponthiere, 2010; Zhou-Richter et al., 2010). For many individuals, old-age dependency represents such a daunting perspective that they’d rather not think about it, and least of all are ready to formally acknowledge such a prospect by subscribing to insurance protection.

Because of these trends the provision of sustainable LTC protection represents a looming challenge for the decades to come. If private markets remain insignificant and family assistance decreases as expected, there appears to be a good case for government intervention. This is even more so as LTC is not only a matter of efficiency (and insurance market failure) but also involves redistributive issues. Individuals who are
sufficiently wealthy can in any event finance LTC expenses from their savings so that the lack of insurance is of limited relevance. However, a large fraction of the population may not have enough savings or retirement income to face their dependency expenses and in the absence of public intervention their failure to insure may have dramatic implications.

The probably most widely debated instruments of public LTC policy are the subsidization of private LTC insurance and the provision of social LTC insurance (with either cash or in-kind benefits). Their appropriate use has received little attention in the literature so far, and the few papers there are do not appear to provide a lot of support for either of these policies. Dependence is known to increase with longevity, which in turn increases with income (Viscusi, 1994; Gerdtham and Johannesson, 2000). This property has two important implications. First, private LTC insurance tends to be a normal or even a luxury good which, from an optimal tax perspective, ought to be taxed rather than subsidized; see e.g., Jonsten et al., (2005); Pestieau and Sato, (2006, 2008) and Cremer and Pestieau (2011). Second, with regard to the results of Rochet (1991) this relationship between income and dependency sheds doubt on the desirability of social LTC insurance. Rochet considers a setting in which individuals differ in risk and productivity (both characteristics being unobservable), where the income tax is optimized and where private insurance markets are actuarially fair. He shows (roughly speaking) that full social insurance is appropriate if (and only if) risk and earning ability are negatively correlated, a property which is commonly thought to be satisfied for various health risks. However, in the case of LTC, we appear to have the opposite pattern of correlation and it follows that full social insurance is certainly not optimal. The question of whether we need social insurance at all under positive correlation is not explicitly addressed by Rochet, but the intuition underlying his other results suggest that this is not likely to be the case.¹

The important policy question is to know if these negative results are mere artifacts of some restrictive assumptions or if they are sufficiently robust to provide a guideline for policy design. One avenue to explore is to relax the assumption that the income tax is optimal and not restricted (except by informational considerations). This approach is taken by Cremer and Pestieau (2011), but they only provide a limited support to

¹See also Cremer and Pestieau (1996).
an active LTC policy. In particular, they show that quite severe restrictions on the income tax are needed to justify social LTC insurance. Specifically, an optimal affine tax function may be sufficient to make social insurance redundant. Furthermore, none of these restrictions justifies a subsidy on private insurance which remains subject to a positive tax (linear or marginal nonlinear).

In this paper we explore an alternative path that one would intuitively expect to provide a rationale for social LTC insurance. Our approach has two main features. First, we assume that some individuals may be myopic in the sense that they underestimate their dependency risk when they make their savings and insurance decisions. Second, to introduce an extra measure of realism, we consider the possibility that private insurance markets may not offer actuarially fair LTC coverage. In other words, private insurance premiums may be subject to a loading factor. Except for these two variations, we remain within the spirit of Rochet (1991) in the sense that we do not arbitrarily restrict the set of available instruments (which is determined solely by the information structure).

We use a two-period model where individuals work in the first period and survive to the second period (old age) with some type-specific probability. Conditional on survival all agents face the same probability of needing LTC. Individuals differ in three unobservable characteristics: level of productivity, survival probability and degree of ignorance concerning the risk of LTC. The former two are perfectly positively correlated. In other words, a unique value of the survival probability is associated with every wage level, and a higher wage also implies a higher survival probability. However, for each pair of these two variables, there may be individuals with different degrees of myopia. In contrast to the government private insurance companies observe an individual’s survival probability implying that high-survival individuals have to pay a higher price for their insurance coverage. Individuals who (partly) ignore their LTC risk make “non-optimal” LTC insurance and savings decisions. In other words, they do not maximize their “true” expected utility and ex post they will regret the decisions they made in the first period. Social welfare, on the other hand, is “paternalistic” in the sense that it depends on individuals’ true preferences. To decentralize a first-best allocation, LTC insurance premiums of the myopic agents can be subsidized (at a “Pigouvian” rate) or alternatively we can have public provision of the appropriate level of LTC. Under full information (and actuarially fair private insurance) these two instruments are perfectly equivalent,
but we need at least one of them. When private insurance is subject to a loading factor (first-best) social LTC provision dominates. Either way, these arguments show that we can expect the two instruments to play a role in our setting but the question is if and how they should be used in a second-best setting.

In accordance with our (second-best) information structure, we consider a nonlinear tax scheme where the policy instruments include a tax on income, savings and private LTC insurance premiums. Additionally, the government may publicly provide LTC. To study this setting we proceed in two steps. First, we abstract from the possibility of public LTC provision and give a characterization for the optimal marginal tax on private LTC insurance premium and old-age savings. We show that for myopic agents, the tax on private LTC insurance premiums is determined by Pigouvian and redistributive elements, where the former calls for subsidization of LTC insurance expenditures, while the latter may well call for a taxation of LTC. In other words, there is a tradeoff between paternalistic and redistributive (incentive) considerations and we may have a tax as well as a subsidy on private LTC insurance. Interestingly, savings (which goes untaxed in the first-best) is also subject to (positive or negative) taxation in the second-best and the expression for the tax rates includes also a Pigouvian and a redistributive term. Observe that in a first-best setting savings and LTC decisions are independent (individuals are fully insured). In the second best this is no longer true; savings plays the role of self-insurance and is treated accordingly by the tax policy.

In a second step, we determine whether it is optimal to use public LTC as an additional instrument. Quite surprisingly, this is never optimal when private insurance markets are fair (irrespective of the degree of the proportion of myopic individuals and their degree of misperception). This result is \textit{not} due to a “cost of public funds” argument but arises because private insurance has an informational advantage over public coverage in our setting.\footnote{Since we have an optimal income tax the cost of public funds is effectively equal to one in the second-best solution; see Jacobs (2010).} At the other extreme, when the loading factor in the private sector is sufficiently high, private coverage is completely crowded out by public provision. For intermediate levels of the loading factors, the solution relies on both types of insurance.

To sum up, while the full information solution to our model provides support for the
subsidization of LTC private insurance and/or for public provision (or insurance), these results have to be qualified in a second-best setting. Surprisingly, myopia (irrespective of its extent) is not per se sufficient to justify public LTC provision; a large private sector loading factor on the other hand does plead for public provision (but this in turn is not surprising). The tax treatment of private insurance is determined by a tradeoff between paternalistic and redistributive considerations. Roughly speaking, misperceptions must be sufficiently severe or widespread to obtain a subsidy. Myopia also affects the tax treatment of savings (which plays the role of self insurance) but here also the sign of the overall tax is ambiguous.

The paper proceeds as follows. Section 2 sets up the model and describes the laissez-faire solution. Section 3 derives the first-best solution. Section 4 analyzes the second-best solution and studies the decentralization (implementation) of both first- and second best solutions. Section 5 studies the tax treatment of private insurance and savings under the restriction that there is no public LTC provision, whereas Section 6 analyzes whether it is optimal to provide public LTC. Numerical examples are presented in Section 7. Section 8 summarizes our results.

2 The economy

2.1 The setup

Consider a two-period model in which all individuals are alive in the first period during which they work. Individuals of type $i = 1, \ldots, N$ live with probability $\varphi_i$ during the second period, where $\varphi_i \leq \varphi_{i+1}$. In this second period they are retired and possibly disabled. Survival probability and earnings ability $w_i$ are perfectly positively correlated. In other words, a single survival probability is associated with every wage level and both variables (weakly) increase with $i$. Conditional on surviving in the second period, all individuals face the same probability of becoming disabled, $\pi$. Lifetime utility $U_i$ is assumed to be additive over time with a zero discount rate. Assuming that utility of being death is zero, expected lifetime utility of individual-$i$ is given by

$$U_i = u(c_i) - v(c_i) + \varphi_i(1 - \pi)u(d_i) + \varphi_i\pi H(m_i),$$
where \( c_i \) denotes first-period consumption while \( d_i \) and \( m_i \) denote second-period consumption in the case of being healthy and in the case of needing long-term care respectively. Utilities \( u \) and \( H \) are such that \( u', H' > 0 \) and \( u'', H'' < 0 \). Labor supplied in the first-period is given by \( \ell_i \) which one can think of as being the retirement age. Gross earnings are thus given by \( y_i = w_i \ell_i \). Labor disutility \( v \) is increasing and strictly convex, i.e., \( v' > 0 \) and \( v'' > 0 \). In the second period individuals live of their savings and, in case of disability, their private and public long-term care insurance coverage. Savings \( s_i \) can be invested on a private annuity market at a zero interest rate and actuarially fair prices implying a return equal to \( s_i/\varphi_i \). Long-term care insurance coverage can be bought in the private market at a price \( \delta \varphi_i \pi \) per unit of coverage, where \( \delta \geq 1 \) represents the private insurance loading factor. Individuals’ insurance premiums \( \theta_i \geq 0 \) depend on their probability of needing long-term care (namely \( \varphi_i \pi \)), that is, insurance companies are able to observe each individual’s type. With this assumption \( \delta = 1 \) yields an actuarially fair private insurance. This is an interesting benchmark which enables us to compare our results to those of Rochet (1991).

Not all individuals make their savings and long-term care insurance decisions according to their true probability of being disabled while old. Some individuals underestimate the risk of needing long-term by a share \( \beta_i \in [0,1] \). These individuals reach their savings and long-term care insurance decisions on the basis of a perceived probability of becoming disabled given by \( \beta_i \pi \). The parameter \( \beta_i \) can be interpreted as the individual’s degree of ignorance or myopia concerning the risk of disability. Formally, savings and insurance decision are chosen according to

\[
U_i = u(c_i) - v(\ell_i) + \varphi_i(1 - \beta_i \pi)u(d_i) + \varphi_i \beta_i \pi H(m_i).
\]

Agents with a very low \( \beta_i \) hardly realize that they may become dependent while old and individuals with a \( \beta_i \) equal to one are perfectly aware of the risk. Any vector \((\varphi, w)\) may be associated with several levels of \( \beta \) and type \( i = 1 \), is defined by the triplet \((\varphi_i, w_i, \beta_i)\). Total population size is normalized to one and the share of type-\( i \) individuals is given by \( \phi_i \). An example of such a setting is one with four types which obtains when \( \varphi_i \in \{\varphi_l, \varphi_h\} \), \( w_i \in \{w_l, w_h\} \) and \( \beta_i \in \{\beta_l, \beta_h\} \); it is depicted in Figure 1. Type-1 and type-3 agents (partly) ignore the risk of needing long-term and either have
low or high ability/survival probability. Whereas type-2 and type-4 individuals have low
or high productivity/survival probability but are less myopic (have a higher $\beta_i$). This
four type specification will be used for the simulations in Section 7 and even simpler
two-type examples will be used to illustrate some results; see Subsections 5.1–5.3.

### 2.2 Laissez-faire

In the *laissez-faire*—that is without government intervention—individuals-$i$ choose their
consumption levels $c_i$, $d_i$, $m_i$, savings $s_i$, labor supply $\ell_i$ and long-term care insurance
expenses $\theta_i$ by solving the following problem

$$\max_{c_i,d_i,m_i,s_i,\ell_i,\theta_i} U_i = u(c_i) - v(\ell_i) + \varphi_i(1 - \beta_i \pi)u(d_i) + \varphi_i \beta_i \pi H(m_i),$$

s.t.

- $c_i = w_i \ell_i - s_i - \theta_i,$
- $d_i = \frac{s_i}{\varphi_i},$
- $m_i = \frac{s_i}{\varphi_i} + \frac{\theta_i}{\delta \varphi_i \pi}.$
Substituting the expression for $c_i$, $d_i$ and $m_i$ into the utility function $U_i$, the first-order conditions with respect to $\ell_i$, $s_i$ and $\theta_i$ are given by

$$w_iu'(c_i) = u'(\ell_i),$$

$$u'(c_i) = (1 - \beta_i \pi)u'(d_i) + \beta_i \pi H'(m_i),$$

$$u'(c_i) = \frac{\beta_i}{\delta} H'(m_i).$$

The first equation simply states that the marginal rate of substitution between consumption and labor is equal to wages. If the individual is perfectly aware of his true probability of needing long-term care ($\beta_i = 1$) and there is no loading ($\delta = 1$), marginal utilities across time and states of nature are equalized, $u'(c_i) = u'(d_i) = H'(m_i)$. Thus, full insurance is optimal. If, however, the agent undervalues the probability of becoming disabled in old age, he is less than perfectly insured against the risk of long-term care. This is because from these individuals’ perspective, the premium of long-term care insurance is not actuarially fair even when $\delta = 1$. For $\beta_i < 1$ and $\delta = 1$ equations (2) and (3) yield $u'(d_i) < u'(c_i) < H'(m_i)$. A high loading factor, $\delta$, also reduces LTC insurance coverage. When $\delta \pi > 1$, the return of private savings is higher than the LTC insurance coverage in case of dependency, that is, private LTC insurance is dominated by savings and we have $\theta_i = 0$. In other words, the individual self-insures through savings rather than buying insurance coverage on the market.

### 3 First-best solution

Throughout the paper we take a paternalistic approach and consider the utilitarian optimum based on individuals’ true probability of needing long-term care and not on the one perceived by myopic individuals. *Ex post* individuals are grateful to the social planner to have forced them to behave according to their true LTC risk. In the first-best setting, the social planner observes the individuals’ types, namely their productivity, survival probabilities and their degree of myopia. Welfare maximization is subject to the resource constraint that aggregate consumption cannot exceed aggregate production.
The corresponding Lagrangian denoted by $L^{FB}$ is

$$L^{FB}(c_i, \ell_i, d_i, m_i) = \sum_i \phi_i \left\{ u(c_i) - v \left( \frac{y_i}{w_i} \right) + \varphi_i (1 - \pi) u(d_i) + \varphi_i \pi H(m_i) \right\}$$

$$+ \mu \sum_i \phi_i \{ y_i - c_i - \varphi_i (1 - \pi) d_i - \varphi_i \pi m_i \}, \quad (4)$$

where $\mu$ is the Lagrangian multiplier of the resource constraint. The FOCs for type-$i$ can be written as

$$w_i u'(c_i) = v'(\ell_i) \quad \text{and} \quad u'(c_i) = u'(d_i) = H'(m_i) = \mu \quad \forall \ i. \quad (5)$$

This implies for labor supply and consumption

$$\ell_i = \ell_j \quad \text{if} \quad w_i = w_j \quad \text{and} \quad \ell_i < \ell_j \quad \text{if} \quad w_i < w_j \quad \forall i, j$$

$$c_i = d_i = \text{const.} \quad \forall i, \quad m_i = \text{const.} \quad \forall i.$$

Individuals with high-productivity should supply more labor (irrespective of their $\beta$). Consumption levels of the young and healthy elderly are equalized across types. In addition, the LTC expenditures $m_i$ are the same for all types. The comparison between $c_i = d_i$ and $m_i$ depends on the functional form of $u$ and $H$ (but marginal utilities are equalized). Given a utilitarian welfare function, resources are transferred not only from high- to low-productivity individuals, but also from the short- to the long-lived ones. As there is a positive correlation between wages and survival probability, the overall direction of redistribution is thus ambiguous. We assume throughout the paper that the redistribution goes from high- to low-wage individuals. In other words, we assume that earning abilities represent the dominating source of heterogeneity.\footnote{Instead we could introduce weights in the social welfare function to insure this outcome. The underlying idea is that redistribution from short- to long-lived (and otherwise identical) individual does not appear to be acceptable; see .} In Section 4.1 we show how the first-best solution can be decentralized.
4 Second-best problem

We now turn our attention to the second-best solution where individuals’ types, that is, \( w_i, \varphi_i \) and \( \beta_i \) are not publicly observable. The distribution of types is common knowledge and this includes the information that productivities and survival probabilities are perfectly positively correlated. Additionally income \( y_i \), savings \( s_i \) and long-term care insurance premiums \( \theta_i \) are observable. Consequently, an individual’s first period consumption level is effectively observable. In the following subsection, we study how the second-best solution can be implemented with the instruments available given this information structure.

4.1 Implementation

Under the considered information structure instruments include a (possibly nonlinear) transfer scheme \( T(y_i, s_i, \theta_i) \) and a payment to the dependent individuals \( D_i \). In other words, income, savings and long-term care insurance premiums can be taxed or subsidized and \( T(y_i, s_i, \theta_i) \) also includes an individualized lump-sum transfer \( T_i \) which can be positive or negative. In addition, public long-term care \( D_i \) can be provided in case of disability.

With such a transfer function the individual’s problem is given by

\[
\max_{y_i, s_i, \theta_i} U_i = u(y_i - s_i - \theta_i - T(y_i, s_i, \theta_i)) - v \left( \frac{y_i}{w_i} \right) + \varphi_i (1 - \beta_i) \pi u \left( \frac{s_i}{\varphi_i} \right) + \varphi_i \beta_i \pi H \left( \frac{s_i}{\varphi_i} + \frac{\theta_i}{\delta \varphi_i \pi} + D_i \right).
\]

This yields the following first-order conditions

\[
\frac{\partial U_i}{\partial y_i} = u'(c_i)(1 - T_y(y_i, s_i, \theta_i)) - v'(\ell_i) \frac{1}{w_i} \leq 0, \tag{7}
\]

\[
\frac{\partial U_i}{\partial s_i} = -u'(c_i)(1 + T_s(y_i, s_i, \theta_i)) + (1 - \beta_i \pi)u'(d_i) + \beta_i \pi H'(m_i) = 0, \tag{8}
\]

\[
\frac{\partial U_i}{\partial \theta_i} = -u'(c_i)(1 + T_\theta(y_i, s_i, \theta_i)) + \frac{\beta_i}{\delta} H'(m_i) \leq 0. \tag{9}
\]

Note that for a high degree of ignorance concerning the need for long-term care (a low
level of $\beta_i$), a high loading, or a high tax on private LTC insurance, the individual may not buy any long-term care insurance coverage at all, that is $\theta_i = 0$. Assuming an interior solution for $y_i$ and $\theta_i$, the above FOCs can be rewritten as

$$\frac{v'(\ell_i)}{w'(c_i)} = w_i(1 - T_y(y_i, s_i, \theta_i))$$

(10)

$$\frac{(1 - \beta_i \pi)u'(d_i) + \beta_i \pi H'(m_i)}{w'(c_i)} = 1 + T_s(y_i, s_i, \theta_i)$$

(11)

$$\frac{H'(m_i)}{w'(c_i)} = \frac{\delta}{\beta_i}(1 + T_\theta(y_i, s_i, \theta_i)).$$

(12)

When $T^i_y, T^i_s, T^i_\theta < (>)0$, type-$i$ faces a marginal subsidy (tax) on income, savings and private long-term care insurance respectively. As $\ell_i$ can be considered as the retirement age of a type-$i$ individual, $T^i_y$ can be also interpreted as an implicit subsidy (tax) on prolonged activity. In our context, savings also act as a self-insurance device. An implicit subsidy (tax) on savings thus increases (reduces) the attractiveness to self-insure against the risk of needing LTC. The same applies for private long-term care insurance. The agent is less than fully insured against the risk of needing LTC whenever the marginal utility in the state of disability is above the marginal utility in the healthy state, i.e., $H'(m) > u'(d)$. Combining equations (11) and (12) yields

$$\frac{H'(m_i)}{w'(d_i)} = \frac{1 - \beta_i \pi}{\frac{\beta_i}{\delta} \frac{1 + T_i^y}{1 + T_i^s} - \beta_i \pi} \geq 1 \quad \text{if} \quad \frac{\beta_i}{\delta} \frac{1 + T_i^s}{1 + T_i^\theta} \leq 1.$$ 

That is—apart from myopia and a positive loading—both the tax on LTC insurance and the tax on savings determine whether the individual is fully insured or not.

Before proceeding with the characterization of the second best, it is useful to discuss the decentralization of the first-best solution. To achieve this, we need—apart from individualized lump-sum transfers—corrective subsidies on long-term care insurance premiums. The required tax rate can be derived by comparing the FOCs of the first-best solution (5) with equations (10) to (12). This yields the following result

$$FB: \quad 1 + T^i_y = 1, \quad 1 + T^i_s = 1, \quad 1 + T^i_\theta = \beta_i.$$ 

(13)

The first-best optimum can be reached with a zero tax on labor and savings, that is, myopia regarding LTC does not distort savings. However, it is optimal to subsidize
agents who undervalue their probability of needing long-term care, $\beta_i < 1$. Specifically, the “Pigouvian subsidy”, $-T_i^P = 1 - \beta_i$, is chosen to exactly offset individuals myopia. Once, we have corrected for myopia with respect to the probability of needing long-term care, we no longer need any subsidies on savings as agents, then, will choose the optimal savings plan by themselves.\(^5\) If there is a positive loading on the private LTC insurance market, $\delta > 1$, the first-best solution can be achieved by providing solely public long-term care. Observe that in a first best setting and when $\delta = 1$, private insurance and public provision are two equivalent ways of implementing the optimum. When $\delta > 1$, this is no longer true and private insurance is dominated by public provision.\(^6\)

### 4.2 General solution

With the considered information structure feasible allocations must satisfy the following incentive constraints

$$
\begin{align*}
&u(c_i) - v \left( \frac{y_i}{w_i} \right) + \varphi_i(1 - \beta_i \pi) u \left( \frac{s_i}{\varphi_i} \right) + \varphi_i \beta_i \pi H \left( \frac{s_i}{\varphi_i} + \frac{\theta_i}{\delta \varphi_i \pi} + D_i \right) \geq \\
&u(c_j) - v \left( \frac{y_j}{w_i} \right) + \varphi_i(1 - \beta_i \pi) u \left( \frac{s_j}{\varphi_i} \right) + \varphi_i \beta_i \pi H \left( \frac{s_j}{\varphi_i} + \frac{\theta_j}{\delta \varphi_i \pi} + D_j \right) \quad \forall \ i \neq j.
\end{align*}
$$

That is any type-\(i\) must be prevented from mimicking any type-\(j\) individual. In addition the resource constraint continues of course to apply. The Lagrangian $L^{SB}(c_i, y_i, s_i, \theta_i, D_i)$ of the second-best problem is then given by

$$
L^{SB} = \sum_i \phi_i \left\{ u(c_i) - v \left( \frac{y_i}{w_i} \right) + \varphi_i(1 - \beta_i \pi) u \left( \frac{s_i}{\varphi_i} \right) + \varphi_i \beta_i \pi H \left( \frac{s_i}{\varphi_i} + \frac{\theta_i}{\delta \varphi_i \pi} + D_i \right) \right\}
$$

$$
+ \sum_{i \neq j} \lambda_{ij} \left\{ u(c_i) - v \left( \frac{y_i}{w_i} \right) + \varphi_i(1 - \beta_i \pi) u \left( \frac{s_i}{\varphi_i} \right) + \varphi_i \beta_i \pi H \left( \frac{s_i}{\varphi_i} + \frac{\theta_i}{\delta \varphi_i \pi} + D_i \right) \right\}
$$

$$
+ \sum_{i \neq j} \lambda_{ij} \left\{ -u(c_j) + v \left( \frac{y_j}{w_j} \right) - \varphi_i(1 - \beta_i \pi) u \left( \frac{s_j}{\varphi_i} \right) - \varphi_i \beta_i \pi H \left( \frac{s_j}{\varphi_i} + \frac{\theta_j}{\delta \varphi_i \pi} + D_j \right) \right\}
$$

$$
+ \mu \sum_i \phi_i \left\{ y_i - c_i - s_i - \theta_i - \varphi_i \pi D_i \right\},
$$

\(^5\)As an alternative to subsidizing private long-term insurance, it could be made compulsory (at the first-best level). This is a feasible way to decentralize the optimum even when $\beta_i = 0$ (in which case marginal subsidization is ineffective).

\(^6\)This is admittedly artificial, though, because one can argue that in a first-best setting we have $\delta = 1$. 

where \( \mu > 0 \) is the multiplier of the resource constraint while \( \lambda_{ij} \geq 0 \) is the multiplier associated with the self-selection constraint from type-\( i \) to type-\( j \). The first-order conditions are given by

\[
\frac{\partial L_{SB}}{\partial c_i} = \left\{ \phi_i + \sum_{j \neq i} \lambda_{ij} - \sum_{j \neq i} \lambda_{ji} \right\} u'(c_i) - \mu \phi_i = 0
\]  
(16)

\[
\frac{\partial L_{SB}}{\partial y_i} = -\left\{ \phi_i + \sum_{j \neq i} \lambda_{ij} \right\} v'((\ell_i) \frac{1}{w_i}) + \sum_{j \neq i} \lambda_{ij}(\ell_{ji}) \frac{1}{w_j} + \mu \phi_i \leq 0
\]  
(17)

\[
\frac{\partial L_{SB}}{\partial s_i} = \left\{ \phi_i(1 - \pi) + \sum_{j \neq i} \lambda_{ij}(1 - \beta_i \pi) \right\} u'(d_i) + \left\{ \phi_i + \sum_{j \neq i} \lambda_{ij} \beta_i \right\} \pi H'(m_i) - \sum_{j \neq i} \lambda_{ji} \left\{ (1 - \beta_j \pi)u'(d_{ji}) + \beta_j \pi H'(m_{ji}) \right\} - \mu \phi_i = 0
\]  
(18)

\[
\frac{\partial L_{SB}}{\partial \theta_i} = \frac{1}{\delta} \left\{ \phi_i + \sum_{j \neq i} \lambda_{ij} \beta_i \right\} H'(m_i) - \frac{1}{\delta} \sum_{j \neq i} \lambda_{ji} \beta_j H'(m_{ji}) - \mu \phi_i \leq 0,
\]  
(19)

\[
\frac{\partial L_{SB}}{\partial D_i} = \varphi_i \pi \left\{ \phi_i + \sum_{j \neq i} \lambda_{ij} \beta_i \right\} H'(m_i) - \sum_{j \neq i} \lambda_{ji} \beta_j \varphi_j \pi H'(m_{ji}) - \mu \phi_i \varphi_i \pi \leq 0,
\]  
(20)

An interior solution for \( c_i \) requires

\[ \phi_i + \sum_{j \neq i} \lambda_{ij} - \sum_{j \neq i} \lambda_{ji} > 0. \]

First, we consider a setting without public provision of long-term care, i.e., we impose \( D_i = 0 \ \forall \ i \) as extra constraint. The remaining questions from our perspective then concern the tax treatment of private LTC insurance and of savings (which has a role of self-insurance). In a second step, we study the optimal level of public LTC. In particular, we examine under what conditions it is optimal for the government to provide public long-term care.

### 5 Second-best solution without public LTC provision

To simplify notation, define

\[
W^i_s = (1 - \beta_i \pi)u'(d_i) + \beta_i \pi H'(m_i) \quad \text{and} \quad W^{ji}_s = (1 - \beta_j \pi)u'(d_{ji}) + \beta_j \pi H'(m_{ji}).
\]  
(21)
Combining and rearranging the first-order conditions equations (16) to (19) yields the following marginal rates of substitution for a type- \(i\) individual

\[
\frac{v'(\ell_i)}{u'(c_i)} = \frac{w_i}{\phi_i + \sum_{j:i\neq j} \lambda_{ij} - \sum_{j:i\neq j} \lambda_{ji}} \frac{v'(\ell_i)w_i}{v'(\ell_i)w_j} \\
(1 - \beta_i \pi)u'(d_i) + \beta_i \pi H'(m_i) = \frac{\phi_i + \sum_{j:i\neq j} \lambda_{ij} - \sum_{j:i\neq j} \lambda_{ji}}{\phi_i 1 + \frac{T_i}{1+T_i^s} + \sum_{j:i\neq j} \lambda_{ij} - \sum_{j:i\neq j} \lambda_{ji} \frac{W_i^j}{W_j}} \\
H'(m_i) = \frac{\delta}{\beta_i \phi_i 1 + \frac{T_i}{1+T_i^s} + \sum_{j:i\neq j} \lambda_{ij} - \sum_{j:i\neq j} \lambda_{ji} \frac{W_i^j}{W_j}} \\
\text{where the “Pigouvian tax” on LTC insurance premiums is defined by (13) as } T_i = \beta_i - 1 \leq 0. \text{ This is the level of subsidy required to correct for individuals myopia. We have seen in the previous section that in the first-best, no tax on savings is required. Once private LTC insurance was at the appropriate level, the saving decision would spontaneously by optimal (even for myopic individuals). This is no longer true in the second-best and we can redefine the “Pigouvian tax” on savings as that which corrects for myopia and is determined by}

\[
1 + T_i^s = \frac{(1 - \beta_i \pi)u'(d_i) + \beta_i \pi H'(m_i)}{(1 - \pi)u'(d_i) + \pi H'(m_i)}.
\]

Observe when \(u'(d_i) = H'(m_i) \) (i.e., there is full insurance) we have \(T_i^s = 0\) which is consistent with our first-best result. However when there is underinsurance \((u'(d_i) < H'(m_i))\), we have \(T_i^s < 0\) and self-insurance is subsidized to compensate for the effect of myopia. Conversely when there is overinsurance a tax on savings is required \((T_i^s > 0)\) to correct for myopia.\(^7\)

When individuals make their consumption, labor and LTC insurance decisions, they equalize these marginal rates with their perceived (tax included) relative prices; see equations (10)–(12). Combining these expressions with equations (22)–(24) then allows us to study the implementing policy. The interesting marginal tax (or subsidy) rates from our perspective are the ones on savings and on private LTC insurance.

First, consider the marginal tax or subsidy on private LTC insurance. Combining

\(^7\)The numerical examples presented in Section 7 show that both cases are possible in the second-best.
equations (12) and (24) yields

\[ 1 + T_i^0 = \frac{\phi_i + \frac{1}{1 + T_{iP}} + \sum_{j: i \neq j} \lambda_{ij} - \sum_{j: i \neq j} \lambda_{ji} \beta_j H'(m_{ji})}{\phi_i + \sum_{j: i \neq j} \lambda_{ij} - \sum_{j: i \neq j} \lambda_{ji}}. \] (26)

Observe that in the full information solution, \( i.e. \) when \( \lambda_{ij} = \lambda_{ji} = 0 \, \forall i, j \), expression (26) yields a Pigouvian subsidy which is positive (and equal to \( 1 - \beta_i \)) for all individuals who undervalue their risk of dependency in old age (see eq. (13)). However, when at least one self-selection constraint is binding, the optimal solution is no longer first-best.

Consider the “top” individual, that is, a person whom nobody mimics implying \( \lambda_{ji} = 0 \, \forall j \), but \( \lambda_{ij} > 0 \) for at least one \( j \). For such a person (26) can be rewritten as

\[ 1 + T_i^0 = \frac{1 + \sum_{j: i \neq j} \lambda_{ij} \phi_i}{1 + T_{iP} + \sum_{j: i \neq j} \lambda_{ji} \phi_i}. \] (27)

If this person is rational and correctly perceives his probability of dependency (so that \( T_{iP} = 0 \)) he faces no distortion because we have \( T_i^0 = 0 \). If, however, this person undervalues the risk of LTC (so that \( T_{iP} < 0 \)), equation (27) implies \( (1 + T_i^0) > (1 + T_{iP}) \) so that the subsidy on LTC is lower than the Pigouvian level. That is for myopic “top” individuals the famous “no distortion at the top” result no longer holds.\(^8\) An observation which is in line with Cremer et al. (1998, 2009) who have pointed out that first- and second-best levels of Pigouvian taxes typically differ. This is because the Pigouvian rule implicitly uses the marginal utility of income to determine the “monetary” (numeraire) equivalent of a utility variation.

Let us now turn to the general case, where individual-\( i \) is not (necessarily) the top individual. Intuitively, one would expect that \( 1 + T_i^0 < 1 \) implying a subsidy on long-term care expenses at the margin for all those individuals who undervalue their probability of becoming dependent when old. However, in the second-best a tax/subsidy on long-term care premiums also has incentive effects. This is shown by the numerator of

\(^8\)Using equation (27) it follows that \( (1 + T_i^0) > (1 + T_{iP}) \) is equivalent to

\[ \frac{1 + \sum_{j: i \neq j} \lambda_{ij} \phi_i}{1 + T_{iP} + \sum_{j: i \neq j} \lambda_{ji} \phi_i} > (1 + T_{iP}) \].

Cross multiplying and rearranging this expression shows that the inequality is implied by \( (1 + T_{iP}) < 1 \).
the LHS of equation (26) where one can think about the first term as the “Pigouvian” (paternalistic) element, while the other terms capture the incentive effects. These incentive effects arise because taxing or subsidizing LTC insurance is not neutral from a redistributive perspective. Observe that while labor supply is separable from the other goods, the Atkinson and Stiglitz result does not hold here because individuals differ in more than one dimensions. This is already true when there is no myopia (or when all individuals are equally myopic) because survival probabilities differ along with productivities. Differences in the degree of myopia introduces yet another dimension of heterogeneity. To get a more precise understanding of the structure of this incentive term and its interaction with the Pigouvian term, we can rearrange equation (26) to obtain the following condition

\[ 1 + T^P_i \geq 1 \iff \frac{(1 - \beta_i)}{\beta_i} \geq \sum_{j \neq i} \frac{\lambda_{ji}}{\phi_i} \left( \frac{\beta_j H'(m_{ji})}{\beta_i H'(m_i)} - 1 \right). \] (28)

The LHS of the second expression is equal to the Pigouvian subsidy (expressed as a tax rate), \(-T^P_\theta/(1 + T^P_\theta) \geq 0\). The RHS (of the second inequality) constrains a sum over all types \(j\), but the corresponding term is nonzero only when \(\lambda_{ji} > 0\), that is when the incentive constraint from type-\(j\) to type-\(i\) is binding. Further, the first term in brackets \((\beta_j H'(m_{ji})/\beta_i H'(m_i))\) represents the ratio of the mimicker’s and the mimicked individual’s marginal rate of substitution between consumption \(c_i\) and LTC expenses \(m_i\); recall that \(c_i\) is effectively observable so that \(c_{ji} = c_i\). When this term is larger than 1, a tax (downward distortion) on \(m_i\) relaxes a binding incentive constraint. Consequently, when the sum of the terms on the RHS is positive, the incentive term supports a tax on \(m_i\) and thus opposes the Pigouvian term. The sign of \(T^P_\theta\) then depends on which of these two terms is the most significant. On the other hand, when the incentive term is negative, both effects go in the same direction and we necessarily have a subsidy on \(m_i\).

When all agents have the same degree of myopia \((\beta_j = \beta_i)\) we have

\[ \frac{H'(m_{ji})}{H'(m_i)} \geq 1 \text{ if } \varphi_j \geq \varphi_i \text{ since } m_{ji} \leq m_i \text{ if } \varphi_j \geq \varphi_i. \] (29)

Thus, if incentive constraints are binding from type-\(h\) to type-\(l\) individuals, this expression is larger than one calling for taxation rather than subsidization of long-term
care insurance premiums. Intuitively this makes sense: compared to type- \( l \) individuals, type- \( h \) individuals put a higher weight on LTC expenses because their survival probability is larger. When the degree of myopia differs between individuals this effect is strengthened when the able tend to be less myopic; otherwise, it is weakened (and may be even reversed).

This discussion is quite general and remains effectively valid for any (finite) number of types. However, it also shows that to predict the sign of the tax/subsidy on LTC insurance one has to make assumption on the pattern of binding incentive constraints which, at this level of generality, is admittedly rather speculative. The assumptions can be better understood when we restrict ourselves to specific distributions of types; see Subsections 5.1–5.3. However, the only way to obtain a precise determination of the pattern of binding incentive constraints is to use simulations. This is what we’ll do in Section 7.

Before turning to these more specific settings let us consider the marginal tax (subsidy) on savings, or self-insurance. Combining equations (11) and (23) and rearranging yields

\[
1 + T^i_s = \frac{\phi_i}{1 + T^i_{sP}} + \frac{\sum_{j:i\neq j} \lambda_{ij} - \sum_{j:i\neq j} \lambda_{ji} W^i_{ji}}{\phi_i + \sum_{j:i\neq j} \lambda_{ij} - \sum_{j:i\neq j} \lambda_{ji}},
\]

where \( T^i_{sP} \) is defined by (25) while \( W^i_s \) and \( W^i_{ji} \) are determined by (21). The structure of this equation very much resembles that of (26). In particular we have Pigouvian and incentive effects which matter. The study of the different terms is, however, considerably more complicated.

As for LTC insurance the no distortion at the top result holds for rational but not for myopic “top” individuals. Specifically, if \( \lambda_{ji} = 0 \) \( \forall j \), but \( \lambda_{ij} > 0 \) for at least one \( j \) equation (30) can be rewritten as

\[
1 + T^i_s = \frac{\phi_i}{1 + T^i_{sP}} + \frac{\sum_{j:i\neq j} \lambda_{ij}}{\phi_i + \sum_{j:i\neq j} \lambda_{ji}},
\]

which is the counterpart to (27) for \( \theta_i \). If this person is fully insured (so that \( T^i_{sP} = 0 \)) he faces no distortion because we have \( T^i_s = 0 \). However, as soon as \( T^i_{sP} \neq 0 \) we will have \( T^i_s \neq T^i_{sP} \). The difficulty is that unlike for \( \theta \), we do not know the sign of the
Pigouvian tax on $s_i$. If the person is less than fully insured against the risk of LTC (so that $T_{sP}^{i} < 0$), equation (31) implies $(1 + T_{s}^{i}) > (1 + T_{sP}^{i})$ so that again the subsidy on LTC is lower than the Pigouvian level. When $T_{sP}^{i} > 0$, we have the opposite result.

For the other types, we can proceed as above and rearrange (30) to obtain the counterpart to (28)

$$1 + T_{s}^{i} \preceq 1 \iff \frac{T_{sP}^{i}}{1 + T_{sP}^{i}} \preceq \sum_{j:i \neq j} \lambda_{ji} \left( \frac{W_{s}^{ji}}{W_{s}^{j}} - 1 \right).$$

(32)

Once again, the LHS of the second part is the Pigouvian subsidy (rate) while the RHS is the incentive term, where $W_{s}^{ji}/W_{s}^{j}$ is the ratio of the mimicker’s and the mimicked individual’s marginal rate of substitution between consumption and saving. The expression is more complicated than that for $\theta_i$ because $s_i$ plays a double role, as consumption in the good state of nature and as self-insurance in case of dependency. When incentive constraints bind from the able to the less able, and when there is no heterogeneity in myopia, we can show that the incentive term is positive. To see this, note that

$$d_{ji} \preceq d_i \iff \varphi_j \preceq \varphi_i \implies \frac{u'(d_{ji})}{u'(d_i)} \preceq 1 \iff \varphi_j \preceq \varphi_i.$$  

With the definition in (21) and (29), we then have

$$\frac{W_{s}^{ji}}{W_{s}^{j}} \preceq 1 \text{ if } \varphi_j \preceq \varphi_i \text{ and } \beta_j = \beta_i.$$  

(33)

For the rest, not much can be said at this level of generality. With both the Pigouvian and the incentive term of ambiguous sign, both positive and negative tax rates on savings are possible (and this is confirmed by the numerical example below).\(^9\)

To gain more insight in the interplay of the Pigouvian tax/subsidy and the redistributive considerations, we study three different 2-type economies in the following subsections. The different configurations are depicted in Figure 2.

\(^9\)When type-$i$ is fully insured against the LTC risk so that $u'(d_i) = H'(m_i)$ we have $1 + T_{sP}^{i} = 1$. If we further assume that incentive constraints are binding according to decreasing ability, it follows from (21) that $W_{s}^{ji}/W_{s}^{j} > 1$ (for any $\beta_j$ and $\beta_i$) which in turn implies $1 + T_{s}^{i} > 1$. In words, a fully insured individual faces a tax on savings. For such an individual saving has no self-insurance role and the tax on savings arises for redistributive reasons.
5.1 No myopic agents-(I)

As a benchmark scenario, consider a society where all agents correctly perceive their dependency probability, i.e., \( \beta_h = 1 \). In other words, there are type-2 and 4 agents (case I in Figure 2). Assume that only the downward self-selection constraint is binding, that is, we want to redistribute to the low ability individuals. Then, type-4 individuals are the “top” agents implying type-2 individuals have no incentive to mimic type-4 individuals and \( \lambda_{24} = 0 \). In this case, the no distortion at the top condition applies for type-4 individuals: \( 1 + T_4^4 = 1 \) and \( 1 + T_\theta^4 = 1 \). For type-2 individuals, we have

\[
1 + T_\theta^2 = \frac{\phi_2 - \lambda_{42}}{\phi_2 - \lambda_{42} \frac{H'(m_{42})}{H'(m_{22})}} > 1 \quad \text{and} \quad 1 + T_s^2 = \frac{\phi_2 - \lambda_{42}}{\phi_2 - \lambda_{42} \frac{W_{42}^P}{W_{22}^P}} > 1.
\]

To establish (??) we use \( \varphi_4 > \varphi_2 \) which implies \( H'(m_{42})/H'(m_{22}) > 1 \) and \( W_{42}^P/W_{22}^P > 1 \) (see eqs. (29) and (33)). Consequently, low ability individuals face a positive marginal tax on both LTC insurance and savings (self-insurance). This does not come as a surprise: individuals of type-4 have a higher survival probability and thus put a higher weight on LTC than type-2 individuals. We then obtain the traditional result that the downward distortion on the low type relaxes an otherwise binding self-selection constraint. And since individuals are rational, no paternalistic considerations come in.
5.2 Perfect correlation between productivity and rationality-(II)

Assume now that productivity and rationality are perfectly positively correlated. In other words, the more able individuals are also the rational ones, while all the less able are myopic (there are only type-1 and 4 individuals). Continue to assume that only the downward incentive constraint is binding so that $\lambda_{14} = 0$, which implies $(1 + T^4_{s}) = 1$ and $(1 + T^4_{\theta}) = 1$ (zero marginal taxes for type 4 individuals). For type-1 individuals, conditions (30) and (26) can be rearranged to yield

$$1 + T^1_{\theta} = (1 + T^1_{\theta}^P) \frac{\phi_1 - \lambda_{41}}{\phi_1 - \lambda_{31} \beta_1 \frac{H(m_{41})}{H(m_{31})}} \lesssim 1 \quad \text{and} \quad 1 + T^1_{s} = \frac{\phi_1 - \lambda_{41}}{\phi_1 \frac{1}{1+T^1_{s}} - \lambda_{41} \frac{W^3}{W^3_s}} \lesssim 1.$$  

The tax on LTC insurance shows that the paternalistic and incentive effects now go in opposite directions. Consequently, for individual 1 the marginal tax on LTC insurance is larger than the Pigouvian tax (which, we know to be negative and equal to $(\beta_1 - 1)$). This means either that the subsidy is reduced for redistributive purposes or even that we switch to a positive tax. Whether a positive tax arises depends on the relative strength of the paternalistic and the redistributive terms. Results concerning the optimal tax/subsidy on savings are not clear-cut.

5.3 All individuals are myopic-(III)

Finally, assume that all individuals underestimate their dependence probability. There are only type-1 and type-3 individuals who differ solely in productivity and survival probability. Continue to assume that the incentive constraints are binding according to wage differentials, i.e., $\lambda_{31} > 0$. For expressions (30) and (26), we then have for type-1 and 3 individuals respectively

$$1 + T^3_{\theta} = (1 + T^3_{\theta}^P) \frac{\phi_3 + \lambda_{31}}{\phi_3 + \beta_3 \lambda_{31}} \lesssim 1 \quad \text{and} \quad 1 + T^3_{s} = \frac{\phi_3 + \lambda_{31}}{\phi_3 \frac{1}{1+T^3_{s}} + \lambda_{31}} \lesssim 1.$$  

The no distortion at the top result no longer holds. As the top agents are ignorant concerning their LTC needs in old age, the government subsidizes their LTC insurance.
This subsidy less than offsets their degree of myopia. Concerning type-1 individuals, results are not clear-cut as both the incentive and Pigouvian terms are at work.

6 Second-best solution with public LTC provision

So far we have restricted $D_i$ to be zero thus ruling out direct provision of LTC services or a system of public LTC insurance (financed by taxation and paying benefits in case of dependency). This assumption has no “direct” impact on the results concerning the tax treatment of $\theta$ and $s$. To be more precise, all the results obtained in Section 5 remain valid when $D_i$ is introduced, as long as the solution for $\theta_i$ remains interior.\footnote{Since individuals work only in the first period the solution for $s_i$ is necessarily interior (under some weak regularity condition on preferences).}

Now, assuming an interior solution for $\theta_i$ appears to be quite natural when the public alternative is ruled out.\footnote{Unless of course $C$ is very large in which case individuals may rely solely on self-insurance.} However, when both $\theta_i$ and $D_i$ are available the existence of an interior solution for all variables is less obvious. As a matter of fact, the first issue that has to be dealt with is whether there is a role for both instruments and, if not, which of the two instruments ought to be used. This is the question to which we now turn.

Combining (19) and (20) shows that to have an interior solution for both $\theta_i$ and $D_i$, the following expression must be zero

$$\delta \frac{\partial L^{SB}_{\theta_i}}{\partial \theta_i} - \frac{1}{\varphi_i} \frac{\partial L^{SB}}{\partial D_i} = \sum_{j \neq i} \lambda_{ji} \beta_j \left( \frac{\varphi_j}{\varphi_i} - 1 \right) H'(m_{ji}) - \mu \phi_i (\delta - 1) = 0. \quad (34)$$

The first term in this equation measures the incentive benefits of $\theta_i$, while the second term represents the cost associated with private insurance (converted into utility terms).

Consider first the case of an actuarially fair private LTC insurance market, $\delta = 1$. One could then be tempted to think that the two instruments would be equivalent (or redundant). Private insurance has no loading cost and since the income tax is optimized the financing of public insurance does not imply any deadweight loss.\footnote{See Jacobs (2010).} However this conjecture does not stand under closer scrutiny. To see this, assume for the time being that at least one incentive constraint toward type-$i$ is binding, i.e. $\lambda_{ji} > 0$ for some $j$. With the assumed pattern of binding incentive constraints (from higher pro-
ductivities to lower productivities and thus from high to low survival probabilities) the mimicker (type-$j$) has a higher survival probability than the mimicked (type-$i$), $\varphi_i < \varphi_j$, and from equation (34) follows immediately $D_i = 0$. In other words, no public LTC benefits should be provided to the mimicked individual. To understand this result, notice first of all that it is not due to a “cost of public funds” argument (which, with an optimal income tax is zero at the second-best solution). Instead, it arises for informational reasons because the two instruments have different impacts on the incentive constraints. By assuming that private insurance markets are actuarially fair, we implicitly assume that private insurance companies observe an individual’s survival probability (and longer lived individuals pay higher premiums). Public authorities, on the other hand, do not observe this survival probability and $D_i$ can be conditioned on the type only through self-selection. Consequently, the mimicker would be better off by just taking $D_i$ (in which case he is treated exactly like type-$i$) than by taking $\theta_i$, as he has to pay a higher price for private LTC insurance coverage. Public provision of LTC would thus only increase incentives for high survival probability types to mimic low survival probability types. In other words it would reinforce (rather than relax) a binding incentive constraint. Interestingly, the fact that some individuals may underestimate their dependence probability has absolutely no bearing on this result.

The argument presented so far applies for $D_i$ as long as there is at least one $\lambda_{ji} > 0$. If not, we are dealing with the “top” individual for which we obtain that private and public LTC are perfect substitutes (as long as private insurance is actuarially fair). In other words, public LTC provision to high types crowds out one-for-one their private LTC insurance coverage and yields no additional welfare gains. To sum up, we can state that with an actuarially fair private LTC insurance market (and an optimal income tax) social LTC insurance is not desirable. This result is not surprising as it is the counterpart to Rochet’s (1991) and confirms the conjecture made in the introduction. What is more surprising, though, is the fact that the presence of myopia does not affect

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13 Specifcally, a small increase in $D_i$ from zero could be financed by a uniform lump-sum tax at no efficiency cost.

14 Rochet (1991) has two parts. In the first part taxes and social insurance are linear and he shows that when the covariance between risk and ability is positive, social insurance is not desirable. However, the part which is closest to our analysis is the second one in which all instruments can be nonlinear. There he focuses on conditions which yield full insurance and does specifically look at the desirability of social insurance.
this result; see equation (34). Myopia comes in only through the Pigouvian term in the
tax/subsidy on \( i \). In other words myopia is taken care of by the marginal tax on \( i \), but
it does not justify social insurance. Even if agents are completely ignorant concerning
their future LTC needs, \( i.e. \beta_i = 0 \), this result remains unaffected. In this case, we need
a mandatory private insurance (rather than just a subsidy), but this is only a matter of
implementation.

So far, we have assumed that the private LTC insurance market offers fair contracts.
When private insurance implies loading costs, the results do of course change and in a
rather intuitive way. Expression (34) shows that when \( \delta \) is sufficiently large, the results
can be completely reversed and we obtain a corner solution with \( \theta_i = 0 \) and \( D_i > 0 \).
Finally, for intermediate level of \( \delta \) we can have an interior solution with both private
and public insurance (\( \theta_i > 0 \) and \( D_i > 0 \)) which strikes a compromise between the costs
and benefits implied by each system (loading factor vs. incentive effect). To be precise,
at this level of generality we can only say that intermediate levels of \( \delta \) are compatible
with an interior solution. The numerical examples presented in the next section show
that such an interior solution can effectively arise.

Before turning to these simulations let us briefly examine the rule for the determ-
ination of \( D_i \) in those cases where the solution is interior, that is \( D_i > 0 \) and \( \theta_i = 0 \).
Combining the first order conditions (16) and (20) yields the following marginal rate of
substitution for a type-\( i \) individual

\[
\frac{H'(m_i)}{u'(c_i)} = \frac{1}{\beta_i} \frac{\phi_i + \sum_{j:i \neq j} \lambda_{ij} - \sum_{j:i \neq j} \lambda_{ji}}{1 + T^P_D + \sum_{j:i \neq j} \lambda_{ij} - \sum_{j:i \neq j} \lambda_{ji} \frac{\partial \phi_j}{\partial \phi_i} H'(m_{ji})},
\]

(35)

where \( T^P_D = T^P_\theta = \beta_i - 1 \) is the Pigouvian tax. We can think of this marginal rate
of substitution as defining an *implicit* marginal tax on \( D_i \), which represents the wedge
between the first- and second best tradeoff. We define this implicit tax \( T^i_D \) as the
marginal tax that decentralizes the second best level of \( D_i \) if public LTC insurance is
otherwise sold at a fair price. Formally, this is given by (12) with \( \delta = 1 \). Combining
this expression with (35) yields

\[
\frac{1}{1 + T^i_D} = \frac{\phi_i + \sum_{j:i \neq j} \lambda_{ij} - \sum_{j:i \neq j} \lambda_{ji} \frac{\partial \phi_j}{\partial \phi_i} H'(m_{ji})}{\phi_i + \sum_{j:i \neq j} \lambda_{ij} - \sum_{j:i \neq j} \lambda_{ji}}
\]

23
Compared to equation (26), the incentive term now includes the ratio $\varphi_j/\varphi_i$ which calls for a higher tax rate if incentive constraints are binding from high to low wage types. This is in line with our discussion above: with only public provision of LTC it becomes less costly to mimic other agents and thus the incentives to do so increase. Consequently, with public provision it is more likely than with private insurance that the incentive effect outweighs the Pigouvian effect so that we end up with a positive tax on LTC. And even when the Pigouvian term dominates, the subsidy will be smaller than with private insurance.\footnote{For a given pattern of binding incentive constraints.}

7 Numerical examples

The results presented so far are fairly general but most of the discussions rely on assumptions on the pattern of binding incentive constraints. We now turn to numerical simulations for which we can effectively determine which incentive constraints are binding. These results are admittedly all for a very specific setting which we cannot claim to be fitted exactly to reality. Still they are useful in several respects. First, they show that the assumptions we make in the general setting are not empty.\footnote{In other words, we do not make assumptions which are inconsistent and can never be satisfied.} Second, they show that the expressions that appeared to be ambiguous in the general model, can effectively produce positive and negative results. Third, they show that the various interior and corner solution alluded to in Section 6 can effectively arise.

In addition, we use the numerical model to study some comparative statics properties. Specifically, we consider variations in the degree of myopia, in the loading factor, and in the proportion of myopic individuals. Due to the restrictive character of the setting our findings have to be qualified accordingly and to be seen merely as an illustration. Simulations are based on the 4-type setting depicted in Figure 1. The proportion of individuals with $\beta_h$ in wage category $w_k$ ($k = l, h$) is denoted $\kappa_k$. With $\beta_h = 1$ this corresponds to the proportion of rational individuals. Utilities and labor disutility are given by

$$u(x_{ij}) = -e^{-x_{ij}} \quad H(m_{ij}) = -e^{-m_{ij} - L} \quad \text{and} \quad v(\ell_{ij}) = \frac{\ell_{ij}^2}{2}$$

The basic parameter values are given in Table 1. With these values, we expect that the
Table 1: Basic parameters

<table>
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<th>$w_l = 2$</th>
<th>$w_h = 5$</th>
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<td>$\varphi_h = 1$</td>
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<td>$0.6\kappa_l$</td>
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</tr>
<tr>
<td>$\beta_h = 0.2$</td>
<td>$0.4(1 - \kappa_h)$</td>
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difference in productivity dominates that in survival probabilities so that the utilitarian solution implies redistribution to type-$l$ individuals (and this conjecture is confirmed by our results). Additionally, we assume $\pi = 0.4$, $L = 3$ and $\delta = 1$ for our benchmark scenario.\(^{17}\)

Table 2 shows the *laissez-faire* solution. As we already know from Section 2.2 even though private LTC care insurance is actuarially fair, myopic agents are not fully insured. In the first-best solution consumption levels across types and time are equalized, \textit{i.e.}, $c_i = d_i = m_i = 1.28$ \forall $i$. Income for type-$l$ individuals is given by $y_1 = y_2 = 1.12$ and for type-$h$ individuals by $y_3 = y_4 = 6.97$.

<table>
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<tr>
<th>Type</th>
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<th>$y_i$</th>
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### 7.1 No public LTC provision

For the remainder of this section, we assume that the share of rational agents is equal among type-$l$ and type-$h$ individuals implying $\kappa_l = \kappa_h$. For the time being we set $D_i = 0$. The second-best solution for the benchmark scenario is reported in Table 3. There turns out to be pooling of individuals with $w_l$; they have the same consumption allocation across time and states irrespective of their level of $\beta_i$. Compared to the *laissez-faire* solution all individuals save less, but demand more private LTC insurance. Only type-4 agents remain fully insured. Due to their high degree of ignorance concerning

\(^{17}\)Brown and Finkelstein (2009) show for the U.S. that the probability of needing long-term care for a 65 year old is between 35 and 50 per cent.
the possibility of dependence, type-3 agents are less than perfectly insured. Type-$l$ individuals are even ‘overinsured’, that is if disabled they have a higher consumption than if they stay healthy in old age. This result can be explained by inspecting the binding incentive constraints (ICs). These are mainly from type-$h$ to type-$l$ individuals.

Inducing type-$l$ individuals to demand more LTC insurance reduces the incentives of type-$h$ individuals to mimic as they have to pay a higher price for LTC insurance.

Now assume $\beta_l = 0.6$ so that myopic agents are less ignorant concerning their possible dependency. Compared to our benchmark scenario, type-$l$ individuals now demand less LTC insurance but self-insure more extensively via private savings. The opposite is true for type-3 agents. Again, this can be explained by the binding ICs. As Table 3 shows type-4 individuals no longer have an incentive to mimic type-$l$ individuals if these are less ignorant concerning their LTC needs. Thus, inducing type-$l$ agents to demand more LTC insurance becomes a less efficient policy.

If private LTC insurance comes with a positive loading factor even type-4 individuals are no longer fully insured against the LTC risk. Specifically, we observe the following changes. The higher costs for LTC insurance reduce the individuals’ consumption while

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Table 3: No public LTC provision: $\kappa_l = \kappa_h = 0.5$
working and in the state of disability in old-age. Their labor supply increases to finance the additional resources required for a possible frailty in old-age. Additionally, a lower tax on private savings enhances the attractiveness for all agents to self-insure via a higher wealth accumulation.

Figures 3 (a)-(g) illustrate how changes in the share of rational agents affect the optimal tax on LTC insurance and savings. In all scenarios the optimal tax on LTC insurance is an increasing function of the share of rational agents. In our benchmark case—Figure 3 (a)—myopic individuals’ LTC insurance is always subsidized. Whether type-2 individuals are taxed or subsidized depends on the share of rational agents. They are taxed for larger shares of rational agents to relax the incentive constraint from type-4 to type-2 agents. A decreasing ignorance concerning LTC in old age reduces the paternalistic objective and thus increases the tax on LTC insurance for all agents (Figure 3 (b)). For a large share of rational agents, also type-3 agents may now face a tax on their private LTC insurance premium. A positive loading factor (Figure 3 (c)) hardly changes the optimal tax on LTC insurance, but reduces the tax on private savings in order to increase all agents’ self-insurance (see Figure 3 (g)). As Figures 3 (b), (e) and (g) illustrate whether an individual faces a tax or subsidy on private savings depends less on the degree of myopia but more on the income type. In all scenarios type-\(l\) individuals face a tax on their savings in order to relax the incentive constraint from type-\(h\) to type-\(l\) individuals. The optimal tax on type-3 agents’ savings very much resembles their tax on LTC insurance. Taking a closer look at the axis of ordinates reveals that the magnitude of the tax on savings is much lower than on LTC insurance, that is compared to the tax on LTC insurance it only plays a minor role in the optimal tax design.

7.2 With public LTC provision

Now assume the government has public long-term care as an additional instrument to redistribute between individuals (\(i.e., \) we no longer impose \(D_i = 0\)). As the binding ICs are those from type-\(h\) to type-\(l\) agents for a given \(\beta\), the utilitarian solution implies redistribution towards low income agents. We know from Section 6 that in this case no public LTC provision is optimal unless \(\delta > 1\). But, a LTC insurance premium that exhibits a mark-up substantially above expected benefits seems to the more realistic
Figure 3: Marginal tax/subsidy on LTC insurance and savings without public LTC.
Table 4: With public LTC provision: $\kappa_l = \kappa_h = 0.5$

<table>
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</tbody>
</table>

scenario (see Brown and Finkelstein, 2007). Table 4 reports the optimal provision of public LTC and each individual’s consumption for loading factors equal to $\delta = 1.05$ and $\delta = 1.2$.

When $\delta = 1.05$ type-$h$ individuals receive solely public LTC. As they are not mimicked by other agents, the negative incentive effects of public LTC care are nil and only the positive efficiency effect prevails. Public provision of LTC to type-$l$ individuals, by contrast, comes along with a higher attractiveness to get mimicked by type-$h$ agents. That is, there is a trade-off between the incentive and the efficiency effect implying only a partial provision of public LTC to type-$l$ individuals. Compared to the scenario with no public LTC provision, type-$l$ individuals are prepared worse for a possible dependency during old-age while the opposite is true for type-$h$ agents.

If the loading on private LTC insurance increases to $\delta = 1.2$, the negative incentive effects of public LTC are offset by its efficiency gains. All individuals rely solely on public LTC. For all individuals but type-3 agents the public system even provides more than full insurance against the LTC risk.

8 Conclusion

This paper has examined whether myopia (misperception of the LTC risk) and private insurance market loading costs provide a possible rationale for social LTC insurance and/or for the subsidization of private insurance. It has employed a model wherein
individuals differ in three unobservable characteristics: level of productivity, survival probability and degree of ignorance concerning the risk of LTC (the former two being perfectly positively correlated). It has shown that to decentralize a first-best allocation, LTC insurance premiums of the myopic agents can be subsidized (at a “Pigouvian” rate) or alternatively we can have public provision of the appropriate level of LTC. Under full information (and actuarially fair private insurance) these two instruments are perfectly equivalent, but we need at least one of them. When private insurance is subject to a loading factor (first-best) social LTC provision dominates.

The support for the considered LTC policy instruments has appeared to be more
mitigated in a second-best setting. When social LTC provision is restricted to zero, a myopic agent’s tax on private LTC insurance premiums involves a tradeoff between paternalistic and redistributive (incentive) considerations and we may have a tax as well as a subsidy on private LTC insurance. Interestingly, savings (which goes untaxed in the first-best) is also subject to (positive or negative) taxation in the second-best and the expression for the tax rates includes also a Pigouvian and a redistributive term. Social LTC provision is never second-best optimal when private insurance markets are fair (irrespective of the degree of the proportion of myopic individuals and their degree of misperception). At the other extreme, when the loading factor in the private sector is sufficiently high, private coverage is completely crowded out by public provision. For intermediate levels of the loading factors, the solution relies on both types of insurance.

To sum up, myopia (irrespective of its extent) is not per se sufficient to justify public LTC provision. Furthermore, misperceptions must be sufficiently severe or widespread to obtain a subsidy on private insurance. Otherwise, the incentive term may well dominate the Pigouvian term so that even myopic individuals may face a (marginal) tax on private LTC insurance.

The main results were obtained for a rather general setting with an arbitrary (but finite) number of types. In particular, our expressions are valid whatever the pattern of binding incentive constraints (which is impossible to assess at this level of generality). For the interpretation we have argued that under “reasonable” assumptions one can expect the incentive term to favor a tax (rather than a subsidy) on private LTC insurance. To shed more light on this issue we have taken different avenues. First, we presented the analytical solution for some special cases. Second, we have presented a number of numerical examples for a four type setting. These additional investigations show that the assumptions we make in the general section are not empty (or mutually inconsistent). They also confirm that the expression for the optimal tax on LTC insurance do not just appear to be ambiguous but that one can effectively have positive and negative taxes as well as corner and interior solutions for social and private LTC insurance.
References


