Macroeconomic Fluctuations with HANK & SAM: an Analytical Approach*

By Morten O. Ravn[1,2,3] and Vincent Sterk[1,2,3]

University College London[1], Centre for Macronomics[2],

and Centre for Economic Policy Research[3]

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Abstract

New Keynesian models with unemployment and incomplete markets are rapidly becoming a new workhorse model in macroeconomics. Such models typically require heavy computational methods which may obscure intuition and overlook equilibria. We present a tractable version which can be characterized analytically. Our results highlight that – due the interaction between incomplete markets, sticky prices and endogenous unemployment risk – productivity shocks may have radically different effects than in traditional NK models, that the Taylor principle may fail, and that pessimistic beliefs may be self-fulfilling and move the economy into temporary episodes of low demand and high unemployment, as well as into a long-lasting "unemployment trap". At the Zero Lower Bound, the presence of endogenous unemployment risk can create inflation and overturn paradoxical properties of the model. We further study financial asset prices and show that non-negligible risk premia emerge.

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*Ravn: Department of Economics, University College London, Drayton house, 30 Gordon Street, London WC1E 6BT, UK. m.ravn@ucl.ac.uk; Sterk: Department of Economics, University College London, Drayton house, 30 Gordon Street, London WC1E 6BT, UK. v.sterk@ucl.ac.uk. We are grateful for comments from seminar participants at the Bank of England, Bocconi University, UCL and at UCLA. Financial support from the ADEMU (H2020, No. 649396) project and from the ESRC Centre for Macroeconomics is gratefully acknowledged. Chanwoo Kim provided superb research assistance.

1 Introduction

The New Keynesian (NK) model has gained widespread use both in academic research and in policy circles. The crux of the model is that nominal frictions induce inefficient fluctuations in the economy that monetary and fiscal policies can be designed to address. Clarida, Galí and Gertler (1999) provide a summary of the model's key insights, based on a highly intuitive, three-equation version. Paradoxically, however, these insights do not pertain to unemployment and distributional issues, two central aspects of many policy discussions.

Recently, a new generation of NK models that addresses these deficiencies has emerged. For example, Gertler and Trigari (2009), Blanchard and Galí (2010), Ravenna and Walsh (2011), and Christiano, Eichenbaum and Trabandt (2016) introduce unemployment by incorporating Search and Matching (SAM) frictions in the labor market. Others have introduced financial market incompleteness, generating inequality in income, wealth and consumption. Kaplan, Moll and Violante (2016) have dubbed such models Heterogeneous Agents New Keynesian (HANK) models.[1] By giving centre stage to HANK and SAM, the new models mark a clear break with the traditional "representative agent" assumption, offer a rich array of cross-sectional predictions, and allow inequality across households to matter in models of aggregate fluctuations.

This paper complements the new vintage of NK models with an analytically tractable counterpart that is as simple as the model in Clarida, Galí and Gertler (2000), but nonetheless features search and matching frictions in the Diamond-Mortensen-Pissarides tradition, and incomplete markets à la Bewley, Huggett and Aiyagari. Our main purpose is to revisit core qualitative results highlighted in the New Keynesian literature and to understand how these results are affected by the interactions between HANK and SAM. We demonstrate profound implications for equilibrium determinacy, the long run equilibrium properties and selection, the response of the economy to shocks, the implications of the Zero Lower Bound (ZLB) on the nominal interest rate, and the determination of asset prices.

The model's tractability derives from a special assumption on households' borrowing limit, see also Krusell, Smith and Mukoyama (2009), Ravn and Sterk (2012) and Werning (2015).[2] The

[1] Papers in this new vein include Auclert (2015), Bayer, Pham-Dao, Luetticke and Tjaden (2015), Beaudry, Galizia and Portier (2014), Berger, Dew-Becker, Schmidt and Takahasi (2016), Challe, Matheron, Ragot and Rubio-Ramirez (2016), den Haan, Rendahl and Riegler (2016), Heathcote and Perri (2015), Gornemann, Kuester and Nakajima (2016), Guerrieri and Lorenzoni (2016), Kekre (2016), Luetticke (2015), McKay, Nakamura and Steinsson (2016), McKay and Reis (2016a,b), and Ravn and Sterk (2012).

[2] Krusell, Smith and Mukoyama (2009) study asset pricing in an endowment economy without nominal rigidities. Werning (2015) derives an aggregate Euler equation under incomplete markets, but does not explicitly model nominal rigidities or search and matching frictions. Ravn and Sterk (2012) study labor market shocks

assumption gives rise to an equilibrium with three groups of households: borrowing-constrained unemployed households, unconstrained but asset-poor employed households, and asset-rich but liquidity-constrained households.[3,4] This assumption, which limits but not eliminates household heterogeneity, allows us to address two limitations of the new generation of NK models. First, they generally require numerical methods along the lines of Krusell and Smith (1998), which makes it hard to understand the economic mechanisms in a clear fashion. Second, a serious issue is the possible emergence of equilibrium multiplicity which may be overlooked in numerical procedures. This possibility is not a mere technical artefact as fluctuations driven by "animal spirits" can arise naturally under incomplete markets and endogenous employment risk.[5]

We demonstrate that the equilibrium outcomes are shaped by the interaction between two endogenous wedges. The first is a standard sticky-price wedge in the labor demand equation (also called the "NK Phillips Curve"). The second wedge appears in the Euler equation, and results from financial market incompleteness. Werning (2015) highlights the emergence of this wedge in an analytically aggregated Euler equation, but does not model explicitly how the wedge is determined in equilibrium. We demonstrate how the incomplete markets wedge, in a model with search and matching frictions, is pinned down as a function of tightness of the labor market, how it interacts with the sticky-price wedge, and how it is affected by policy. This labor market wedge produces endogenous unemployment risk which feeds back though price setting and savings decisions to produce a powerful amplification mechanism. We also provide a micro-foundation for exogenous discount factor shocks that are often introduced in NK models, either to better fit the data (see e.g. Smets and Wouters, 2007) or to drive down the interest rate to a point at which the ZLB becomes binding (see e.g. Christiano, Eichenbaum and Evans, 2013).

We present six sets of results. The first concerns the steady-state properties of the model. As in the basic NK model there is an "intended" steady state as well as an unintended "liquidity

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using numerical simulations.

[3] See Kaplan, Violante and Weidner (2014) for empirical evidence on the asset distribution across households, emphasizing the empirical importance of households with large wealth but few liquid assets. In our model, the wealthy households endogenously face a low degree of consumption risk, which weakens their desire to accumulate bonds for precautionary reasons, relative to the poor households. As a result, the wealthy households are unwilling to invest in bonds at prevailing market interest rates, causing them to be liquidity-constrained in equilibrium.

[4] Like Clarida, Galí and Gertler (1999), we abstract from physical capital for reasons of tractability. Our model, however, does have a form of investment, namely investment in vacancies. In an extension of the model with capital, the precautionary savings effects would create only weak spillovers, or no spillovers at all, to capital investment, for the precise same reason that in the present model these effects do not spill over to vacancy investment: the owners of the firms are rich, which shields them from idiosyncratic risk.

[5] Intuitively, a wave of pessimism among households about their employment prospects could be self-fulfilling as the increased desire to build precautionary savings reduces aggregate demand, causing firms to hire fewer workers when prices are sticky and stabilization policy is insufficiently responsive.

trap". In the latter steady state, the ZLB binds and output is relatively low, as in Benhabib, Schmitt-Grohé and Uribe (2001, 2002). Unlike the standard NK model, however, our model may have a third steady state, which we label the "unemployment trap", in which aggregate demand is depressed to a level at which it is no longer profitable for firms to invest in vacancies, and in which inflation is moderately smaller than in the intended steady-state. In this equilibrium, hiring declines to a minimum, which perpetuates high job uncertainty and hence low demand.[6]

Next, we study local determinacy properties, exploring the scope for belief-driven dynamics around steady states. We first present an analytical determinacy condition for the intended steady state. We show that local indeterminacy can arise even when the "Taylor Principle" is satisfied (i.e. when the interest rate rule coefficient on inflation is larger than one, see e.g. Woodford, 2003, Chapter 2). This result derives from the presence of the endogenous incomplete markets wedge, and depends crucially on its interaction with the sticky-price wedge. Additionally, we show that the unemployment trap is determinate under a standard rule which responds more than one-for-one to inflation. Around this steady state, the monetary policy rule determines the rate of inflation, but has no grip on unemployment.

Our third set of results concerns the responses to fundamental and non-fundamental shocks. We present an analytical formula for the local response to a productivity shock around the intended steady state and show that the presence of incomplete markets can create significant amplification, see also Ravn and Sterk (2012) for numerical results. When the steady state is locally indeterminate, pessimistic belief shocks generate joint declines in employment, inflation and the real interest rate. The persistence of the effects is endogenously determined, and is maximized at degrees of price stickiness and market incompleteness that are just strong enough to generate local indeterminacy, but are otherwise relatively moderate.

Fourth, we revisit the role of the ZLB. We show that, due to the interaction sticky prices and incomplete markets and in contrast to representative-agent NK models, a negative productivity shock may bring the nominal interest rate to the ZLB. Further we show that ZLB episodes are not necessarily deflationary under incomplete markets. This happens as the real interest rate declines when unemployment increases, due to a heightened demand for precautionary savings. At the ZLB, a decline in the real interest rate implies an increase in inflation via the Fisher relation.

Additionally, we revisit "paradoxes" that arise in the representative-agent NK model when

[6]Kaplan and Menzio (2016) and Schaal and Taschereau-Duchoumel (2016) analyze multiplicity of equilibria and steady states in models with unemployment and demand externalities, but abstract from sticky prices and precautionary savings effects.

the ZLB binds, see e.g. Eggertsson (2010), Eggertsson and Krugman (2012) and Werning (2012). Specifically, we show that the precautionary savings mechanism in our model can overturn the paradox that, at the ZLB, positive productivity shocks may be contractionary, as well as the paradox that greater price flexibility may lead to larger drops in output. Both paradoxes arise from the fact that, at the ZLB, a transitory decline in inflation increases the real interest rate temporarily. In a representative agent model, this can only be consistent with a macroeconomic contraction, reducing household consumption and hence weakening the desire to save. With incomplete markets, however, an increase in the real rate can be consistent with a macroeconomic expansion, since the desire to build precautionary savings is dampened as the economy expands.

Fifth, we study the determination of risk premia. There is little existing research on this topic within the context of the NK model, since under complete markets the model does not generate first-order risk premia. At the same time, monetary policy is widely believed to have a large impact on financial markets. We show that under incomplete markets, the NK model can generate substantial risk premia and we provide an analytical formula for their magnitudes. The formula reveals a close connection between risk premia, the business cycle, and monetary policy. This results from the fact that idiosyncratic unemployment risk co-moves negatively with aggregate demand, causing households to dislike risky assets which pay off relatively little after an adverse shock hits the macro economy. Monetary policy has a dual effect on risk premia, since more stable fluctuations in aggregate demand reduce both fluctuations in asset payoffs and fluctuations in households' stochastic discount factors.

Finally, we propose a simple way to confront the model with the data. Clearly, the purpose of our study is qualitative in nature rather than quantitative. Nonetheless, it turns out that the model has a key prediction that can be checked directly in the data, which distinguishes the model from a representative-agent counterpart. Specifically, our model predicts that the real interest rate declines during times when the labor market becomes less tight, which increases unemployment risk and strengthens the precautionary savings motive. Under complete markets, the real interest rate *increases* when tightness of the labor market weakens, as income declines temporarily, encouraging households to save *less*. We show that in the data, there is a striking, positive co-movement between the real interest rate and the vacancy-unemployment ratio, providing direct support for the precautionary savings mechanism. Further, we show how the observed variances of the real interest rate and the vacancy-unemployment ratio can be used to discipline the values of the model parameters controlling the strength of the precautionary savings effects.

2 The Model

We construct a model which combines nominal rigidities in price setting as in the NK tradition, labor market matching frictions in the Diamond-Mortensen-Pissarides (DMP) tradition, and incomplete asset markets in the Aiyagari-Bewley tradition. The economy is made up of households who consume and work, firms which produce output, and a monetary authority in charge of the nominal interest rate. We allow for both aggregate and idiosyncratic uncertainty and assume lack of household insurance against idiosyncratic income risk.

2.1 Preferences and Technologies

Preferences: There is a continuum of mass 1 of infinitely lived single-member households indexed by $i \in (0,1)$. Households consume goods, $c_{i,s}$, have disutility of work and maximize the expected discounted present value of their utility streams:

$$\mathcal{V}_{i,t} = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{c_{i,s}^{1-\mu} - 1}{1-\mu} - \zeta n_{i,s} \right), \tag{1}$$

where $\mathbb{E}_t x_s = E\left(x_s | I_t\right)$ is the date t conditional expectation of x_s, $\beta \in (0,1)$ the subjective discount factor, $\mu > 0$ the measure of relative risk aversion, $n_{i,s}$ the household's employment status, and $\zeta > 0$ is a parameter that measures the disutility of market work. An individual household is either employed and works full-time or does not work at all:

$$n_{i,s} = \begin{cases} 0 \text{ if not employed at date } s \\ 1 \text{ if employed at date } s \end{cases}. \tag{2}$$

The consumption level of an individual household is a constant-elasticity-of-substitution aggregator of a basket of consumption goods, c_i^j:

$$c_{i,s} = \left(\int_j \left(c_{i,s}^j\right)^{1-1/\gamma} dj \right)^{1/(1-1/\gamma)}, \tag{3}$$

where $\gamma > 1$ is the elasticity of substitution between goods varieties. Workers who are not employed produce ϑ units of the aggregate consumption good at home.

Households decide on consumption, savings, on the financial portfolio, and on whether or not to participate in the labor force. A household not in the labor force cannot search for jobs in the market. Households who stand to lose on the net from employment declare themselves out of the labor force. We discuss the savings and portfolio problems later.

Production technology: Market goods are produced by a continuum of monopolistically competitive firms, indexed by $j \in (0, M)$, that each supply a differentiated good. The technology is:

$$y_{j,s} = \exp(A_s)\, n_{j,s}, \tag{4}$$

where $y_{j,s}$ is firms j's output and $n_{j,s}$ its employment. A_s is an *aggregate* stochastic productivity shock which follows a first-order autoregressive process:

$$A_s = \rho A_{s-1} + \sigma_A \varepsilon_s^A, \tag{5}$$

where $\rho \in (-1, 1)$, $\sigma_A > 0$ and $\varepsilon_s^A \sim \mathcal{N}(0, 1)$.

The law of motion of employment of firm j is:

$$n_{j,s} = (1 - \omega)\, n_{j,s-1} + h_{j,s}, \tag{6}$$

where ω is a constant employment separation rate and $h_{j,s}$ denotes hiring by firm j. Firms hire workers by posting and fill each posted job vacancy with probability q_s. We take firms to be sufficiently large that q_s is also the fraction of vacancies that are filled.[7] Thus, the total number of vacancies posted by firm j is given by $h_{j,s}/q_s$.

Matching technology: Agents receive information about current productivity shocks at the beginning of each period. Existing worker-firm relationships are resolved at the end of the period and new ones are formed at the beginning of the next period. Job separations are exogenous and affect existing hires randomly so that employees perceive ω to be the risk that they lose their current job.

New hires are produced by a matching function which relates the measure of newly formed worker-firm matches to the aggregate measures of vacancies, v_s, and job searchers, e_s, as:

$$\mathbf{M}(e_s, v_s) = \psi e_s^{\alpha} v_s^{1-\alpha}, \tag{7}$$

where $\psi > 0$ indicates match efficiency, $\alpha \in (0, 1)$, and $v_s = \int_j (v_{j,s} + \widetilde{v})\, dj$ is the aggregate measure of vacancies. Here, $v_{j,s}$ denotes the number of "formal" vacancies posted by firm j, which

[7] This is useful because we will later assume symmetry across firms and the large firm assumption avoids having to consider that the number of vacancies filled are stochastic.

come at a flow cost $\kappa > 0$ per unit. Further, $\widetilde{v} \geq 0$ is a fixed amount of "informal" vacancies, available to each firm, which come at no cost. The idea behind the latter type of vacancies is that even without devoting any resources to recruitment, firms would be able to hire some workers via word-of-mouth channels. The idea that not all hiring requires costly investments on the firm side is in line with empirical findings of Davis, Haltiwanger and Faberman (2013), who report that a substantial fraction of hiring takes place at establishments with no reported vacancies.[8]

The job filling probability, q_s, and the job finding rate, η_s, i.e. the probability that a jobless worker finds a new employer, depend on market tightness, $\theta_s \equiv \frac{v_s}{e_s}$, as:

$$q_s = \frac{\mathbf{M}(e_s, v_s)}{v_s} = \psi\theta_s^{-\alpha}, \tag{8}$$

$$\eta_s = \frac{\mathbf{M}(e_s, v_s)}{e_s} = \psi\theta_s^{1-\alpha}, \tag{9}$$

Note that the job filling rate and the job finding rate are related as $q_s = \psi^{\frac{1}{1-\alpha}}\eta_s^{\frac{\alpha}{\alpha-1}}$. It turns out that ψ and κ enter the model equations in a way that is observationally equivalent for our purpose. Hence we normalize ψ to one from now on.

2.2 Price and Wage Setting

Prices: Firms set prices of their products, $P_{j,s}$, subject to a quadratic price adjustment cost as in Rotemberg (1982). The extent of nominal rigidities in price setting is parameterized by $\phi \geq 0$ which determines the size of the price adjustment costs. Let w_s denote the average real wage, y_s aggregate output, and P_s be the aggregate price level. We anticipate that in equilibrium wages are the same for all workers and hence exclude worker- and firm specific indices for the wage. Firms maximize:

$$\mathbb{E}_t \sum_{s=t}^{\infty} \Lambda_{j,t,t+s} \left[\frac{P_{j,s}}{P_s} y_{j,s} - w_s n_{j,s} - \kappa v_{j,s} - \frac{\phi}{2} \left(\frac{P_{j,s} - P_{j,s-1}}{P_{j,s-1}} \right)^2 y_s \right], \tag{10}$$

[8] In much of the analysis, the informal vacancies play no role since in most equilibria new workers are –at the margin– hired via costly vacancies. However, we will show that there may also be equilibria in which firms are unwilling to invest any resources in vacancies. In those cases, the informal vacancies become relevant as they avoid a complete collapse of employment.

subject to (6) as well as to a demand constraint which derives from the consumers' decision problems:

$$y_{j,s} = \left(\frac{P_{j,s}}{P_s}\right)^{-\gamma} y_s, \tag{11}$$

where $y_s = \int_j y_{j,s} dj$ denotes aggregate output and $\Lambda_{j,t,t+s}$ is the discount factor of the firm's owners (discussed below). We also impose that investment in formal vacancies cannot be negative, i.e.

$$v_{j,s} \geq 0. \tag{12}$$

Real marginal costs is the sum of the wage and hiring costs of a marginal worker (relative to productivity). To hire a marginal additional worker at date s, firms must spend κ/q_s but since matches persist, hiring today brings about future hiring cost savings $(1-\omega)\,\kappa/q_s$ (discounted at the appropriate rate). Real marginal costs are therefore:

$$mc_{j,s} = \frac{w_s}{\exp(A_s)} + \frac{\kappa}{q_s} - \lambda_{v,j,s} - (1-\omega)\,\mathbb{E}_s\Lambda_{j,s,s+1}\left\{\frac{\kappa}{q_{s+1}} - \lambda_{v,j,s+1}\right\}, \tag{13}$$

where $\lambda_{v,j,s} \geq 0$ is the Kuhn-Tucker multiplier on Equation (12), which satisfies the complementary slackness condition $\lambda_{v,j,s} v_{j,s} = 0$. Exploiting symmetry across firms, marginal costs equalize across firms and hence we drop the firm subscript from now on. The firms' price-setting problems deliver the following first-order condition:[9]

$$1 - \gamma + \gamma mc_s = \phi\,(\Pi_s - 1)\,\Pi_s - \phi\mathbb{E}_s\Lambda_{s,s+1}\left[\frac{y_{s+1}}{y_s}\,(\Pi_{s+1} - 1)\,\Pi_{s+1}\right]. \tag{14}$$

Wages: Because of the matching friction, worker-firm matches produce surpluses which need to be divided between firms and workers. We assume that real wages are determined by Nash bargaining between workers and firms. As discussed by Krusell, Mukoyama and Sahin (2010), financial market incompleteness and risk aversion jointly imply that the surpluses that households derive from employment generally depend on their wealth levels. Hence we label the households' value and surplus functions by i. Firms are symmetric and hence we do not include a firm index

[9]Note that in the absence of price rigidities and search and matching frictions, the marginal cost equals $mc_s = \frac{w_s}{\exp(A_s)} = \frac{\gamma-1}{\gamma}$. To avoid trivial equilibria in which market work can generate no surplus to workers, even without labor market and price setting frictions, we assume that $\frac{\vartheta^{1-\mu}-1}{1-\mu} + \zeta < \frac{\left(\frac{\gamma-1}{\gamma}\exp(A_s)\right)^{1-\mu}-1}{1-\mu}$. Strictly speaking, this requires a bound on the support of the stochastic productivity process.

in the bargaining equations. The wage solves the following maximization problem:

$$\max \left(S_{i,s}^{e}\right)^{\upsilon} \left(S_{s}^{f}\right)^{1-\upsilon}, \tag{15}$$

where $S_{i,s}^{e}$ is the worker's surplus, S_{s}^{f} is the firm's surplus and $\upsilon \in (0,1)$ is the worker's bargaining weight. We assume that were negotiations to fall through, the worker becomes unemployed while the firm can attempt to hire a new worker in the same period. The employed worker's surplus ($S_{i,s}^{e}$), the difference between the value of being employed ($V_{i,s}^{e}$) and unemployed ($V_{i,s}^{u}$), is then:

$$\begin{aligned}
S_{i,s}^{e} &= V_{i,s}^{e} - V_{i,s}^{u}, \\
V_{i,s}^{e} &= \frac{c_{i,e,s}^{1-\mu}}{1-\mu} - \zeta + \beta \mathbb{E}_s \omega \left(1-\eta_{s+1}\right) V_{i,s+1}^{u} + \beta \mathbb{E}_s \left(1-\omega\left(1-\eta_{s+1}\right)\right) V_{i,s+1}^{e}, \\
V_{i,s}^{u} &= \frac{c_{i,u,s}^{1-\mu}}{1-\mu} + \beta \mathbb{E}_s \left(1-\eta_{s+1}\right) V_{i,s+1}^{u} + \beta \mathbb{E}_s \eta_{s+1} V_{i,s+1}^{e},
\end{aligned}$$

where $c_{i,e,s}$ ($c_{i,u,s}$) is the consumption level optimally chosen by the household in case of employment (unemployment). Recall that separations take place at the very end of the period whereas new matches are formed at the very beginning. Accordingly, the term $\omega\left(1-\eta_{s+1}\right)$ in the second equation is the probability that an employed worker in period s is still employed in period $s+1$, either because the current match remains in tact, or because the current breaks down but the worker immediately finds a new job in the beginning of the next period.

Since the firm will post vacancies to hire a replacement worker should the current negotiations fail, the surplus of the match to the firm satisfies:

$$S_{s}^{f} = \frac{\kappa}{q_s}. \tag{16}$$

2.3 Monetary Policy

The monetary authority follows an interest rate rule. Specifically, the interest rate responds to inflation, given by $\Pi_s \equiv \frac{P_s}{P_{s-1}}$, and to labor market tightness. The latter variable naturally captures, inversely, the degree of labor market slack. The interest rate rule is given by:

$$R_s = \max\left\{\overline{R}\left(\frac{\Pi_s}{\overline{\Pi}}\right)^{\delta_\pi}\left(\frac{\theta_s}{\overline{\theta}}\right)^{\delta_\theta}, 1\right\}, \tag{17}$$

where $\overline{R}$, $\overline{\Pi}$, $\overline{\theta}$, $\delta_\pi \geq 0$ and $\delta_\theta \geq 0$ are policy parameters and the max operator captures the zero lower bound on the net nominal interest rate, $R_s - 1$.

2.4 Financial Markets

In NK models with unemployment it is typically assumed that individual households are insured against idiosyncratic earnings shocks within large diversified families or, alternatively, that households can purchase unemployment insurance contracts at actuarially fair prices. Whilst this conveniently allows one to use a representative agent framework, it also has the unfortunate consequence that individuals' consumption streams do not depend on their idiosyncratic earnings shocks - including those related to unemployment - which raises questions on the empirical relevance of the model.

Here we instead assume that households live in single-member families and cannot purchase unemployment insurance contracts, c.f. Challe and Ragot (2016) and Ravn and Sterk (2012). Households can attempt to self-insure against job uncertainty through savings in a zero-dividend one-period nominal bond purchased at price $1/R_s$ units of currency at date s. Let the household's purchases of bonds at date s be given by $b_{i,s}$. Households must observe a liquidity constraint:

$$b_{i,s} \geq \underline{b}. \tag{18}$$

A second asset that is available to households is firm equity. In Section 5 we discuss asset pricing implications and introduce additional risky and riskless assets.

2.5 Conditions for a Tractable Equilibrium

Without further assumptions, the model above can only be solved numerically. In this paper we aim at an analytical characterization of the equilibrium. It turns out that this can be attained by imposing two assumptions. First, we impose the following borrowing constraint:

$$\underline{b} = 0, \tag{19}$$

see also see also Krusell, Smith and Mukoyama (2009), Ravn and Sterk (2012) and Werning (2015). Second, we assume limited participation in the market for firm ownership. Specifically, only a fraction $\xi \in (0, 1)$ of the households has the ability to invest in firm equity.

To appreciate why the model now simplifies very considerably, consider first the households who cannot invest in firm equity ($i \geq \xi$). Amongst these households, employed workers have an incentive to save while unemployed workers have an incentive to dissave or borrow. The borrowing constraint above, however, implies that households cannot borrow and therefore there is no supply of bonds. In equilibrium, these households are therefore unable to accumulate any savings. We therefore refer to the households who cannot invest in firm equity as the "asset poor". In the absence of savings, they consume their current incomes, i.e.

$$\begin{aligned} c_{i \geq \xi, e, s} &= w_s, \\ c_{i \geq \xi, u, s} &= \vartheta. \end{aligned}$$

Next, consider the households who can invest in firm equity ($i < \xi$), who end up being asset rich as they receive the monopoly profits of the firm. These households will typically receive higher levels of income than those who are unable to invest in equity, and therefore may be unwilling to work depending on the level of the disutility parameter ζ and the fraction of households that can invest (which determines the amount of profits per household). For simplicity we will assume that parameter values are such that investing households declare themselves out of the labor force, but this is not important for the key results. The consumption levels of the asset-rich, $c_{i<\xi,s}$, equalizes across all agents $i \in (0, \xi)$, and is given by:

$$c_{i<\xi,u,s} = \vartheta + \frac{1}{\upsilon}\left(y_s - \kappa v_s - w_s n_s - \frac{\phi}{2}\left(\Pi_s - 1\right)^2 y_s\right).$$

It follows that firms discount profits at a common rate $\Lambda_{t,t+s} = \beta\left(c_{i<\xi,u,t}/c_{i<\xi,u,t+s}\right)^{\mu}$.

All workers (asset-poor households) have the same wealth (zero) and therefore bargain the same wage because their outside options are identical. Similarly, all investors (asset-rich households) consume the same amounts. Since there is no heterogeneity across households conditional on their type and employment status, we drop the i-subscript and denote consumption levels as $c_{e,s} = c_{i \geq \xi, e, s}$, $c_{u,s} = c_{i \geq \xi, u, s}$, and $c_{r,s} = c_{i<\xi,u,s}$, where subscript r denotes the asset-rich households. The first-order condition for bonds delivers the following Euler equations for the three types of households:

$$
\begin{aligned}
c_{e,s}^{-\mu} &= \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} \left(\omega \left(1-\eta_{s+1}\right) c_{u,s+1}^{-\mu} + \left(1-\omega\left(1-\eta_{s+1}\right)\right) c_{e,s+1}^{-\mu}\right) + \lambda_{e,s}, && (20)\\
c_{u,s}^{-\mu} &= \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} \left(\left(1-\eta_{s+1}\right) c_{u,s+1}^{-\mu} + \eta_{s+1} c_{e,s+1}^{-\mu}\right) + \lambda_{u,s}, && (21)\\
c_{r,s}^{-\mu} &= \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} c_{r,s+1}^{-\mu} + \lambda_{r,s}, && (22)
\end{aligned}
$$

where $\lambda_{e,s}$, $\lambda_{u,s}$, and $\lambda_{r,s}$ are the Kuhn-Tucker multipliers on the liquidity constraints of the employed asset-poor households, the unemployed asset-poor households and the asset-rich households, respectively.

Since $w_s > \vartheta$ and $\omega > 0$, the liquidity constraint always binds for the asset-poor unemployed households, i.e. $\lambda_{u,s} > 0$ at all times. Further, it is straightforward to verify that in any steady state without aggregate uncertainty it holds that (i) $\lambda_{r,s} > 0$, i.e. the asset rich households are at the liquidity constraint, and (ii) $\lambda_{e,s} = 0$, i.e. the employed asset-poor households are not at the liquidity constraint, (iii) and $\mathbb{E}_s \frac{R_s}{\Pi_{s+1}} < \beta$. Intuitively, the asset-rich are not exposed to idiosyncratic risk and hence are unwilling to take a positive position in bonds, which pay a real interest rate that lies below their subjective discount rate.[10] The employed asset-poor households, by contrast, are exposed to idiosyncratic risk. This gives rise to a precautionary savings motive which makes them willing to invest in bonds at a return that is lower than their subjective discount rate. Therefore, when analyzing steady-state equilibria or in their vicinity, we can drop Equations (21) and (22), as well the variables $\lambda_{u,s}$, and $\lambda_{r,s}$.

Finally, consider the equilibrium labor market flows. Provided that the asset-poor are unwilling to leave the labor force, the labor market participation rate is constant and given by $1-\xi$. In that case, the aggregate unemployment rate is given by:

$$u_s = 1 - n_s, \tag{23}$$

where $n_s = \frac{1}{1-\xi} \int_j n_{j,s} dj$ is the aggregate employment rate, as a fraction of the labor force. The law of motion of unemployment is given as

$$u_s = u_{s-1} + \omega n_{s-1} - h_s, \tag{24}$$

[10] Note that even in scenarios in which the asset-rich would be willing to work, they would still be better insured against idiosyncratic income risk than the asset-poor households.

where $h_s = \frac{1}{1-\xi} \int_j h_{j,s} dj$ is the number of new hires as a fraction of the labor force. The aggregate number of job searchers is given by $e_s = (1-\xi)(u_{s-1} + \omega n_{s-1})$.

3 Non-stochastic Equilibria

This section discusses the set of equilibria that can arise, absent aggregate shocks. We explore the nature of the steady-state equilibria that can arise and whether such equilibria are locally unique in the vicinity of those steady states.

3.1 Global Determinacy

Consider of a version of the model without aggregate productivity shocks ($\sigma_A = 0$) and without potential belief shocks. A crucial difference vis-à-vis the extant complete markets NK literature is that although the aggregate allocation (and prices) are constant in the steady state, asset-poor households face idiosyncratic risk in the incomplete markets model due to lack of insurance against job uncertainty. This has fundamental consequences for the properties of the model, which we now consider.

We indicate steady-state values by removing time subscripts from variables. Define for convenience $R^* \equiv \overline{R}\,\overline{\Pi}^{-\delta_\pi}\overline{\theta}^{-\delta_\theta}$. The solution to the steady-state wage can be expressed as function of the job finding rate, $\mathbf{w}(\eta)$. This function is derived in the Appendix, in which we also discuss some of its basic properties. Steady-state equilibria can be characterized by the solutions to:

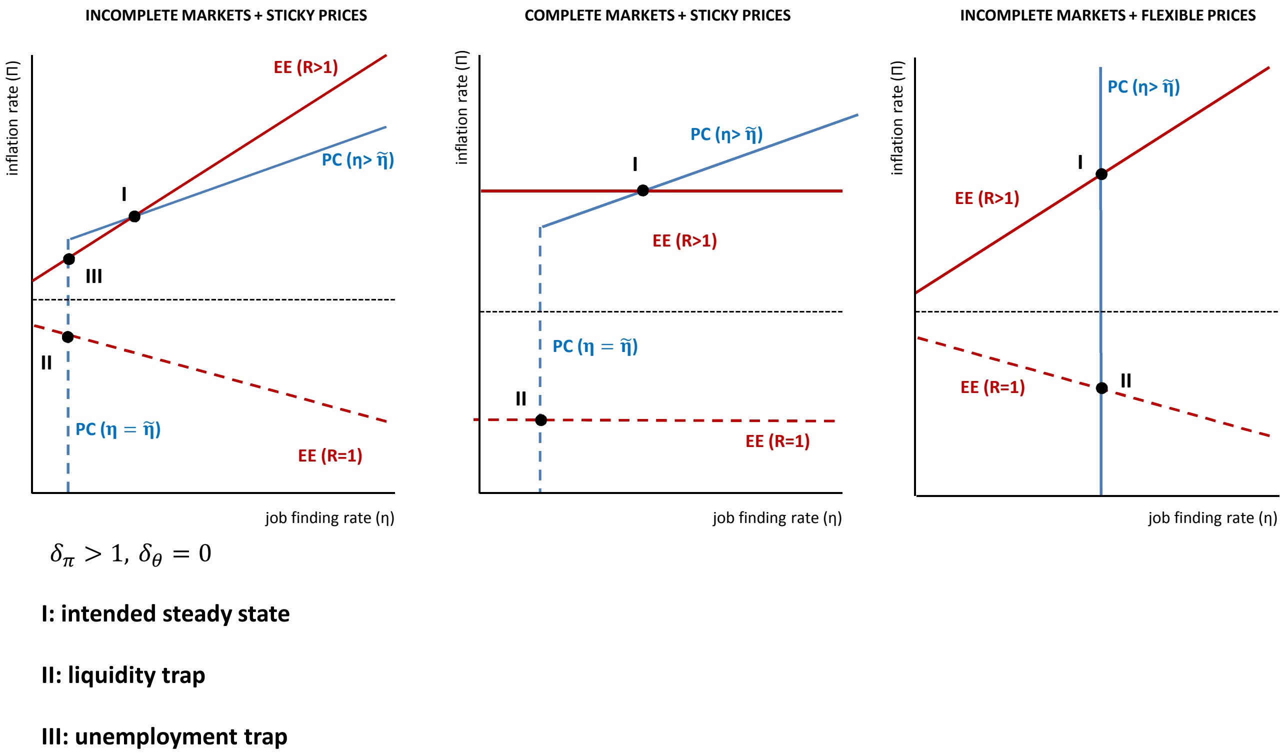
$$\underbrace{\phi(1-\beta)(\Pi-1)\Pi}_{\text{sticky-price wedge}} = 1 - \gamma + \gamma\left(\mathbf{w}(\eta) + \left(\kappa\eta^{\alpha/(1-\alpha)} - \lambda_f\right)(1-\beta(1-\omega))\right), \qquad (\mathbf{PC})$$

$$1 = \beta\frac{\max\{R^*\Pi^{\delta_\pi}\eta^{\delta_\theta/(1-\alpha)}, 1\}}{\Pi}[\underbrace{\omega(1-\eta)(\vartheta/\mathbf{w}(\eta))^{-\mu} + 1 - \omega(1-\eta)}_{\text{incomplete-markets wedge}}], \qquad (\mathbf{EE})$$

where $\eta \geq \widetilde{\eta}$, $\lambda_f \geq 0$, and $\lambda_f(\eta - \widetilde{\eta}) = 0$. Here, $\widetilde{\eta}$ is the job finding rate that would prevail in a steady-state in which firms are unwilling to invest in formal vacancies thus only hire via costless informal channels. In such a steady-state, the job finding rate is given as $\widetilde{\eta} = \frac{\mathbf{M}(\widetilde{e},\widetilde{v})}{\widetilde{e}}$, where $\widetilde{e} = (1-\xi)\left(\frac{\omega(1-\widetilde{\eta})}{\omega(1-\widetilde{\eta})+\widetilde{\eta}}(1-\omega)+\omega\right)$.

Equation (**PC**) is the steady-state version of (14), the optimality condition for prices in the symmetric equilibrium ("the Phillips Curve"), and defines a relationship between inflation, Π,

Figure 1: Illustration of steady-state equilibria.



and the job finding rate, η, provided that $\eta > \widetilde{\eta}$. The left-hand side of the equation is a standard sticky-price wedge which vanishes in the absence of price adjustment costs ($\phi = 0$).

Equation (**EE**) is the steady-state version of the employed households' Euler equation (20) which also defines a relationship between Π and η. The term between square brackets arises due to incomplete markets, and would collapse to one when either $\vartheta = \mathbf{w}(\eta)$, i.e. when consumption is fully insulated against job loss, or when $\mu = 0$, i.e. when households are risk neutral. Note that the wedge is a function of the job finding rate and that the two schedules are non-linear. Two sources of non-linearity are particularly important in paving the way for multiple steady states. The first is the ZLB on the short term nominal interest rate, whereas the second is the non-negativity constraint on investment in costly vacancies.

Solutions to the above system give the steady-state outcomes for inflation and the job finding rate. Figure 1 illustrates the steady-state schedules. For simplicity, we consider a case in which the interest rate rule only reacts to inflation (i.e. $\delta_\theta = 0$). There are two **EE** schedules, one at which the ZLB binds ($\mathbf{EE}(R = 1)$) and one at which the ZLB does not bind ($\mathbf{EE}(R > 1)$). The $\mathbf{EE}(R > 1)$ implies a positive relation between η and Π. Intuitively, when the job finding rate is high, the precautionary savings motive is weak, implying a relatively high real interest rate. Since monetary policy responds more than one-for-one to inflation, a high real interest rate implies a high rate of inflation. The $\mathbf{EE}(R=1)$, on the other hand, implies a negative relation between η and Π, since at the ZLB a high real interest rate implies a low rate of inflation. The $\mathbf{PC}$ schedule slopes upward as long as the job finding rate is positive ($\mathbf{PC}(\eta > \widetilde{\eta})$) but becomes vertical at the point the job finding rate hits zero and the non-negativity constraint on vacancies becomes binding ($\mathbf{PC}(\eta = \widetilde{\eta})$).

Consider first the left panel of Figure 1, which illustrates a case with incomplete markets and sticky prices.[11] Three possible steady states emerge:[12]

I *Intended steady state.* This steady state occurs at the intersection of the $\mathbf{PC}(\eta > \widetilde{\eta})$ and the $\mathbf{EE}(R > 1)$ schedule. This is the "intended" steady state at which the ZLB does not bind and the job finding rate is relatively high.

II *Liquidity trap.* This steady state arises because of the ZLB on the nominal interest rate and occurs at the intersection of the $\mathbf{PC}(\eta = \widetilde{\eta})$ and the $\mathbf{EE}(R = 1)$ schedule. This is the "liquidity trap" examined by Benhabib, Schmitt-Grohé and Uribe (2001, 2002) and Mertens and Ravn (2013). This steady state features a lower rate of inflation than the intended steady state, as well as a lower job finding rate. In fact, the job finding rate is zero in the illustration.

III *Unemployment trap.* This steady state occurs at the intersection of the $\mathbf{PC}(\eta = \widetilde{\eta})$ and the $\mathbf{EE}(R > 1)$ schedule. In this equilibrium, investment in vacancies comes to a complete standstill despite the fact that the ZLB on the nominal interest rate does not bind. Thus, any hiring takes place occurs informally. Note that the inflation rate in this steady state lies in between those in the intended steady state and the liquidity trap.[13]

The first two of these types of equilibria occur also in standard complete-markets representative-agent NK models. There are, however, important differences between the properties of the equilibria under complete and incomplete markets. With full insurance, the steady-state real interest rate needs to equal $1/\beta$ in order to be consistent with constant consumption. Without

[11] The Figure assumes that $\delta_\theta = 0$, i.e. monetary policy only reacts to inflation, and a sticky real wage, i.e. $\upsilon = 0$.

[12] We ignore the possibility of an additional equilibrium that occurs due to the quadratic price adjustment term, or due to non-linearities in $\mathbf{w}(\eta)$.

[13] We have assumed that firms always use the costless informal vacancies, given by $\widetilde{\upsilon}$. Allowing firms to potentially leave informal vacancies unused, may given rise to an additional steady state.

full insurance, the wedge in (**EE**) is positive, which reduces the equilibrium real interest rate below the inverse of the discount rate, $\frac{R}{\Pi} < \frac{1}{\beta}$. Intuitively, the consumption loss associated with job loss creates a precautionary savings motive. Since the net-supply of bonds is zero, the real interest rate adjusts downward to restore equilibrium.

In the incomplete markets economy, the equilibrium real interest rate depends on the job finding rate and on the consumption loss that a worker experiences in case of job loss. It therefore follows that the equilibrium long run real interest rate depends on economic policy to the extent that policy choices influence the job finding rate and/or the consumption loss. Secondly, whilst the aggregate quantities and prices are constant in the steady states, the combination of unemployment risk and incomplete markets imply that individual households are subject to idiosyncratic risk in the steady state in the incomplete markets model. The liquidity trap generated by the model have very interesting properties which we discuss in detail in Section 5.

The possible emergence of a third steady state depends critically on the interaction between sticky prices and incomplete markets. The middle and right panels of Figure 1 illustrate, respectively, a case with complete markets (but sticky prices) and a limit case with flexible prices (but incomplete markets). Under complete markets, the **EE** schedules become horizontal, because the steady-state real interest rate equals the households' subjective discount rates. This rule out a third steady state.[14] In the limit case with flexible prices, the $\mathbf{PC}(\eta > \widetilde{\eta})$ schedule becomes vertical, as inflation no longer affects firms' marginal costs. As a result, the $\mathbf{PC}(\eta = \widetilde{\eta})$ schedule vanishes, thus allowing for only two steady states. Thus, without the interaction between sticky prices and incomplete markets, the third steady state cannot not exist.[15] However, in the presence of both sticky prices and incomplete markets, this steady-state equilibrium outcome can arise and its likelihood is higher when markets are more incomplete, when monetary policy reacts little to inflation and/or labor market tightness, and hiring costs are limited.

The unemployment trap is an intriguing outcome. The slow recovery after the Great Recession and the very protracted nature of the surge in unemployment observed in the U.S. (and many other OECD economies) have spurred a renewed interest in "secular stagnation," equilibrium outcomes consistent with long periods of low activity and high unemployment. Hansen (1939) argued that such outcomes (with negative natural real interest rates) were most likely

[14] In the left and middle panel of Figure 1, the liquidity trap does not feature any hiring. For different parameter configurations, the liquidity trap can occur at a positive job finding rate. This, however, would rule out the third steady state.

[15] Note further that this combination is a necessary but not sufficient condition for the emergence of the third steady state, since the complete markets version is the limit of the incomplete markets version.

produced by a combination of low rate of technological progress and population ageing implying high savings rates and low investment rates. Recently, Eggertsson and Mehrotra (2014) have argued that deleveraging may lead to secular stagnation and exacerbate the problems that follow from an ageing population and falling investment goods prices.

The unemployment trap that can arise in the incomplete markets NK model offers an alternative perspective of secular stagnation which ties together low real interest rates, high unemployment and low activity. In this steady state, hiring is at a minimum and unemployment therefore potentially very high. Moreover, because of the low job finding rate, there is a strong incentive for precautionary savings which drives down the real interest rate. Intriguingly, the unemployment trap can occur in our model purely because of expectations and thus does not rely on sudden changes in population growth, technological progress or financial tightening. Furthermore, while the nominal interest rate may be low in the unemployment trap, its root cause does not derive from the ZLB on nominal interest rates. Therefore, the ongoing discussions about the design of monetary policy to deal with the ZLB – such as increasing the inflation target and allowing for negative nominal interest rates on central bank deposits – may be in vain.

3.2 Local Determinacy

The log-linearized model: We now log-linearize the model in order to study the local stability properties of the equilibria. Let a hat denotes a log deviation from the intended steady state, i.e. $\widehat{x}_s = \ln x_s - \ln \overline{x}^I$, where $\overline{x}^I$ denotes the value of x_s in the intended steady-state (discussed above). We assume that monetary policy parameters are such that $\overline{R}$, $\overline{\theta}$ and $\overline{\Pi}$ correspond to the levels of, respectively, R, θ and Π, in the intended steady state.

The log-linearized Euler equation of the employed households, (20), can be expressed as (see the Appendix for details):

$$\begin{aligned} -\mu\widehat{c}_{e,s} + \mu\beta\overline{R}\mathbb{E}_s\widehat{c}_{e,s+1} &= \widehat{R}_s - \mathbb{E}_s\widehat{\Pi}_{s+1} - \underbrace{\beta\overline{R}\Theta\mathbb{E}_s\widehat{\eta}_{s+1}}_{\text{incomplete-markets wedge}}, \\ \Theta &\equiv \omega\eta\left((\vartheta/w)^{-\mu} - 1\right) - \chi\mu\omega(1-\eta) \end{aligned} \tag{25}$$

$\widehat{R}_s - \mathbb{E}_s\widehat{\Pi}_{s+1}$ is the real interest rate while the last term on the right-hand side is the incomplete-markets wedge, which fluctuates proportionally with the expected job finding rate. Its strength is determined by Θ, a convolution of parameters which consists of two parts. The first part, $\omega\eta\left((\vartheta/w)^{-\mu} - 1\right) > 0$, represents the impact of job loss on the marginal utility of consumption. If home production equals the steady-state wage, i.e. if $\vartheta = w$, or if the household is risk neutral ($\mu = 0$), this part of the wedge collapses to zero. An expected decline in the job finding rate increases the probability of unemployment and strengthens the households precautionary savings motive. The second part, $-\chi\mu\omega(1-\eta) < 0$, derives from the fact that, under incomplete markets, expected wage growth transmits only partially to expected consumption growth, since the worker may be unemployed in the next period. This part of the wedge vanishes if wages are fully sticky ($\chi = 0$) or when households are risk neutral ($\mu = 0$). Note that when $\Theta = 0$, the incomplete-markets wedge vanishes and the above equation reduces to the log-linearized Euler equation obtained in standard representative-agent models.[16]

Next, we log-linearize the firms' price-setting condition, Equation (14), around the intended steady state:

$$\underbrace{\frac{\phi}{\gamma}\widehat{\Pi}_s - \beta\frac{\phi}{\gamma}\mathbb{E}_s\widehat{\Pi}_{s+1}}_{\text{sticky-price wedge}} = w\left(\widehat{w}_s - A_s\right) + \frac{\kappa}{\overline{q}}\frac{\alpha}{1-\alpha}\widehat{\eta}_s - \beta(1-\omega)\frac{\kappa}{\overline{q}}\left(\frac{\alpha}{1-\alpha}\mathbb{E}_s\widehat{\eta}_{s+1} + \widehat{\Lambda}_{s,s+1} - (1-\rho)A_s\right), \tag{26}$$

where we have exploited that $q_s = \eta_s^{-\frac{\alpha}{1-\alpha}}$. For now, we abstract from productivity shocks, setting $A_s = 0$ at any date s. The left-hand side of the above equation is the sticky-price wedge, which vanishes in the absence of price adjustment costs ($\phi = 0$) or in the limit with perfect competition ($\gamma \to \infty$). The right-hand side is the log-linearized marginal cost, which is standard given the presence of search and matching frictions.

The policy rule reads, log-linearized around the intended steady state, reads:

$$\widehat{R}_s = \delta_\pi\widehat{\Pi}_s + \delta_\theta\widehat{\theta}_s. \tag{27}$$

In the Appendix, we further show that the log-linearized bargaining equations imply that:

$$\widehat{w}_s = \chi\widehat{\eta}_s, \tag{28}$$

where χ is a convolution of the model's deep parameters, which captures the sensitivity of the wage to fluctuations the job finding rate and depends critically on the bargaining parameter υ. Finally, note that in equilibrium the employed households consume their wage, i.e. $\widehat{c}_{e,s} = \widehat{w}_s$.

Reducing the model to a single equation: For maximal tractability, we introduce two

[16] It can be verified that in that case also $\beta\overline{R} = 1$.

further assumptions which allow us to reduce the model to a single equation. First, we set the monetary policy coefficient equal to $\delta_\pi = \frac{1}{\beta} > 1$. This is inconsequential, since the coefficient on tightness, δ_θ, is left unrestricted.[17] Second, we assume that the households who can invest in equity (i.e. those with index $i < \xi$) are risk neutral. In this case, the log-linearized model has no endogenous state variables. In the appendix, we relax this assumption. The results suggest that allowing for risk-averse equity investors has only very limited implications for the model dynamics.

The log-linearized model can now be reduced just one dynamic equation for the job finding rate (see the Appendix for a derivation):

$$\begin{aligned} \mathbb{E}_s \widehat{\eta}_{s+1} &= \Psi \widehat{\eta}_s \\ \Psi &\equiv \frac{\phi\gamma^{-1}\mu\chi\beta + \phi\gamma^{-1}\frac{\beta\delta_\theta}{1-\alpha} + w\chi + \frac{\kappa}{q}\frac{\alpha}{1-\alpha}}{\frac{\kappa}{q}\frac{\alpha\beta(1-\omega)}{1-\alpha} + \phi\gamma^{-1}\mu\beta^2\overline{R}\chi + \phi\gamma^{-1}\beta^2\overline{R}\Theta} \end{aligned} \tag{29}$$

Under conventional parameter values, both the numerator and the denominator of Ψ are positive, and we will proceed under the assumption that this is the case. While the expression for Ψ seems complicated at a first glance, it turns out to deliver very intuitive results, which we present below.

Determinacy around the intended steady state: rigid real wages: How does the presence of incomplete markets impact on the possibility of local self-fulfilling equilibria? Intuitively, an increase in job uncertainty reduces aggregate demand, which in turn reduces the incentives to post vacancies. The reduction in vacancies in turn reduces the job finding rate, further increasing unemployment risk. It is precisely this feedback spiral that opens up the possibility that exogenous changes in beliefs, or "sunspot fluctuations" , are a source of macroeconomic fluctuations, as the equilibrium is no longer uniquely determined.

The model formalizes the condition under which such fluctuations can occur. For simplicity, we start with a version with sticky wages ($\chi = 0$). Since market tightness is not a state variable, the equilibrium is locally determinate if and only if $\Psi > 1$, i.e. if and only if:

$$\phi\gamma^{-1}\left(\beta^2\overline{R}\Theta - \frac{\beta\delta_\theta}{1-\alpha}\right) < \frac{\kappa}{q}\frac{\alpha}{1-\alpha}\left(1-\beta\left(1-\omega\right)\right).$$

The above equation makes clear how the occurrence of local indeterminacy depends on five types of market frictions present in the model, as well as on monetary policy:

[17] The log-linearized model contains no endogenous state variables and hence for any desire pair of values δ_π and δ_θ one can find a value δ_θ^* such that the same solution is obtained under the restriction that $\delta_\pi = \frac{1}{\beta}$.

(i) *Price rigidity.* If prices are fully flexible ($\phi = 0$) the equilibrium is always determinate since the left-hand side collapses to zero and the right-hand side is strictly positive.

(ii) *Imperfect competition.* Under perfect competition ($\gamma \to \infty$) the equilibrium is always determinate, for the same reason as above.

(iii) *Incomplete markets.* Under sticky wages, the incomplete-markets parameter collapses to $\Theta = \omega\eta\left((\vartheta/w)^{-\mu} - 1\right)$. The case $\vartheta = w$ corresponds to full insurance, in which case the incomplete markets term collapses to zero. In this case, the equilibrium is always determinate. The same is true when households are risk neutral ($\mu = 0$).

(iv) *Monetary policy.* The more aggressively monetary policy responds to tightness, i.e. the higher δ_θ, the less likely indeterminacy is to occur.

(v) *Labor adjustment cost.* The term $\frac{\kappa}{q}\frac{\alpha}{1-\alpha}\left(1 - \beta\left(1 - \omega\right)\right)$ denotes the steady-state marginal cost of hiring a worker today rather than tomorrow, so we can think of it as a labor adjustment cost, i.e. a real labor rigidity. Note that this cost is proportional to the steady-state hiring cost $\frac{\kappa}{q}$.

There are two main differences between the incomplete markets model and the standard model with insurance against idiosyncratic risk. The first is simply that the conditions for determinacy are more stringent under incomplete markets. With complete markets, a sufficient conditions for local determinacy is that $\delta_\pi > 1$ as we have assumed (notice that the left hand side of the inequality is negative when $\Theta = 0$). Under incomplete markets this is no longer a sufficient condition.

Secondly, there is an important interaction between market incompleteness, sticky prices, and risk aversion due to the multiplicative nature of the coefficient on the left hand side. Specifically, price rigidities only make indeterminacy more likely if the incomplete markets effect dominates the monetary policy effect, i.e. if $\Theta > \beta\delta_\theta$. Moreover, less complete financial markets, i.e. higher Θ, make indeterminacy more likely, but only if prices are sticky and the goods market is imperfectly competitive. However, if $\Theta > \beta\delta_\theta$, market incompleteness, nominal rigidities and risk aversion are complements making local indeterminacy increasingly likely in combination.

An intriguing insights regards the impact of labor market frictions. According to the condition above, the higher is the labor adjustment cost, the *less* likely it is for indeterminacy to happen. Thus, less flexible labor markets imply less amplification. The reason for this is that when it is costly for firms to adjust on the labor margin, they are more likely to adjust prices which neutralizes the feedback mechanism.

Determinacy around the intended steady state: flexible real wages: The determinacy condition becomes somewhat more involved when we introduce wage flexibility ($\chi > 0$):

$$\phi\gamma^{-1}\left(\beta^2\overline{R}\Theta - \frac{\beta\delta_\theta}{1-\alpha}\right) - w\chi - \phi\gamma^{-1}\mu\beta\left(1-\beta\overline{R}\right)\chi < \frac{\kappa}{q}\frac{\alpha}{1-\alpha}\left(1-\beta\left(1-\omega\right)\right).$$

Wage flexibility affects determinacy via three channels. First, it does so via an *incomplete markets channel*. Recall that $\Theta \equiv \omega\eta\left(\left(\vartheta/w\right)^{-\mu}-1\right) - \chi\mu\omega\left(1-\eta\right)$, so wage flexibility reduces the incomplete markets wedge. Second, wage flexibility creates a *marginal cost channel*, as it pushes down wage costs during times of low market tightness, pushing up vacancy posting. This channel comes in via the term $-w\chi$. Finally, wage flexibility generates an *intertemporal substitution* channel, as a decline in wages reduces employed households' incentives to save. This channel enters via the term $-\phi\gamma^{-1}\mu\beta\left(1-\beta\overline{R}\right)\chi$. Finally, note that through all three channels wage flexibility pushes the model towards the determinacy region of the parameter space. In conclusion, real wage flexibility is stabilizing in the vicinity of the intended steady-state.

Determinacy around the unemployment trap: We now consider local determinacy around the unemployment trap. To this end, we exploit that the non-negativity constraint on vacancies binds. Hence, we can drop Equation (26) and set η_s equal to $\widetilde{\eta}$. Thus, the job finding rate is trivially determined. The Euler equation, log-linearized around the unemployment trap, is given by:

$$0 = \delta_\pi\widehat{\Pi}_s - \mathbb{E}_s\widehat{\Pi}_{s+1}.$$

It follows immediately that the equilibrium is unique if and only if $\delta_\pi > 1$, i.e. the interest rate elasticity with respect to inflation exceeds unity. Thus, the unemployment trap is determinate under a standard Taylor rule which responds more than one-for-one to inflation.

4 Fluctuations

4.1 Local Shocks

Belief shocks: We now explicitly solve for the local dynamics in the vicinity of the intended steady state in response to shocks. We first focus on "belief shocks" starting with a version of the model without productivity shocks. From Equation (29) it follows that if the equilibrium is locally determinate ($\Psi > 1$), then the only stable solution is given by $\widehat{\eta}_s = 0$ at all times. When equilibria are locally indeterminate, the solution is given by

$$\widehat{\eta}_{s+1} = \Psi \widehat{\eta}_s + \Upsilon^B \varepsilon^B_{s+1},$$

where ε^B_s is an i.i.d. belief shock with mean zero and a standard deviation normalized to one, and Υ^B is a parameter. Thus, in a model with only belief shocks the job finding rate follows an AR(1) process. While the magnitude of the belief shocks, captured by Υ^B, is not pinned down in the model, the persistence of the effects of belief shocks on the job finding rate is captured by Ψ, and thus endogenously determined. Persistence is maximal at $\Psi = 1$, i.e. exactly at the border between the determinacy and indeterminacy region of the parameter space.

Productivity shocks: We now consider the effects of technology shocks.[18] In the Appendix, we show that the model with productivity shocks can be written as:

$$\begin{aligned}
\mathbb{E}_s \widehat{\eta}_{s+1} &= \Psi \widehat{\eta}_s - \Omega A_s, \\
A_s &= \rho A_{s-1} + \sigma_A \varepsilon^A_s, \\
\Omega &\equiv \frac{w}{\frac{\kappa}{q}\frac{\alpha\beta(1-\omega)}{1-\alpha} + \frac{\phi}{\gamma}\mu\beta^2 \overline{R}\chi + \frac{\phi}{\gamma}\beta^2 \overline{R}\Theta}.
\end{aligned}$$

where under conventional parameter values $\Omega > 0$.

Consider first the determinate case ($\Psi > 1$). We apply the method of undetermined coefficients and guess a solution of the form $\widehat{\eta}_s = \Gamma_\eta A_s$. Plugging this guess into the above system of equations yields the following solution:

$$\Gamma_\eta = \frac{\Omega}{\Psi - \rho}. \tag{30}$$

It can now be shown that, in the determinacy region of the parameter space, the job finding rate responds positively to productivity shocks, i.e. $\Gamma_\eta > 0$. To see why, recall that the numerator of Equation (30) is positive and note that for the denominator to be positive as well, it is required that $\Psi > \rho$. In the determinacy region, this is the case since determinacy requires $\Psi > 1$ and it further holds that $\rho < 1$.

[18] For an analysis of technology shocks in the standard New Keynesian model, see Galí (1999).

Writing out the solution for Γ_η explicitly gives:

$$\Gamma_\eta = \frac{w}{\phi\gamma^{-1}\beta\left(\frac{\delta_\theta}{1-\alpha} - \rho\beta\overline{R}\Theta\right) + \frac{\kappa}{q}\frac{\alpha(1-\rho\beta(1-\omega))}{1-\alpha} + \left(w + \phi\gamma^{-1}\mu\beta\left(1-\beta\overline{R}\rho\right)\right)\chi}.$$

Given $\Gamma_\eta > 0$, it holds that $\frac{\partial\Gamma_\eta}{\partial\Theta} \geq 0$, i.e. a higher value of the market incompleteness parameter Θ amplifies the impact of productivity on the job finding rate. The amount of amplification, however, depends critically on the amount of price stickiness, since $\frac{\partial\Gamma_\eta}{\partial\Theta} = 0$ when $\phi = 0$. Similarly, more aggressive monetary policy dampens the response, since $\frac{\partial\Gamma_\eta}{\partial\delta_\theta} \leq 0$, but only when prices are sticky. Real wage flexibility dampens the response of the job finding rate to productivity shocks, i.e. $\frac{\partial\Gamma_\eta}{\partial\chi} < 0$, since $\beta\overline{R} \leq 1$, $\rho \in (0,1)$ and $\frac{\partial\Theta}{\partial\chi} < 0$.

We can now solve for the inflation rate, guessing a solution of the form $\widehat{\Pi}_s = \Gamma_\Pi A_s$. Plugging this guess into the log-linearized Euler equation gives:

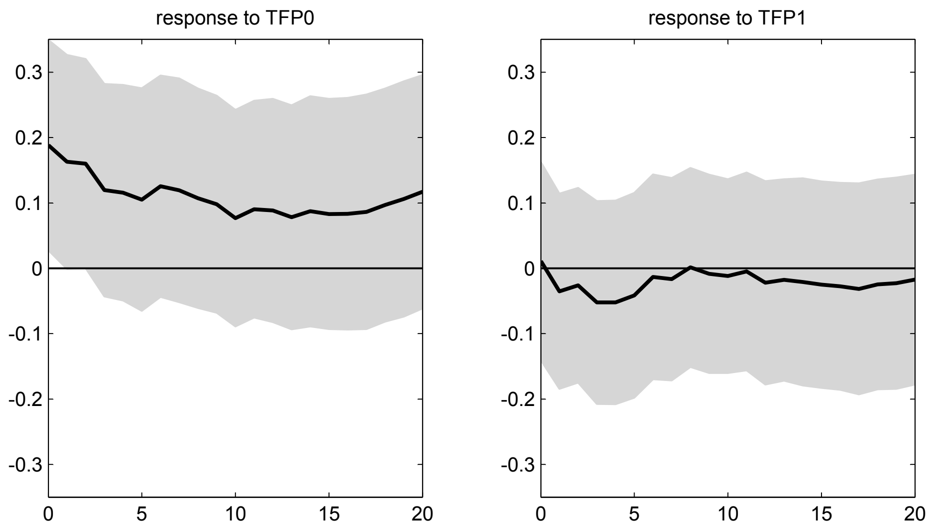
$$\Gamma_\Pi = \frac{\beta^2\overline{R}\Theta\rho - \frac{\beta\delta_\theta}{1-\alpha} - \mu\chi\beta\left(1-\rho\beta\overline{R}\right)}{1-\beta\rho}\Gamma_\eta.$$

It follows that inflation increases following a positive technology shock (i.e. $\Gamma_\Pi > 0$) if and only if $\beta^2\overline{R}\Theta\rho > \frac{\beta\delta_\theta}{1-\alpha} + \mu\chi\beta\left(1-\rho\beta\overline{R}\right)$. Thus, unlike the response of the job finding rate, the sign of the inflation response is ambiguous. Without the incomplete markets wedge ($\Theta = 0$), inflation declines following positive technology shocks, as long as either $\delta_\theta > 0$ or $\chi > 0$. The reason why prices may increase when the incomplete markets wedge is active comes from a demand channel: the increase in vacancy posting pushes up job finding rates, reducing the precautionary savings motive. This creates a boom in demand which pushes up prices, which may more than offset the direct effect of the technology shock, which is to reduce prices by reducing costs. Finally, note that when $\Theta = \frac{\frac{\delta_\theta}{1-\alpha} + \mu\chi\left(1-\rho\beta\overline{R}\right)}{\beta\overline{R}\rho}$, inflation does not respond to productivity shocks.

The possibility that higher productivity produces higher inflation is not a mere theoretical curiosity. In Figure 2 we show the impulse response of CPI inflation to TFP shocks where the latter correspond to those estimated by Fernald and Wang (2016). Using local projection, we regressed (400 times) quarterly (log) changes in the CPI on TFP (log) growth (times 100) for a sample that starts in 1980. Depending on whether one controls for movements in factor utilization or not, higher TFP either leaves inflation unchanged or gives rise to higher inflation. While the empirical results come with a fair amount of uncertainty, they do suggest that a positive inflation response is not simply an odd feature of our model.[19]

[19] The result holds also for the core PCE and here the positive response holds regardless of the TFP measure.

Figure 2: Response of CPI inflation to a positive TFP shock.



Notes: IRF of 400*log(cpit/cpit-1) to change in log TFP as estimated by Fernald http://www.frbsf.org/economic-research/publications/working-papers/2016/wp2016-07.pdf using local projection. The sample starts in 1980 and we included 4 lags. TFP0 (TFP1) refers to Fernald estimate for Total Factor Productivity without (with) control for factor utilization. Shaded areas denote error bands of two standard deviations.

Finally, consider the local responses to productivity shocks in case the model parameters are in the indeterminacy region ($\Psi \leq 1$). This is fundamentally complicated by the fact that, within the model, it is not pinned down to what extent fundamental shocks change beliefs. Many different assumptions on this are possible. To understand the issue at hand, let us express Equation (29) as:

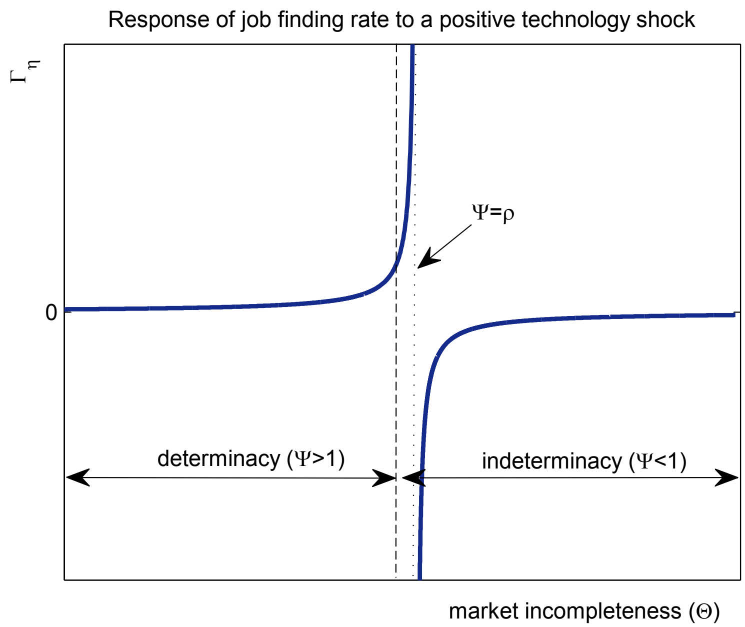
$$\widehat{\eta}_{s+1} = \Psi\widehat{\eta}_s - \Omega A_s + \Upsilon^A \varepsilon^A_{s+1} + \Upsilon^B \varepsilon^B_{s+1}.$$

It is straightforward to verify that in the determinacy region ($\Psi > 1$) it holds that $\Upsilon^A = \Gamma_\eta$ and $\Upsilon^B = 0$. In the indeterminacy region, however, Υ^A and Υ^B are not pinned down.

Lubik and Schorfheide (2004) suggest to assume that the responses to fundamental shocks do not "jump" as the parameters are pushed from the determinacy region into the indeterminacy region. In our case, this boils down to assuming $\Upsilon^A = \Gamma_\eta$. Equation (30) makes clear that under this assumption, the response of tightness to a productivity shock may be either positive or negative, since $\Psi - \rho$ can have either sign when $\Psi < 1$. A singularity occurs at $\Psi = \rho$. Letting Ψ approach ρ from the left, the response approaches infinity. Thus, in the indeterminacy region

The results also hold true for a sample period that starts in 1984, the sample split that Fernald and Wang (2016) focus upon.

Figure 3: Illustration: amplification and determinacy.



Notes: the blue line illustrates the contamporaneous response of the job finding rate to a positive productivity shock, as a function of the market incompleteness parameter Θ.

amplification is possibly infinitely large. Figure 3 illustrates the amplification.

5 Implications for the Zero Lower Bound

5.1 Contractionary Shocks and the ZLB

In recent years, a large literature has emerged on the effects of Zero Lower Bound (ZLB) in New Keynesian model, see e.g. Christiano, Eichenbaum and Rebelo (2011), Krugman and Eggertsson (2012) and Farhi and Werning (2013). Often, these analyses start off from a premise that some exogenous and transitory shock brings the economy to the temporarily to the ZLB. The specific shock introduced for this purpose is typically an exogenous shock to the discount factor, making agents temporarily more patient. The shock thus directly increases the agents' willingness to save and drives down aggregate demand. This puts downward pressure on inflation and the real interest rate. Via the interest rate rule, this results in a decline in the nominal interest rate, which may hit the ZLB if the shock is large enough. Arouba, Cuba-Borda and Schorfheide (2016) estimate a NK model and find that such discount factor shocks were responsible for taking the

US economy to the ZLB.

The exogenous discount factor shock used in standard analyses may strike one as unsatisfactory, leaving unexplained what created households' increased desire to save in the first place. But to appreciate the purpose of this specific shock, it helps to note that more conventional recessionary shocks, such as negative productivity shocks, may not lead to a decline in the nominal interest rate. There are two reasons for this. First, recessionary shocks reduce aggregate income. In a representative-agent model, aggregate income moves in tandem with individual income, and the desire to smooth consumption implies that households' desire to save declines in a recession. This puts upward pressure on the real interest rate and, when prices are sticky, also increases the nominal interest rate, see e.g. Galí (2015, Chapter 3). A negative technology shock additionally increases costs, which puts further upward pressure on inflation and, via the Taylor rule, also the nominal interest rate. Thus, in a standard NK model without other sources of shocks, *expansionary* rather than *recessionary* technology shocks are required to produce a decline in the nominal interest rate. For that reason, much research in the NK literature has introduced discount factor shocks when studying ZLB dynamics.

The precautionary savings mechanism that arises under incomplete markets can radically alter the cyclicality of the real interest rate, avoiding the need for discount factor shocks. Mechanically, the incomplete-markets wedge acts as a shock to the discount factor in the Euler equation, but is determined endogenously rather than exogenously. As economic conditions worsen, the risk of becoming unemployed increases, driving down aggregate demand and increasing agents' desire to save. If the precautionary savings mechanism is strong enough, the nominal interest rate declines.

To illustrate the above points more formally, consider the log-linearized interest rate rule, for simplicity assuming that the monetary authority only responds to inflation ($\delta_\theta = 0$). The solutions derived in the previous section implies that the nominal interest rate, in log deviations from the intended steady state, is given by $\widehat{R}_s = \delta_\pi \widehat{\Pi}_s = \delta_\pi \Gamma_\Pi A_s$. Recall that Γ_Π is negative when $\Theta = 0$. It immediately follows that under complete markets the nominal interest rate responds positively to a negative technology shock. However, when Θ is sufficiently large, Γ_Π is positive. In that case, a negative technology shock drives down the nominal interest rate, which hits the ZLB if the shock is large enough.

5.2 Understanding Missing Deflation

Although inflation has been moderate in the aftermath of the financial crisis, no country has experienced persistent deflation. This is not easy to reconcile with the standard NK model since it implies deflation in a liquidity trap steady state. Under the assumption of complete markets (CM), the deterministic steady-state version Euler equation reads:

$$c^{-\mu} = \beta \left(\frac{R}{\Pi}\right)^{CM} c^{-\mu}$$

which implies that the real interest rate is given by $\left(\frac{R}{\Pi}\right)^{CM} = 1/\beta$. It follows that when the ZLB binds in a steady state, the gross inflation rate must equal β which implies that liquidity traps must be deflationary. Temporary episodes at the ZLB will be even more deflationary than this since the stochastic Euler equation in that case will only be satisfied as long as $\Pi < \beta$ during the ZLB regime.[20] It is important to notice that these implications are independent of the arguments that enter the interest rate rule.

The incomplete markets NK model has different implications. As explained earlier, the relevant steady-state condition for the real interest rate under incomplete markets (IM) is:

$$\left(\frac{R}{\Pi}\right)^{IM} = \frac{1}{\beta}\frac{1}{(1-\omega(1-\eta)) + \omega(1-\eta)(\vartheta/w(\eta))^{-\mu}} < \frac{1}{\beta},$$

given that $\vartheta < w(\eta)$, which implies that the steady-state real interest rate depends on labor market conditions. When the ZLB binds, the steady-state Euler equation and the policy rule for the interest rate imply that the following two conditions must be satisfied in a liquidity trap (LT):

$$\begin{aligned}
\Pi^{LT} &= \beta\left[\left(1-\omega\left(1-\eta^{LT}\right)\right) + \omega\left(1-\eta^{LT}\right)\left(\vartheta/w\left(\eta^{LT}\right)\right)^{-\mu}\right] > \beta \\
\Pi^{LT} &< \overline{\Pi}\overline{\theta}^{\delta_\theta/\delta_\pi}\overline{R}^{-1/\delta_\pi}\left(\eta^{LT}\right)^{-(\delta_\theta/\delta_\pi)/(1-\alpha)}
\end{aligned}$$

Notice that if $\delta_\theta = 0$, the policy rule implies that $\Pi^{LT} < \overline{\Pi}\overline{R}^{-1/\delta_\pi} < 1$ so that the liquidity trap is deflationary, given that in the intended steady state $\Pi = \overline{\Pi} = 1$ and $R = \overline{R} > 1$. When

[20] Suppose that the ZLB regime persists with probability p while the intended steady-state is absorbing. In that case, the inflation rate during the ZLB episode is determined as $\Pi^{LT} = \beta\left(p + (1-p)\left(c^I/c^{LT}\right)^{-\mu}\right)$ where Π^{LT} is the inflation rate during the liquidity trap, c^I is consumption in the intended steady-state and c^{LT} is consumption in the liquidity trap. This condition implies $\Pi^{LT} < \beta$ as long as $c^I > c^{LT}$.

$\delta_\theta > 0$, however, inflation may be positive or negative in the liquidity trap. In particular, steady-state inflation is likely to be positive if $\left(\vartheta/w\left(\eta^{LT}\right)\right)^{-\mu} \gg 1$ and wages are not too responsive to the job finding rate, i.e. when the incomplete markets wedge is sufficiently strong and not moderated strongly by wage adjustments.

5.3 Paradoxes at the Zero Lower Bound

It is well known that at the ZLB, the representative-agent NK model has some paradoxical properties, see e.g. Eggertsson (2010), Eggertsson and Krugman (2012) and Werning (2012). Two paradoxes have gained special attention. The first is a "supply shock paradox": at the ZLB, positive shocks to the supply side of the economy can be contractionary. The second is a "paradox of flexibility", and is associated to the finding that, at the ZLB, a higher degree of price flexibility creates a larger drop in output.[21]

The flexibility paradox originates from the fact that at the ZLB, firms are cutting prices, i.e. inflation is negative. The lower price adjustment costs, the more willing firms are to cut prices and hence the lower is the rate of inflation. The supply shock paradox arises from the fact that a positive supply shock pushes down production costs and hence inflation.

The paradoxical effects of a decline in expected inflation can be understood from the consumption Euler equation. Consider, for simplicity, the complete-markets Euler equation under perfect foresight at the ZLB:

$$\left(\frac{c_{s+1}}{c_s}\right)^{\mu} = \beta\frac{1}{\Pi_{s+1}}$$

The effect of a decline in expected inflation, at the ZLB, is that the real interest rate, $\frac{1}{\Pi_{s+1}}$, increases. The above Euler equation makes clear that this implies an increase in expected consumption growth, c_{s+1}/c_s. Given that the decline in inflation is transitory however, an increase in expected consumption growth implies a *decline* in the current level of consumption, i.e. an economic contraction.

The joint presence of incomplete markets and search and matching frictions, however, can mitigate or even overturn these results. Mechanically, the endogenous incomplete-markets wedge in the Euler equation can absorb the effect of a decline in the real interest rate. Intuitively, an increase in output implies an increase in hiring, which reduces the precautionary savings motive. The reduced desire to save makes an expansion in output compatible with an increase in the

[21] Throughout this subsection, we consider equilibria which ultimately lead to the intended steady state. Properties of equilibria leading to the liquidity trap steady state can be very different, see e.g. Mertens and Ravn (2014).

real interest rate.

We now formalize these arguments. Suppose that the economy fluctuates discretely between a "depressed state" at which the ZLB binds, and a "normal state" which coincides with the intended steady state. Let p be the probability that the ZLB regime persists and let the normal state be absorbing. In the appendix we show that in this setting, the elasticity of the job finding rate with respect to expected inflation, at the ZLB, is given by

$$\frac{d\widehat{\eta}_s^{ZLB}}{d\mathbb{E}_s\widehat{\Pi}_{s+1}^{ZLB}} = \frac{1}{\mu\chi\left(1-\beta\overline{R}p\right)-\beta\overline{R}\Theta p}.$$

Under complete markets ($\Theta = 0$), the elasticity is positive since $\mu\chi > 0$ and $\beta\overline{R}p < 1$. Thus, any additional shock which reduces expected inflation creates a labor market contraction. As explained above, this is the source of the two paradoxes. However, when $\Theta > \frac{\mu\chi}{p}(\beta^{-1}\overline{R}^{-1} - p)$, i.e. when markets are sufficiently incomplete, the elasticity is negative. In that case, a reduction in expected inflation creates a labor market expansion.

6 Pricing Risky Assets

This section explores asset pricing implications of the model. We show that the model generates a positive risk premium, but only if markets are incomplete. Intuitively, agents dislike asset with returns that co-move negatively with the probability of becoming unemployed, and hence require a discount relative to asset with acyclical returns.

For simplicity, consider the model with sticky wages ($\chi = 0$) and no sunspots. We focus on equilibria around the intended steady state. The stochastic discount factor of an employed household is given by $\Lambda_{e,s,s+1} = \beta\omega\left(1-\eta_{s+1}\right)\left(\vartheta/w\right)^{-\mu} + \beta\left(1-\omega\left(1-\eta_{s+1}\right)\right)$. Note that the period-$s$ conditional correlation between $\Lambda_{s,s+1}$ and η_{s+1} (and hence between $\Lambda_{s,s+1}$ and A_{s+1}) is perfectly negative, due to the fact that $\vartheta < w$. The appendix shows that the conditional variance of the stochastic discount factor is given by:

$$Var_s\left\{\Lambda_{e,s,s+1}\right\} = \beta^2\Theta^2\Gamma_\eta^2\sigma_A^2.$$

Note that under complete markets ($\Theta = 0$), we obtain $Var_s\left\{\Lambda_{e,s,s+1}\right\} = 0$, i.e. the stochastic discount factor is constant. Intuitively, when agents' income is fully insured against unemployment risk and wages are sticky, their income, and hence their desire to save, is completely constant.

When markets are incomplete, the precautionary savings motive emerges and fluctuates with the cycle since the amount of unemployment risk varies over the business cycle.

Exogenous payoffs: We now use the model to price risky assets with simple payoff structures. First, consider a risky asset that pays off $1 + A_{s+1} - \rho A_s$ in period $s+1$. We choose this payoff structure as it has the simplifying property that the expected payoff is one, while at the same time payoffs increase after an expansionary shock to productivity.

To obtain analytical tractability, we again assume that the asset is in zero net supply and that households cannot go short in the asset. As a result, the employed asset-poor households are the ones pricing the asset at the margin, whereas the other two types of households are in equilibrium at the no-short sale constraint. Krusell, Smith and Mukoyama (2011) exploit a similar setup to price risky asset under incomplete markets, but in an economy with exogenous endowments. Here, we analyze the importance of the endogenous feedback mechanism created by HANK and SAM, and study the effects of monetary policy on asset prices.

In the appendix, we show that the employed households' stochastic discount factor and the solution of the log-linearized model imply that the price of the risky asset, denoted z_s, is given by:

$$z_s = \mathbb{E}_s \Lambda_{e,s,s+1} - \beta \Theta \Gamma_\eta \sigma_A^2.$$

In the above equation, the term $\beta \Theta \Gamma_\eta \sigma_A^2$ is the discount relative to a risky asset. To see this, consider a riskless asset that pays out one unit of goods in the next period regardless of the state of the world (i.e. a real bond). Again imposing the no-shortsale constraint, it follows immediately from the households' discount factor that the price of the riskless asset is given by $\mathbb{E}_s \Lambda_{e,s,s+1}$.

The above equation thus makes clear that if markets are incomplete, i.e. $\Theta > 0$, there is a risk premium, which emerges despite the fact that the above equation is based on the solution of the log-linearized model.[22] Further, recall that Γ_η is the response of the job finding rate to a productivity shock. The magnitude of Γ_η depends on the strength of the endogenous interaction between HANK and SAM, as well as on the monetary policy rule. By responding more aggressively to economic shocks, the central bank stabilizes the economy, reducing the strength of the precautionary savings mechanism and thereby the risk premium. Finally, note that without shocks, i.e. $\sigma_A = 0$, there is no risk premium.

[22] In representative agent models risk premia typically vanish after log-linearization since in the steady state there is no risk. Recall that in our model, by contrast, there is still idiosyncratic risk in the steady state.

Endogenous payoffs: Consider now another risky asset with an payoff equal to $1+\widehat{\eta}_{s+1}-\rho\widehat{\eta}_{s}$. Note that, again, the expected payoff is one and that the payoff is increasing in next period's job finding rate. Again, we impose the no-shortsale constraint. The appendix shows that price of the asset is given by:

$$z_s = \mathbb{E}_s \Lambda_{e,s,s+1} - \beta \Theta \Gamma_\eta^2 \sigma_A^2$$

Note that in the return of the risky asset we now observe Γ_η^2 rather than Γ_η. This reflects the fact that the payoff of the asset is now endogenous. As a result, market frictions and monetary policy affect the risk premium via two channels: through he households' stochastic discount factor (via their unemployment risk) and through the asset payoff (via the equilibrium effects of household demand).

7 An Empirical Perspective

In this final section we propose a simple way of confronting the model with the data. Using the log-linearized Euler equation, we can obtain the following expression for the real interest rate,

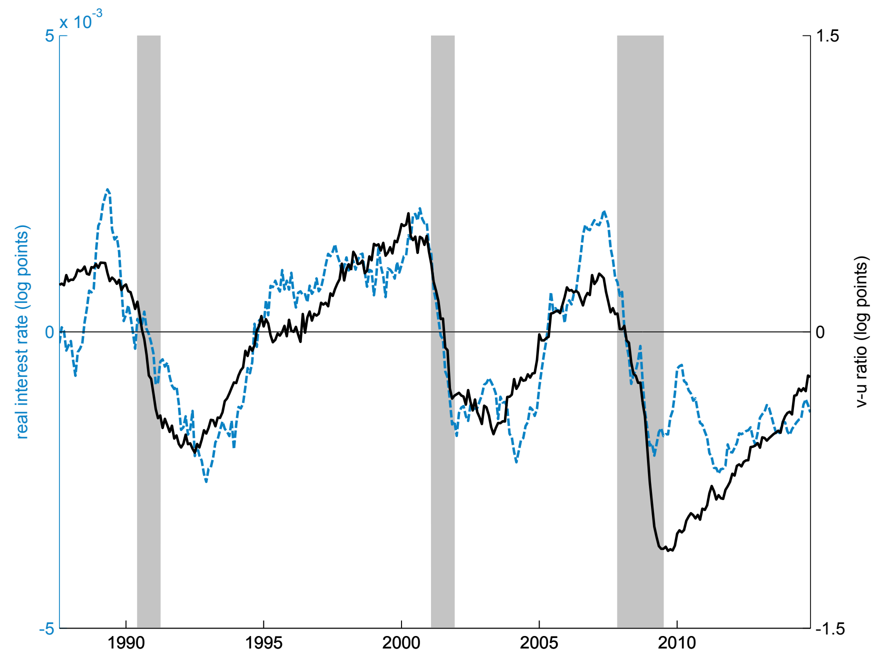
$$\widehat{R}_s^r = (1-\alpha)\left(-\mu\chi + (\mu\chi + \Theta)\,\beta\overline{R}\rho\right)\widehat{\theta}_s$$

where $\widehat{R}_s^r \equiv \widehat{R}_s - \mathbb{E}_s\widehat{\Pi}_{s+1}$. The above equation provides a direct relation between market tightness and the real interest rate, which we can confront with the data.

Note that under complete markets ($\Theta = 0$) we obtain $\frac{d\widehat{R}_s^r}{d\widehat{\theta}_s} = (1-\alpha)\,\mu\chi\,(\rho-1) < 0$. Thus, the complete markets model predicts that in a recession, when the labor market is less tight, the real interest rate increases. Intuitively, a transitory decline in income motivates households to borrow, pushing up the equilibrium real interest rate.

When Θ is sufficiently high, however, the relation between the real interest rate and market tightness is positive. Under incomplete markets, low labor market tightness strengthens the precautionary savings motive and hence pushes down the equilibrium real interest rate. Figure 4 presents the relation between the two variables over the period since Paul Volcker left the Federal Reserve. Data are expressed in percentage deviations from a linear trend, estimated over the period up to the end of 2007. The two series display a striking positive correlation, with a coefficient of 0.84. During all three recessions, indicated by shaded areas, the two variables jointly decline. Thus, the data appear to strongly favor the incomplete-markets model over its complete-markets counterpart.

Figure 4: Real interest rate (R^r) and labor market tightness (v/u) in the data.

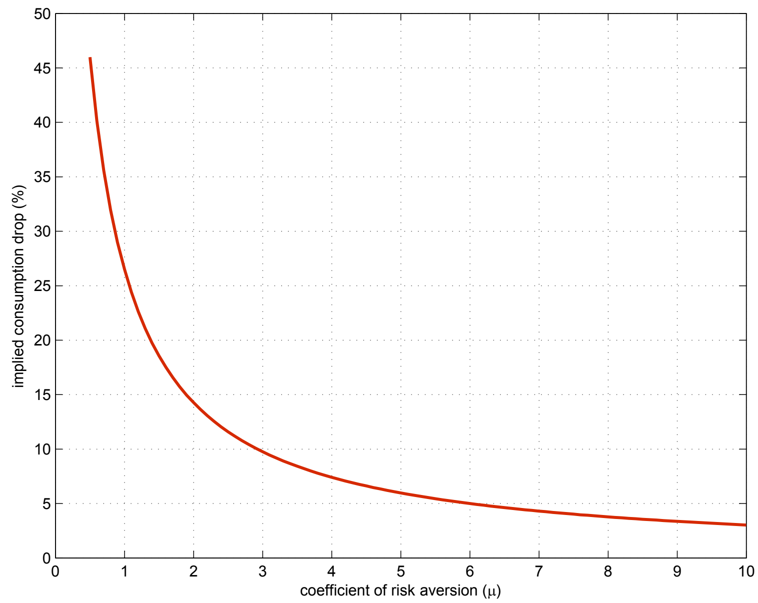


Notes: Real interest rate and labor market tightness (vacancy-unemployment ratio) in the United States; deviations from trend. The real interest rate is expressed on a monthly basis and is computed as the Federal Funds rate minus a six-month moiving average of CPI inflation. Vacancies are measured as the composite Help Wanted index from Barnichon (2010). Data series were logged and de-trended using a linear trend estimated over the period up to the end of 2007.

The data can also be used to directly parameterize Θ and get a sense of the quantitative importance of the key mechanism in the model. For simplicity, consider a model with sticky wages ($\chi = 0$). The log-linearized Euler equation implies that the ration of unconditional variances of the two variables are given by $\frac{Var(\widehat{R}^r_s)}{Var(\widehat{\theta}_s)} = (1-\alpha)^2 \Theta^2 \beta^2 \overline{R}^2 \rho^2$. Suppose for example that $\alpha = \frac{1}{2}$, $\rho = 0.99$ and $\beta\overline{R} = 1$.[23] Given the ratio of variances observed in the data, this implies that $\Theta = 0.0061$. To facilitate interpretation of this number, Figure 4 plots for a range of assumptions on the coefficient of risk aversion μ, the implied consumption loss upon unemployment (ϑ/w). For example, for a coefficient of risk aversion of 2, the calculation implies a consumption loss of about 15 percent, which seems reasonable in the light of empirical evidence.

[23] Under incomplete markets it typically holds that $\beta\overline{R} < 1$. Even then, however, the number tends to be close to one, and lowering it has very limited effects on the results.

Figure 5: Consumption loss upon unemployment.



8 Conclusion

We have proposed a simple and intuitive heterogeneous-agents New Keynesian (NK) model with endogenous unemployment, and highlighted that the interaction between market frictions can give rise to belief-driven fluctuations. Moreover, the interaction between these frictions produces potentially a significant amount of amplification of shocks to the economy. The essence of the interaction is that incomplete markets produces movements in aggregate demand in response to fluctuations in the job finding rate which impact on the supply side when there are nominal rigidities and creates a feedback mechanism. In particular, weak labor demand produces low goods demand which in itself produces low labor demand. The combination of HANK and SAM therefore has fundamental consequences and puts labor markets in the centre of the amplification and transmission mechanism.

We have also shown that the new NK model can resolve a large number of puzzles that have arisen in the macroeconomic literature. These involve the existence of persistent low growth equilibria with low but positive inflation, the impact of supply shocks on inflation dynamics, and various paradoxes at the ZLB. Intriguingly, the model can also provide a coherent framework for understanding the positive relationship between real interest rates and labor market tightness which can be observed in the US.

We have demonstrated that under incomplete markets the NK model becomes useful to analyze the link between monetary policy and financial asset prices. While we have limited the analysis to simple analytical exercises, it would be interesting to evaluate the extent to which a full-scale heterogeneous-agents NK can explain observed asset prices. Vice versa, financial markets data may be useful to impose empirical discipline on the new generation of NK models.

Throughout the analysis, we have assumed that government policies are summarized by a simple interest rate rule, subject to the zero lower bound. It would be interesting to think use the framework to obtain insights into the stabilization effects of other government policies, such as fiscal policy or labor market policies. Also, the framework could be used to consider optimal policies. We leave these issues for future research.

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Appendix

Steady-state Nash bargaining solution

The steady-state expressions of the asset-poor households' surplus and value functions are:

$$\begin{aligned} V^e\left(1-\beta\left(1-\omega\left(1-\eta\right)\right)\right) &= \frac{w^{1-\mu}}{1-\mu}-\zeta+\beta\omega\left(1-\eta\right)V^u, \\ V^u\left(1-\beta\left(1-\eta\right)\right) &= \frac{\vartheta^{1-\mu}}{1-\mu}+\beta\eta V^e, \end{aligned}$$

where we have exploited that in equilibrium the asset-poor households are the same and consume their incomes. Now substitute out V^u in the first equation:

$$V^e\left(1-\beta\left(1-\omega\left(1-\eta\right)\right)\right)=\frac{w^{1-\mu}}{1-\mu}-\zeta+\frac{\beta\omega\left(1-\eta\right)}{1-\beta\left(1-\eta\right)}\left(\frac{\vartheta^{1-\mu}}{1-\mu}+\beta\eta V^e\right).$$

$$V^e\left(1-\beta\left(1-\omega\left(1-\eta\right)\right)-\frac{\beta\omega\left(1-\eta\right)}{1-\beta\left(1-\eta\right)}\beta\eta\right)=\frac{w^{1-\mu}}{1-\mu}-\zeta+\frac{\beta\omega\left(1-\eta\right)}{1-\beta\left(1-\eta\right)}\frac{\vartheta^{1-\mu}}{1-\mu},$$

We can now express the two values as functions of η and w:

$$\begin{aligned} V^e\left(\eta,w\right) &= \frac{\frac{w^{1-\mu}}{1-\mu}-\zeta+\frac{\beta\omega(1-\eta)}{1-\beta(1-\eta)}\frac{\vartheta^{1-\mu}}{1-\mu}}{1-\beta\left(1-\omega\left(1-\eta\right)\right)-\frac{\beta\omega(1-\eta)\beta\eta}{1-\beta(1-\eta)}} \\ V^u\left(\eta,w\right) &= \frac{\frac{\vartheta^{1-\mu}}{1-\mu}+\beta\eta V^e\left(\eta,w\right)}{1-\beta\left(1-\eta\right)} \end{aligned}$$

The first-order condition to the Nash Bargaining problem is given by

$$\left(1-\upsilon\right)S^e=\upsilon S^f,$$

or,

$$\left(1-\upsilon\right)\left(V^e\left(\eta,w\right)-V^u\left(\eta,w\right)\right)=\upsilon\kappa\eta^{\alpha/(1-\alpha)}.$$

$$\left(V^e\left(\eta,w\right)-V^u\left(\eta,w\right)\right)=\frac{\upsilon}{1-\upsilon}\kappa$$

The above is an equation in two variables, which implicitly defines the wage as a function of the job finding rate, i.e the function $w(\eta)$.

Basic properties: Consider the special case in which $\eta=0$. From the Nash bargaining solution it follows that the wage must satisfy $V^e\left(0,w(0)\right)=V^u\left(0,w(0)\right)=\frac{\frac{\vartheta^{1-\mu}}{1-\mu}}{1-\beta}$. It follows that

$\frac{w(0)^{1-\mu}}{1-\mu} = \frac{\vartheta^{1-\mu}}{1-\mu} + \zeta$ and hence $w(0) > \vartheta$ whenever $\zeta > 0$.

At the other extreme, under $\eta = 1$ we get from the Nash Bargaining solution $V^e(1,w) = V^u(1,w) + \frac{\upsilon\kappa}{1-\upsilon}$. Also, the worker value functions imply that $V^e(1,w) - V^u(1,w) = \frac{w(1)^{1-\mu}}{1-\mu} - \zeta - \frac{\vartheta^{1-\mu}}{1-\mu}$. It follows that $\frac{w(1)^{1-\mu}}{1-\mu} = \frac{\vartheta^{1-\mu}}{1-\mu} + \zeta + \frac{\upsilon}{1-\upsilon}\kappa$ and hence $w(1) > w(0)$, $V^e(1,w(1)) > V^e(0,w(0))$ and $V^u(1,w) > V^u(0,w)$.

Finally, consider a case in which the worker has no bargaining power ($\upsilon = 0$). It follows from the Nash bargaining solution that in this case $V^e(\eta,w) = V^u(\eta,w)$ which implies that $\frac{w(\eta)^{1-\mu}}{1-\mu} = \frac{\vartheta^{1-\mu}}{1-\mu} + \zeta$. As a result, the real wage does not depend of η, i.e. the real wage is sticky.

Log-linearizing the model

Nash Bargaining block

The first-order condition to the Nash bargaining problem, together with the asset-poor workers' value functions are given by:

$$
\begin{aligned}
(1-\upsilon)\left(V_s^e - V_s^u\right) &= \upsilon\kappa\eta_s^{\alpha/(1-\alpha)}, \\
V_s^e &= \frac{w_s^{1-\mu}}{1-\mu} - \zeta + \beta\mathbb{E}_s\omega\left(1-\eta_{s+1}\right)V_{s+1}^u + \beta\mathbb{E}_s\left(1-\omega\left(1-\eta_{s+1}\right)\right)V_{s+1}^e, \\
V_s^u &= \frac{\vartheta^{1-\mu}}{1-\mu} + \beta\mathbb{E}_s\left(1-\eta_{s+1}\right)V_{s+1}^u + \beta\mathbb{E}_s\eta_{s+1}V_{s+1}^e.
\end{aligned}
$$

After log-linearization, the above system can be written in the following form:

$$
\mathbf{A}\begin{bmatrix}\widehat{V}_s^e \\ \widehat{V}_s^u \\ \widehat{w}_s\end{bmatrix} + \mathbf{B}\widehat{\eta}_s = \mathbb{E}_s\mathbf{C}\begin{bmatrix}\widehat{V}_{s+1}^e \\ \widehat{V}_{s+1}^u \\ \widehat{w}_{s+1}\end{bmatrix} + \mathbb{E}_s\mathbf{D}\widehat{\eta}_{s+1}
$$

where $\mathbf{A}$ and $\mathbf{C}$ are 3×3 matrices and $\mathbf{B}$ and $\mathbf{D}$ are 3×1 vectors, all consisting of parameter values. Note that none of the variables $\widehat{V}_s^e$, $\widehat{V}_s^u$ and $\widehat{w}_s$ is a state variable. Provided that $\widehat{\eta}_s$ follows some linear law of motion and given the law of motion for A_s, we can apply the method of undetermined coefficients to find solutions for $\widehat{V}_s^e$, $\widehat{V}_s^u$ and $\widehat{w}_s$ as linear functions of $\widehat{\eta}_s$. We denote the solution for the wage as $\widehat{w}_s = \chi\widehat{\eta}_s$, where it follows that χ is a function of the parameters that enter $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$.

Monetary Policy rule, Euler equation, Phillips Curve

The log-linerarized monetary policy rule is given by:

$$\widehat{R}_s = \delta_\pi \widehat{\Pi}_s + \delta_\theta \widehat{\theta}_s.$$

Next, consider the Euler equation of the employed households. Exploiting the fact that in Equilibrium $c_{e,s} = w_s$ and $c_{u,s} = \vartheta$, we can express the employed workers' Euler equation, Equation (20), as:

$$w_s^{-\mu} = \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} \left(\omega \left(1 - \eta_{s+1}\right) \vartheta^{-\mu} + \left(1 - \omega \left(1 - \eta_{s+1}\right)\right) w_{s+1}^{-\mu} \right)$$

and note that in the intended steady state we obtain $w^{-\mu} = \beta \overline{R} \left(\omega \left(1 - \eta\right) \vartheta^{-\mu} + \left(1 - \omega \left(1 - \eta\right)\right) w^{-\mu} \right)$. Log-linearizing the above equation around the intended steady state gives:

$$\begin{aligned} -\mu \widehat{w}_s &= \widehat{R}_s - \mathbb{E}_s \widehat{\Pi}_{s+1} - \beta \overline{R} \omega \eta \left(\vartheta / w\right)^{-\mu} \mathbb{E}_s \widehat{\eta}_{s+1} + \beta \overline{R} \omega \eta \mathbb{E}_s \widehat{\eta}_{s+1} - \mu \beta \overline{R} \left(1 - \omega \left(1 - \eta\right)\right) \mathbb{E}_s \widehat{w}_{s+1} \\ &= -\mu \beta \overline{R} \mathbb{E}_s \widehat{w}_{s+1} + \widehat{R}_s - \mathbb{E}_s \widehat{\Pi}_{s+1} - \beta \overline{R} \omega \eta \left(\left(\vartheta / w\right)^{-\mu} - 1 \right) \mathbb{E}_s \widehat{\eta}_{s+1} + \mu \beta \overline{R} \omega \left(1 - \eta\right) \mathbb{E}_s \widehat{w}_{s+1} \\ &= -\mu \beta \overline{R} \mathbb{E}_s \widehat{w}_{s+1} + \widehat{R}_s - \mathbb{E}_s \widehat{\Pi}_{s+1} - \beta \overline{R} \Theta \mathbb{E}_s \widehat{\eta}_{s+1} \end{aligned}$$

where $\Theta = \omega \eta \left(\left(\vartheta / w\right)^{-\mu} - 1 \right) - \chi \mu \omega \left(1 - \eta\right)$. Exploiting that $\widehat{w}_s = \chi \widehat{\eta}_s$ gives:

$$-\mu \chi \widehat{\eta}_s = \widehat{R}_s - \mathbb{E}_s \widehat{\Pi}_{s+1} - \beta \overline{R} \left(\Theta + \mu \chi\right) \mathbb{E}_s \widehat{\eta}_{s+1}.$$

Next, consider the firms' price setting condition, which can be written as:

$$\begin{aligned} & \phi \left(\Pi_s - 1\right) \Pi_s - \phi \mathbb{E}_s \Lambda_{s,s+1} \frac{y_{s+1}}{y_s} \left(\Pi_{s+1} - 1\right) \Pi_{s+1} \\ = \;& 1 - \gamma + \gamma \left(w_s + \kappa \eta_s^{\alpha/(1-\alpha)} - \left(1 - \omega\right) \kappa \mathbb{E}_s \Lambda_{s,s+1} \eta_{s+1}^{\alpha/(1-\alpha)} + \lambda_{v,s} \right). \end{aligned}$$

and note that at the intended steady state $\lambda_{v,s} = 0$ and $\Lambda_{s,s+1} = \beta$. Log-linearizing the equation around the intended steady state with $\Pi = 1$ gives:

$$\frac{\phi}{\gamma} \widehat{\Pi}_s - \frac{\phi}{\gamma} \beta \mathbb{E}_s \widehat{\Pi}_{s+1} = w \chi \widehat{\eta}_s - w A_s + \frac{\kappa}{q} \left(\frac{\alpha}{1 - \alpha} \widehat{\eta}_s - \frac{\alpha \beta \left(1 - \omega\right)}{1 - \alpha} \widehat{\eta}_{s+1} - \beta \left(1 - \omega\right) \widehat{\Lambda}_{s,s+1} \right)$$

where we have substituted out the wage using $\widehat{w}_s = \chi \widehat{\eta}_s$.

Reducing the model

Under the the two assumptions ($\delta_\pi = \frac{1}{\beta}$ and risk-neutrality of the equity investors) and in the absence of productivity shocks, the log-linearized Euler equation and pricing condition become:

$$\begin{aligned}
-\mu\chi\beta\widehat{\eta}_s + \mu\beta^2\overline{R}\chi\mathbb{E}_s\widehat{\eta}_{s+1} &= \widehat{\Pi}_s - \beta\mathbb{E}_s\widehat{\Pi}_{s+1} + \frac{\beta\delta_\theta}{1-\alpha}\widehat{\eta}_s - \beta^2\overline{R}\Theta\mathbb{E}_s\widehat{\eta}_{s+1} \\
w\chi\widehat{\eta}_s + \frac{\kappa}{q}\left(\frac{\alpha}{1-\alpha}\widehat{\eta}_s - \frac{\alpha\beta\left(1-\omega\right)}{1-\alpha}\mathbb{E}_s\widehat{\eta}_{s+1}\right) &= \frac{\phi}{\gamma}\left(\widehat{\Pi}_s - \beta\mathbb{E}_s\widehat{\Pi}_{s+1}\right)
\end{aligned}$$

where in the first equation we have substituted out the interest rate using $\widehat{R}_s = \delta_\pi\widehat{\Pi}_s + \delta_\theta\widehat{\theta}_s$, and tightness using $\widehat{\theta}_s = \frac{\widehat{\eta}_s}{1-\alpha}$. Using the first equation to substitute out $\widehat{\Pi}_s - \beta\mathbb{E}_s\widehat{\Pi}_{s+1}$ in the second equation gives:

$$w\chi\widehat{\eta}_s + \frac{\kappa}{q}\left(\frac{\alpha}{1-\alpha}\widehat{\eta}_s - \frac{\alpha\beta\left(1-\omega\right)}{1-\alpha}\mathbb{E}_s\widehat{\eta}_{s+1}\right) = \frac{\phi}{\gamma}\left(-\mu\chi\beta\widehat{\eta}_s + \mu\beta^2\overline{R}\chi\mathbb{E}_s\widehat{\eta}_{s+1} - \frac{\beta\delta_\theta}{1-\alpha}\widehat{\eta}_s + \beta^2\overline{R}\Theta\mathbb{E}_s\widehat{\eta}_{s+1}\right).$$

Collecting terms gives:

$$\mathbb{E}_s\widehat{\eta}_{s+1} = \Psi\widehat{\eta}_s,$$

where

$$\Psi = \frac{\frac{\phi}{\gamma}\mu\chi\beta + \frac{\phi}{\gamma}\frac{\beta\delta_\theta}{1-\alpha} + w\chi + \frac{\kappa}{q}\frac{\alpha}{1-\alpha}}{\frac{\kappa}{q}\frac{\alpha\beta(1-\omega)}{1-\alpha} + \frac{\phi}{\gamma}\mu\beta^2\overline{R}\chi + \frac{\phi}{\gamma}\beta^2\overline{R}\Theta}.$$

Adding productivity shocks

With productivity shocks the model becomes:

$$\begin{aligned}
& w\chi\widehat{\eta}_s - wA_s + \frac{\kappa}{q}\left(\frac{\alpha}{1-\alpha}\widehat{\eta}_s - \frac{\alpha\beta\left(1-\omega\right)}{1-\alpha}\mathbb{E}_s\widehat{\eta}_{s+1}\right) \\
&= \frac{\phi}{\gamma}\left(-\mu\chi\beta\widehat{\eta}_s + \mu\beta^2\overline{R}\chi\mathbb{E}_s\widehat{\eta}_{s+1} - \frac{\beta\delta_\theta}{1-\alpha}\widehat{\eta}_s + \beta^2\overline{R}\Theta\mathbb{E}_s\widehat{\eta}_{s+1}\right), \\
A_s &= \rho A_{s-1} + \sigma_A\varepsilon_s^A,
\end{aligned}$$

which we can rewrite as

$$\begin{aligned}
\mathbb{E}_s\widehat{\eta}_{s+1} &= \Psi\widehat{\eta}_s - \Omega A_s, \\
A_s &= \rho A_{s-1} + \sigma_A\varepsilon_s^A,
\end{aligned}$$

where

$$\Omega = \frac{w}{\frac{\kappa}{q}\frac{\alpha\beta(1-\omega)}{1-\alpha} + \frac{\phi}{\gamma}\mu\beta^2\overline{R}\chi + \frac{\phi}{\gamma}\beta^2\overline{R}\Theta}.$$

The Euler equation at the ZLB

Consider the setup described in Section 5.3. At the ZLB, it holds, for $x = \{\eta, \Pi\}$, that $\mathbb{E}_s x_{s+1} = p\mathbb{E}_s x_{s+1}^{ZLB} + (1-p)x$, where x is the level at the indended steady state. Log-linearization of this equation around the intended steady state gives $\mathbb{E}_s \widehat{x}_{s+1} = p x_{s+1}^{ZLB}$. Note further that at the ZLB, $R_s = 1$ and hence hence $\widehat{R}_s = -\ln \overline{R}$.

Plugging these results into the Euler equation, log-linearized around the steady state and as derived above, gives:

$$\left(\mu\chi\left(1-\beta\overline{R}p\right) - \beta\overline{R}\Theta p\right)\widehat{\eta}_s^{ZLB} = \ln\overline{R} + \mathbb{E}_s\widehat{\Pi}_{s+1}^{ZLB}.$$

where we have used that, conditional on staying at the ZLB, it holds that $\mathbb{E}_s\widehat{\eta}_{s+1}^{ZLB} = p\mathbb{E}_s\widehat{\eta}_s^{ZLB}$. After differentiation, we obtain

$$\frac{d\widehat{\eta}_s^{ZLB}}{d\mathbb{E}_s\widehat{\Pi}_{s+1}^{ZLB}} = \frac{1}{\mu\chi\left(1-\beta\overline{R}p\right) - \beta\overline{R}\Theta p}.$$

Pricing risk assets

Consider the stochastic discount factor of the employed, asset-poor households:

$$\Lambda_{e,s,s+1} = \beta\omega\left(1-\eta_{s+1}\right)(\vartheta/w)^{-\mu} + \beta\left(1-\omega\left(1-\eta_{s+1}\right)\right)$$

Given the solution, the job finding rate is –up to a first-order approximation– given by $\eta_s = \eta + \eta\Gamma_\eta A_s$. We exploit this to write the period$-s$ conditional expectation and variance of $\Lambda_{e,s,s+1}$, respectively, as:

$$\begin{aligned}\mathbb{E}_s\Lambda_{e,s,s+1} &= \beta\omega\left(1-\mathbb{E}_s\eta_{s+1}\right)(\vartheta/w)^{-\mu} + \beta\left(1-\omega\left(1-\mathbb{E}_s\eta_{s+1}\right)\right) \\ &= \beta\omega\left(1-\eta-\rho\eta\Gamma_\eta A_s\right)(\vartheta/w)^{-\mu} + \beta\left(1-\omega\left(1-\eta-\rho\eta\Gamma_\eta A_s\right)\right)\end{aligned}$$

and

$$
\begin{aligned}
Var_s\left\{\Lambda_{e,s,s+1}\right\} &= \beta^2\omega^2\left(1-(\vartheta/w)^{-\mu}\right)^2 Var_s\left\{\eta_{s+1}\right\}, \\
&= \beta^2\omega^2\left(1-(\vartheta/w)^{-\mu}\right)^2 \eta^2\Gamma_\eta^2 Var_s\left\{\rho A_s+\sigma_A\varepsilon_{s+1}^A\right\}, \\
&= \beta^2\omega^2\left(1-(\vartheta/w)^{-\mu}\right)^2 \eta^2\Gamma_\eta^2\sigma_A^2 \\
&= \beta^2\Theta^2\Gamma_\eta^2\sigma_A^2
\end{aligned}
$$

Exogenous payoffs: The pricing equation for the asset that pays off $1+A_{s+1}-\rho A_s$ in period $s+1$ reads:

$$
\begin{aligned}
z_s &= \mathbb{E}_s\left\{\Lambda_{e,s,s+1}\left(1+A_{s+1}-\rho A_s\right)\right\} \\
&= \mathbb{E}_s\Lambda_{e,s,s+1}\mathbb{E}_s\left(1+A_{s+1}-\rho A_s\right)+Cov_t(\Lambda_{e,s,s+1},1+A_{s+1}-\rho A_s) \\
&= \mathbb{E}_s\Lambda_{e,s,s+1}-\sqrt{Var_s\left\{\Lambda_{e,s,s+1}\right\}Var_s\left\{1+A_{s+1}-\rho A_s\right\}} \\
&= \mathbb{E}_s\Lambda_{e,s,s+1}-\beta\Theta\Gamma_\eta\sigma_A^2
\end{aligned}
$$

where we exploited the fact that the $Cor_s\left\{\Lambda_{s,s+1},A_{s+1}\right\}=-1$, that $1+\mathbb{E}_sA_{s+1}-\rho A_s=1$, and that $Var_s\left\{1+A_{s+1}-\rho A_s\right\}=\sigma_A^2$.

Endogenous payoffs: Consider now another risky asset with an payoff equal to $1+\widehat{\eta}_{s+1}-\rho\widehat{\eta}_s$. The pricing equation for this asset reads:

$$
\begin{aligned}
z_s &= \mathbb{E}_s\left\{\Lambda_{e,s,s+1}\left(1+\widehat{\eta}_{s+1}-\rho\widehat{\eta}_s\right)\right\} \\
&= \mathbb{E}_s\left\{\Lambda_{e,s,s+1}\left(1+\Gamma_\eta A_{s+1}-\rho\Gamma_\eta A_s\right)\right\} \\
&= \mathbb{E}_s\Lambda_{e,s,s+1}-\sqrt{Var_s\left\{\Lambda_{e,s,s+1}\right\}Var_s\left\{1+\Gamma_\eta A_{s+1}-\rho\Gamma_\eta A_s\right\}} \\
&= \mathbb{E}_s\Lambda_{e,s,s+1}-\beta\Theta\Gamma_\eta^2\sigma_A^2
\end{aligned}
$$

Risk-averse investors

When we log-linearized the model, we have assumed for simplicity that the asset-rich firm owners are risk neutral. The reason is that, technically, the unemployment rate becomes a state variable for inflation and the job finding rate, once we assume risk averse investors. With an additional state variable, the analytical solution of the model becomes more cumbersome, detracting from the key intuitions of the model.

Below, we use numerical simulations to compare versions with risk-neutral and risk-averse investors, showing only very small differences. We parametrize the model as follows. We choose the subjective discount factor β target a steady-state interest rate of 3 percent per annum. The coefficient of risk aversion, μ, is set to 2, whereas the elasticity of substitution between goods, γ, is set to 6. To calibrate the price-stickiness parameter ϕ, we exploit the observational equivalence between the Calvo and Rotemberg versions of the log-linearized New Keynesian model, and target an average price duration of 5 months. The home production parameter, ϑ, is set to imply a 15 percent consumption drop upon unemployment.

The vacancy cost is parametrized to target a stead-state hiring cost of about 4 percent of the quarterly wage, following Silva and Toledo (2009). We further target a monthly job finding rate of 0.3 and set the job loss rate, ω, to 2 percent. The matching function elasticity parameter, α, is set to 0.5. Regarding the monetary policy rule, we set $\delta_\pi = 1.5$ and $\delta_\theta = 0$. The persistence parameter of the technology shock is set to $\rho = 0.95$. For simplicity we assume sticky wages ($\chi = 0$).

The left panel of the figure below ("incomplete markets") plots the response of the job finding rate to a positive technology shock. On impact, the response is larger with risk-averse investors. In subsequent periods, the pattern reverses and the response is smaller with risk-averse investors. Quantitatively, however, the differences are very small.

Next, we consider a version of the model in which we set the home production parameter ϑ such that there is no consumption loss upon job loss. Effectively, this removes the incomplete-markets wedge from the model. The right panel of the figure below ("complete markets") again compares the versions with risk-averse and risk-neutral investors. The differences are similar to the complete markets case. Most importantly, differens are again very small.

Figure 6: Responses to a positive technology shock.