Intermediation in Networks

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Abstract

This paper studies bargaining and exchange with intermediation in a network setting. Possibilities to trade are restricted through a network of existing relationships and traders bargain over the division of available gains from trade along feasible routes. Using a stochastic model of bargaining, I characterize stationary equilibrium payoffs as the fixed point of a set of intuitive value function equations and study the relationship between network structure and payoffs. The model shows how with competing trade routes agents that are not essential to a trade opportunity receive a payoff of zero as trade frictions go to zero.

1 Introduction

This paper presents a model of bargaining and exchange to study intermediation in networks. The network perspective focuses on the study of markets in which existing relationships matter for the interaction of economic agents. Such networked markets appear in many settings, including for example markets for agricultural goods in developing countries as well as over-the-counter (OTC) markets in financial assets. Recent years have seen a significant increase in such off-exchange trading[1] which often involves brokers and market makers that provide intermediation services. The network structure of trade relationships amongst banks is documented by Upper and Worms (2004) and Craig and von Peter (2010) who report a tiered structure in the German interbank market. Their data match a core-periphery structure with many peripheral banks that do not trade directly with others but only through the well-connected intermediaries of the core.

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[1]The Bank of International Settlements in quarterly data reports the total amount of OTC derivative contracts outstanding increasing from about US$ 80,000bn at the end of 1998 to over US$ 600,000bn at the end of 2010, with a peak of US$ 673,000bn in June 2008. Source: BIS Quarterly Review, September 2011

In this paper I employ an explicit network perspective on exchange with intermediation to focus attention on the role and value of relationships used to facilitate transactions between parties that otherwise might lack the opportunity to conduct trade directly. Such reliance on existing relationships might arise from reputational concerns, trust or the need for collateral provisions to be in place, for example from previous transactions. Existing relationships may also help in overcoming significant search costs involved in identifying trade opportunities or finding a suitable counterparty for more specialized asset classes. It is the provision of intermediation in settings where such relationships are critical to trade that this paper deals with. I present a model of bargaining and exchange with intermediation in a network setting, investigating the patterns to intermediation and their dependence on the network structure as well as the payoffs for intermediaries and trading parties.

The paper is structured as follows. Section 2 provides the literature context for the research questions investigated. Section 3 sets out the main model and Section 4 provides equilibrium analysis as well as the key results of the paper. Section 5 concludes.

2 Literature Context

This paper contributes to the literature on intermediaries as well as the the growing literature on networked markets and financial networks in particular.

The provision of intermediation services and middlemen activities which this paper investigates in a network setting has been investigated in other non-structural frameworks by several authors, with overviews provided in Bose (2001) and Spulber (1999). Intermediaries have been credited with a number of different functions, including the provision of immediacy (Demsetz, 1968) or acting as a screening device between different types of traders that might be prevented from engaging with each other as in Bose and Pingle (1995). A seminal contribution in this literature is provided by Rubinstein and Wolinsky (1987), who investigate a setting with three types of agents: buyers, sellers and middlemen. Trade is conducted on the basis of stochastic pairwise matching and a steady state equilibrium is derived.[2] A key insight of that paper is that the outcome of trade and the terms of trade depend on whether the middleman take ownership of the good from sellers or work on a consignment basis. In the first case, the market is biased in favor of buyers, whereas in the second case symmetry between parties is restored.

In the financial markets literature on intermediation in markets, classic contributions include Glosten and Milgrom (1985) and Kyle (1985) who consider the impact of private information in asset markets with intermediaries acting as market makers. An explicit discussion of OTC markets is provided by Duffie et al. (2005) who construct a model of search and bargaining with buyers, sellers and market-makers. As the other papers cited above, their model is free of explicitly

[2] In steady state equilibrium the outflow of pairs of traders which conclude a trade is exactly balanced by an exogenously given inflows of agents.

structural features and thus does not provide insights into the role of structural patterns, which are the focus of this paper.

In contrast to the work cited above, structural features are at the core of a growing literature on exchange in networks, which has been a very active field recently with numerous contributions. Seminal early works in this field include Corominas-Bosch (2004) on bargaining in networks and the exchange model in Kranton and Minehart (2001). Both adopt a bipartite networks approach, precluding an analysis of intermediation. More recent contributions in this direction include Manea (2011) and Elliott (2011). Models which take explicit account of intermediation are provided by Gale and Kariv (2007) and Blume et al. (2009). The latter is probably closest in outlook to the present paper. There the authors investigate a trading network with price setting traders. Traders set bid and ask prices and buyers and sellers choose from the offered menu. The authors establish existence and efficiency of trade equilibria and link payoffs to network structure, showing that positive payoffs depend on the traders adding marginal value to the network. However, their model cannot show that such traders indeed extract positive surplus as their framework permits multiple equilibria, which limits predictive power. In contrast to their work, I consider a setting with explicit bargaining in which surplus is allocated in a dynamic setting.

The literature on financial networks employs network tools to analyze various aspects of financial markets, including risk sharing and contagion amongst financial institutions. An overview is provided in Allen and Babus (2009). Babus (2010) provides a network perspective to OTC trading and investigates the incentives for financial institutions seeking to exchange assets to form relationships. In her model, links describe relationships which allow banks to use repeated interactions instead of costly collateral to implement and enforce exchange agreements.

Finally, at a technical level, this paper employs the framework of stochastic bargaining games with perfect information analyzed in detail in Merlo and Wilson (1995, 1998) and extends it for use in analyzing games on networks.

3 The Model

This section presents a model of exchange with intermediation on a network. We consider a setting in which agents' interactions are restricted by a *network* of relationships. Agents have access to trade opportunities that generate surplus, e.g. an asset trade between a buyer and a seller. Agents are matched according to the network of existing relationships and bargain over the allocation of the available surplus within feasible trade routes. The bargaining protocol allows for the random selection of trade routes as well as the identity of proposer, incorporating the notion of competition between different alternative routes.

Players and Network Let the set $N = \{1, 2, ..., n\}$ denote a set of agents. Agents interact according to a network denoted by $G = (N, E)$ where the set of undirected edges E describes the set of feasible bilateral trades. Agents can trade with each other directly only if there exists a link between them. As will be described in greater detail below, trade between two nodes that are only indirectly connected is feasible through intermediaries if there exists at least one path between them.

Trade opportunities We assume there is an agent $A \in N$ – the seller – who holds a single, indivisible good that she can sell to each of a set of other nodes, $B = \{B_1, B_2, \dots\}$, who are characterized by their valuation of the good b_i. Remaining nodes in N have zero valuation for the good but may act as intermediaries. We model exchange using buyer/seller pairs but it may also reflect some other value adding interaction between two parties, such as liquidity provision, R & D cooperation or joint entrepreneurial efforts. We thus focus on a single trade opportunity specific to a given seller, reflecting the notion of *thin markets*. This assumption approximates trade in highly individualized products such as the complex financial securities commonly traded in OTC markets. This is in contrast to *thick* markets of more generic assets such as commodities or standard financial products where there may be many buyers and sellers in the market at the same time.

Routes For each buyer/seller pair, trade is feasible if there exists one path in the given network G connecting them. We label the set of paths connecting A and a given buyer B_i as *routes*. Depending on the network N for each given buyer-seller pair there may be several feasible routes.

Matching and bargaining protocol The model operates in discrete time. In each period, traders are matched and bargain under a stochastic route selection and bargaining protocol adapted from Merlo and Wilson (1995) as follows. At the beginning of each period, a stochastic process σ determines both a trade route and an order of play for agents on this route. Based on this draw, players that are on the route bargain according to the order prescribed within the state.

Specifically, σ generates a state $s \in S$ in each period characterized by three elements:

1. A buyer $B(s) \in B$ and associated valuation $b(s)$, representing the surplus available if trade is concluded in this state.
2. A route $R(s) \subset N$ connecting the pair of agents who have the trade opportunity. Note that any $R(s)$ contains A and $B(s)$. The route is drawn from the set of shortest paths between seller and buyer.
3. A permutation $\rho(s)$ on $R(s)$ which denotes the order in which the traders move in the bargaining protocol. $\rho_i(s)$ denotes the player moving in ith position. Following Merlo and Wilson (1995) we denote by $\kappa(s) \equiv \rho_1(s)$ the first mover in the order, labeled "proposer".

To simplify the analysis, I assume that the process σ is independent across periods, such that each period's draw is independent of the previous period's state. This assumption allows me to dispense with conditioning on the current state whenever expectations about future realizations are formed and is in the spirit of standard alternating bargaining games. However, much of the analysis will carry through to more general stochastic processes with suitable modifications.

On realization of state s, trader $\kappa(s)$ may propose an allocation or pass. If a proposal is made, this takes the form of an n-dimensional vector x such that $\sum_{i \in N} x_i \leq b(s)$ which represents a split of available surplus amongst all players, allocating a share x_i of the good to each trader in N. The other traders on the route then respond sequentially in order given by $\rho(r)$ by accepting or rejecting the proposal. This process continues until either (i) one player rejects proposal x or (ii) all players in $R(s)$ have accepted it.

If all responders accept x, the proposed split is implemented and the game ends. If the proposer passes or at least one responder rejects the proposed split, bargaining terminates and the game moves to the next period in which a new state – s' – consisting of both a route $r(s')$ and a new order of play $\rho(s')$, is redrawn and the bargaining process is repeated. This sequence is continued until an allocation is accepted by all players.

Payoffs Payoffs are linear in the share of surplus allocated, with common discount factor $\beta \in (0, 1)$. If proposal x is accepted in period t, player i receives utility:

$$u_i(x) = \beta^t x_i \tag{1}$$

We assume that the surplus to be allocated is bounded above by $\bar{b} \geq b(s) \forall s$ and thus $u_i(x) \to 0$ as $t \to \infty$ without agreement being reached.

The model describes an infinite horizon dynamic game. Players have to take a decision in two distinct roles: as proposer and as responder. As proposer, a player suggests a split of surplus on a given route conditional on the route selected and being selected as proposer. As responder, players have to decide whether to accept or reject a proposed surplus division. This decision is conditioned on the selected route and proposer as well as the suggested division of surplus.

Note that bargaining in the model is multilateral and the good remains with the seller unless agreement with all steps on the route to the buyer has been reached. Potentially interesting considerations which arise from the good "traveling" along the route, such as the hold-up power that lower links have over intermediaries[3] or counterparty risk associated with disappearing resale opportunities, thus remain outside the model. The setting here is more directly applicable to intermediators acting as a broker rather than a market-maker.[4]

[3] See the discussion in Rubinstein and Wolinsky (1987) concerning the difference between middlemen taking ownership of the good and acting on consignment.

[4] In the wake of the 2008 financial crisis brokers in corporate bond markets appear to increasingly show the behavior implied in the model: *"In the wake of the financial crisis and ahead of tighter regulatory constraints,*

Histories and strategies are defined as usual. A history is a sequence of realized states and actions taken by players. A strategy specifies a feasible action at every history when a player must act. We restrict attention to pure strategies.

4 Equilibrium Analysis

This section develops the equilibrium analysis of the model. We restrict attention to Markov perfect equilibria (MPE) in pure strategies, that is, subgame perfect equilibria consisting of strategies which are history independent and condition only on the drawn state, that is, the selected route and the order of proposals, and the offer on the table in the given period.

4.1 Equilibrium Payoff Characterization

A unique MPE in pure strategies exists and can be characterized through an intuitive set of recursive equations using results derived in Merlo and Wilson (1998) and extending the analysis to the setting of networked markets.

As additional notation, denote by (S^μ, μ) a stationary *outcome* where $S^\mu \subseteq S$ denotes the set of states in which agreement is struck and $\mu(s)$ specifies the allocation that is proposed and accepted if $s \in S^\mu$ is reached. Associated with outcome (S^μ, μ) is a stopping time τ (a random variable dependent on the realization of σ) and an expected payoff vector for each state s defined by $\upsilon^\mu(s) = E\left[\beta^\tau \mu(\sigma_\tau) | \sigma_0 = s\right] = E\left[\beta^\tau \mu(\sigma_\tau)\right]$, where the last step uses independence in the stochastic process.

Proposition 1. *The bargaining game has a unique MPE, characterized by payoff function $f(s)$ as follows:*

a. If $b(s) \geq \beta \sum_{j \in R(s)} f_j(s')$:

$$f_i(s) = \begin{cases} b(s) - \beta E\left[\sum_{j \in R(s)\setminus i} f_j(s')\right] & \text{for Proposer } i = \kappa(s) \\ \beta E\left[f_i(s')\right] & \text{for Responder } i \in R(s) \setminus \kappa(s) \\ 0 & \text{for Excluded } i \notin R(s) \end{cases} \tag{2}$$

b. If $b(s) < \beta \sum_{j \in R(s)} f_j(s')$:

$$f_i(s) = \beta E\left[f_i(s')\right] \ \forall i \in N \tag{3}$$

The proof has been relegated to Appendix A.1. The equilibrium payoff function distinguishes two cases:

large Wall Street dealers have become far less willing to hold the risk of owning corporate bonds, known in market parlance as 'inventory,' in order to facilitate trading for their clients. Instead, they are increasingly trying to match buyers and sellers, acting more as a pure intermediary, rather than stockpiling bonds and encouraging a liquid market for secondary trading." Source: Financial Times, November 8, 2011

1. If available surplus exceeds the total expected value of moving to the next stage for players on the selected route, then f indicates that the proposer extracts from responding parties on the selected route all surplus over and above their outside option value given by $E[f(s')]$, leaving zero to traders not included on the route.

2. If available surplus in a given state s is less than the expected value of moving to the next stage for players on the selected route, f assigns that payoff to each player.

Note that whilst excluded players receive a zero payoff in the first case, in the latter case they have a positive expected payoff reflecting the fact that they may be included in negotiations in the next period. The result allows the analysis of equilibrium outcomes and payoffs for a wide range of possible trade networks and buyer valuations on the basis of a set of equations describing value functions in a recursive manner.

For simplicity, we restrict attention in the following sections to cases in which all buyers have the same valuation b. Note that this restriction removes from the analysis a number of potentially interesting features such as instances where players might want to hold out from agreeing on certain routes in order to wait for a more advantageous state involving a buyer with a higher valuation.

4.2 Example

As an illustration consider the simple example in Figure 1. There is a single buyer with a valuation of one and two possible intermediation routes. The state space thus contains 12 elements: for each route three possible proposers and for each proposer two possible orders for the remaining two players on the relevant route. For the purposes of this example we assume that each state is equally likely under σ. The states are listed in Table 1. Note that for the purposes of the equilibrium characterization, many states fall into pairs that are equivalent. For example, states 1 and 2 share the same buyer, the same route as well as the same partition of agents into proposer and responders. Proposition 1 implies that responders receive equal payoffs irrespective of their position in the order of play amongst the group of responders.

By Proposition 1 equilibrium payoffs are characterized by the following conditions, where $V_i(j)$ describes expected payoff for player i in a state where player j is proposer.

- For the seller A:

$$f_A(A) = 1 - f_{I_1}(A) - f_{B_1}(A) \tag{4}$$

$$= 1 - f_{I_2}(A) - f_{B_1}(A) \tag{5}$$

$$f_A(I_1) = \frac{\beta}{6}\left[2f_A(A) + f_A(I_1) + f_A(I_2) + 2f_A(B_1)\right] \tag{6}$$

$$f_A(B_1) = \frac{\beta}{6}\left[2f_A(A) + f_A(I_1) + f_A(I_2) + 2f_A(B_1)\right] = f_A(I) \tag{7}$$

- For the buyer B_1 analogous to the seller.

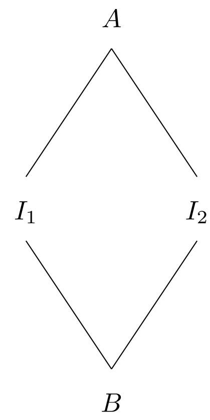

Figure 1: An example with two intermediaries

State s	$R(s)$	$\rho(s)$	$\kappa(s)$	$P(s)$
1	$\{A, I_1, B_1\}$	$\{A, I_1, B_1\}$	A	$\frac{1}{12}$
2	$\{A, I_1, B_1\}$	$\{A, B_1, I_1\}$	A	$\frac{1}{12}$
3	$\{A, I_1, B_1\}$	$\{I_1, A, B_1\}$	I_1	$\frac{1}{12}$
4	$\{A, I_1, B_1\}$	$\{I_1, B_1, A\}$	I_1	$\frac{1}{12}$
5	$\{A, I_1, B_1\}$	$\{B_1, I_1, A\}$	B_1	$\frac{1}{12}$
6	$\{A, I_1, B_1\}$	$\{B_1, A, I_1\}$	B_1	$\frac{1}{12}$
7	$\{A, I_2, B_1\}$	$\{A, I_2, B_1\}$	A	$\frac{1}{12}$
8	$\{A, I_2, B_1\}$	$\{A, B_1, I_2\}$	A	$\frac{1}{12}$
9	$\{A, I_2, B_1\}$	$\{I_2, A, B_1\}$	I_2	$\frac{1}{12}$
10	$\{A, I_2, B_1\}$	$\{I_2, B_1, A\}$	I_2	$\frac{1}{12}$
11	$\{A, I_2, B_1\}$	$\{B_1, I_2, A\}$	B_1	$\frac{1}{12}$
12	$\{A, I_2, B_1\}$	$\{B_1, A, I_2\}$	B_1	$\frac{1}{12}$

Table 1: State space for example with two intermediaries

- For the intermediary I_1 (and analogous for I_2):

$$f_{I_1}(I_1) = 1 - f_A(I_1) - f_{B_1}(I_1) \tag{8}$$

$$f_{I_1}(A) = \frac{\beta}{6}\left[f_{I_1}(A) + f_{I_1}(I_1) + f_{I_1}(B_1)\right] \tag{9}$$

$$f_{I_1}(B_1) = \frac{\beta}{6}\left[f_{I_1}(A) + f_{I_1}(I_1) + f_{I_1}(B_1)\right] \tag{10}$$

Solving this system payoffs can be computed for each player:

- For the seller A:

$$f_A(A) = \frac{2-\beta}{2} \tag{11}$$

$$f_A(I_1) = \frac{(2-\beta)\beta}{2(3-2\beta)} \tag{12}$$

$$f_A(I_2) = f_A(I_1) \tag{13}$$

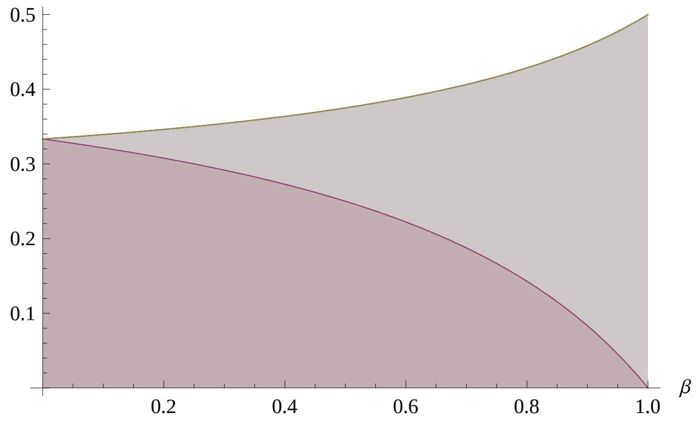

Figure 2: Expected payoffs for traders with two competing intermediaries

- For the buyer B_1 analogous to the seller.
- For the intermediary I_1 (and analogous for I_2):

$$f_{I_1}(I_1) = \frac{(3-\beta)(1-\beta)}{3-2\beta} \tag{14}$$

$$f_{I_1}(A) = \frac{(1-\beta)\beta}{2(3-2\beta)} \tag{15}$$

$$f_{I_1}(B_1) = f_{I_1}(A) \tag{16}$$

From these expressions ex ante expected payoffs are computed by multiplying each term with the probability of the relevant state occurring. Expected payoffs are presented in Figure 2 for different values of β, with the top line representing payoffs for buyer and seller and the bottom line representing those for the intermediaries. Note that expected payoffs for buyer and seller are identical, reflecting their symmetry within the network structure and the stochastic process. Similarly, payoffs for the two intermediaries are identical.

The example also illustrates how competition between the two intermediaries results in their payoffs being lower. In particular, as trade frictions vanish with $\beta \to 1$, payoffs for intermediaries tend towards zero, replicating the outcome of a Bertrand-type setting with simultaneous offers being made by intermediaries. I return to this feature of the model in Section 4.4.

4.3 Efficiency

This section considers the efficiency properties of the equilibrium. Given available surplus is the same for each buyer, efficiency requires that agreement be concluded instantly, that is in every state.

Proposition 2. *The MPE of the game is efficient. Let (S^μ, μ) be an outcome in the MPE of the game. Then $S^\mu = S$ and agreement is struck in all states.*

Proof. We proof by contradiction. Assume $\exists \tilde{s}$ in which no agreement is struck. Then by Proposition 1:

$$b - \beta \sum_{j \in R(\tilde{s} \setminus i)} < \beta E\left[f_i(s')\right] \tag{17}$$

Rearranging and combining expected payoffs from delay:

$$\begin{aligned} b &< \beta \sum_{j \in R(\tilde{s})} E\left[f_j(s')\right] && (18) \\ &\leq \beta \sum_{j \in N} E\left[f_j(s')\right] && (19) \\ &= \beta b && (20) \end{aligned}$$

where the last line delivers the contradiction. □

Proposition 2 implies that trade is concluded even along intermediation routes which may involve relatively large numbers of intermediaries when shorter, more direct routes are available. Thus, the intuitive prediction that it may be better for buyer and seller to delay trade in such situations does not hold. This is due to the fact that rents for intermediaries on the longer route are adjusted downwards and reflect the constraint exerted by the presence of the shorter route. Delay may be beneficial in settings where each stage of intermediation involves a cost or where different routes involve different amounts of total surplus. Neither feature is included in the model at this stage but may provide insights with further work.

4.4 Structural Features and Equilibrium Payoffs

Next we consider the relationship between structural features of the trade network and equilibrium payoffs. One implication of Proposition 1 is that players excluded in a state where agreement is struck receive a zero payoff, which is unsurprisingly given that such players are not involved in decision making in those settings. As a consequence, on a forward looking basis, players that may find themselves in such situations may be expected to have their bargaining power reduced. I investigate this question first by considering the way in which payoffs change as the number of competing intermediaries increases before deriving a more general result by considering the impact of being "essential" to a trade on payoffs.

4.4.1 The effect of additional intermediation routes

To investigate the impact the number of intermediaries has on payoffs, consider a simple setting with a single buyer with valuation one and a set of k intermediaries that directly link to both the seller and the single buyer for the asset (see Figure 3). Expected equilibrium payoffs for the end-nodes A and B and any intermediary I are then given by f_B and f_I respectively:

$$f_B = \frac{k-\beta}{k\,(3-\beta)-2\beta} \tag{21}$$

$$f_I = \frac{1-\beta}{k\,(3-\beta)-2\beta} \tag{22}$$

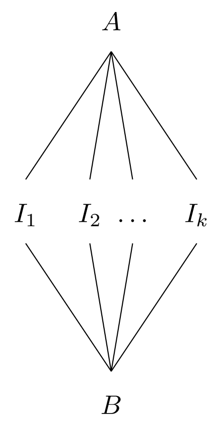

Figure 3: A setting with k intermediaries

As expected, payoffs for end-nodes increase with the entry of additional intermediaries. Also, as previously observed in Section 4.2 for the case $k=2$, as $\beta \to 1$, payoffs for intermediaries go to zero. The ratio of the payoffs is given by $\frac{f_B}{f_I} = 1 + \frac{k-1}{1-\beta}$. At $k=1$, the relative shares are equal and as k increases the ratio increases linearly at rate $\frac{1}{1-\beta}$.

4.4.2 The effect of structural features on payoffs in the limit

The analysis in the previous section illustrates the impact of competition in a simple setting with simple, competing intermediaries. One result of this analysis is that as trade frictions vanish in the limit intermediaries receive an expected payoff of zero. This section shows how the intuition derived from this simple example carries through to general structures.

Definition 1. *A player i is essential to a trade opportunity if $i \in R(s) \forall s \in S$.*

This definition formally captures the approach adopted in Goyal and Vega-Redondo (2007). Structurally speaking, a player is essential if he is located on all possible trade routes between the buyer and the seller of the good. As such, non-essential traders are competing for the business of intermediating the trade opportunity.

Proposition 3. *In an MPE of the game implementing outcome (η, τ) the payoff of trader i is strictly greater than zero as $\beta \to 1$ if and only if the trader is essential.*

Proof. We establish the result by considering first the payoffs of essential players as $\beta \to 1$. Let i be essential, then by Proposition 1, for states s in which i is responding, $f_i(s) \to E\left[f_i(s')\right]$. Adding across states and noting that by being essential i is either proposing or responding, this implies equalization of payoffs across states, i.e. $f_i(\tilde{s}) \to E\left[f_i(s')\right]$ for states $\tilde{s}$ in which i is proposing.

Now consider a non-essential player k involved in two states s and $\tilde{s}$ which share the same route such that $R(s) = R(\tilde{s}) = R$ and $k \in R$. Furthermore, let $k = \kappa(s)$ and $i = \kappa(\tilde{s})$ with i essential. Then as $\beta \to 1$, payoffs for i tend to the same amount across s and $\tilde{s}$. Furthermore, all other responding players will receive equal payouts on the route by Proposition 1. This implies that also for k payoffs will equalize, i.e. $f_k(s) \to f_k(\tilde{s}) \to E\left[f_k(s')\right]$.

Finally, given that by Proposition 1 $f_k(s) = 0$ for s in which k is excluded and such states arrive with positive probability, equalization is only feasible if $E\left[f_k(s')\right] = 0$ as required.

□

Intuitively, the key distinction between essential and non-essential players is the the latter have a positive probability of being excluded. This means that in the limit their implicit discount factor remains strictly below one whilst for essential players it converges to one. The result then reflects the basic intuition of standard alternating bargaining models.

Proposition 3 provides microfoundations for an analysis of competing intermediaries on networks and maps the intuitive Bertrand outcome into the bargaining setting investigated here. As such it provides a justification for the payoff structure used in Goyal and Vega-Redondo (2007), who investigate incentives for network formation in a setting with intermediation rents. Whilst they assume that non-essential players receive zero payoff, justifying it as the kernel and core in a cooperative bargaining setup, the present analysis may provide some grounding for this assumption in a non-cooperative bargaining setting.

5 Conclusion

In this paper, I study a model of bargaining and exchange with intermediation on networks, extending the Merlo and Wilson (1995) framework as a tool to analyze stochastic bargaining games into a network setting. I characterize payoffs with a simple set of value function equations allowing the analysis of efficiency and the impact of structure on payoffs in equilibrium outcomes.

A interesting and potentially insightful extension to the model would allow for heterogeneity in surplus available across states capturing both variation in buyer valuations and capacity of intermediaries to facilitate trade. This would allow for example an analysis of the question of efficiency in the presence of states in which agreement would result in lower surplus being realized than in others. In addition, such an extension would allow an analysis of the competitive constraint exerted by imperfect substitutes, namely alternative trade routes which would result in payoffs that are lower than on the preferred route.

A Appendix

A.1 Proof of Equilibrium Characterization Result

This section presents the proof of Proposition 1. The approach taken employs a fixed point argument adapted from Merlo and Wilson (1998).

Consider the family of bounded measurable functions F^n from S into R^n. An element f of this family in our context represents a value function, with $f_i(s)$ giving the value of agent i of being in state s.

We first establish that for any stationary outcome (S^μ, μ) the payoff vector $v^\mu(s)$ is characterized by a natural recursive formulation. The proof is in Merlo and Wilson (1998).

Lemma 1. *(Merlo and Wilson, 1998) If (S^μ, μ) is a stationary outcome, then v^μ is the unique element of F^n for which $v^\mu(s) = \mu(s)$ for $s \in S^\mu$ and $v^\mu(s) = \beta\left[v^\mu(s')\right]$ for $s \in S - S^\mu$.*

Using Lemma 1 we proceed to the proof of Proposition 1 itself. Consider an operator A on the payoff function f defined as follows.

$$A_i(f)(s) = \begin{cases} \max\left\{b(s) - \beta E\left[\sum_{j \in R(s)\setminus i} f_j(s')\right], \beta E\left[f_i(s')\right]\right\} & \text{for } i = \kappa(s) \\ \beta E\left[f_i(s')\right] & \text{for } i \in R(s) \setminus \kappa(s) \\ 0 & \text{for } i \notin R(s) \end{cases} \tag{23}$$

The proof of the proposition then requires demonstrating f is an MPE payoff *if and only if* $A(f) = f$.

- $\Rightarrow$ *"f is an MPE payoff" implies "$A(f) = f$"*

  Consider an MPE payoff f and fix a state s with $i = \kappa(s)$. The best reply for responder j to a given proposal x is to reject if $x_j < \beta E\left[f_j(s')\right]$ and to accept if $x_j > \beta E\left[f_j(s')\right]$. This implies that i can earn $b(s) - \beta E\left[\sum_{j \in R(s), j \neq i} f_j(s')\right]$ from making a proposal that is accepted and $E\left[f_i(s')\right]$ from passing. Thus, if $c < \beta E\left[\sum_{j \in R(s)} f_j(s')\right]$, the proposer will pass and $f(s) = E\left[f(s')\right]$. If $b(s) > \beta E\left[\sum_{j \in R(s)} f_j(s')\right]$, i will make a proposal that is accepted, earning $b(s) - \beta E\left[\sum_{j \in R(s), j \neq i} f_j(s')\right]$ for i,$\beta E\left[f_j(s')\right]$ for $j \in R(s) \setminus i$ and 0 for $k \notin R(s)$. If $b(s) = \beta E\left[\sum_{j \in R(s)} f_j(s')\right]$, the proposer is indifferent with $f(s) = \beta E\left[f(s')\right]$ again. Thus $A(f) = f$.

- $\Leftarrow$ *"$A(f) = f$" implies "f is an MPE payoff"*

  Assume $A(f) = f$. We need to show that f is an MPE outcome described by (S^μ, μ) with $f = v^\mu$. Define a candidate outcome by $S^\mu = \left\{s \in S : \beta \sum_{j \in R(s)} f_j(s) \leq b(s)\right\}$ and $\mu(s) = f_(s)$ for $s \in S$.

First, we show that $f = \upsilon^\mu$. For this we use Lemma 1 and show that f meets the properties stated there. Consider state s with proposer $i = \kappa(s)$. Then $f = A(f)$ implies $f_j(s) = \beta E\left[\sum_{j \neq i} f_j(s')\right]$. This yields $f_i(s) = max\left\{b(s) - \beta E\left[\sum_{j \in R(s), j \neq i} f_j(s')\right], E\left[f_i(s')\right]\right\}$. In the first case, $b(s) = \beta \sum_{j \in R(s)} f_j(s)$ and thus $s \in S^\mu$. Otherwise, $b(s) < \beta \sum_{j \in R(s)} f_j(s)$ and thus $s \in S - S^\mu$ and $f(s) = \beta E\left[f(s')\right]$. This shows that $f(s)$ meets the criteria of Lemma 1 and thus $f(s) = \upsilon^\mu$.

Second, we show that (S^μ, μ) is indeed an MPE outcome, by defining a suitable strategy profile and demonstrating that no player can be better off by unilaterally deviating. The strategy profile instructs proposers to pass unless $s \in S^\mu$ in which case the proposer offers μ. Responders will then accept, which yields $\upsilon_j^\mu(s) = \beta E\left[f_i(s')\right]$. Now, given future payoffs are given by υ^μ, there is no incentive for any j to deviate and reject. For player i, there is no incentive to deviate as $\mu_i \geq \beta E\left[f_i(s')\right]$ for $s \in S^\mu$. Finally, for $k \notin R(s)$, the rules of the do not permit an action and thus no possibility for deviation. $\square$

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