Rational blinders: is it possible to regulate banks using their internal risk models?*

Job Market Paper

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Abstract

Financial institutions use quantitative risk models not only to manage their risks, but also to communicate information. The Basel regulation in particular uses banks' own estimates to make capital requirements more sensitive to each bank's risks, and both the models and the regulation have been blamed for their over-optimism. I link over-optimism to a hidden information problem between a regulator and a bank who knows better which models are correct. If the regulator treats this problem as "model risk" and only uses tighter capital requirements (e.g. switches from Basel II to Basel III), a wider adoption of optimistic models to bypass the regulation and an increase of banks' risk can follow. On the other hand, there is a cost of ensuring banks use adequate models, which increases with the extent to which internal models are used to compute finer capital requirements. Informational constraints thus make the case for a model-based regulation much weaker. More broadly, this paper shows how economic incentives can impact the development of new predictive models.

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1 Introduction

The recent financial crisis has highlighted an important discrepancy between the latest advances in economics and finance and the practice of financial institutions and regulators. The inability of the latter to take into account extreme risks is denounced precisely as new generations of models using fat-tailed distributions or extreme value theory flourish. Dowd, Cotter, Humphrey, and Woods (2008) illustrate the extent to which some older models used in practice were flawed: some "25-sigmas events" happening several times in a row in August 2007 were "supposed" to occur once in every 10^{135} years! Since the regulation of financial institutions increasingly relies on internal models, it is urgent to understand why many agents seem to have chosen internal models that were too optimistic regarding extreme risks, and how this can be avoided.

More generally, internal models have been increasingly used in the past years for "external purposes": communicating information about an institution's risks to creditors, shareholders, regulators, or rating agencies. The regulation of banks in particular relies heavily on internal models to compute capital requirements more in line with a bank's risk: "*because the most accurate information regarding risks is likely to reside within a bank's own internal risk measurement and management systems, supervisors should utilize this information to the extent possible*" (Federal Reserve System Task Force on Internal Credit Risk Models (1998)). Daníelsson (2008), Rochet (2010) or Eichengreen (2011) argue that the Basel regulation, by allowing banks to use internal models to compute regulatory capital, has given them incentives to use optimistic models to increase leverage, or at least has not encouraged them to produce cautious estimates of risk.

The first part of this paper analyzes this argument formally. I consider financial intermediaries with limited liability, competing both to attract investors and to lend to final borrowers. Intermediaries have to choose a risk model which determines the regulatory capital they have to maintain. I model this situation as a hidden information problem in which a bank reports an internal model to the regulator, who chooses a capital constraint depending on the report. Adopting an overoptimistic model allows a bank to understate its risk and lend more. In the current state of the regulation, banks face little penalties for reporting over-optimistic models for some types of risk. The regulatory response has been to tighten capital requirements and set aside provisions for "model risk". But model risk arises when models may be wrong due to unbiased mistakes, not to bad incentives. The first part of this paper shows that such a regulatory tightening, like the transition from Basel II to Basel III, can counter-intuitively *increase* the risk that a bank defaults. When the regulation tightens, the supply of bank loans decreases and the interest rate on loans increases. As a result, using optimistic models to bypass the regulation becomes more profitable, and the wider adoption of over-optimistic models can lead to an increase in the average risk of banks.

This result means that the regulator cannot substitute a "naive" regulatory tightening for a regulation optimally solving the hidden information problem. Is such a regulation achievable? In the second part of this paper, I look at two possible solutions: a menu of capital constraints and penalties punishing intermediaries who had reported optimistic models when high levels of losses occur, and an auditing mechanism where the regulator spends more time checking the robustness of more optimistic models. The task of the regulator is complicated by the intermediaries' limited liability and the fact that they can opt out of the mechanism and remain unleveraged, or choose Basel's "standard approach", both options being type dependent. A trade-off appears between costs for the regulator and the risk-sensitiveness of the regulation. With a menu of penalties, the main difficulty for the regulator is that internal models typically differ in their predictions about extreme levels of losses: she would need to punish an optimistic intermediary if extreme levels of losses realize but, if regulation is very sensitive to the model reported, a bank with a very optimistic model is allowed a leverage so high that it can be already in default when it should be punished. A similar problem arises with auditing: to reduce auditing costs, banks with low risk have to face higher capital constraints than in the first-best regulation, such that high risk banks do not want to use optimistic models and pretend they are low risk banks, and high risk banks face lower capital constraints, so that they make a higher profit when they tell the truth. Thus the second best regulation is less risk sensitive, to save on auditing costs.

Ultimately, in both cases ensuring truthful revelation has a cost that reduces the desirability of a regulation based on internal models. Depending on the regulator's information and on informational costs, the second-best regulation can be a capital ratio that is computed using the regulator's information but where banks' internal models play no role.

Let me illustrate in more concrete terms the issue of model uncertainty and strategic model selection with the following example. Gordy (2000) compares two popular commercial models to compute the VaR of a credit portfolio, CreditRisk+ and CreditMetrics. Results from CreditRisk+ depend very much on a parameter, denoted σ, controlling for the sensibility of different bonds to systemic risk. With the appropriate parameterizations, both models can actually be quite close, but since the parameters are difficult to estimate, many specifications appear plausible. The following table is a part of table 3 in his article, and shows the values of the 95% and 99.5% VaRs for a low quality portfolio computed with CreditMetrics, and with CreditRisk+ under three different parameterizations. In brackets is the VaR as a percentage of the VaR given by CreditMetrics (CM):

Prob. threshold	CM	CR+, $\sigma = 1$	CR+, $\sigma = 1.5$	CR+, $\sigma = 4$
0.95	1.989 (100%)	2.044 (102.8%)	2.041 (102.6%)	1.586 (79.7%)
0.995	3.124 (100%)	3.321 (106.3%)	3.664 (117.3%)	4.504 (144.2%)

The different models and parameterizations (except $\sigma = 4$) lead to similar conclusions for the 0.95 threshold, but much less so for the 0.995 threshold[1]. While the regulator would like a financial institution to have enough capital to cover losses at the 0.995 level, an institution with limited liability typically has a much lower target[2]. If the regulator were to set regulatory capital based on the VaR produced by the institution only, the institution would have to keep 17% more capital if it uses CreditRisk+ with $\sigma = 1.5$ than if it uses CreditMetrics, while both models' predictions for the 0.95 level are similar. This opens the possibility for the institution to use the more optimistic model to relax the regulatory constraint without making forecasting mistakes about risk levels it cares about, or having to use "double accounting" (section 5.2 deals with this last issue in more detail). Also note that since both models differ markedly at the 0.995 level only, and since these models have a one-year horizon, it takes an average of 200 years for an observation to occur that gives information about which model is the true one[3]. Thus the regulator cannot backtest credit risk models but can only check their methodology ex-ante, which is a difficult task (see Jackson and Perraudin (2000)).

Clearly, it is possible to use optimistic models without the regulator noticing, and there are incentives to do so. The results in the first part of the paper give clear-cut empirical predictions about which models will be used. The main one is that an exogenous increase of demand for banks' loans or other products should cause more banks to use more optimistic models, as measured for instance by the VaRs given by the different models for a common representative portfolio. More banks should also abandon the "standardized" approach of the Basel regulation for the "internal ratings based approach" allowing the use of internal models to compute capital requirements. Finally, since the implementation of Basel III leads to much heavier capital charges, the development and adoption of more optimistic models should be used to mitigate the additional costs of complying with the regulation.

An implication of the second part is that good candidate models to bypass the regulation are those which are proven to be over-optimistic only when the situation is so bad that the regulator cannot be too harsh on banks, by fear of deepening an economic downturn. Interestingly, it has indeed taken the recent financial crisis to reconsider the validity of risk models, and the pace at which new regulations are implemented partly reflects concerns about the effects of increased capital requirements on economic activity.

Finally, the hidden information problem I consider here also turns up in other forms of "regulation". An external rating agency estimating the creditworthiness of a firm or the risk of a pool of loans similarly has to rely partly on internal models chosen by an agent with

[1] Koyluoglu, Bangia, and Garside (2002) and Tarashev (2005) also illustrate this point.

[2] For instance, on the basis of a study by the Federal Survey, Mingo (2000) reports the case of a bank using a capital allocation rule covering 95% of the loss distribution (prior to the implementation of the Basel framework).

[3] More advanced tests can be used, such as the one proposed by Lopez and Saidenberg (2000), but the problem of insufficient data cannot be completely bypassed, see also Kupiec (2002).

an incentive to cheat. Inside a firm, the models a team or a desk use to manage risk may partly determine the funds the desk manages and the compensation its members receive. With slight amendments, the model I develop here can be used to analyze this larger class of problems.

Related literature

There is a huge literature on the regulation of banks and on the Basel framework in particular. Rochet (1992) shows that risk-based capital requirements are necessary to control banks' risk without inducing inefficient choices of assets. The use of VaR for market risk and internal ratings for credit risk has been seen as a way to implement capital requirements responding finely to a bank's risk. Dangl and Lehar (2004) for instance have analyzed the risk-taking of a bank under VaR based regulation. Several papers like Daníelsson, Shin, and Zigrand (2004), Heid (2007) or Kashyap and Stein (2004) have criticized the pro-cyclical effects of using risk-based capital requirements when the focus shifts from an individual bank to a market equilibrium. The first part of this paper also focuses on equilibrium effects but without the assumption that risk-based requirements stem from a correct representation of risk. This part can be linked to several models of competition between leveraged banks, for instance Herring and Vankudre (1987), Matutes and Vives (2000), Bolt and Tieman (2004). Closely related are also recent papers studying the strategic choice of banks between Basel's "standardized approach" and the "internal ratings based" approach; these include Antão and Lacerda (2011), Hakenes and Schnabel (2011) and Ruthenberg and Landskroner (2008). The framework of this paper is in general simpler so as to allow for a discussion of the effects of a more risk-sensitive regulation, additional uncertainty on internal models and other variables that were not the focus of the aforementioned papers.

Several papers have considered the possibility of biased models for the regulation of *market risk*. An interesting difference with credit risk models is that market risk is evaluated on a *daily* basis, typically by the value at risk at the 99% level, such that after 100 days it is already possible to detect blatant over-optimism. Incentives not to use optimistic market risk models have been carefully provided in the Basel framework (and actually since the 1996 amendment to the Basel I capital accord), as studied theoretically by Lucas (2001) and Cuoco and Liu (2006). Empirical studies by Berkowitz and O'Brien (2002), Pérignon, Deng, and Wang (2008) and Pérignon and Smith (2010) show that VaRs reported for market risk are actually *too conservative*, implying that the penalty for under-reporting the VaR is probably too high and has an unwanted impact on the way banks evaluate their market risk. These papers show that banks respond to incentives to choose pessimistic market risk models - thus it wouldn't be surprising if they also responded to incentives to choose optimistic credit risk models.

The second part focusing on how the regulator could elicit the revelation of the true model echoes papers like Chan, Greenbaum, and Thakor (1992) and Freixas and Rochet (1998) on the problem of fairly priced deposit insurance. In my paper the hidden information is about "models", or distributions of losses, which typically give similar predictions except for high levels of losses. This adds an interesting difficulty to the design of the optimal regulation: the regulator can use the observed level of losses as a signal on the agent's information, but only in extreme cases. The auditing problem generalizes the one considered by Prescott (2004) and shows an incentive to bias the required capital ratios *downwards* and not only upwards, such that in the end capital ratios are not necessarily higher than in the first-best, but less sensitive to the intermediary's type.

This paper contributes to yet another strand of the literature, concerned with "markets for models/theories". Banks in my paper are on the demand side of such a market. Examples include Hong, Stein, and Yu (2007), who study agents relying on partial models and shifting from one model to the other depending on their observations, and Cogley, Colacito, and Sargent (2007) who study rational learning of macroeconomic models with a feedback from learning on economic variables. Fewer papers look at situations where the demand for models is not directly derived from their predictive power only. Exceptions include Millo and MacKenzie (2009) who study the usefulness of simple risk management models for communicating and reducing complexity, and Gosh and Masson (1994), who suggest that governments could in fact pretend to believe in economic models they know to be false so as to gain in bargaining power when meeting with other countries' representatives.

The remaining of the paper is organized as follows: section 2 describes the framework, solves the maximization program of an intermediary with a given model and given prices, and derives the optimal risk-sensitive regulation under complete information. Section 3 studies the problem of a regulator whose only tool is model-sensitive capital requirements. Section 4 studies how the regulator could use penalties contingent on the realized level of losses and auditing schemes to reveal the true model. Finally, section 5 discusses extensions of the model to account for other possible incentives to develop credit risk models.

2 Framework

2.1 Agents and assets

In order to study how market prices depend on the models chosen by financial intermediaries, and how these prices in turn determine the incentives to choose a given model, I need to introduce at least three classes of agents:

Borrowers need to finance risky projects. They have a demand for loans $D(r_L)$ as a function of the gross interest rate r_L. D is decreasing in r_L, $D(1) = +\infty$ and $\lim_{+\infty} D(r_L) = 0$. I denote $r_L(L)$ the inverse demand function. A random proportion t of borrowers will default, where t is taken from a distribution $f(t, \sigma)$ with support over $[0, 1]$, $F(t, \sigma)$ being the cumulative. σ is a parameter of the distribution, a higher σ being associated with more default risk (more on this below). Finally, I assume that a defaulting loan yields 0 (failure of the borrower's project)[4].

Investors with a large initial wealth W may invest in a safe asset yielding the exogenous riskless rate r_0 with certainty, or they can lend to financial intermediaries at a rate r_D, but not directly to borrowers.

Financial intermediaries can lend to borrowers, invest in the safe asset, and borrow M from investors at rate r_D. They initially own K (equity) and are protected by limited liability. Finally, I assume that a debt contract between an investor and an intermediary cannot be made contingent on the intermediary's subsequent choice of leverage or assets.

All agents of a given type are homogenous. All are risk-neutral and act as price-takers on a perfect competitive market (see section 5 for a discussion of this assumption). Finally a benevolent *regulator* can set limits to intermediaries' leverage and aims at maximizing social welfare. Throughout the paper a female pronoun refers to the regulator, and a male pronoun to an intermediary. Figure 1 sums up the market structure.[5]

[4]As I do not wish to address the problem of adverse selection, default is assumed to be independent of the interest rate and the amount lent. Relaxing this assumption would make the analysis much more cumbersome without altering the main results.

[5]All figures are in the text, the notations used are summed up in A.1.

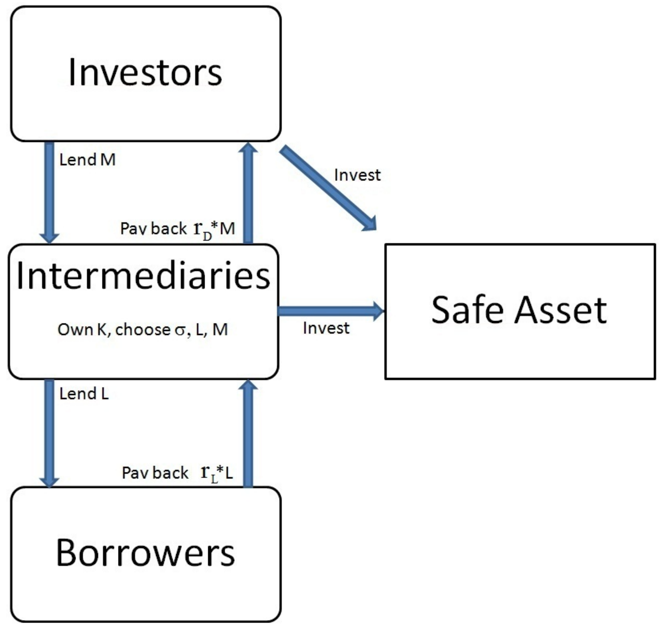

Figure 1: Market structure

2.2 Model uncertainty

The intermediary chooses how much to lend based on his estimation of the probability distribution of defaults among borrowers. This estimation may come from different methods, but is typically given in large banks by an internal credit risk model. Due to the difficulty of backtesting internal models, there is important model uncertainty. Moreover, there is asymmetric information about models since an intermediary is likely to have more information than outsiders about which models are more reliable. Assumptions on model uncertainty are the following:

- **M1**: Let $\{F(.,\sigma), \sigma \in [\sigma_{min}, \sigma_{max}]\}$ be a family of cumulative distributions over $[0,1]$, parameterized by σ, twice-continuously differentiable in both arguments. Denote $f(.,\sigma)$ the corresponding densities. The family of distributions has the monotone likelihood ratio property:
$$\forall t_0, t_1, \sigma_0, \sigma_1 \text{ with } t_1 \geq t_0,\ \sigma_1 \geq \sigma_0,\ \frac{f(t_1,\sigma_1)}{f(t_1,\sigma_0)} \geq \frac{f(t_0,\sigma_1)}{f(t_0,\sigma_0)}$$
- **M2**: A given σ is randomly selected in $[\sigma_{min}, \sigma_{max}]$ according to some distribution $\Psi(.)$, density $\psi(.)$. Intermediaries observe σ before they take any decision, but σ remains hidden to the regulator. t, the proportion of defaulting loans, is drawn from the cumulative distribution function $F(.,\sigma)$.

M1 amounts to assuming there exists a set of different plausible models indexed by σ, with enough models and parameterizations available for the family to be continuous and twice differentiable. Moreover, models are ranked in terms of likelihood ratios: models with a low σ give risk estimates unambiguously more optimistic than models with a high σ. This assumption will play a role mostly in section 4. The family $F(.,.)$ can be interpreted as one model with different parameterizations, or as different models from different families, where each model is indexed by some σ[6]. Finally, **M2** means that intermediaries know the true value of the parameter σ while the regulator does not, thus an extreme form of asymmetric information.

2.3 The intermediary's program

Taking the model chosen and prices as given, I derive the demand for funds and the supply of credit by financial intermediaries. In the next section I will endogenize the choice of a model and study the equilibrium of the market. Take r_0, r_D, r_L as given with $r_L \geq r_D \geq r_0$. In this setup it never pays off for a financial intermediary to borrow $M > 0$ and invest at the riskless rate r_0 since investors necessarily ask for an interest $r_D \geq r_0$. Thus we must have either $L = M + K$ with M possibly zero, or $L = M = 0$ (intermediaries invest their equity at the riskless rate).

Due to limited liability, an intermediary's realized profit if he lends L and a proportion t of borrowers do not repay can be written as $\max\big(0, r_L(1-t)(M+K)-r_D M\big)$. The intermediary cannot reimburse his creditors if there have been too many defaults in his portfolio, that is if:

$$t > 1 - \frac{r_D}{r_L}\left(1 - \frac{K}{L}\right) = \theta$$

θ is the *maximum proportion of sustainable losses*, the maximum losses an intermediary can bear without defaulting. It is of course inversely related to leverage L/K and to r_D/r_L. If he chooses $L > 0$, the intermediary's expected profit is

$$\int_0^\theta (r_L(1-t)L - r_D(L-K))f(t,\sigma)dt = r_L L \int_0^\theta (\theta - t)f(t,\sigma)dt$$

Thus, denoting $\pi(\theta,\sigma)$ the intermediary's expected profit, we have

$$\begin{aligned} \pi(\theta,\sigma) &= r_L L \times s(\theta,\sigma) \qquad (1) \\ \text{with } s(\theta,\sigma) &= F(\theta,\sigma)\mathbb{E}_\sigma(\theta - t | t \leq \theta) \end{aligned}$$

Profit is thus the product of two terms. $r_L L$ are the revenues if all borrowers repay their

[6]See Gordy (2000), or Koyluoglu, Bangia, and Garside (2002) who compare the popular commercial models CreditMetrics, KMV's Portfolio Manager and CreditRisk+, and show that these models share a common structure and give the same results if their parameterizations are consistent with each other, but the standard parameterizations are such that the estimates drawn from these models can widely differ.

debt. The second term is the probability that the intermediary survives, times the expected difference between the maximum proportion of defaults the intermediary can handle, and the expected proportion of defaults given that the intermediary survives. Assume that all borrowers between θ and 1 repay their debt. These repayments do not bring any profit to the intermediary but enable him to repay his own debt. Then all further repayments are profit. Thus the second term is the probability of survival times the expected proportion of "surplus" repayments, with the convention that this proportion is 0 if the intermediary defaults. I denote this quantity $s(\theta, \sigma)$. Finally the operator $\mathbb{E}_\sigma$ denotes an expectation according to the distribution $f(., \sigma)$.

The intermediary will either invest all equity in the riskless asset and not borrow, or maximize $\pi(\theta, \sigma)$ in θ, taking prices as given. As is detailed in the next subsection, the intermediary also faces a regulatory constraint on the ratio K/L, which has to be larger than some α. The maximum proportion of sustainable losses θ thus has to be higher than $1 - \frac{r_D}{r_L}(1-\alpha)$ and the intermediary's program can be written as:

$$\max_{\theta} \pi(\theta, \sigma) \text{ s.t. } \theta \geq 1 - \frac{r_D}{r_L}(1-\alpha) \tag{2}$$

A decrease in θ increases L and expands the scale of operation, bringing more profit for a given proportion of expected surplus repayments. But this proportion itself is increasing in θ (we have $s_1'(\theta, \sigma) = F(\theta, \sigma)$): using less leverage means a lower probability of default and less debt to repay. I show in Appendix A.2 that the profit function is decreasing and then increasing in θ as on figure 3. Thus only three choices make sense: (i) investing K in loans without using any debt ($\theta = 1, L = K$), (ii) investing K in the safe asset without using any debt ($L = 0$), (iii) borrowing until the leverage constraint binds and investing everything in loans ($K/L = \alpha$).

Lemma 1 (Demand for funds and supply of loans)**.** *Let (M^*, L^*) be the profit-maximizing choice of the intermediary. There exists $\underline{r_L}$ such that:*

- *If $r_L \geq \underline{r_L}$ then $L^* = \frac{K}{\alpha}, M^* = \frac{K(1-\alpha)}{\alpha}$.*
- *If $r_L < \underline{r_L}$ then $M^* = 0$. $L^* = K$ if $r_L \geq \frac{r_0}{\mathbb{E}_\sigma(1-t)}$, and $L^* = 0$ otherwise.*

The value of $\underline{r_L}$ and the proof are in Appendix A.2. The intermediary uses the maximum leverage allowed by the regulation if r_L is high enough to compensate for the high risk of defaulting, and otherwise doesn't borrow but invests in loans or in the safe asset depending on which one has the higher expected return.

2.4 Regulation under complete information

I first analyze the optimal capital ratio the regulator can set if she also knows σ. Without intermediation, the optimal amount of loans would maximize the sum of investors' gains and the surplus of the $(1-t)$ surviving borrowers. The optimal L in this case is such that at the interest rate $r_L(L)$ investors would exactly break even if they could lend directly to borrowers:

$$r_L(L) = r_L^e = \frac{r_0}{\mathbb{E}_\sigma(1-t)} \tag{3}$$

When intermediation is necessary and for a given level of capital however, reaching the optimal level of loans may require a high leverage, and thus the possibility that an intermediary defaults. One of the traditional rationales for banking regulation is that banks are indebted towards small retail depositors unable or unwilling to monitor the bank's riskiness. Without regulation, investors would expect banks to use infinite leverage and default with a probability close to one, such that they would not be ready to lend at finite interest rates. The regulator can improve on this situation by insuring deposits. For simplicity I do not consider the cases of partial insurance or insurance with a non-fixed premium, but only complete deposit insurance[7]. Then we have $r_D = r_0$ since loans to intermediaries are now riskless, and investors' welfare is constant. The regulator will consider the surplus of borrowers, the profit of intermediaries, and the cost of repaying losses to investors. I assume a deadweight cost $c > 0$ from taxation, such that each unit of repayment with taxpayers' money costs $1 + c$ to taxpayers and gives 1 to investors.

Under complete information, the regulator can use "model-sensitive" capital ratio constraints $K/L \geq \alpha(\sigma)$. It will be convenient to translate this constraint on the capital ratio into a constraint on the maximum sustainable losses of the intermediary:

$$K/L \geq \alpha(\sigma) \Leftrightarrow \theta \geq 1 - \frac{r_D}{r_L}(1-\alpha(\sigma)) = \theta(\sigma)$$

As $\theta(\sigma)$ reflects $\alpha(\sigma)$, the constraint faced by the intermediary is just $\theta \geq \theta(\sigma)$, and as shown in the previous subsection this constraint will be binding for a high enough interest rate r_L. Expressed in terms of θ, the regulatory constraint means that the intermediary has enough capital to bear at least $\theta(\sigma)$ losses in his portfolio. We thus have the following objective

[7] If investors are partially or not insured but know the true risk, and can fully monitor the bank, then in equilibrium $r_L = r_L^e$. But interestingly intermediaries can still choose over-optimistic models, see section 5. The most realistic assumption would be to have partially insured and partially informed investors, the result would combine insights from the two extreme cases I consider.

function for the regulator, to maximize in θ for a given σ:

$$
\begin{aligned}
V(\theta,\sigma) &= r_0 W + \int_0^{\theta} (r_L L(1-t) - r_0(L-K)) f(t,\sigma)dt + \mathbb{E}_\sigma(1-t)\left(\int_0^L r_L(u)du - r_L(L)L\right) \\
&- (1+c)\int_\theta^1 (r_0(L-K) - (1-t)r_L L) f(t,\sigma)dt \\
&= \underbrace{r_0(W+K-L)}_{\text{Safe asset}} + \underbrace{\mathbb{E}_\sigma(1-t)\int_0^L r_L(u)du}_{\text{Surplus from loans}} - \underbrace{c\int_\theta^1 (r_0(L-K) - (1-t)r_L L) f(t,\sigma)dt}_{\text{Deadweight costs}} \qquad (4)
\end{aligned}
$$

Both r_L and L will depend on the level θ chosen by the regulator. To keep things simple, I assume demand to be very elastic, such that the effect of θ on r_L can be considered as negligible. Otherwise increasing θ could lead to an increase in r_L, making losses lower and less probable, which can lead to multiple local optima and an optimal θ increasing in σ only by parts. As a result:

Lemma 2 (First-best)**.** *For a given level of capital* K[8]*:*

1. *If* $D(r_L^e) \geq K$ *the first-best is to set* $\theta^* = 1$ $(L = K)$ *and let intermediaries invest in loans up to the point where* $r_L = r_L^e$*.*
2. *If* $D(r_L^e) < K$*,* c *is high enough and demand* $D(.)$ *is elastic enough,* $V(\theta,\sigma)$ *is concave and the optimal regulatory threshold* $\theta^*(\sigma) = argmax_\theta V(\theta,\sigma)$ *is increasing in* σ*.*

The case for a model-based regulation here is straightforward: when the true model is more pessimistic (σ higher), there is less surplus to gain by expanding credit and more risks of default for a given level of θ, hence the regulator wants to restrict leverage more when σ is higher.

2.5 Numerical example

Consider the following numerical example, to be kept throughout the paper to illustrate the main results. The proportion of defaults follows a Beta distribution with parameters a, b. a measures the slope at the origin of the distribution, b is inversely related to the fatness of the tail. Assume a is known to be 3.5, b is equal to 31.5 but many values are possible for this parameter, up to $b = 50$ which will be our most optimistic model. The true model and the most optimistic one yield of course very different predictions about defaults, not only in terms of expected defaults but also in terms of extreme events, much more likely to occur with $b = 31.5$. Take σ to be $1/b$ so that assumption **M1** is satisfied.

[8] The result depends on the assumption that K is fixed in the short-run, otherwise it would be optimal for the regulator to impose that intermediaries are entirely financed by equity. The only thing I need here is some informational cost of levying capital (Myers and Majluf (1984)) that the regulator cannot suppress, in which case it will be optimal to allow a limited leverage and r_L higher than r_L^e.

In practice, banking regulators aim at capping the probability that each intermediary defaults, typically 0.1% in the Basel framework. Assume this is an approximate solution of the regulator's program. Then θ^* is easy to compute: since an intermediary will default with probability $1 - F(\theta, \sigma)$ when the true model is σ, to ensure a probability lower than p that the intermediary defaults, $\theta^*(\sigma)$ has to be such that $F(\theta^*(\sigma), \sigma) = 1 - p$. For this example to be easier to visualize I assume $p = 0.05$ (5% probability to default). Plotting the CDFs we can easily see $\theta^*(1/31.5)$ and $\theta^*(1/50)$ graphically on Fig. 2. The cumulative in solid line corresponds to the true model, and is associated with a higher minimum default point.

Figure 3 plots the profit as a function of θ, with $r_L = 1.1, r_D = 1, K = 1$ and when defaults follow the "true" Beta distribution with $a = 3.5, b = 31.5$. For θ high enough (low leverage), the profit is increasing in θ. Notice that if the regulator knows the true model and imposes $\theta \geq \theta^*(1/31.5)$, the intermediary prefers not to use any leverage because $r_L < r_L^e$. But if the regulator falsely believes that the true model is $b = 50$ and imposes $\theta \geq \theta^*(1/50)$, the intermediary chooses maximum leverage. This result illustrates the adverse selection problem associated to regulation based on internal models: intermediaries have incentives to pretend the true model is more optimistic than it really is, or to spend resources on lobbying the regulator to be allowed to use optimistic models. In this rather extreme example, if the intermediary is successful at convincing the regulator that $b = 50$ he will increase his expected profit by 10% and default with a 25% probability, five times higher than the regulator's objective.

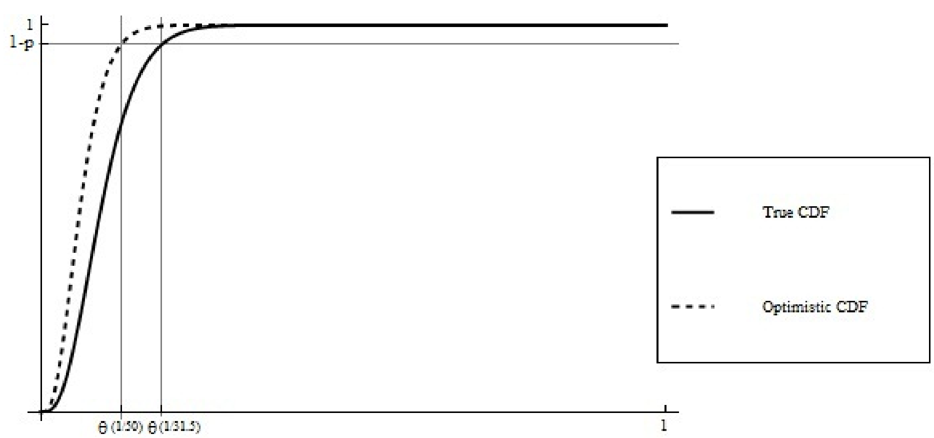

Figure 2: Cumulatives and minimum default points

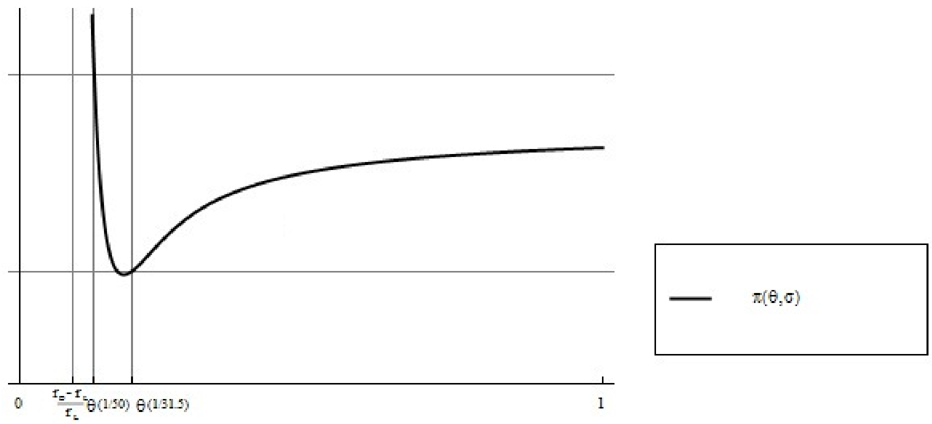

Figure 3: Profit as a function of the default point

3 Model choice and market equilibrium under incomplete regulation

3.1 Market equilibrium under incomplete regulation

I now go back to the general case to study how intermediaries choose their models in equilibrium when regulation is "incomplete", in the sense that it doesn't ensure intermediaries truthfully report the value of σ. The goal is to show that when the use of the correct model is not warranted, a simple tightening of capital ratios can actually *increase* the number and severity of intermediaries' defaults, because it can lead to a wider adoption of optimistic models. A possible interpretation is that the shift from Basel II to Basel III requirements can lead to a wider adoption of optimistic models to bypass the regulation, making the total effect on risk ambiguous. The problem of information revelation cannot be naively side-stepped by tightening ratios to "compensate" for possibly over-optimistic models.

To study the role of market prices it will be convenient to assume there is a representative intermediary taking prices as given, or equivalently a continuum $[0, 1]$ of intermediaries. The timeline of the game is as follows:

T=0 the regulator specifies a rule linking any model σ to a capital ratio $\alpha(\sigma)$, and a number of requirements that an intermediary's internal model has to satisfy to be used for regulatory purposes. These requirements define a set of models accepted by the regulator. For simplicity assume all models with a positive probability to be the true model are accepted, such that this set of models is the interval $[\sigma_{min}, \sigma_{max}]$.

T=1 σ is drawn from the distribution $\Psi(.)$, each intermediary observes σ and reports a model σ'[9]. If $\sigma' \in [\sigma_{min}, \sigma_{max}]$ the regulator constrains the intermediary to choose K, L satisfying $K/L \geq \alpha(\sigma')$. Otherwise the intermediary gets 0.

T=2 r_L, r_D, M and L are simultaneously determined by competitive equilibrium conditions. $D(r_L)$ must be equal to the aggregate supply of loans. An intermediary who has reported a model σ' chooses a supply of loans and a demand for deposits maximizing his profit taking r_L and r_D as given under the constraints $K/L \geq \alpha(\sigma')$ and $L \leq M+K$. Finally, investors must be indifferent at the margin between lending to intermediaries at rate r_D or investing in the safe asset at rate r_0.

T=3 a proportion t of borrowers default, where t is drawn from the distribution $F(., \sigma)$.

In practice, banks using the "advanced internal ratings based approach" (IRB) for credit risk have to provide the regulator with a given number of parameters estimated by an internal model (probability of default, loss given default, exposure at default, effective maturity), then these parameters enter a formula based on a Merton-type model which is used to compute regulatory capital. This is what happens at time $T = 1$ in the model. A key assumption is the absence of transfers designed to reveal the intermediary's information. No payment is asked from the intermediary if he uses a very optimistic model, no penalties are set. This assumption fits the letter of the Basel agreements where little, if anything, is designed to ensure that banks use correct models for credit risk, except the requirement that the bank has used the model for internal purposes for several years before it can use it for regulatory purposes (as shown in section 5.2, this is unlikely to be enough). Although these assumptions are meant to provide a stylized view of the Basel advanced IRB approach, the framework could be applied to other situations: an intermediary may wish to keep a good rating, and thus must prove to a rating agency that his probability of default is low for instance. But here again the rating will probably partly depend on the intermediary's own risk assessments.

Intermediaries' choice at $T = 1$. Solving the model backwards, I first define more formally what is the equilibrium of the subgame starting at $T = 1$ when a given σ is realized:

Definition 1 (Equilibrium with choice of a risk model)**.** *For an increasing $\alpha(.)$ and a given realization of σ, an equilibrium is a 5-uple $(r_L, r_D, \mu_l, \mu_r, \mu_s)$ and a function $h : (\sigma_{min}, \sigma_{max}] \rightarrow [0, 1]$ where a proportion $h(\sigma')$ of intermediaries choose σ', μ_l choose σ_{min} and $K/L = \alpha(\sigma_{min})$, μ_r choose $K = L$, μ_s choose $L = 0$ and invest K in the safe asset, and*

[9]The assumption that different intermediaries know the same σ is not key to the main results but simply allows to define easily a competitive equilibrium. It is possible to consider that for each σ there are $\psi(\sigma)$ intermediaries present on a submarket with a specific type of borrowers. The regulator knows that all types are represented, but does not know which banks lend to which type of borrowers. The model would be equivalent but more cumbersome, as the market I consider in this section for a given σ would be one in a continuum of submarkets for all possible values of σ.

1. *Each intermediary's choice given his model, r_L and r_D is a solution to the intermediary's program of lemma 1, the supply of loans by intermediaries is equal to $D(r_L)$ and funds borrowed by intermediaries equal funds supplied by investors at an interest rate r_D.*

2. *Investors are indifferent between lending to intermediaries and investing in the safe asset.*

3. *No intermediary has an incentive to choose a different σ' or change his investment strategy.*

This definition simply enlarges the standard concept of competitive equilibrium by requiring that no intermediary wants to choose a different model. Solving the equilibrium is easy when investors are fully insured. Since they face no risk when they lend to intermediaries, it must be the case by condition 2 that $r_D = r_0$. This equality implies that if $r_L \geq r_L^e$ it always pays to borrow at least a little, whereas if $r_L < r_L^e$ an unleveraged intermediary prefers the safe asset to loans, so in both cases $\mu_r = 0$. We know from lemma 1 that, depending on r_L, either an intermediary uses no leverage at all or his capital constraint is binding. Then for any $\sigma \in (\sigma_{min}, \sigma_{max}]$ we have $h(\sigma) = 0$. Thus only two strategies may be used by intermediaries: not borrowing and investing in the safe asset (proportion μ_s of intermediaries), or choosing the most optimistic model and using maximum leverage (proportion μ_l). In equilibrium all intermediaries will use the same strategy, or mix between the two strategies. The result depends on how strong the final demand for loans is. To see this, assume the demand function depends positively on a parameter η and consider a family of demand functions $\{\tilde{D}(r_L) = \eta D(r_L), \eta \in \mathbb{R}^{+*}\}$.

Proposition 1 (Equilibrium with insured investors)**.** *For a given σ, starting at $T = 1$ there exists a unique equilibrium with choice of a risk model, in which μ_l intermediaries choose the most optimistic model and maximum leverage, $\mu_s = 1 - \mu_l$ intermediaries do not borrow and invest in the safe asset. Assuming demand is $\eta D(r_L)$, μ_l is increasing in η and decreasing in σ. $\mu_l = 1$ for η high enough or σ low enough.*

Corollary 1. *Over-optimism and the expected proportion of defaulting intermediaries are procyclical (increase in η).*

See appendix A.4 for the proof. This proposition illustrates the role of demand in giving incentives to choose an optimistic model: when demand is high, all intermediaries choose the most optimistic model and use maximum leverage. When all intermediaries use a very optimistic model, they are able to use a high leverage and the supply of loans is high, thus the interest rate on loans has to be low. This situation is an equilibrium if and only if the interest rate is not so low as to make it more profitable not to borrow, that is if demand is high enough. Conversely, if few intermediaries use leverage, the supply of loans will be low and the interest rate high, and to have an equilibrium the interest rate must be low enough so that it doesn't pay to use even the most optimistic model, thus demand has to be low.

The regulator's choice at $T = 0$. The regulator anticipates that intermediaries will always choose either $L = 0$ or $\sigma' = argmin\ \alpha(.)$. The regulator's choice thus amounts to choosing the minimum of the function $\alpha(.)$, which I denote $\bar{\alpha}$. The regulation here cannot be model-sensitive since the choice of σ' by intermediaries does not depend on the realization of σ, and the regulator may just as well choose the function $\alpha(.)$ constant and equal to $\bar{\alpha}$. The equilibrium level of r_L and μ_l however depend both on $\bar{\alpha}$ and σ, I denote them $r_L(\bar{\alpha}, \sigma)$ and $\mu_l(\bar{\alpha}, \sigma)$. Then the objective of the regulator can be written as:

$$\begin{aligned} \max_{\bar{\alpha}} \quad & \int_{\sigma_{min}}^{\sigma_{max}} (\gamma(\bar{\alpha}, \sigma) - \delta(\bar{\alpha}, \sigma))\psi(\sigma) d\sigma \qquad (5) \\ \gamma(\bar{\alpha}, \sigma) \quad & = r_0(W - D(r_L(\bar{\alpha}, \sigma)) + K) + \mathbb{E}_\sigma (1-t) \int_0^{D(r_L(\bar{\alpha},\sigma))} r_L(u) du \\ \delta(\bar{\alpha}, \sigma) \quad & = \mu_l(\bar{\alpha}, \sigma) c \int_{1-\frac{r_D}{r_L}(1-\bar{\alpha})}^{1} (r_0((K/\bar{\alpha}) - K) - (1-t) r_L(\bar{\alpha}, \sigma)) f(t, \sigma) dt \end{aligned}$$

The regulator has to select a single $\bar{\alpha}$ to solve the trade-off between surplus $\gamma(\bar{\alpha}, \sigma)$ and deadweight losses from taxation $\delta(\bar{\alpha}, \sigma)$ for all realizations of σ. The following observation shows this task is quite subtle:

Remark 1 (Counterproductive tightening)**.** *If a small enough number of intermediaries use the most optimistic model, then: for a low enough elasticity of the demand for loans, tightening capital requirements increases taxpayers' losses.*

Proof: notice that the number of intermediaries using the most optimistic model is small when σ is high, thus tightening capital requirements increases banks' default probability and losses to taxpayers precisely when borrowers' risk is already high. Losses $\delta(\bar{\alpha}, \sigma)$ are the product of two terms: the proportion of intermediaries choosing the most optimistic model and a high leverage, μ_l, and the expected costs of repaying one intermediary's creditors. When μ_l is low, the effect of increasing $\bar{\alpha}$ on the second term is negligible compared to the effect on μ_l. Thus the only thing to show is that increasing $\bar{\alpha}$ leads to an increase of μ_l when demand is rigid enough. The equilibrium is defined by μ_l and r_2 satisfying the two following equations:

$$\mu_l K = D(r_2)\bar{\alpha} \qquad (6)$$

$$r_0 \bar{\alpha} = r_2 s \left(1 - \frac{r_0}{r_2}(1-\bar{\alpha}), \sigma\right) \qquad (7)$$

Equation 7 derives from the fact that r_2 is the interest rate r_L such that an intermediary is indifferent between choosing $L = K/\bar{\alpha}$ and $L = 0$. When $\bar{\alpha}$ increases there are two effects: r_2 increases so that $D(r_2)$ decreases, and for a given r_2 the product $D(r_2)\bar{\alpha}$ increases. If demand is rigid enough the first effect is negligible, so $D(r_2)\bar{\alpha}$ increases in equation 6 and μ_l has to increase. ■

More intuitively, there are three effects when the regulator increases $\bar{\alpha}$. First, an intermediary already using the most optimistic model will have less leverage than before, which decreases losses to taxpayers. When the true model is quite pessimistic and few intermediaries use the optimistic model this effect is small. Second, choosing the most optimistic model is less profitable because it allows less leverage. Third, since intermediaries have a tighter capital constraint, the supply of loans decreases and the interest rate r_L goes up. This increase makes it *more* profitable to use the most optimistic model. When demand is rigid enough, the third effect is stronger than the second, so an increase in $\bar{\alpha}$ leads more intermediaries to adopt the most optimistic model, which in turn increases risk and losses to taxpayers. Thus a naive tightening of the regulation can counter-intuitively increase risk precisely in those states of the world where the true model is quite pessimistic and risk is already high, as on figure 5.

3.2 Discussion: market and regulation, from Basel II to Basel III

In the extreme case where investors are fully insured, there is no market discipline since intermediaries' creditors no longer care about the probability that an institution defaults. Intermediaries then face important incentives to use the most optimistic model possible. The market still gives a natural counterweight to this effect however: when more intermediaries adopt optimistic models and use a high leverage, the interest rate on loans goes down and increasing leverage becomes less profitable. Thus a high proportion of intermediaries using over-optimistic models is possible only if the final demand for loans is high enough (proposition 1): a model-based regulation with no penalties for choosing optimistic models is necessary to explain how over-optimistic models can be used in equilibrium, but not sufficient. High demand for loans or low risk-free rates have to be part of the story.

Remark 1 shows that market and regulation are partial substitutes in limiting intermediaries' use of over-optimistic models. A tighter regulation restricts leverage, tends to increase the interest rate on loans, and thus incentives to use optimistic models and high leverage. Tightening the regulation can thus be counterproductive because it alters market counterweights. In particular, the strong increase of capital requirements with the transition from Basel II to Basel III gives incentives to develop and use the most optimistic models that the regulator will accept, so as to minimize the impact of increased capital requirements.

The following two figures illustrate this section. I use the same example as in section 2 and add a demand for loans equal to $D(r_L) = \frac{\eta}{r_L - 1}$. Letting η vary between 0 and 5, I compute the value of μ_l, r_L and the average proportion p_d of defaulting intermediaries for $\bar{\alpha} = 0.05$. We see on figure 4 that when demand is low enough an increase in demand leads to more intermediaries adopting the most optimistic model while r_L is constant, until all intermediaries use the most optimistic model and r_L adjusts supply and demand. On figure 5 I set η equal to 1 and I plot the same variables as a function of $\bar{\alpha}$. We see that tightening the regulation leads to more intermediaries adopting the most optimistic model in this example, and as a result, for low levels of $\bar{\alpha}$, the default probability increases when regulation tightens.

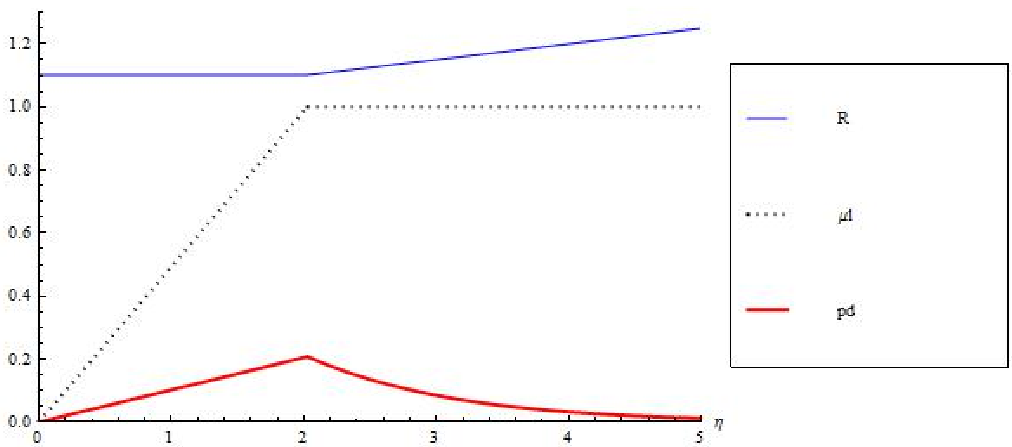

Figure 4: Intermediaries using the most optimistic model, interest rate on loans and intermediaries' default probability as demand increases

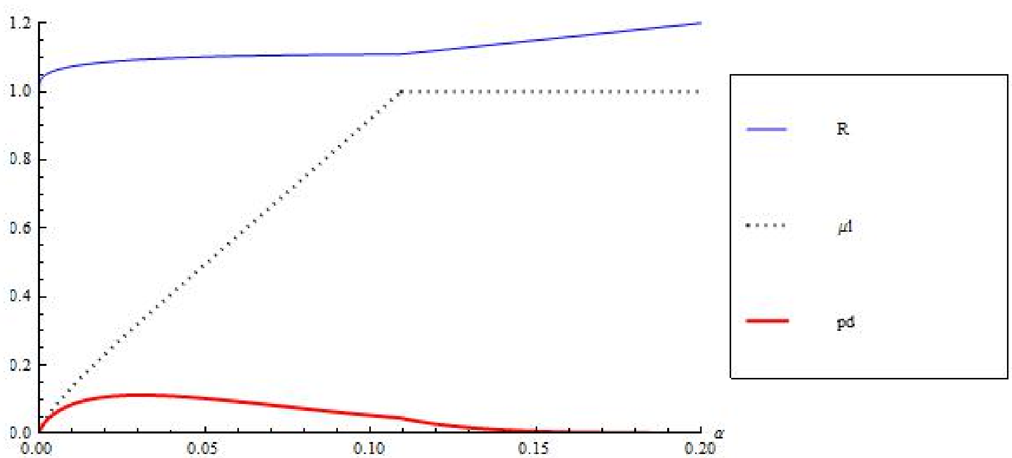

Figure 5: Intermediaries using the most optimistic model, interest rate on loans and intermediaries' default probability as regulation tightens

A natural question is of course why the regulator would naively let banks choose any internal credit risk model. In practice the regulator defines a certain number of characteristics that an eligible model must have; it must also have been used by the bank for two years before it can serve the computation of regulatory capital; and finally the model is backtested and audited to check it meets "industry standards". These rules prevent banks from using models totally off the mark, but cannot prevent them from using models slightly more optimistic than the model they think the most plausible. Backtesting for example does not often lead to the rejection of a model for credit risk given the low power of the tests.

I see four why regulators may emphasize more the necessity to rely on sophisticated and up-to-date models than the hidden information problem: (i) when Basel II was put in place, internal models were already in use and had no reason to be biased, hence regulators thought they could rely on them but neglected to take into account incentives to tweak the models in the future. (ii) The regulator can consider that the priority is to give incentives to use quantitative internal models and increase the transparency of the institution, in the hope that the market will penalize banks using unrealistic models. (iii) Banks' defaults affect non-national investors, such that national regulators use their discretion in allowing more or less optimistic models to favor their national banks at the expense of global stability, which is exactly what the Basel framework was supposed to avoid (Rochet (2010)). (iv) The next section shows that giving incentives to use the correct models and using model-based capital requirements is a difficult task for the regulator, who may prefer to deal with this problem by using more cautious capital requirements. But remark 1 shows that increasing capital ratios to "compensate" for possible over-optimism or adding capital requirements to cover "model risk" is not sufficient, and can actually increase risk instead of reducing it. It is therefore necessary to develop a mechanism giving incentives to use the correct model.

4 Optimal regulation with hidden model

It is natural to ask how regulation could prevent financial intermediaries from picking optimistic models. Checking ex-post that the model reported by the regulated meets industry standards can only lead to a regression toward the most optimistic models when interest rates are high enough. What is needed is a mechanism inducing truthful revelation of the regulated institution's private information. To concentrate on information problems and abstract from market equilibrium effects, I assume from now on that r_L is given, or equivalently that demand is very elastic. Moreover I assume $r_L > r_0/E_{\sigma_{max}}(1-t)$ such that, even if the true model is the most pessimistic one, any capital requirement will be binding. As a consequence, when she sets $\alpha(\sigma)$ the regulator equivalently sets a constraint of the form $\theta \geq \theta(\sigma)$, which will be easier to work with. Moreover, L is determined by θ as follows:

$$L(\theta) = \frac{r_0 K}{r_0 - r_L(1-\theta)} \tag{8}$$

Assume the regulator wants to implement a leverage constraint dependant on the intermediary's type and expressed as $\theta = \theta(\sigma)$. The goal is to maximize under incentive compatibility constraints:

$$\mathbb{E}_\psi(V(\theta(\sigma), \sigma)) - \text{Expected costs of the regulation}$$

where V is the social welfare function studied in section 2.4. In particular the best $\theta(.)$ the regulator can implement is the first-best $\theta^*(.)$. The difficulty is that $\pi_1' \leq 0$ and thus the intermediary wants to use optimistic models. There are of course a number of constraints for the regulator in addition to incentive compatibility. I will focus on two of them. The first one, that also motivates the need for regulation, is limited liability: by the time the regulator realizes a financial institution has been over-optimistic, it may be too late for punishment. Moreover, the regulated institution's outside option is typically type-dependent (Jullien (2000)). I assume that an institution can opt out of the mechanism and then get $\bar{\pi}(\sigma)$ if σ is the true parameter, with $\bar{\pi}' \leq 0$. For instance an institution could choose not to borrow at all and earn $r_L K(1 - \mathbb{E}_\sigma(t))$, or in the Basel regulation a bank can opt for the "standard approach", not use any internal model and earn a profit that will depend on the true state of the economy.

4.1 Menu of capital ratios and state-contingent transfers

I first study a mechanism in which an intermediary observes the true model σ, announces some parameter σ' and faces the constraint $\theta \geq \theta(\sigma')$. Given the assumption $r_L > r_0/E_{\sigma_{max}}(1-t)$, the intermediary chooses $\theta = \theta(\sigma')$, then suffers some level of defaults t in his portfolio and finally pays a transfer $T(\sigma', t)$ if $t \leq \theta(\sigma')$. It will be useful to denote $u(\theta, t)$ the profit before transfers of an intermediary choosing θ when t defaults realize:

$$u(\theta, t) = r_L L(\theta)(1-t) - r_0(L(\theta) - K) \tag{9}$$

The regulator's program is the following:

$$\max_{\theta(.), T(.,.)} \mathbb{E}_\psi(V(\theta(\sigma), \sigma) + E_\sigma(T(\sigma, u))) \text{ with:} \tag{10}$$

$$
\begin{array}{rlr}
\forall\sigma,\sigma',\ \pi(\theta(\sigma),\sigma)-\mathbb{E}_\sigma(T(\sigma,t)) & \geq \pi(\theta(\sigma'),\sigma)-\mathbb{E}_\sigma(T(\sigma',t)) & (IC)\\
\forall\sigma,\ \pi(\theta(\sigma),\sigma)-\mathbb{E}_\sigma(T(\sigma,u)) & \geq \bar{\pi}(\sigma) & (IR)\\
\forall\sigma,t,\ u(\theta(\sigma),t) & \geq T(\sigma,t) & (LL)
\end{array}
$$

Constraint (IC) is a standard incentive-compatibility constraint, an intermediary has to be better off telling the truth about the model. Constraint (IR) requires that the mechanism gives each type of intermediary at least what he would get by opting out of the mechanism (e.g. keeping Basel's standardized approach). (LL) is the limited liability constraint: the regulator cannot tax more than what the intermediary has earned. In particular it is impossible to "punish" a defaulting intermediary. The spirit of such a regulation is easy to understand: the regulator offers a profile of transfers $T(\sigma,t)$ such that an intermediary reporting σ will be heavily taxed if the realized level of defaults is relatively unlikely given the model announced. In the extreme case where for each model σ there exist some levels of default that have positive probability if and only if σ is the true parameter, the regulator could leave some money to the regulated only in those states and tax everything in all other states. More generally, due to the constraints (IR) and (LL) the regulator faces a complicated problem, and may have either to leave some surplus to intermediaries, not be able to implement $\theta^*(.)$, or shut down types with the highest outside options.

Remark 2. *Any menu $T(.,.)$ satisfying (IC) and such that $\mathbb{E}_\sigma(T(\sigma',t))$ is twice differentiable in σ and σ' must be such that for any σ:*

$$
\begin{array}{rcl}
\dfrac{\partial\pi(\theta(\sigma'),\sigma)}{\partial\sigma'}|_{\sigma'=\sigma} & = & \dfrac{\partial\mathbb{E}_\sigma(T(\sigma',t))}{\partial\sigma'}|_{\sigma'=\sigma}\\
\dfrac{\partial^2\pi(\theta(\sigma'),\sigma)}{\partial\sigma'^2}|_{\sigma'=\sigma} & \leq & \dfrac{\partial^2\mathbb{E}_\sigma(T(\sigma',t))}{\partial\sigma'^2}|_{\sigma'=\sigma}
\end{array}
$$

The proof follows straightforwardly from the first-order condition of (IC) and the local second-order condition. Incentive compatibility requires that at the margin reporting a lower σ' to increase leverage will be exactly compensated by additional expected taxes/penalties. Moreover, it must be the case that these taxes increase more rapidly than profit when the reported σ' decreases. This can be a problem since the reservation utility is type-dependent. Consider a simpler example with only two types σ_1,σ_2, $\sigma_2>\sigma_1$, two possible realizations of defaults $\underline{t},\bar{t}$, $\bar{t}>\underline{t}$, and $\Pr(t=\underline{t}|\sigma_i)=p_i$, $p_1>p_2$. To satisfy (IC) and bind (IR) the regulator can give $\bar{\pi}(\sigma_1)/p_1$ to a type reporting σ_1 if $\underline{t}$ realizes, 0 otherwise, and $\bar{\pi}(\sigma_2)$ to type σ_2 irrespective of the realization. (IC) for type σ_2 gives $\bar{\pi}(\sigma_2)/\bar{\pi}(\sigma_1)\geq p_2/p_1$. If the outside option of type σ_1 is much higher than the outside option of type σ_2 and the likelihood ratios of the different states under both models are not different enough, then it is impossible to bind (IR) while satisfying (IC). In other words, if profit decreases quickly in σ it has to be the case that the different models give predictions different enough, otherwise some inefficiency appears or a rent has to be left to the regulated. The following proposition generalizes this idea to a continuum of types:

Proposition 2. *If, in addition to **M1**, $F(.,.)$ is log-concave in its second argument and $\bar{\pi}$ is log-convex, then with $\theta(.) = \theta^*(.)$ there exists a menu of transfers $T(.,.)$ satisfying (IC) and (LL) and such that (IR) is binding for every σ. It is possible in particular to use the following menu:*

$$T(\sigma,t) = \begin{cases} \max(0, u(\theta^*(\sigma),t)) & \text{if } t > a(\sigma) \\ u(\theta^*(\sigma),t) - \frac{\bar{\pi}(\sigma)}{F(a(\sigma,\sigma))} & \text{if } t \leq a(\sigma) \end{cases}$$

with $a(\sigma)$ increasing and such that:

$$\frac{F_2'(a(\sigma),\sigma)}{F(a(\sigma),\sigma)} = \frac{\bar{\pi}'(\sigma)}{\bar{\pi}(\sigma)}$$

With the proposed menu, if he reports model σ, an intermediary is taxed such such that he gets $\frac{\bar{\pi}(\sigma)}{F(a(\sigma),\sigma)}$ if the realized level of defaults is less than $a(\sigma)$, and gets zero otherwise. By definition such a mechanism satisfies the limited liability condition. Moreover, if he reports truthfully the model σ the intermediary gets exactly $\bar{\pi}(\sigma)$ in expectation, thus the mechanism binds condition (IR). We only have to find $a(\sigma)$ such that the incentive compatibility condition holds for all types. Under **M1** we can induce truthful revelation with an increasing $a(.)$: intermediaries announcing a low σ get a high payoff but only if the level of defaults is under a very low threshold (which will be crossed only with a small probability if their report is truthful), intermediaries announcing a higher σ get a lower payoff but for higher levels of default. The two other assumptions ensure that this particular mechanism satisfies (IR) and (IC). Since $F(.,.)$ is decreasing in its second argument and $\bar{\pi}$ decreasing, the log-concavity of $F(.,.)$ in σ expresses the idea that the different distributions do not give too similar predictions as σ increases, and on the contrary the log-convexity of $\bar{\pi}$ implies that the outside option does not decrease too quickly as σ increases. The intuition is the same as in the binary example above. See Appendix A.5 for the full proof.

The logic behind this result is simple: if risk is really low, then a financial intermediary is ready to pay high taxes or penalties if high default levels realize in his portfolio, because this event is unlikely. In principle, observing the level of defaults ex-post gives a powerful tool to the regulator to detect and punish the users of over-optimistic models. Remember however that credit risk models are typically difficult to backtest because they give similar predictions for not too extreme levels of default. As a consequence, the optimal menu of penalties may include transfers for levels of default above those at which a bank defaults itself. Since limited liability may prevent the regulator from taxing enough for high levels of default, it will be necessary to *subsidize* intermediaries with high default levels who announced very pessimistic parameters. This happens with the proposed mechanism if $a(\sigma) > \theta^*(\sigma)$, and in particular when backtesting is difficult, in the following sense:

Definition 2. *Models $\sigma \in [\sigma_{min}, \sigma_{max}]$ are **distinguishable only above** $\hat{t}$ if for any $(\sigma, \sigma') \in [\sigma_{min}, \sigma_{max}]^2$ and for any $t < \hat{t}$ we have $f(t, \sigma) = f(t, \sigma')$.*

This definition reflects the idea that for low levels of risk there is a lot of historical data to calibrate different models, such that they tend to deliver similar predictions, while for extreme levels of risk data is much more sparse. This fact is modeled in a stylized way here since I assume the different models are perfectly equivalent up to a given level of defaults. Now take any two models σ, σ' with $\sigma < \sigma'$ such that $\theta(\sigma) \leq \theta(\sigma')$. Assume σ is so low that the regulator wants to implement $\theta(\sigma) < \hat{t}$, in which case a bank using model σ will default for levels of losses that give no information on which is the true model. I prove that in this case it will be necessary to subsidize a bank using model σ after it defaults.

By contradiction, assume it is not the case. Then for any $t \geq \theta(\sigma)$ we have $u(\theta(\sigma), t) = T(\sigma, t) = 0$. To bind the constraint (IR) for type σ we thus need:

$$\int_0^{\theta(\sigma)} [u(\theta(\sigma), t) - T(\sigma, t)] f(t, \sigma) dt = \bar{\pi}(\sigma)$$

If he reports truthfully, an intermediary with type σ' will get $\bar{\pi}(\sigma')$. If he lies and reports σ he will get:

$$\int_0^1 [u(\theta(\sigma), t) - T(\sigma, t)] f(t, \sigma') dt = \int_0^{\theta(\sigma)} [u(\theta(\sigma), t) - T(\sigma, t)] f(t, \sigma) dt = \bar{\pi}(\sigma)$$

where the second term is implied by $f(t, \sigma) = f(t, \sigma')$ for $t \leq \theta(\sigma) \leq \hat{t}$. Since $\sigma < \sigma'$ we have $\bar{\pi}(\sigma) > \bar{\pi}(\sigma')$, which violates incentive compatibility for type σ', a contradiction. It is thus necessary to have $T(\sigma, t) < 0$ at least for some $t > \hat{t}$: to have incentive compatibility and bind the individual rationality constraints, it must be the case that a type with a low σ gets a positive payoff for some realizations of t that have a higher probability when the true model is σ than when it is a more pessimistic model.

Having to subsidize defaulting banks is of course a bad property of the optimal menu of penalties. It is politically difficult ex-post to use taxpayers' money to give a subsidy to the shareholders of a defaulting bank, and the regulator may be unable to commit to such a mechanism. If the regulator is unwilling or unable to commit to a $T(\sigma, t) < 0$ for $t > \theta(\sigma)$, the above reasoning shows that she has to use transfers such that for any σ with $\theta(\sigma) < \hat{t}$, an intermediary truthfully reporting σ' must get at least $\bar{\pi}(\sigma)$. Hence the following proposition:

Proposition 3. *If models $\sigma \in [\sigma_{min}, \sigma_{max}]$ are distinguishable only above $\hat{t}$ and $\theta(\sigma_{min}) \leq \hat{t}$:*

- *To satisfy (IC), (LL) and bind (IR) for all types, then for all models σ with $\theta(\sigma) < \hat{t}$ the regulator needs to set $T(\sigma, t) < 0$ for some $t > \hat{t}$. She must be able to commit to subsidizing the intermediary after he defaults.*

- *If she is unable to commit, the regulator has to set $T(.,.)$ such that each type gets at least $\bar{\pi}(\sigma_{min})$, the highest reservation value, in order to induce the truthful revelation by all types.*

Remark 3. *If the regulator is unable to commit to subsidizing an intermediary after he defaults, if $\theta^*(\sigma_{min}) < \hat{t}$ there is a trade-off between extracting the intermediaries' surplus and how model-sensitive the regulation can be.*

When the realized level of defaults gives information about which is the true model, it is so high than an intermediary having reported an optimistic model defaults. Thus it is impossible to "punish" the use of optimistic models, the only possibility is to give a "bonus" for the use of pessimistic models. This "bonus" can be very costly here since all intermediaries have to get the reservation value of the most optimistic type. To avoid these costs, the only solution is to increase the capital requirements of the most optimistic types such that there is no $\theta(\sigma)$ below $\hat{t}$. But this means reducing the model-sensitiveness of the regulation compared to the first-best solution. Moreover, when backtesting is more difficult ($\hat{t}$ is higher) model-sensitiveness has to decrease more.

4.2 Auditing internal models

Since it is difficult to backtest models ex-post, another option is to try to detect over-optimistic models ex-ante by searching for strange parameterizations, extreme assumptions, theoretical flaws and so on. In the current state of the banking regulation, internal models have to be audited before their approval by the regulator. This auditing procedure is meant to check that a bank's model meets industry standards, that the model is sophisticated enough without being grossly over-optimistic. This is what I have modeled by assuming any $\sigma \in [\sigma_{min}, \sigma_{max}]$ is admissible: there is a range of models meeting industry standards.

Can the regulator also use auditing to encourage truthful revelation? Assume she has the following auditing technology $P(.)$: if an intermediary announces parameter σ', the regulator hires auditors to check the model during $H(\sigma')$ hours, each hour costing $w > 0$. Auditing will be assumed to be the search for mistakes in the model specification: intuitively, if an intermediary knowingly uses an over-optimistic model he has to use a false assumption or a strange parametrization at some point, something that can possibly be uncovered by an auditor. If the reported model is the correct one ($\sigma' = \sigma$) then the auditors never find the model to be wrong. Otherwise there is a probability $P(H(\sigma'))$ that they find a mistake and declare the model to be wrong, where $P' \geq 0, P'' \leq 0, P(0) = 0, \forall H \geq 0 \; P(H) < 1$.

Since I assume no type 1 error when auditing a model[10], a regulator who wants all types of intermediaries to report truthfully can punish as much as possible an intermediary caught with a wrong model. Thus an intermediary reporting the model σ' when the true model is σ gets:

$$U(\sigma',\sigma) = \begin{cases} (1-P(H(\sigma')))\pi(\theta(\sigma'),\sigma) + P(H(\sigma')) \times 0 & \text{if } \sigma' \neq \sigma \\ \pi(\theta(\sigma),\sigma) & \text{if } \sigma' = \sigma \end{cases}$$

By definition, such a scheme satisfies limited liability and individual rationality constraints. To satisfy incentive compatibility it must be the case that for any σ and σ' we have $\pi(\theta(\sigma),\sigma) \geq (1-P(H(\sigma')))\pi(\theta(\sigma'),\sigma)$. In words, for any model σ', $H(\sigma')$ has to be set such that no type has an incentive to falsely report model σ'. The regulator tries to maximize in $\theta(.)$ and $H(.)$ the social welfare $V(\theta(\sigma),\sigma)$ minus auditing costs subject to incentive compatibility:

$$\max_{\theta(.),H(.)} \int_{\sigma_{min}}^{\sigma_{max}} V(\theta(\sigma),\sigma) - w(1+c)H(\sigma)\psi(\sigma)d\sigma \text{ s.t. } \forall\sigma,\ 1-P(H(\sigma)) = \min_{\sigma'} \frac{\pi(\theta(\sigma'),\sigma')}{\pi(\theta(\sigma),\sigma')} \tag{11}$$

Since auditing is costly, the regulator wants to audit models just enough to elicit truthful revelation while minimizing the expected cost of auditing. The incentive compatibility constraint is difficult to study. Take a given value of σ; $\theta(\sigma)$ appears several times in the program. First, when σ realizes type σ has to be truthful. Second, there may be several other types σ' whose best deviation is to falsely report model σ. Denote:

$$m(\sigma) = argmin_{\sigma'} \frac{\pi(\theta(\sigma'),\sigma')}{\pi(\theta(\sigma),\sigma')} \tag{12}$$

$$d(\sigma) = \{\sigma_0 \in [\sigma_{min},\sigma_{max}],\ \sigma \in m(\sigma_0)\} \tag{13}$$

$m(\sigma)$ is the set of types who bind the constraint associated with σ, that is a set of "mimickers", just indifferent between telling the truth and mimicking σ. Conversely $d(\sigma)$ is the set of models for which σ is a mimicker: all models in this set are also potential deviations of type σ. First, notice that $m(\sigma) \neq \emptyset$ if $\theta(\sigma)$ is not the maximum of θ, and $m(\sigma) = \emptyset$ if $\theta(\sigma)$ maximizes θ. If $m(\sigma)$ is the empty set then it is optimal not to audit this model at all, if $\theta(\sigma)$ is lower than the maximum θ, the type with the maximum θ has a strict incentive to mimic σ. Conversely, nobody wants to mimic the type with the highest θ, hence it is never optimal to audit this model. Second, $m(\sigma)$ has to be a set of null measure in $[\sigma_{min},\sigma_{max}]$, otherwise it would be optimal to set $\theta(\sigma)$ equal to one, which would have a welfare cost in a state with probability $\psi(\sigma)d\sigma$ but make it possible to reduce distortions compared to the first-best on a set with positive measure. But if $\theta(\sigma) = 1$ then $m(\sigma) = \emptyset$, a contradiction. Third, $d(\sigma)$ has

[10] In practice it is of course possible that the regulator wrongly rejects a good model. In the Basel regulation the bank can typically revert to the standard approach and get $\overline{\pi}(\sigma)$ instead of 0, which makes it more difficult to ensure truthful reports. I abstract from this problem here, but taking this into account would make auditing even more costly.

to be a set of null measure, since otherwise it would be optimal to set $\theta(\sigma) = 0$, which would save auditing costs on the whole set $d(\sigma)$. But if $\theta(\sigma) = 0$ then $d(\sigma) = \emptyset$, a contradiction.

For the exposition, consider a given σ, and assume $m(\sigma) = \sigma_m, d(\sigma) = \sigma_d$: each set is a singleton[11]. Then, denoting $\lambda(\sigma)$ the multiplier associated to the incentive compatibility constraint for type σ, the first-order conditions with respect to $H(\sigma)$, $H(\sigma_d)$ and $\theta(\sigma)$ give:

$$\begin{aligned} V_1'(\theta(\sigma),\sigma)\psi(\sigma) &= \lambda(\sigma_d)\pi_1'(\theta(\sigma),\sigma) - \lambda(\sigma)(1-P(H(\sigma)))\pi_1'(\theta(\sigma),\sigma_m) \\ \lambda(\sigma)P'(H(\sigma))\pi(\theta(\sigma),\sigma_m) &= -(1+c)w\psi(\sigma) \\ \lambda(\sigma_d)P'(H(\sigma_d))\pi(\theta(\sigma_d),\sigma) &= -(1+c)w\psi(\sigma_d) \end{aligned}$$

Lastly, using the two incentive compatibility constraints, we have:

$$\frac{dH(\sigma_d)}{d\theta(\sigma)} = \frac{1}{P'(H(\sigma_d))} \times \frac{dP(H(\sigma_d))}{d\theta(\sigma)} = \frac{-\pi_1'(\theta(\sigma),\sigma)}{P'(H(\sigma_d))\pi(\theta(\sigma_d),\sigma)} \quad (14)$$

$$\frac{dH(\sigma)}{d\theta(\sigma)} = \frac{1}{P'(H(\sigma))} \times \frac{dP(H(\sigma))}{d\theta(\sigma)} = \frac{\pi_1'(\theta(\sigma),\sigma_m)\pi(\theta(\sigma_m),\sigma_m)}{P'(H(\sigma))\pi(\theta(\sigma),\sigma_m)^2} \quad (15)$$

which finally gives us the following first-order condition:

$$V_1'(\theta(\sigma),\sigma)\psi(\sigma) = (1+c)w\left(\underbrace{\frac{dH(\sigma_d)}{d\theta(\sigma)}\psi(\sigma_d)}_{\geq 0} + \underbrace{\frac{dH(\sigma)}{d\theta(\sigma)}\psi(\sigma)}_{\leq 0}\right) \quad (16)$$

This condition can be interpreted as follows: the first-best $\theta(\sigma)$ would satisfy $V_1'(\theta(\sigma),\sigma) = 0$, however auditing costs impose two distortions. First, when the true model is σ, the intermediary must be given incentives to report σ and not σ_d. If $\theta(\sigma)$ is high, the profit from reporting σ is low, and thus σ_d must be audited more. If the regulator biases $\theta(\sigma)$ downwards compared to the first-best, she increases the intermediary's profit if he tells the truth, and thus saves on auditing costs when the true model is σ_d. Biasing $\theta(\sigma)$ downwards means that $V_1'(\theta(\sigma),\sigma)$ is positive. Second, when the true model is σ_m, the intermediary would like to report σ. To prevent him from doing so, σ has to be audited more. A way to save on auditing costs when σ is the true model is to bias $\theta(\sigma)$ upwards, thus reducing the profit from misreporting σ. Interestingly, which effect dominates depends on $\psi(\sigma_d)$ and $\psi(\sigma)$. Notice that the auditing costs to ensure truthful revelation by type σ are effectively paid when σ_d is the true model, hence the downward bias is strong when $\psi(\sigma_d)$ is high. Conversely, when σ is the true model, the auditing costs come from incentive compatibility for type σ_m, the

[11] Otherwise it is necessary to sum the constraints over all elements in sets $d(\sigma)$ and $m(\sigma)$, but since they must have null measure the first-order condition is similar. The complete equation is given in proposition 4.

upward bias is strong when $\psi(\sigma)$ is high. Another interesting consequence is that if a model σ has a high prior probability to be the true model, such that $H(\sigma)$ will have to be paid often, in order to reduce auditing costs the regulator needs to look not at the incentives of type σ, but at the incentives of the type who would like to misreport σ. In other words, if a model is more likely than the others, it is necessary to make sure that, *when this model is wrong*, the intermediary does not want to use it.

From these two effects we can deduce important properties of the second-best regulation (θ^{**}, H^{**}):

Lemma 3. *1. $\theta^{**}(.)$ is increasing.*

2. $\theta^{**}(\sigma_{max}) \leq \theta^*(\sigma_{max})$ and conversely $\theta^{**}(\sigma_{min}) \geq \theta^*(\sigma_{min})$.

Proof of the first part (see Appendix A.6 for the detailed proof): intuitively, the main problem for a regulator facing a high σ is to prevent this type from deviating, and for a low σ to discourage other types to mimic σ. Take two models σ_0, σ_1 with $\sigma_1 > \sigma_0, \theta(\sigma_1) < \theta(\sigma_0)$. Type σ_0 knows profits will be higher from a given θ, and faces a tighter constraint if he tells the truth, hence he always has strictly more incentives to lie than σ_1. Thus there is no reason to bias $\theta(\sigma_1)$ downwards. Moreover since $\theta^*(.)$ is increasing it's more costly in terms of welfare to bias σ_0 upwards. And finally more auditing costs can be spared by decreasing $\theta(\sigma_0)$ than by decreasing σ_1, thus overall it cannot be the case that $\theta(\sigma_1) < \theta(\sigma_0)$ at an optimum. ■

Proof of the second part: consider figure 6, and assume (θ^{**}, H^{**}) are such that $\theta^{**}(\sigma_{max}) > \theta^*(\sigma_{max})$ as on the black curve. We want to show there exists $(\tilde{\theta}, \tilde{H})$ that improve welfare without increasing auditing costs. Since θ^{**} is increasing, if $\theta^{**}(\sigma_{min}) > \theta^*(\sigma_{max})$ it is possible to improve over θ^{**} by setting $\tilde{\theta}$ constant and equal to $\theta^*(\sigma_{max})$, such that $\tilde{\theta}(.)$ is closer to the first-best and induces zero auditing costs. If $\theta^{**}(\sigma_{min}) \leq \theta^*(\sigma_{max})$, there exists $\bar{\sigma}$ such that $\theta^{**}(\bar{\sigma}) = \theta^*(\sigma_{max})$. Then we can set $\tilde{\theta}$ constant and equal to $\theta^*(\sigma_{max})$ for $\sigma \in [\bar{\sigma}, \sigma_{max}]$, and $\tilde{\theta} = \theta^{**}$ otherwise. No type has an incentive to misreport a model in $[\bar{\sigma}, \sigma_{max}]$ since it cannot allow a higher leverage. Thus it is no longer required to audit models in this interval. For $\sigma < \bar{\sigma}$ we can use the same auditing intensity $H^{**}(\sigma)$, all types below $\bar{\sigma}$ still report truthfully because they face the same incentives as before, and all types above $\bar{\sigma}$ make more profit than before by telling the truth and thus also report truthfully. Overall $(\tilde{\theta}, \tilde{H})$ is both more efficient and less costly than (θ^{**}, H^{**}), a contradiction. The symmetric reasoning shows we must have $\theta^{**}(\sigma_{min}) \geq \theta^*(\sigma_{min})$. ■

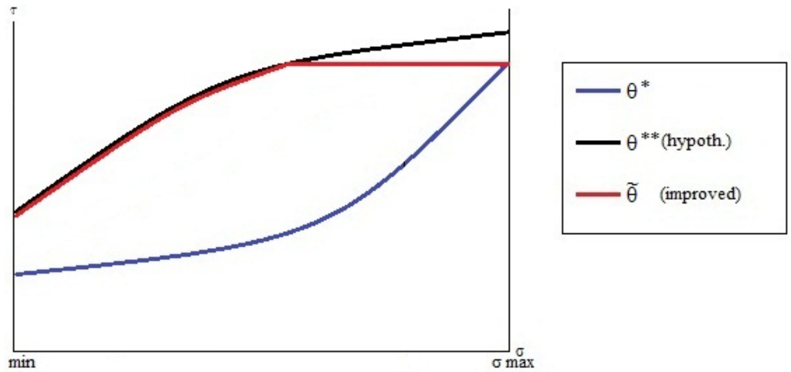

Figure 6: $\theta^{**}(\sigma_{max}) \geq \theta^{*}(\sigma_{max})$

I summarize all the results on auditing in the next proposition:

Proposition 4 (Model-sensitivity/auditing costs trade-off)**.** *The optimal regulation with costly auditing satisfies:*

1. *θ^{**} is increasing in σ. Thus $\theta^{**}(\sigma_{max}) - \theta^{**}(\sigma_{min})$ is lower than $\theta^{*}(\sigma_{max}) - \theta^{*}(\sigma_{min})$, thus the second-best regulation is on average less model-sensitive than the first-best regulation.*

2. *H^{**} is decreasing in σ. $H^{**}(\sigma_{max}) = 0$ and $P(H^{**}(\sigma_{min})) < 1$.*

3. $$V_1'(\theta^{**}(\sigma), \sigma)\psi(\sigma) = (1+c)w\left(\sum_{\sigma_d \in d(\sigma)} \left(\frac{dH^{**}(\sigma_d)}{d\theta(\sigma)}\psi(\sigma_d)\right) + \frac{dH^{**}(\sigma)}{d\theta(\sigma)}\psi(\sigma)\right)$$

Remark 4. *When auditing costs w tend to $+\infty$, the optimal regulation features a constant $\theta^{**}(.)$.*

This proposition shows that the more the regulator tries to use the information conveyed by models to compute capital requirements, the higher the incentives to misreport the model and thus the auditing costs are. As a result, the regulator should ask higher capital requirements than the first-best when the report is optimistic, and lower capital requirements than the first-best when the report is pessimistic. The first-order condition helps us to understand why: if the true model is very optimistic the intermediary does not have much incentives to lie, since by telling the truth he already gets a low capital requirement and a high profit. But such a model is a tempting deviation for an intermediary with a high risk. To discourage misreporting, capital requirements for the most optimistic types have to be higher. Conversely, the possibility that a type may want to misreport a very pessimistic model is not a concern. But an intermediary who knows the true model is very pessimistic has high incentives to lie, and to decrease them it is necessary to lower capital requirements for such types. Because of these two effects, the second-best regulation makes less use of the models' information than the first-best. Finally, when auditing costs are infinitely high even ensuring that the most constrained type does not want to mimic the least constrained one has an infinite cost, so that the regulator chooses a constant $\theta^{**}(.)$ and does not use the models' information at all.

4.3 Discussion

This section concludes on a rather negative note. In principle the regulator could use the observation of the realized level of default to detect intermediaries using over-optimistic models, she could offer a menu of capital requirements and penalties inducing truthful revelation. This is the spirit of the penalties used in the Basel regulation when market losses exceed the VaR produced by an internal market risk model too often. Credit risk however is likely to be different: different models typically yield different predictions for tail values only, and when high levels of default are reached it is quite possible that the institution will already be at risk, such that punishing over-optimism will be impossible ex-post, unless the regulator can commit to giving positive transfers to the shareholders of defaulting institutions. If this is impossible, it limits how sensitive to the intermediary's model the regulatory constraint can be. Even when the true model is the most optimistic one, the regulated should not be allowed too high a level of leverage, otherwise we will be in the case of models distinguishable only above the intermediary's default point. But if the regulation has to be less reactive to the intermediary's report, using internal models for regulatory purposes is also less useful.

The idea of auditing models seems easier to apply in practice. But the same trade-off appears, although for different reasons. If the purpose of a model-based regulation is to obtain finely risk-sensitive capital requirements, then capital requirements will also be *model*-sensitive. Then there are incentives to report a false model, and a costly auditing procedure is necessary to ensure truthful revelation. Proposition 4 shows that the second-best regulation will always be less risk-sensitive than the first-best. Two effects play a role: intermediaries knowing the true model is pessimistic must be allowed not too low a leverage to have less incentives to misreport, and intermediaries knowing the true model is optimistic must be allowed not too high a leverage, otherwise mimicking them would be too profitable. If auditing costs are high, these effects may make the second-best regulation much less risk-sensitive, such that the use of a model-based regulation is an unnecessary complication. For prohibitively high auditing costs by definition the regulator cannot use the models' information at all. The auditing procedure is close to the one studied in Prescott (2004), but in his paper the amount a bank can invest is fixed, thus a lower capital constraint does not allow to lend more, but only to get funding at a lower cost. As a result the type with the highest risk is always the one with the highest incentives to misreport and the schedule of second-best capital ratios is above the first-best, but not necessarily less risk-sensitive.

Models of adverse selection in a banking context similar to the menu of transfers discussed above have been used to study deposit insurance premia (Chan, Greenbaum, and Thakor (1992) and Freixas and Rochet (1998)). In these models, a bank is offered either a low insurance premium and a high capital requirement, or a high insurance premium and a low capital requirement, the low-risk bank selects the first offer and the high-risk bank the second one. But it has been assumed throughout this paper that, typically because of informational frictions, equity is more costly than debt, even without deposit insurance. Increasing capital requirements when the true σ is low (low risk) would thus decrease the amount of loans in the economy precisely when they have a higher social value. It is in principle possible to do better by using transfers to banks depending on the model they reported and on the level of defaults that realizes, which gives information about whether the model used is realistic or not.

An interesting extension of this section is to take into account that developing models is also long and costly. Assume for instance an intermediary has to pay a cost C in order to learn which one is the true model. The regulator has a new constraint linked to "information gathering" as in Crémer, Khalil, and Rochet (1998): she must make sure that the intermediary searches for the true model and pays the cost C. If C is high enough, more rent has to be left to intermediaries in the case of a menu of penalties. But if auditing is used, it may become necessary for the regulator to *increase* the risk-sensitiveness of the regulation. The intuition is the following: if auditing costs are very high, the second best θ^{**} is almost constant, so incentives to lie are low and few auditing hours are required. But an intermediary reporting σ_{max} gets $E_\psi(\pi(\theta^{**}(\sigma_{max}), \sigma))$, while if he searches he gets $E_\psi(\pi(\theta^{**}(\sigma), \sigma)) - c$. If θ^{**} is exactly constant, the first option is necessarily better. If we take into account this new constraint, the regulator can no longer get truthful revelation and arbitrarily low auditing intensity. Thus, for high enough auditing costs for the regulator and research costs for the intermediary, the second-best regulation will be not to use internal models at all. Conversely a model-based regulation is useful when intermediaries are very efficient at developing models and the regulator has a cheap auditing technology.

5 Extensions: possible countervailing forces to over-optimism

5.1 Gradual adoption of new models

It is certainly not always the case that banks or other financial intermediaries consciously choose risk models to bypass regulatory constraints. More plausibly, there is a process in which new models are developed, and since model uncertainty is hard to resolve, more "useful" models have a competitive advantage in the process. Either their users tend to favor useful models, or their "suppliers", often specialized firms, realize that models both plausible and not too pessimistic are more likely to become popular. The equilibrium of section 3.1 can be seen as a steady-state of such a process. It is first useful to make the following remark:

Remark 5. *With insured investors and incomplete regulation, the strategies μ_l chosen by two intermediaries are strategic substitutes.*

The proof follows from proposition 1: incentives to choose the most optimistic model are higher when r_L is higher, and r_L is higher when less intermediaries choose the most optimistic model. This fact has interesting dynamic implications. Imagine the following process: at the beginning intermediaries use all available models in the same proportions, or invest in the safe asset only. In each subsequent period, each intermediary has the opportunity to choose a new risk model/a new strategy. Intermediaries are not capable of computing precisely which model is the best to use, so they tend to adopt models which seem widely used and profitable: if $n_{i,t}$ is the number of intermediaries using strategy i in period t, $\pi(i,t)$ the profit made by an intermediary adopting this strategy in period t, and $\bar{\pi}_t$ the average profit in this period, I assume:

$$\forall i, \forall t \geq 0,\ n_{i,t+1} = \frac{\pi(i,t)}{\bar{\pi}_t} n_t$$

This process of "replicator dynamics" has the property that the total number of intermediaries stays constant, a strategy yielding more profit than the average is more and more adopted, and if the process converges to some distribution of strategies in the population then this distribution and the associated market prices form an equilibrium in the sense of definition 1. I simulate such a process with the same parameters for the distribution of defaults and demand as in the illustration of section 3.1. At period 0, I assume 90% of intermediaries invest in the safe asset only, and the others are using in the same proportions 1000 models giving them values of θ between $\theta(\sigma_{min})$ and 1. Figure 7 shows the evolution over 10 periods of the proportion of intermediaries choosing the most optimistic model, and the distribution of intermediaries over the different models available in period 10. The figure shows the proportion of intermediaries choosing all models between 2 and 1000, 24% of intermediaries choose the most optimistic one, and 72% invest in the safe asset only (which is consistent with figure 4).

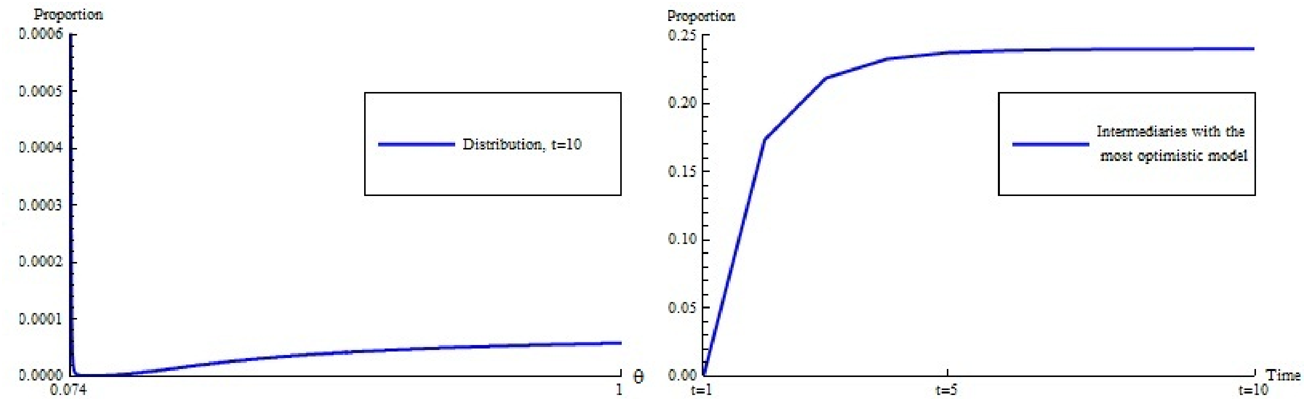

Figure 7: Distribution of intermediaries using the different models under incomplete regulation after 10 periods, and use of the most optimistic model over time.

At the beginning, most intermediaries do not invest in the risky asset, hence r_L is high and larger than r_L^e. It is thus extremely profitable to use the most optimistic models, and many intermediaries switch to them. This behavior increases the supply of loans and decreases the interest rate, such that in period 2 already the increase in the number of intermediaries using the most optimistic model is much smaller. r_L continues to shrink gradually, but as r_L decreases it becomes profitable to use low levels of leverage, and we obtain a process in two waves: forerunners rush to very optimistic models and the interest rate on loans drops, then these first models are gradually abandoned and replaced by more conservative ones. In the end the process will converge to an asymmetric situation with intermediaries using either maximum leverage or no leverage at all.

5.2 The cost of bad forecasts

I have made the rather extreme assumption that an intermediary could choose a very over-optimistic model with no more cost than if he chose a more realistic one. In a more general framework with several types of final borrowers for instance, the model could have two different functions: assessing the intermediary's risk and evaluating relative risks of defaults for two claims. It could be costly, or even impossible, to choose a very optimistic model to bypass regulation and at the same time allocate between the several types of borrowers as the true model advises. This would add a countervailing force giving incentives to stay closer to the true model and intermediaries would face a more complicated tradeoff.

However, for this countervailing force to be of any importance, it must be the case that optimistic models are too optimistic *for default levels below the maximum sustainable losses.* If the model just underestimates the probability of extreme events and the intermediary will default even for events less extreme, the forecasting mistake is privately irrelevant. Assume the true model is σ and the intermediary reports σ', the interest rate on loans is multiplied if the level of realized defaults is t by $\epsilon(|f(t,\sigma') - f(t,\sigma)|)$, with ϵ a decreasing function and $\epsilon(0) = 1,\ \epsilon(d) \geq 0\ \forall d$. In words, the return on each loan (conditional on repayment) is discounted, and the discount is higher when the probability of the realized default level was more badly forecast (because loans were not properly monitored, or losses were not properly hedged for instance). Equation 1 can be rewritten:

$$\pi(L) = \int_0^{\theta^C(L,\sigma')} \left(r_L L(1-t)\epsilon(|f(t,\sigma') - f(t,\sigma)|) - r_D(L-K)\right) f(t,\sigma)dt$$

where

$$\theta^C(L,\sigma')\ s.t.\ r_L L(1-\theta^C(L))\epsilon(|f(t,\sigma') - f(t,\sigma)|) - r_D(L-K) = 0$$

Assume the most optimistic model σ_{min} and the true model σ are distinguishable only in the tail above $\theta^C(L,\sigma_{min})$. Then $\theta^C(L,\sigma_{min}) = \theta(\sigma_{min})$ and the profit of the intermediary is exactly the same as without costs for forecast errors since below $\theta^C(L,\sigma_{min})$ the optimistic model's predictions are correct. If an intermediary chooses the most optimistic model, he will default for relatively low levels of losses in his portfolio, and he doesn't care about forecasting errors concerning higher levels of losses.

Thus, the incentive to adopt overoptimistic models is robust to the introduction of costs associated to forecasting errors if some models are available which are both optimistic regarding the probability of extreme events, and realistic everywhere else, which is precisely the fact highlighted in proposition 3. This reasoning also shows that a less model-sensitive regulation is a way to ensure all types of intermediaries care about forecasting mistakes.

5.3 Other extensions

The model is flexible enough to allow for a number of other extensions, which can be used to analyze other incentives to adopt more or less optimistic models.

A possible countervailing force to the selection of over-optimistic models would be of course to assume risk-adverse intermediaries, who wouldn't take too much risk and thus wouldn't choose the most optimistic models. This would probably not cancel incentives to be over-optimistic however, as the parallel with Gollier, Koehl, and Rochet (1997) suggests. It is also possible to assume some depositors will randomly withdraw their deposits before loans are reimbursed, or that banks have a "charter value" that limits their risk-taking. In all cases the choice of the intermediary would be made smoother, at the cost of additional complexity of the model, but the main results would not change.

It is interesting to consider what happens if investors are no longer insured. If they also know the true model and the regulator can commit on not bailing out defaulting institutions, investors charge higher interest rates to banks adopting more optimistic models. The financial structure of intermediaries becomes irrelevant and in equilibrium we have $r_L = r_L^e$. Regulation then relies mainly on Basel's third pillar (market discipline): the regulator makes sure that banks provide investors with quantitative estimates of the risk they incur and that the methodology used is clear enough for investors to detect over-optimistic models. Investors ask higher interest rates to lend to banks using more optimistic models, market discipline then limits the incentives to use over-optimistic models and to use more leverage. This is optimal if the regulator cares only about protecting investors. But if a bank's default has some external costs not borne by its creditors, then risk is too high. Thus another interpretation is that the possibility to freely choose the risk model on which regulation is based enables intermediaries and their creditors to entirely bypass the regulation and reach the level of leverage maximizing their joint profit. An interesting application is the case of an originator of securitized products (intermediary) who faces investors who can by law only invest in products with a sufficiently high grade. This model could be used to investigate the claim that many originators used the rating agencies to label their products "investment grade" and be able to sell them to regulated investors, while the latter may have been aware that the ratings were unrealistic but wanted to bypass the regulation (see for instance Pagano and Volpin (2009) ; and Bolton, Freixas, and Shapiro (2009)).

Another extension would be to assume incomplete competition among intermediaries. As we have seen, choosing a risk model is very similar in my framework to building capacity in an industrial organization model, with the important difference that usable capacity will depend both on the model chosen and on the interest rate r_L. Intuitively, we would expect competition in strategic substitutes to lead intermediaries to use optimistic models ("top dog" strategy), whereas competition in strategic complements could lead them to use pessimistic models ("puppy dog"): keeping a pessimistic model and being heavily regulated could be a way to commit to keeping the interest rate high. Moreover, intermediaries being of different sizes, since adopting new internal models and being able to satisfy the requirements of the internal model based regulation is costly and involves fixed costs, a side-effect might be to favor larger institutions at the expense of smaller ones.

Other possible mechanisms to reveal the intermediaries' private information could be analyzed. In particular in this model it is formally possible to use the fact that intermediaries are symmetric and know the same σ; hence a straightforward way to reveal their information would be to use the mechanism proposed in Maskin (1999): ask all intermediaries to report a model at the same time, and punish them if they don't all report the same one. Even if intermediaries have different beliefs about which is the true model, it would still be possible for the regulator to do better by exploiting the correlation between the different intermediaries' types as in Crémer and McLean (1988). Collusion among regulated intermediaries is likely with this mechanism and would limit the revelation of information (Laffont and Martimort (2000)). A further limitation is that the model can be reinterpreted as featuring institutions specialized in different segments of the credit market (see footnote 9): on each segment banks face specific problems and have to use models that are not directly comparable, which is one of the reasons why the regulator uses internal estimates and not a one-size-fits-all methodology. However, studying how the banking regulation could more generally use information from all banks at the same time and not from each institution separately, as is currently the case, is an interesting topic for further research.

A last interesting extension would be to model the fact that externalities arising from an intermediary's default are partly transnational, such that a national regulator can deliberately allow the use of optimistic models to favor domestic institutions. Rochet (2010) sees this as one of the main problems arising from the internal ratings based approach of Basel: it becomes impossible for agents outside the regulator-regulated relationship to check that the regulation is adequate, and the outcome is the race to the bottom in regulatory standards that Basel sought to prevent.

6 Conclusion

"Model-based" regulation is a sensible way to exploit banks' better information about their own risk to compute risk-based capital ratios. This information however is private, and financial intermediaries cannot be expected to develop the best models if they only face incentives to do otherwise. The choice of a risk model involves costs and benefits. In many cases the most "useful" model is also the correct one, but when a model has an "external use", its popularity won't depend on its validity only, but also on other characteristics. This paper shows how a regulation failing to give the proper incentives can lead to the wide adoption of over-optimistic risk models.

This work should not be interpreted as meaning that banks always choose internal models to maximize leverage, regardless of their plausibility; but rather as suggesting that the whole process of elaboration/adoption of new models may be biased towards models more useful to some agents. I don't mean either that some people deliberately elaborate wrong models: when one tries to improve on the best models already in use, it is tempting to relax the assumptions making the model too pessimistic first. The new model will be more general than the previous one and in this sense "better", but if such incentives influence the whole process there can be a drift toward over-optimism.

It is in principle possible to use a model-based regulation and ensure that financial institutions choose the correct models. This requires the regulator to be able to commit to charging important penalties for the use of over-optimistic models, which can be difficult, or to be proficient at auditing internal models. In both cases a more risk-sensitive regulation, for which using internal models would be the most relevant, makes it harder for the regulator to reveal an intermediary's true model, for three reasons. First, a more risk-sensitive regulation gives more incentives to use slightly over-optimistic models as it will enable the intermediary to increase leverage. Second, if intermediaries are allowed to use more leverage it is more likely that they will default for high levels of losses. Since these high levels are the ones that enable the regulator to tell whether a model was too optimistic, it becomes more difficult to punish over-optimistic intermediaries. Lastly, if intermediaries default for lower levels of losses they care about less states of the world, and hence using a model that is over-optimistic about tail risk is less damaging for their profit.

The conclusion is that banking regulation should be either even more complex, or much simpler. A first possibility is to continue using internal models and take into account the hidden information problem. This option should be chosen if benefits from a risk-sensitive regulation are high (e.g. very heterogeneous banks), and if for the regulator it is much easier to audit banks' risk models rather than developing some of her own (otherwise it would be better to use a regulatory model). Intermediaries must be very efficient at developing accurate models, and the regulator at checking their validity. The opposite solution is to simply abandon the use of internal models for regulatory purposes and have a regulator prompt to intervene when losses or capital ratios hit certain thresholds instead of trying to manage finely the banks' risks ex-ante, as advocated in Dewatripont and Tirole (1994), Decamps, Rochet, and Roger (2004) or Rochet (2010). In both cases the issue of how banks select their model has to be addressed carefully by the regulation.

Finally, although I developed the example of banks, there are other instances in which such a strategic use of models can take place. The regulation of insurance companies in the European Solvency II framework is comparable for what concerns risk model choice, with model uncertainty perhaps even more severe. Another interesting phenomenon along the same lines is the use of internal models to measure the performance of employees, desks, and departments. More generally, many firms use internal models to convey information from one hierarchic level to another, to rating agencies, shareholders... Sibbertsen, Stahl, and Luedtke (2008) quote a report according to which some investors "tend to apply an across-the-board discount of about 20% to the published numbers" and present it as an evidence of model risk. But this negative discount cannot stem from honest and random mistakes about the true model, much rather from the suspicion that models are deliberately chosen to bias the reported information. Hence the relevant problem here may not be model risk, but "model moral hazard".

A Appendix

A.1 Notations

r_L	gross interest rate on loans to borrowers
r_D	gross interest rate on loans to intermediaries
r_0	risk-free rate, normalized to 1
r_L^e	first-best interest rate on loans, equals $r_0/(1-\mathbb{E}(t))$
$D(.)$	demand for loans by final borrowers
$r_L(.)$	inverse demand function for loans
L	amount lent by an intermediary/the representative intermediary
M	amount borrowed by an intermediary
K	capital owned by an intermediary
W	investors' wealth
t	random proportion of defaulting loans
$f(.,\sigma), F(.,\sigma)$	family of pdf and cdf, parameterized by σ, modeling the proportion of defaulting loans
$\psi(.)$	distribution from which the true σ is drawn
θ	default point, maximum proportion of defaults an intermediary can suffer in his portfolio
$s(\theta,\sigma)$	expected proportion of surplus repayments in an intermediary's portfolio
$\pi(\theta,\sigma)$	expected profit of an intermediary
$V(\theta,\sigma)$	social welfare
$\alpha(\sigma)$	minimum capital ratio required from an intermediary reporting model σ
$\theta(\sigma)$	minimum θ allowed by the regulation if the intermediary reports model σ
η	parameter of the demand function used in simulations
μ_l, μ_r, μ_s	proportions of intermediaries with max. leverage / investing K in loans / K in the safe asset

A.2 Proof of Lemma 1

For given prices r_L, r_D and a given constraint $K/L \geq \alpha$, the intermediary's program if he invests in loans can be written as:

$$\max_{\theta} r_L L s(\theta,\sigma), \text{ s.t. } \theta \geq 1 - \frac{r_D}{r_L}(1-\alpha) = \underline{\theta}$$

Notice in particular that we must have $\theta \geq 1-(r_D/r_L)$, or equivalently $r_D - r_L(1-\theta) \geq 0$. It is easy to compute that $s'(\theta,\sigma) = F(\theta,\sigma)$, moreover L can be reexpressed as $r_D K/(r_D - r_L(1-\theta))$. Then we have

$$\pi_1'(\theta,\sigma) = \frac{r_L L(F(\theta,\sigma)(r_D - r_L(1-\theta)) - r_L s(\theta,\sigma))}{r_D - r_L(1-\theta)}$$

Denoting $G(\theta) = F(\theta,\sigma)(r_D - r_L(1-\theta)) - r_L s(\theta,\sigma)$, we have $G'(\theta) = F(\theta,\sigma)(r_D - r_L(1-\theta))$, thus G' is always positive. This implies that $\pi(\theta,\sigma)$ is either decreasing and then increasing in θ, always increasing or always decreasing. Finally, we have $\pi_1'(1,\sigma) = \frac{r_L K}{r_D}(r_D - r_L(1-\mathbb{E}(t))$, $\lim_{\theta\to 1-\frac{r_D}{r_L}} \pi_1'(\theta,\sigma) = -\infty$. Using equation 3 we have:

- If $r_L \geq (r_D/r_0)r_L^e$, then $\pi_1'(1,\sigma) \leq 0$ and π_1' is negative for every θ, thus if he invests in loans the intermediary chooses $\theta = \underline{\theta}$. Notice that we also have $r_L(1-\mathbb{E}(t)) \geq r_0$, hence the intermediary prefers investing in loans to investing in the safe asset.

- If $r_D r_L^e > r_L > r_L^e$ then the intermediary chooses $\theta = \underline{\theta}$ or $\theta = 1$, since profit is either first decreasing and then increasing in θ, or always increasing. A comparison shows that he will choose $\theta = \underline{\theta}$ if and only if

$$r_L \geq r_D \left(\frac{1-\mathbb{E}(t)-s(\underline{\theta},\sigma)}{(1-\underline{\theta})(1-\mathbb{E}(t))} \right) = r_1(r_D,\underline{\theta})$$

- If $r_D r_L^e > r_L^e > r_L$ profit is decreasing and then increasing in θ, but investing K in the safe asset yields more than in loans. The intermediary chooses $\theta = \underline{\theta}$ over $L = 0$ if and only if

$$r_L \geq \frac{r_0 r_D}{r_D s(\underline{\theta},\sigma) + r_0(1-\underline{\theta})} = r_2(r_D,\underline{\theta})$$

- If $r_D r_L^e > r_L^e = r_L$ the previous condition applies, except that the intermediary is indifferent when he doesn't borrow between investing in the safe asset or in loans.

We have to compare the different thresholds for r_L. First, we have:

$$\begin{aligned}
& r_1(r_D,\underline{\theta}) && > r_D r_L^e \\
\Leftrightarrow\quad & \underline{\theta} && > \mathbb{E}(t) + s(\underline{\theta},\sigma) \\
\Leftrightarrow\quad & \underline{\theta} && > \int_0^1 tf(t,\sigma)dt + \underline{\theta}F(\underline{\theta},\sigma) - \int_0^{\underline{\theta}} tf(t,\sigma)dt \\
\Leftrightarrow\quad & \underline{\theta}(1-F(\underline{\theta},\sigma)) > \textstyle\int_{\underline{\theta}}^1 tf(t,\sigma)dt \Leftrightarrow \underline{\theta} > \mathbb{E}(t|t \geq \underline{\theta}) &&
\end{aligned}$$

The last inequality is obviously false. Next, developing and rearranging $r_1(r_D,\underline{\theta})$ and $r_2(r_D,\underline{\theta})$, it is easy to show that

$$\begin{aligned}
r_1(r_D,\underline{\theta}) > r_2(r_D,\underline{\theta}) \quad &\Leftrightarrow \quad r_1(r_D,\underline{\theta}) > r_L^e \Leftrightarrow r_2(r_D,\underline{\theta}) > r_L^e \\
&\Leftrightarrow \quad r_D > \frac{r_0(1-\theta)}{1-\mathbb{E}(t)-s(\underline{\theta},\sigma)} \qquad (17)
\end{aligned}$$

This last inequality may be true or false depending on r_D. Thus we have two cases to consider and the conditions above prove the following:

If $r_1(r_D,\underline{\theta}) > r_L^e$: when $r_L < r_L^e$ the intermediary chooses $r_L^* = 0$, when $r_1(r_D,\underline{\theta}) > r_L \geq r_L^e$ he chooses $L^* = K$, when $r_L > r_1(r_D,\underline{\theta})$ he chooses $\theta^* = \underline{\theta}$. Now if $r_1(r_D,\underline{\theta}) < r_L^e$: if $r_L < r_2(r_D,\underline{\theta})$ the intermediary chooses $L^* = 0$, if $r_L \geq r_2(r_D,\underline{\theta})$ he chooses $\theta^* = \underline{\theta}$. This implies the proposition where I denote $\underline{r_L} = \max(r_1(r_D,\underline{\theta}), r_2(r_D,\underline{\theta}))$.

A.3 Proof of Lemma 2

When demand is close to perfectly elastic r_L does not depend on α, such that choosing α is equivalent to choosing $\theta = 1 - \frac{r_D}{r_L}(1-\alpha)$. Moreover we can write:

$$L(\theta) = \frac{r_0 K}{r_0 - r_L(1-\theta)}$$

L is obviously decreasing in θ. To prove that θ^* is decreasing we need to look at the derivatives of V_1':

$$\begin{aligned} V_1'(\theta,\sigma) &= L'(\theta)\left(r_L \mathbb{E}_\sigma(1-t) - r_0 - (c-1)\int_\theta^1 (r_0 - r_L(1-t))f(t,\sigma)dt\right) \\ &= L'(\theta)\left(r_L \mathbb{E}_\sigma(1-t) - r_0 - (c-1)(1-F(\theta,\sigma))(r_0 - r_L \mathbb{E}_\sigma(1-t|t>\theta))\right) \end{aligned}$$

Assumption **M1** implies that a distribution with a higher σ dominates a distribution with a lower one in the sense of first-order stochastic dominance. As a result when σ increases $\mathbb{E}_\sigma(1-t)$ and $\mathbb{E}_\sigma(1-t|t>\theta)$ decrease, $(1-F(\theta,\sigma))$ increases, such that $V_1'(\theta,\sigma)$ increases. Hence $V_{1,2}''(\theta,\sigma) \geq 0$. We can then compute:

$$V_{11}''(\theta,\sigma) = L''(\theta)(r_L\mathbb{E}_\sigma(1-t)-r_0)-(c-1)L''(\theta)\int_\theta^1(r_0-r_L(1-t))f(t,\sigma)dt+(c-1)L'(\theta)(r_0-r_L(1-\theta))$$

L'' is positive, thus the first term can be positive since in general the regulator will allow less leverage than what would lead to $r_L = r_L^e$. Intuitively when the regulator reduces the leverage further the supply of loans decreases but at a declining speed, hence welfare losses due to credit restriction increase more slowly, which gives some convexity in θ to V. When costs are high enough however this effect can be compensated by the two other terms. By definition of θ we have $r_0 - r_L(1-t) \geq 0$ for $t \geq \theta$, such that the two other terms are negative. Hence if c is high enough V_{11}'' is negative and $V_{1,2}''(\theta,\sigma)$ positive, such that for every σ there is a unique maximum of V for $\theta = \theta^*(\sigma)$, with θ^* increasing.

A.4 Proof of Proposition 1 and Corollary 1

When $r_D = r_0$, inequality 17 is equivalent for any θ to $\theta > \mathbb{E}(t) + s(\theta)$, which was proven to be wrong in the proof of lemma 1. Thus we have $r_L^e \geq r_2(r_0, \theta(\sigma_{min})) \geq r_1(r_0, \theta(\sigma_{min}))$. Thus we know that an intermediary will choose either $L = 0$ or $\theta = \theta(\sigma_{min})$. Thus in equilibrium a proportion μ_l of intermediaries choose model σ_{min} and $L = K/\alpha(\sigma_{min})$, and the others invest only in the safe asset. If the interest rate on loans is r_L, the supply of loans must equal the demand:

$$\mu_L \frac{K}{\alpha(\sigma_{min})} = D(r_L)$$

This equation implicitly defines $r_L(\mu_l)$, r_L is decreasing and $\lim_{\mu_l \to 0} r_L(\mu_l) = +\infty$ since when

μ_l is equal to zero there is no supply of credit. For brevity I denote $r_2 = r_2(r_0, \theta(\sigma_{min}))$. Clearly it is impossible to have $\mu_l = 0$ in equilibrium. Thus there are only two possibilities: either $\mu_l = 1$, from lemma 1 we know this is possible if and only if $r_L(1) \geq r_2$, or $\mu_l > 0, \mu_s > 0$, in which case intermediaries must be indifferent between $L = 0$ and $L = K/\alpha(\sigma_{min})$, and we need $r_L(\mu_l) = r_2$. Compute $r_L(1)$. If $r_L(1) \geq r_2$ then $\mu_l = 1$ is an equilibrium, moreover it is unique: if we decrease μ_l r_L will increase and it is impossible to find μ_l such that $r_L(\mu_l) = r_2$. If $r_L(1) < r_2$ we cannot have $\mu_l = 1$ in equilibrium, but since $r_L(.)$ is continuous and increasing and goes to $+\infty$ as μ_l approaches 0, there is a unique μ_l such that $r_L(\mu_l) = r_2$ which corresponds to an equilibrium.

Finally when we increase η we shift the demand function to the right and thus we shift $r_L(.)$ upwards, which will increase the equilibrium μ_l. When σ increases the only effect is an increase in r_2, in an interior equilibrium since demand is $D(r_2)$ it decreases, hence the supply has to decrease as well and thus μ_l decreases. The corollary follows from the fact that the expected proportion of defaulting intermediaries is $\mu_l(1 - F(\theta(\sigma_{min}), \sigma))$.

A.5 Proof of Proposition 2

Defining $U(\sigma', \sigma)$ the expected profit of an intermediary reporting σ' when the true parameter is σ, we have:

$$\begin{aligned} U(\sigma', \sigma) &= F(a(\sigma'), \sigma)\frac{\bar{\pi}(\sigma')}{F(a(\sigma'), \sigma')} \\ U_1'(\sigma', \sigma) &= \frac{(\bar{\pi}'(\sigma')F(a(\sigma'), \sigma) + a'(\sigma')f(a(\sigma'), \sigma)\bar{\pi}(\sigma'))F(a(\sigma'), \sigma')}{F(a(\sigma'), \sigma')^2} \\ &- \frac{(a'(\sigma')f(a(\sigma'), \sigma') + F_2'(a(\sigma'), \sigma'))\bar{\pi}(\sigma')F(a(\sigma'), \sigma)}{F(a(\sigma'), \sigma')^2} \end{aligned} \tag{18}$$

The first-order condition gives for every σ:

$$U_1'(\sigma, \sigma) = 0 \Leftrightarrow \frac{F_2'(a(\sigma), \sigma)}{F(a(\sigma), \sigma)} = \frac{\bar{\pi}'(\sigma)}{\bar{\pi}(\sigma)} \tag{19}$$

Notice first that under our assumptions the left-hand side is increasing in a (MLRP), decreasing in σ, and the right-hand side is increasing in σ. This ensures that if there exists a solution $a(.)$ it is increasing in σ. Is it always possible to find such an $a(\sigma)$? Notice that whether $\bar{\pi}$ is the payoff of an intermediary not using any leverage or allowed some default point θ independent of σ, it can be written as $\bar{\pi}(\sigma) = r_L K L(\theta) s(\theta, \sigma)$, with $\theta = 1$ if no leverage is allowed. Moreover, $s(\theta, \sigma)$ can be rewritten as $s(\theta, \sigma) = \int_0^\theta F(t, \sigma)dt$. Thus we can rewrite equation 19 as:

$$\frac{F_2'(a(\sigma), \sigma)}{F(a(\sigma), \sigma)} = \frac{\int_0^\theta F_2'(t, \sigma)dt}{\int_0^\theta F(t, \sigma)dt}$$

$a(\sigma)$ can take values between 0 and 1. We have $F_2'(1, \sigma)/F(a, \sigma) = 0$. If $\lim_{a \to 0} \frac{F_2'(a,\sigma)}{F(a,\sigma)} = -\infty$ we know there exists a value a to satisfy the required equality. If this limit is some negative number $-k$, then we know that for every t in $[0,1]$ we have $F_2'(t,\sigma) \geq -kF(t,\sigma)$. Integrating this inequality we find:

$$\frac{\int_0^\theta F_2'(t,\sigma)dt}{\int_0^\theta F(t,\sigma)dt} \geq -k$$

hence for a given σ the right-hand side is always lower than $\frac{F_2'(1,\sigma)}{F(1,\sigma)}$ and greater than $\lim_{a\to 0}\frac{F_2'(a,\sigma)}{F(a,\sigma)}$. Since $\frac{F_2'(a(\sigma),\sigma)}{F(a(\sigma),\sigma)}$ is increasing in a there is always a unique a satisfying the inequality for a given σ, and hence there always exists a unique solution $a(.)$ increasing and satisfying the first-order condition.

We now have to show that the second-order condition is met. We can use equation 19 to replace $F_2'(a(\sigma'),\sigma')$ in equation 18. After some rearrangements this gives us:

$$U_1'(\sigma',\sigma) \geq 0 \Leftrightarrow \frac{f(a(\sigma'),\sigma)}{f(a(\sigma'),\sigma')} \geq \frac{F(a(\sigma'),\sigma)}{F(a(\sigma'),\sigma')}$$

by the monotone likelihood ratio property we thus have $U_1'(\sigma',\sigma) \geq 0 \Leftrightarrow \sigma' \leq \sigma$, hence truthfully reporting σ globally maximizes $U(.,\sigma)$. Finally the set of families of distributions satisfying MLRP and the assumptions of the proposition is not empty, for instance a family of truncated Gaussian distributions differing in their means satisfies all the required properties when the variance of the underlying Gaussian distributions is 1 and the means are not too far from 5.

A.6 Proof of Proposition 4

The only part not proven in the text is that $\theta^{**}(.)$ is increasing. To prove this we will have to use the following lemma:

Lemma 4. *For any θ and σ we have $\pi_{11}''(\theta,\sigma) \geq 0$ and $\pi_{12}''(\theta,\sigma)\pi(\theta,\sigma) - \pi_1'(\theta,\sigma)\pi_2'(\theta,\sigma) \geq 0$.*

To prove the first part is is enough to differentiate π twice and get:

$$\pi_{11}''(\theta,\sigma) = r_L L f(\theta,\sigma) - 2\frac{r_L^2 L^2 F(\theta,\sigma)}{r_0 K} + 2\frac{r_L^3 L^3 s(\theta,\sigma)}{r_0^2 K^2}$$

using the fact that π_1' is negative and equal to:

$$\pi_1'(\theta,\sigma) = -\frac{r_L^2 L^2 s(\theta,\sigma)}{r_0 K} + r_L L F(\theta,\sigma)$$

shows that the the expression of $\pi_{11}''(\theta,\sigma)$ above is positive.

For the second part of the lemma, simple derivations and rearranging yields:

$$\begin{aligned} &\pi_{12}''(\theta,\sigma)\pi(\theta,\sigma) - \pi_1'(\theta,\sigma)\pi_2'(\theta,\sigma) \geq 0 \\ \Leftrightarrow\quad & r_L^2 L^2 (F_2'(\theta,\sigma)s(\theta,\sigma) - F(\theta,\sigma)s_2'(\theta,\sigma)) \geq 0 \end{aligned}$$

Remember $s'_1 = F$, thus we have to show that

$$s''_{12}(\theta,\sigma)s(\theta,\sigma) \geq s'_1(\theta,\sigma)s'_2(\theta,\sigma) \tag{20}$$

Since $F(.,.)$ has the MLRP property we have for $\sigma_1 \geq \sigma_0$:

$$\frac{F(x,\sigma_1)}{f(x,\sigma_1)} \leq \frac{F(x,\sigma_0)}{f(x,\sigma_0)}$$

This can be rewritten as:

$$\frac{s''_{11}(x,\sigma_1)}{s'_1(x,\sigma_1)} \geq \frac{s''_{11}(x,\sigma_0)}{s'_1(x,\sigma_0)}$$

this implies in particular that $s'_1(x,\sigma_1)/s'_1(x,\sigma_0)$ increases in x. From which we deduce that for any x, y, σ_0, σ_1 with $x \leq y, \sigma_0 \leq \sigma_1$ we have:

$$s'_1(y,\sigma_1)s'_1(x,\sigma_0) \geq s'_1(y,\sigma_0)s'_1(x,\sigma_1)$$

Integrating both sides with respect to x between 0 and y and rearranging we get:

$$\frac{s'_1(y,\sigma_1)}{s(y,\sigma_1)} \geq \frac{s'_1(y,\sigma_0)}{s(y,\sigma_0)}$$

which means that s'_1/s is increasing in σ, which is equivalent to inequality 20 and implies the second part of the lemma.

We can now prove the proposition by contradiction. Assume there are two models σ_0, σ_1 with $\sigma_1 > \sigma_0, \theta(\sigma_1) < \theta(\sigma_0)$. The second part of the lemma implies that $\pi'_1(\theta,\sigma)/\pi(\theta,\sigma)$ is increasing in σ, which means that for any θ we have:

$$\frac{\pi'_1(\theta,\sigma_1)}{\pi(\theta,\sigma_1)} \geq \frac{\pi'_1(\theta,\sigma_0)}{\pi(\theta,\sigma_0)}$$

this implies that $\pi(\theta,\sigma_1)/\pi(\theta,\sigma_0)$ is increasing in θ, from which we deduce that for any θ, θ' with $\theta' < \theta$ we have:

$$\frac{\pi(\theta,\sigma_1)}{\pi(\theta',\sigma_1)} \geq \frac{\pi(\theta,\sigma_0)}{\pi(\theta',\sigma_0)}$$

and finally this implies that for any model σ' such that $\theta(\sigma') < \theta(\sigma_1)$ we have:

$$\frac{\pi(\theta(\sigma_1),\sigma_1)}{\pi(\theta(\sigma'),\sigma_1)} > \frac{\pi(\theta(\sigma_0),\sigma_0)}{\pi(\theta(\sigma'),\sigma_0)}$$

such that $d(\sigma_1) = \emptyset$, for any model that type σ_1 wants to mimic, type σ_0 has strictly more incentives to deviate to the same model. Now assume for clarity that $m(\sigma_i) = \sigma_{mi}, d(\sigma_0) = \sigma_{d0}$, the result remains unchanged if these sets have zero or more than one element. The first-order conditions for models σ_0 and σ_1 give:

$$V_1'(\theta(\sigma_0),\sigma_0)\psi(\sigma_0) - (1+c)w\left(\psi(\sigma_0)\frac{dH(\sigma_0)}{d\theta(\sigma_0)} + \psi(\sigma_{d0})\frac{dH(\sigma_{d0})}{d\theta(\sigma_0)}\right) = 0 \tag{21}$$

$$V_1'(\theta(\sigma_1),\sigma_1)\psi(\sigma_1) - (1+c)w\left(\psi(\sigma_1)\frac{dH(\sigma_1)}{d\theta(\sigma_1)}\right) = 0 \tag{22}$$

Since we have $V_{11}'' \leq 0, V_{12}'' \geq 0$, it must be the case that $V_1'(\theta(\sigma_0),\sigma_0) \leq V_1'(\theta(\sigma_1),\sigma_1)$. Using equations 21 and 22:

$$\frac{dH(\sigma_0)}{d\theta(\sigma_0)} + \frac{\psi(\sigma_{d0})}{\psi(\sigma_0)}\frac{dH(\sigma_{d0})}{d\theta(\sigma_0)} \leq \frac{dH(\sigma_1)}{d\theta(\sigma_1)} \tag{23}$$

since the second term on the left-hand side is positive, to contradict 23 it is enough to show the following:

$$\frac{dH(\sigma_0)}{d\theta(\sigma_0)} > \frac{dH(\sigma_1)}{d\theta(\sigma_1)}$$

since $\theta(\sigma_1) < \theta(\sigma_0)$ it must be the case that $H(\sigma_1) > H(\sigma_0)$ and hence $P'(H(\sigma_1)) > P'(H(\sigma_0))$. Using the expression of $\frac{dH(\sigma)}{d\theta(\sigma)}$ in equation 15it is enough to contradict the assumption $\theta(\sigma_1) < \theta(\sigma_0)$ to show that:

$$\frac{\pi_1'(\theta(\sigma_0),\sigma_{m0})}{\pi(\theta(\sigma_0),\sigma_{m0})} \geq \frac{\pi_1'(\theta(\sigma_1),\sigma_{m1})}{\pi(\theta(\sigma_1),\sigma_{m1})} \tag{24}$$

using the lemma we have $\pi_{11}'' \geq 0, \pi_{12}'' \geq 0$, hence we have to show that $\sigma_{m0} \geq \sigma_{m1}$. By definition of σ_{m0} and σ_{m1} it has to be the case that σ_{m0} has a higher incentive to mimic σ_0 than does σ_{m1} and conversely. Thus we must have:

$$\frac{\pi(\theta(\sigma_{m1}),\sigma_{m1})}{\pi(\theta(\sigma_0),\sigma_{m1})} \geq \frac{\pi(\theta(\sigma_{m0}),\sigma_{m0})}{\pi(\theta(\sigma_0),\sigma_{m0})},\ \frac{\pi(\theta(\sigma_{m0}),\sigma_{m0})}{\pi(\theta(\sigma_1),\sigma_{m0})} \geq \frac{\pi(\theta(\sigma_{m1}),\sigma_{m1})}{\pi(\theta(\sigma_1),\sigma_{m1})}$$

from these two inequalities we deduce:

$$\frac{\pi(\theta(\sigma_1),\sigma_{m1})}{\pi(\theta(\sigma_1),\sigma_{m0})} \geq \frac{\pi(\theta(\sigma_0),\sigma_{m1})}{\pi(\theta(\sigma_0),\sigma_{m0})}$$

and if $\sigma_{m0} < \sigma_{m1}$ this last inequality is incompatible with the second part of lemma 4. As a conclusion $\sigma_{m0} \geq \sigma_{m1}$, thus inequality 24 is true and hence the first-order conditions 21 and 22 are not logically consistent, which implies that $\theta(\sigma_1)$ cannot be lower than $\theta(\sigma_0)$.

A.7 References

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