

# Reputation and Securitization\*

Keiichi Kawai<sup>†</sup>

Northwestern University

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## Abstract

We analyze a dynamic market with a seller who can make a one-time investment that affects the returns of tradable assets. The potential buyers of the assets cannot observe the seller's investment prior to trade, nor verify it in anyway after trade. The market faces two types of inefficiency: the *ex-ante* inefficiency, i.e., the seller's moral hazard problem; and the *ex-post* inefficiency, i.e., inefficient ex-post allocations due to the adverse selection problem. We analyze how the observability of information by future buyers through which the seller builds a reputation affects the two types of inefficiency, and the interplay between them. Our conclusions are as follows: If the cost of investment is small, then the observability of such information mitigates both types of inefficiency. If the cost of investment is not small, however, then it may not have any effect on them. Moreover, even when the observability of such information improves the ex-post efficiency, it does so only at an offsetting cost in terms of the ex-ante efficiency. A simple regulation based on prices observable in the markets can enhance the role of such information and achieve the social welfare which is arbitrarily close to the first-best.

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<sup>†</sup>E-mail: keiichikawai2008@u.northwestern.edu. Address: Department of Economics, Northwestern University, Evanston IL, 60208, USA.

# 1 Introduction

The advent of securitization – converting illiquid assets into liquid securities – has transformed the traditional role of financial intermediaries in the mortgage market from the traditional relationship banking model to an “originate-to-distribute” model. The potential benefits of securitization include, among others, improving risk sharing and reducing banks’ cost of capital.

In the aftermath of the 2008 financial crisis, however, a lot of attention has been brought to the potential moral hazard problem that arises from the separation of a loan’s originator and the bearer of the loan’s default risk.<sup>1)</sup> Since the investors do not observe the quality of the mortgages they are buying, the separation of the loan’s originator and the bearer of the loan’s default risk reduces the incentives of financial intermediaries to carefully screen borrowers. As a result, the securitization process endogenously creates the adverse selection problem – an impediment to efficient trade – via the moral hazard problem the originators face. Several recent empirical studies – e.g., Mian and Sufi (2009) and Keys et al. (2010) – support this view.

The proponents of securitization do not concur. Indeed, they point out that if potential investors believe that loans are originated without careful screening, then the originators face the risk of losing future investors. Therefore, loan originators are mindful of building a reputation through the information that potential investors have on them, including previous transactions and the performance of loans originated. The proponents of securitization argue that such reputational incentives should mitigate the moral hazard problem and the resulting adverse selection problem, if not solve them completely.

In this paper, we show that such an argument misses the mark in regard to the effect of the originator’s reputational incentive on the unique interplay between the *ex-ante* inefficiency, i.e., the originator’s moral hazard problem, and the *ex-post* inefficiency, i.e., inefficient ex-post allocation due to the adverse selection problem.

To see this point, we analyze a three-period game with a bank and short-lived buyers. The bank lives for three periods,  $t = 0, 1, 2$ . In  $t = 0$ , the bank decides whether or not to undertake an investment with cost  $c > 0$ . In each period  $t = 1, 2$ , the bank is endowed with a loan, which matures at the end of period  $t$ . If the bank chooses to invest, then the loans with which it is endowed

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<sup>1)</sup>For example, Geithner and Summers (2009) write: “In theory, securitization should serve to reduce credit risk by spreading it more widely. But by breaking the direct link between borrowers and lenders, securitization led to an erosion of lending standards, resulting in a market failure that fed the housing boom and deepened the housing bust.”

are *good loans*, i.e., loans with a low expected default rate. In contrast, if the bank decides not to undertake the investment, then the loans with which it is endowed are *bad loans*, i.e., loans with a high expected default rate.

Now suppose that there is no securitization, i.e., no trade in the secondary markets. The bank is then the sole bearer of the loan's default risk. As a result, the bank undertakes the socially efficient investment, i.e., there is no moral hazard problem. Therefore, *the efficiency gains from securitization* in our model are measured by the difference between the two terms: (i) *the gains from trade*, that is, the *ex-post efficiency* created by the transfer of loans; and (ii) *the losses from moral hazard*, that is, the *ex-ante inefficiency* in the investment caused by the unobservability of the investment.

Next, suppose that the bank can sell its loans in the secondary markets, but future buyers cannot observe any information about the quality of the past loans. The bank then has no reputational incentives. We show that the gains from trade are completely offset by the losses from moral hazard, i.e., there are no efficiency gains from securitization. The objective of this paper is to show how the information about the quality of the past loans affects the two types of inefficiency and their interplay.

To consider the model with reputational incentives, we assume that buyers in  $t = 2$  can observe (i) the highest offer in  $t = 1$ , and (ii) the performance of the loan sold in  $t = 1$ . They cannot, however, observe the performance of any loans that the bank chooses not to sell.<sup>2)</sup> There are then two ways the bank sends a signal that it has invested.

One way is to refuse to sell even for a high price. Then, any bank that does sell its loan in  $t = 1$  is perceived as a bad bank by the future buyers, irrespective of the performance of the loan sold in  $t = 1$ . In other words, a good bank strives to build a reputation by holding loans. In such a case, we say a good bank has a *negative reputational incentive* because such a reputational incentive hinders the trade in  $t = 1$ .

The efficiency gains from securitization in an equilibrium where a good bank has a negative reputational incentive turn out to be zero. This is because under such an equilibrium, a good bank can send a signal if and only if the buyers in  $t = 1$  make high offers. However, it never has an opportunity to do so on any equilibrium path because buyers never make such an offer knowing it will be accepted only by a bad bank. Therefore, the information that the buyers in  $t = 2$  have is no finer than the information that buyers in  $t = 1$  have. Consequently, the buyers who face the bank with a negative reputational

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<sup>2)</sup>See Chari et al. (2010) for a similar assumption.

incentive are in the same situation as the buyers who face the bank without any reputational incentives.

The other way that a good bank sends a signal is to sell its loan in  $t = 1$  even at a current loss in order to generate public information about the loan's performance. In this case, any bank with good performance is likely to be perceived as a good bank by future buyers, and hence to receive high offers in  $t = 2$ . In such a case, we say a good bank has a *positive reputational incentive* because such a reputational incentive facilitates the trade.

In an equilibrium where a good bank has a positive reputational incentive, the buyers in  $t = 2$  have finer information than the buyers in  $t = 1$ . When a high offer – at which a good bank is willing to sell – is made in  $t = 1$ , the highest offer a bank receives in  $t = 2$  depends on the performance of the loan it sold in  $t = 1$ . If the loan did not default, then the highest offer the bank receives exceeds the reservation value of holding a good loan. On the other hand, in the case of default, the bank receives a low offer – i.e., an offer below the reservation value of holding a good loan – with a positive probability.

We show that whether efficiency gains from securitization exist in a positive reputational equilibrium depends on the cost of investment. If the cost of investment is small, then the higher probability of facing a default is sufficient to induce the bank to invest with a higher probability than the case without any reputational incentive. Consequently, the losses from moral hazard shrink, and the gains from trade increase. In this case, there are efficiency gains from securitization.

If the cost of investment is sufficiently large, however, then the higher probability of facing a default in itself is not sufficient to induce the bank to invest any more. As a result, the bank's reputational incentive creates more gains from trade, but only at the cost of offsetting losses from moral hazard. Therefore, in contrast to what proponents of securitization argue, the reputational incentive can exacerbate the moral hazard problem.

We then show a simple regulation based on prices observable in the market can achieve the social welfare arbitrarily close to the first best by enhancing the role of the information through which the bank build its reputation.

The rest of the paper proceeds as follows. Section 2 reviews the relevant literature. Section 3 provides the model. Section 4 discusses the case where there is no information through which the bank can build reputation. Section 5 analyzes the case where there is such information. Section 6 provides the discussion the simple regulation that achieves an outcome arbitrarily close to the first-best outcome. Section 7 provides concluding remarks.

## 2 Related Literature

**Adverse Selection in Asset Markets with Reputational Considerations** Our paper is closely related to the literature on adverse selection in asset markets where the asset holders have reputational incentives.

Among others, Chari et al. (2010) analyze a model that is very close to the interim stage of our game, i.e., the game that starts from  $t = 1$ . As in our model, they find that a dynamic model with adverse selection and reputational incentives generates multiple equilibria: namely, a “good” reputational equilibrium in which loans are actively traded, and a “bad” reputational equilibrium in which good loans are held by the bank and the market freezes.<sup>3)</sup> They regard the multiplicity of equilibria as the source of “fragility” of outcomes, which could explain a sudden collapse of trading in the secondary markets.

Other papers on dynamic adverse-selection models with reputational incentives include Hartman-Glaser (2011) and Daley and Green (2010). Hartman-Glaser (2011) considers a repeated security-issuance game, and analyzes the relationship between the issuer’s reputational incentives and the costly signaling via retention of the asset. Daley and Green (2010) studies the strategic issues that arise when informational asymmetries dissipate gradually over time.

In those papers, unobservable ex-ante types are assumed to be exogenous.<sup>4)</sup> Therefore, the interplay between ex-ante and ex-post inefficiency is absent in their models. In contrast, there is no ex-ante incomplete information in our model, and unobservable types are endogenously determined by the bank’s choice.

**Ex-ante Inefficiency vs. Ex-post Inefficiency** The types of investment involving the tension between ex-ante inefficiency and ex-post inefficiency discussed in this paper are also analyzed in Hermalin (2011), Kawai (2011a), and Kawai (2011b). Kawai (2011a) analyzes a dynamic market for lemons in which the quality of the good is endogenously determined by the seller’s investment, and potential buyers sequentially submit offers to the seller. It then shows that the competition among the buyers drives the efficiency gain down to zero. Hermalin (2011) analyzes the static version of Kawai (2011a) and shows how

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<sup>3)</sup>The model by Hörner and Vieille (2009), which analyzes a dynamic market for lemons, also shows why and how reputational incentives may hinder trade. See also Ely and Välimäki (2003) and Ely et al. (2008) for models in which reputational concern can worsen outcomes.

<sup>4)</sup>For example, in Chari et al. (2010) and Daley and Green (2010), the probability that the bank owns a good loan is exogenously given. In Hartman-Glaser (2011), the issuer is some behavioral type with an exogenous probability.

the distribution of bargaining power affects the efficiency gain created by the possibility of trade.

Kawai (2011b) analyzes a bilateral bargaining problem involving one seller and one buyer for a single object, the quality of which is endogenously determined by the seller's investment. If the seller chooses to invest in the good, then it becomes a high-quality good, with each individual's valuation of which is unknown to the other, as in Myerson and Satterthwaite (1983). However, if the seller chooses not to invest in the good, then the good is a low-quality good. It then shows that the surplus-maximizing, voluntary trading mechanism brings balance to the trade-off between providing investment incentives (the ex-ante efficiency), and creating the gain from trade (the ex-post efficiency).

**Regulation** A policy proposal similar to ours is known to be effective when the investment only affects the buyers' willingness to pay.<sup>5)</sup> Atkeson et al. (2011) study a market with free entry and exit of firms, which can produce a high-quality experience good only if making they make a costly but efficient, initial unobservable investment. They find that an entry tax can enhance the role of reputation but at the expense of reducing the volume of trade.

### 3 Model

The game consists of three periods,  $t = 0, 1, 2$ . There is a bank that lives for three periods, and there are two short-lived buyers who live only in period 1, and two who live only in period 2.

In  $t = 0$ , the bank decides whether or not to undertake an investment with cost  $c > 0$ , which is commonly known. However, whether or not the bank has invested is the bank's private information. Therefore, the buyers do not know the bank's *unobservable interim type*. We say the bank's unobservable interim type is *bad* if the bank did not invest, and is *good* if the bank invested. The bank's interim type is persistent throughout the game.

The first period consists of four stages. At the first stage, the bank is endowed with a loan which matures at the end of period 1. A good bank is endowed with a *good loan*, which yields a return of 1 with probability  $\pi_g \in (0, 1)$ , and 0 with probability  $1 - \pi_g$ . On the other hand, a bad bank is endowed with a *bad loan*, which yields a return of 1 with probability  $\pi_b \in (0, \pi_g)$ , and 0 with probability  $1 - \pi_b$ .

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<sup>5)</sup>Note that in our model, the bank's investment changes the reservation value of the loans to the bank as well as the value of loans to potential buyers.

At the second stage, two buyers simultaneously offer prices to purchase the loan. The purchaser of a loan is entitled to the resulting return.

At the third stage, the bank decides whether to sell the loan in the secondary market or hold the loan. We assume that the buyers have a comparative advantage in holding loans for some exogenous reason. We model this by assuming that the bank discounts an end-of-period payoff by a discount factor  $\gamma < 1$ , while the short-lived buyers do not discount at all, i.e., the bank is more impatient.<sup>6)</sup>

Therefore, if the bank holds its loan to its maturity, then the payoff for  $t = 1$  is  $\gamma\pi_g$  when the loan is good, and  $\gamma\pi_b$  when it is bad. Alternatively, if the bank decides to sell when the buyers' highest offer is  $p_1$ , then its payoff is  $p_1$ .

At the fourth stage of  $t = 1$ , the loan matures.

To summarize, the four stages of period 1 are as follows:

1. The bank is endowed with a loan.
2. The buyers simultaneously offer prices to purchase the loan.
3. The bank decides either to sell the loan to one of the buyers or hold the loan to maturity.
4. The loan matures.

The second period consists of the same four stages as the first period. As in  $t = 1$ , a good bank will be endowed with a good loan, and a bad bank will be endowed with a bad loan.

The difference between the buyers in  $t = 1$  and these in  $t = 2$  is that the buyers in  $t = 2$  can observe (i) the highest offer in  $t = 1$ , (ii) whether the bank has sold a loan or not at the highest offer, and (iii) the performance of loan sold in the secondary market. They cannot, however, observe the performance of any loan that the bank chooses not to sell.<sup>7)</sup>

**Remark 1** *One might think that the performance of each loan is easily verifiable, and therefore that the bank and a buyer should sign a contingent contract instead of relying*

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<sup>6)</sup>This assumption is commonly used in the literature. See, e.g., DeMarzo and Duffie (1999). This assumption is a proxy for the bank's (within-period) preference for cash, which arises from regulatory capital requirements.

<sup>7)</sup>This assumption could be seen as a proxy for a high cost of finding out the true performance of any loans that the bank decided not to sell. For example, the bank can easily make it harder to find out the true performance of loans it has decided not to sell by means of creative accounting. This again is the assumption used in the much of literature (see Chari et al. (2010)).

on the spot market, as in this paper.<sup>8)</sup> However, such a contract would be complicated for securitized assets. Moreover, any such contingent contract requires a deferred payment, or equivalently requires the bank to hold a portion of the proceeds from a loan sale until the loan's maturity. No such contract, however, is renegotiation-proof.<sup>9)</sup>

The equilibrium concept we use is that of a perfect Bayesian equilibrium. In the analysis below, we impose the following assumptions:

**Assumptions:**

1.  $2\gamma\pi_g - c > 2\pi_b$ .
2.  $2\pi_g - c < 2\gamma\pi_g$ .
3. Buyers in a same period use a symmetric strategy.

The first assumption concerns the scenario in which the buyers believe that the bank is bad. In such a scenario, the buyers' willingness to pay for the loans is  $\pi_b$ . The bank then invests and holds rather than not investing and selling the loans.

To understand the second assumption, note that the buyers never offer prices higher than  $\pi_g$ . The LHS is thus the highest payoff a good bank can obtain. In contrast, the RHS represents a bad bank's payoff when it expects to receive the reservation value of holding a good loan, i.e.,  $\gamma\pi_g$ , in both periods. Therefore, as the second assumption states, if the bank expects to receive high offers in both periods, it prefers not to invest.

The third assumption is an innocuous assumption because the set of equilibrium outcomes will not change with this restriction.

## 4 Benchmark Cases

### 4.1 Preliminaries

**No Secondary Market** Suppose that there is no secondary market, that is, the bank holds its loans in both  $t = 1$  and  $t = 2$ . Then, Assumption 1 implies that

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<sup>8)</sup>See Hartman-Glaser et al. (2011) for the analysis of optimal contracts that are not renegotiation-proof.

<sup>9)</sup>Recall that the bank and potential buyers have different within-period discount factors, and the liquidity created by the difference among within-period discount factors is the source of the gains from trade. Any contract which forces the bank to hold a portion of the proceeds from a loan sale shrinks the liquidity available to the bank at the time of trade. This implies that the buyer – who agreed to such a contract, believing that the bank was a good bank – has every incentive to offer a new contract which gives both the bank and him higher payoffs.



the bank strictly prefers to invest, i.e.,

$$\underbrace{2\gamma\pi_g - c}_{\text{payoff of a good bank}} > \underbrace{2\gamma\pi_b}_{\text{payoff of a bad bank}}. \quad (4.1)$$

**Gains from Trade** In our model, the secondary loan markets enable the transfer of loans to buyers who have a comparative advantage in holding the loans. This implies that the transfer of loans from the bank is always *ex-post efficient*. We call the gains in social welfare associated with this *ex-post efficiency* the *gains from trade*.

Formally, when the bank invests with probability  $\mu$ , and a good bank and a bad bank sell their loans in period  $t$  with probability  $\sigma_t^G$  and  $\sigma_t^B$ , respectively, the gains from trade are defined by

$$\underbrace{\mu(\sigma_1^G + \sigma_2^G)(1 - \gamma)\pi_g}_{\text{gains from trade with a good bank}} + \underbrace{(1 - \mu)(\sigma_1^B + \sigma_2^B)(1 - \gamma)\pi_b}_{\text{gains from trade with a bad bank}}. \quad (4.2)$$

**Losses from Moral Hazard** Note that when loans are always traded, the gains from trade represented by (4.2) are increasing in the probability  $\mu$  of the bank being good. Therefore, the investment is *ex-ante efficient*.

However, whether or not the bank has invested is unobservable to the buyers. Therefore, the possibility of loan trade in the secondary markets may create the moral hazard problem, i.e., the bank may not invest with probability one. We call the losses in social welfare associated with this *ex-ante inefficiency* the *losses from moral hazard*. Formally, when the bank invests with probability  $\mu \in [0, 1]$ , the losses from moral hazard are defined by

$$(1 - \mu)\left((2\gamma\pi_g - c) - 2\gamma\pi_b\right). \quad (4.3)$$

**Efficiency Gains from Securitization** The *efficiency gains from securitization* are then measured as the difference between the gains from trade and the losses from moral hazard.

In the *first-best outcome*, i.e., the outcome that maximizes the efficiency gains from securitization, the bank invests with probability one, and the loans are sold in both periods. Thus, under the first-best outcome, the gains from trade are  $2(1 - \gamma)\pi_g$ , and the losses from moral hazard are zero.

## 4.2 No Reputational Incentive

We now analyze the situation in which the buyers in  $t = 2$  cannot observe any information through which the bank can build its reputation. To be precise, neither (i) the performance of the loan that is sold in the secondary market, nor (ii) whether or not the bank has sold its loan in the secondary market is observable to the buyers in  $t = 2$ .

In such a case, the bank does not have any reputational incentive. We show that there are no efficiency gains from securitization in any equilibrium. One might expect this to occur because the absence of information about the bank's interim type leads to a complete moral hazard, i.e., no investment; or to a complete market breakdown via adverse selection problem, i.e., no trade in both periods. If the buyers believe that the bank does not invest, however, then the highest offer the bank receives is so low that the bank prefers to invest and hold loans. Therefore, in any equilibrium, the bank invests with a positive probability, and good loans are traded at least once with a positive probability. As a result, there are gains from trade. However, they are completely offset by the losses from moral hazard.

**Bank's Behavior in  $t = 1$  and  $t = 2$**  First, we characterize the bank's behavior in periods 1 and 2. Since the bank does not have any reputational incentive, the bank accepts the highest offer if it exceeds the reservation value of the loan it holds, and rejects it if it is below that.

**Claim 1** *In period  $t = 1, 2$ , a good bank accepts  $p_t$  if  $p_t > \gamma\pi_g$ ; rejects  $p_t$  if  $p_t < \gamma\pi_g$ ; and is indifferent between accepting and rejecting if  $p_t = \gamma\pi_g$ . Similarly, a bad bank a bad bank accepts  $p_t$  if  $p_t > \gamma\pi_b$ ; rejects  $p_t < \gamma\pi_b$ ; and is indifferent between accepting and rejecting if  $p_t = \gamma\pi_b$ .*

**Buyers' Behavior** Next, we characterize the buyers' behavior. To facilitate the discussion, we introduce a few notations. Let  $p(\mu)$  be the expected value of a loan to a buyer when he believes the loan is good with probability  $\mu$ , i.e.,

$$p(\mu) \equiv \mu\pi_g + (1 - \mu)\pi_b.$$

Also let  $\bar{\mu}$  be the unique value of  $\mu$  that solves the following equation  $p(\bar{\mu}) = \gamma\pi_g$ . Therefore, when a buyer believes a loan is good with probability  $\bar{\mu}$ , then the expected value of the loan to him is the reservation value of the bank holding a good loan. We then have the following claim.

**Claim 2** Suppose that the buyers believe that the bank is good with probability  $\mu$ . In any equilibrium, their highest offer is (i)  $p(\mu)$  if  $\mu > \bar{\mu}$ ; (ii) either  $\gamma\pi_g$  or  $\pi_b$  if  $\mu = \bar{\mu}$ ; and (iii)  $\pi_b$  if  $\mu < \bar{\mu}$ .

**Proof.** In the Appendix. ■

**Remark 2** The assumption that (two) buyers run a Bertrand competition plays an important role. One may wonder if it should suffice to simply require that market prices satisfy the zero-payoff condition for the buyers, as in a competitive equilibrium. If there is no competition among the buyers and  $\mu > \bar{\mu}$ , however, then we have two market clearing prices:  $p(\mu)$  and  $\pi_b$ . We find such multiplicity unattractive because the ex-post inefficient allocation at a market clearing price  $\pi_b$  is not a direct consequence of the adverse selection problem in the secondary loan markets, and because obvious gains from trade are not being exploited. Indeed, if a buyer offers  $p_1 \in (\gamma\pi_g, p(\mu))$ , then a good bank accepts  $p_1$ ; and both a good bank and the buyer make positive payoffs.

**Equilibria** Now we are ready to characterize the equilibria.

**Lemma 1** There exists an equilibrium. In any equilibrium, (i) the bank invests with probability  $\bar{\mu}$ , and (ii) the ex-ante probability that a good bank sells its loans at least once is positive, but the ex-ante probability that it sells its loans in both periods is less than one.

**Proof.** In the Appendix. ■

Parts (i) and (ii) of Lemma 1 say, respectively, that the losses from moral hazard and the gains from trade are both positive.

Note that as a result of competitions among buyers, the bank is the sole beneficiary of the efficiency gains from securitization. Therefore, the efficiency gains from securitization are also represented by the increase in the bank's payoff after the securitization. However, part (i) of Lemma 1 and Claim 2, taken together, suggest that the highest offers the bank receives are at most  $\gamma\pi_g$ , i.e., the reservation value of holding a good loan. Therefore, the payoff of a good bank in any equilibrium is the same as the payoff of holding good loans in both periods. This leads to the following proposition:

**Proposition 1** There are no efficiency gains from securitization if the bank does not have any reputational incentive.

When the bank does not have any reputational incentive, the bank invests with a positive probability only when it faces the risk of holding a loan to its

maturity. In other words, the possibility of the ex-post inefficiency (inefficient ex-post allocations) is what mitigates the ex-ante inefficiency (the bank's moral hazard problem). This is why the gains from trade in the equilibrium cannot be "too large," and hence are completely offset by the losses from moral hazard.

The objective of this paper is to show how the observability of such information affects the two types of inefficiency, as well as the interplay of them.

## 5 Reputational Incentives

Recall that the buyers in  $t = 2$  can observe (i) the highest price offered in  $t = 1$ , (ii) whether or not the bank has sold a loan at the highest offer, and (iii) the performance of any loan sold in  $t = 1$ . They cannot, however, observe the performance of any loan the bank decided not to sell. There are then two ways the bank sends a signal that it has invested.

In one way, a good bank tries to send a signal by refusing to sell even for a high price. Then, any bank that sells its loan in  $t = 1$  at the high price is perceived as a bad bank by future buyers, irrespective of the performance of the loan sold in  $t = 1$ . As will be formally defined in the next subsection, we say a good bank has a *negative reputational incentive* in this case because the bank's reputational incentive hinders trade.

The other way that a good bank sends a signal is by selling a loan in  $t = 1$ , even at a current loss, in order to generate public information about the loan's performance. In this case, any bank that sold at  $p_1$  with the high return is likely to be perceived as a good bank by future buyers, and hence to receive a high offer in  $t = 2$ . As will be formally defined in the next subsection, we say a good bank has a *positive reputational incentive* in this case because the bank's reputational incentive facilitates trade.

### 5.1 Negative and Positive Reputational Equilibria

In this subsection, we formally define the two types of the bank's reputational incentive discussed above. In order to do so, we first observe that if buyers believe that the bank invests with a very small probability  $\mu$ , then a good bank never accepts  $p(\mu)$  in any equilibrium because the higher price in  $t = 2$  cannot compensate for the loss in  $t = 1$ , i.e.,  $\gamma\pi_g - p(\mu)$ .

**Claim 3** *Suppose that the bank is good with some exogenous probability  $\mu$ . Then there exists a  $\underline{\mu} \in (0, \bar{\mu})$ , and there is an equilibrium in which a good bank sells in  $t = 1$  if and only if  $\mu \geq \underline{\mu}$ .*

**Proof.** In the Appendix. ■

As an immediate corollary, we can conclude that there is no equilibrium in which the bank invests with probability  $\mu < \underline{\mu}$ . Indeed, if there were such an equilibrium, then a bad bank's equilibrium payoff would be at most  $2\pi_b$ . Then, the bank would choose to invest with probability one, which is a contradiction.

Based on these observations, we define a negative reputational equilibrium and a positive reputational equilibrium as follows.

**Definition 1** *A negative reputational equilibrium is an equilibrium in which (a) the bank invests with probability  $\mu \in [\underline{\mu}, 1)$ ; and (b) a good bank rejects  $p_1$  in  $t = 1$  if  $p_1 \leq p(\mu)$ . In this case, we say that a good bank has a negative reputational incentive.*

**Definition 2** *A positive reputational equilibrium is an equilibrium in which (a) the bank invests with probability  $\mu \in [\underline{\mu}, 1)$ ; and (b) a good bank accepts  $p_1$  in  $t = 1$  if  $p_1 \geq p(\underline{\mu})$ . In this case, we say that a good bank has a positive reputational incentive.*

In a negative reputational equilibrium, any bank that sells at  $p(\mu)$  is perceived as a bad bank by the buyers in  $t = 2$ , irrespective of the performance of the loan sold in  $t = 1$ . As a result, a good bank does not sell at  $p(\mu)$  in  $t = 1$ . Therefore, even though the buyers in  $t = 1$  are competing in making offers, that competition does not lead them to make offers as high as  $p(\mu)$ . Since the bank's reputational incentive impedes trade, we call this type of equilibrium *a negative reputational equilibrium*, following the terminology in Chari et al. (2010).

In contrast, in a positive reputational equilibrium, a good bank tries to send a signal by selling a loan in  $t = 1$ , even at a current loss, in order to generate public information about the loan's performance. Note that in a positive reputational equilibrium with  $\mu \in (\underline{\mu}, 1)$ , the buyers in  $t = 1$  believe that a good bank will accept any price above  $p(\underline{\mu})$ . Hence, the competition among buyers in  $t = 1$  leads them to raise offers to  $p(\mu)$ .<sup>10</sup> Since the bank's reputational incentive facilitates trade, we call this type of equilibrium *a positive reputational equilibrium*.

We note that as in any other Bayesian game, there is a plethora of belief-driven equilibria in our model, whose equilibrium outcomes cannot be supported by either a negative reputational incentive or a positive reputational incentive. However, under the presumption that the buyers are competing in making offers, we find restricting attention to those two types of equilibria is reasonable.

<sup>10</sup>When the bank invests with probability  $\mu$ , the offer  $p(\mu)$  is not necessarily a result of competition because a good bank never accepts  $p_1 < p(\underline{\mu})$  by the construction of  $\underline{\mu}$ .

The first reason for this is that the competition among buyers drives their equilibrium payoffs down to zero in any negative or positive reputational equilibrium.<sup>11)</sup> The second reason is that, as we have argued above, buyers' on-the-equilibrium offers in  $t = 1$  that give a good bank a payoff higher than the payoff from holding loans in both periods are always supported by "competition-driven" beliefs.<sup>12)</sup>

Moreover, the objective of this paper is not to characterize all reputational equilibria, but rather to analyze the effect of a bank's reputational incentive on the ex-ante and ex-post inefficiency, and on their interplay, when buyers are competing in making offers. For these reasons, we limit our attention to negative and positive reputational equilibria.

## 5.2 Negative Reputational Equilibria

We first analyze negative reputational equilibria. We show that a negative reputational equilibrium exists if and only if the cost of investment is large. Moreover, we show there are no efficiency gains from securitization in any negative reputational equilibrium.

**Lemma 2** *There exists a  $\bar{c}^N$ ; and a negative reputational equilibrium exists if and only if  $c \geq \bar{c}^N$ . Moreover, (i) if  $c > \bar{c}^N$ , then  $\mu = \bar{\mu}$  in any negative reputational equilibrium; and (ii) if  $c = \bar{c}^N$ , then for any  $\mu \in [\bar{\mu}, 1)$  there is a negative reputational equilibrium with  $\mu$ , and there is no negative reputational equilibrium with  $\mu \notin [\bar{\mu}, 1)$ .*

**Proof.** In the Appendix. ■

In a negative reputational equilibrium, no loan is traded in  $t = 1$ . Therefore, Lemma 2 states that a negative reputational incentive impedes trade in  $t = 1$  if the cost of investment  $c$  is not small, that is,  $c > \bar{c}^N$ .<sup>13)</sup> Now we discuss the efficiency gain from securitization in a negative reputational equilibrium.

We know that a good loan is traded in  $t = 2$  with a positive probability, and therefore there are gains from trade. Also, since the bank does not invest with probability one, the losses from moral hazard exist as well.

In order to derive the efficiency gains from securitization, note that the bank invests with probability  $\bar{\mu}$  in any negative reputational equilibrium. Moreover,

<sup>11)</sup>We can construct an equilibrium in which a good bank accepts some  $p_1 < p(\mu)$ , but rejects any  $p'_1 \in (p_1, \pi_g)$  in  $t = 1$ .

<sup>12)</sup>Recall that in a positive reputational equilibrium with  $\mu \in (\underline{\mu}, 1)$ , the buyers in  $t = 1$  believe that a good bank accepts any price above  $p(\underline{\mu})$ . Therefore, the competition among buyers leads them to raise their offers to  $p(\mu)$ .

<sup>13)</sup>Here, we ignore a non-generic investment cost  $c = \bar{c}^N$  for the simplicity of argument.

note that a bad bank also holds its loan in  $t = 1$  on the equilibrium path. This implies that the highest offer made in  $t = 2$  is  $\gamma\pi_g$ , i.e., the reservation value of holding a good loan. Hence, a good bank's equilibrium payoff coincides with the payoff from holding its loans in  $t = 1$  and  $t = 2$ . This leads to the following proposition:

**Proposition 2** *Suppose  $c > \bar{c}^N$ . There are then no efficiency gains from securitization in any negative reputational equilibrium.*

Recall that the bank invests with probability  $\bar{\mu}$  when it does not have any reputational incentive. Therefore, with a negative reputational incentive, the losses from moral hazard neither increase nor decrease. Then by Proposition 2, we know that there is no change in gains from trade. In other words, a negative reputational incentive has no effect on either the losses from moral hazard or the gains from trade.

This is because under a negative reputational equilibrium, a good bank can send a signal if and only if buyers in  $t = 1$  make high offers. However, the bank never has an opportunity to send a signal on any equilibrium path because buyers never make such an offer, knowing that only a bad bank would accept it. In other words, the information that buyers in  $t = 2$  face is no finer than the information that buyers in  $t = 1$  face. Consequently, the buyers in a negative reputational equilibrium are in the same situation as the buyers who face the bank that has no reputational incentive. This is why there are no efficiency gains from securitization even though the bank has a reputational incentive.

### 5.3 Positive Reputational Equilibria

In this subsection, we analyze positive reputational equilibria. We show that there is a positive reputational equilibrium for any  $c$ , but losses from moral hazard increase with a positive reputational incentive when  $c$  is large.

**Lemma 3** *For any  $c$ , there is a positive reputational equilibrium. Moreover, there are  $\underline{c}^P$  and  $\bar{c}^P > \underline{c}^P$ ; and the following statements follow:*

1. *If  $c < \underline{c}^P$ , then  $\mu = \bar{\mu}^P$  for some  $\bar{\mu}^P > \bar{\mu}$  in any positive reputational equilibrium.*
2. *If  $c \in [\underline{c}^P, \bar{c}^P]$ , then there exists a  $\bar{\mu} \in [\underline{\mu}, \bar{\mu}^P]$  and for any  $\mu' \in \{\underline{\mu}, \bar{\mu}, \bar{\mu}^P\}$  there is a positive reputational equilibrium with any  $\mu = \mu'$ . Moreover, there is no positive reputational equilibrium with  $\mu \notin \{\underline{\mu}, \bar{\mu}, \bar{\mu}^P\}$ .*

3. If  $c > \bar{c}^P$ , then  $\mu = \underline{\mu}$  in any positive reputational equilibrium.

**Proof.** In the Appendix. ■

As one might expect, a positive reputational incentive creates more gains from trade. However, whether or not there are efficiency gains from securitization depends on the cost of investment.

**Proposition 3** *There is a positive reputational equilibrium in which (i) there are efficiency gains from securitization if  $c < \bar{c}^P$ ; and (ii) there are no efficiency gains from securitization if  $c \geq \bar{c}^P$ .*

**Proof.** Note that if there is a positive reputational equilibrium with  $\mu = \underline{\mu}$ , then the payoff of a good bank is the same as its payoff from holding loans in both periods  $t = 1$  and  $t = 2$ . Therefore, there are no efficiency gains from securitization in a positive reputational equilibrium with  $\mu = \underline{\mu}$ .

In contrast, in a positive reputational equilibrium with  $\mu > \underline{\mu}$ , the payoff of a good bank strictly exceeds its payoff from holding loans in both periods. ■

To see how a positive reputational incentive affects the gains from trade and the losses from moral hazard, suppose that the buyers are expecting that the bank invests with probability  $\bar{\mu}$ , i.e., the probability of the bank's investment without any reputational incentive. This in turn implies the gains from trade are higher than in the case without any reputational incentive.

If the cost of investment  $c$  is small, then the higher probability of facing a default is sufficient to induce the bank to invest. As a result, the bank invests with a probability higher than  $\bar{\mu}$ . Then, the buyers in  $t = 1$ , as well as the buyers in  $t = 2$  who make offers to a bank with the high return, all increase their offers. Such higher offers creates the incentive not to invest. Therefore, in the equilibrium, the offers increase to the level at which the bank is indifferent between investing and not investing.

Consequently, the losses from trade shrink, and the gains from trade further increase. In other words, there are efficiency gains from securitization as the proponents of securitization argue, but these gains occur only when the cost of investment is small.

If the cost of investment  $c$  is not small, however, then the higher probability of facing a default in itself is not sufficient to induce the bank to invest any more. Therefore, the bank now invests with a smaller probability. As a result, the buyers in  $t = 1$ ; as well as the buyers in  $t = 2$  who make offers to a bank with good performance lower their offers. Such lower offers induce the bank to invest more. Hence, in the equilibrium, the offers decrease to the level at which



the bank are indifferent between investing and not investing. This implies that the gains from trade are larger than the case without any reputational incentive, but so are the losses from moral hazard.

For a sufficiently large  $c$ , the highest offers the bank receives in  $t = 1$ , and  $t = 2$  in case of its loan yields the low return become sufficiently low. As a result, the payoff of a good bank is the same as the payoff it attains from holding the loans in both periods. This implies that the larger gains from trade in the positive reputational equilibrium are completely offset by the larger losses from moral hazard. In other words, there are no efficiency gains from securitization. Hence, in contrast to what proponents of securitization argue, our results suggest that the reputational incentive can exacerbate the moral hazard problem.

To summarize the analysis so far, we have shown that when the cost of investment  $c$  is large, there are no efficiency gains from securitization. There are two reasons for this result. First, just as in a negative reputational equilibrium, buyers may not observe any information about the quality of loans on the equilibrium path. As a result, the bank invests only because it faces the threat of ex-post inefficiency, as in the case without any reputational incentive. Hence, the gains from trade cannot be “too large” and are completely offset by the losses from moral hazard. Second, just as in a positive reputational equilibrium with a large investment cost, the larger gains from trade exacerbate the moral hazard problem, and hence are completely offset by the increased losses from moral hazard.

## 6 Policy Implication

As we have seen above, when the buyers are short lived, the mere existence of the secondary market does not guarantee the efficiency gains from securitization even when the bank has a reputational incentive.

In this section, we show that a simple regulation based on prices observable in the markets can enhance the role of such information and achieve the social welfare which is arbitrarily close to the first-best. To see this, suppose tentatively that buyers are long-lived, and the performance of loans are ex-post verifiable.

In this case, a long-term contract can improve ex-ante efficiency without undermining the ex-post efficiency, i.e., the potential efficiency gains from securitization can be fully realized.

Indeed, consider the following long-term contract between the bank and a long-lived buyer: (i) in  $t = 1$ , the (long-lived) buyer pays  $\pi_g - \tau$  for some  $\tau > 0$ ;

and (ii) in  $t = 2$ , the buyer pays  $\pi_g + \tau/\pi_g$  if the loan in  $t = 1$  yielded the high return; and pays  $\pi_g$  if the loan in  $t = 1$  yielded the low return.

Under the contract, the bank is willing to invest if and only if  $\tau \geq \tilde{\tau} \equiv \frac{\pi_g c}{\pi_g - \pi_b}$ .<sup>14)</sup> Also, the payoff of the buyer is exactly zero. Hence, this contract is mutually agreeable. More importantly, the first-best outcome is achieved because the bank invests with probability one, and trade always occurs in both periods.

In the long-term contract described above, the bank's moral hazard problem is solved by a direct reward for good performance, not by the threat of ex-post inefficiency.

Now suppose buyers are short-lived as in our original model, but there is a regulator who can observe the quality of loans. If he imposes an entry tax  $\tilde{\tau}$  for the bank in  $t = 0$ , and gives the bank  $\tilde{\tau}/\pi_g$  in  $t = 2$  if and only if the loan performs well in  $t = 1$ , then the first-best outcome is achieved as in the long-term contract. Moreover, his expected payoff is zero, i.e., the regulation is budget-balancing.

Even when the regulator cannot observe the quality of loans, he can achieve the efficiency gains from securitization arbitrarily close to the first-best level by using the prices observable in the markets.

To see this, consider the following regulation: (i) in  $t = 0$ , the bank pays  $\tau$  to the regulator; and (ii) in  $t = 2$ , the bank receives  $s$  if and only if the price in the secondary market strictly exceeds the price at which the loan is traded in  $t = 1$ . Then, the regulation can induce the bank to invest with any probability arbitrarily close to one.

**Lemma 4** *For any  $\mu$  close to one, there is an equilibrium with a pair of tax and subsidy in which (i) the bank invests with probability  $\mu$  in  $t = 0$ ; (ii) the trade always occurs in both  $t = 1$  and  $t = 2$ ; and (iii) the regulator's expected revenue is zero.*

**Proof.** In the Appendix. ■

As the next proposition shows, a proper regulation can solve both the moral hazard problem and the adverse selection problem. To be precise, recall that the efficiency gains from securitization under the first-best outcome are  $E \equiv 2(1 - \gamma)\pi_g$ . We then have the following proposition:

**Proposition 4** *For any  $\epsilon > 0$ , there is a regulation that achieves the efficiency gains from securitization at least as large as  $E - \epsilon$ .*

**Proof.** In the Appendix. ■

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<sup>14)</sup>Note that while the payoff of a bad bank is  $2\pi_g - \left(1 - \frac{\pi_b}{\pi_g}\right)\tau$ , the payoff of a good bank is  $2\pi_g - c$ .

As we saw, the bank's reputational incentive is not sufficiently strong by itself to mitigate the moral hazard without any regulation. As a result, the bank invests with a positive probability only because it faces the chance of holding its loans until their maturity. Therefore, even with the reputational incentive, the ex-ante efficiency and the ex-post efficiency are at odds.

The regulation described above resolves the tension between the ex-ante and ex-post inefficiency by enhancing the role of information through which the bank builds its reputation. Hence, the outcome that is arbitrarily close to the first-best outcome can be achieved.

## 7 Concluding Remarks

In the aftermath of the financial crisis, various policy proposals have been made concerning how to address the two types of inefficiency associated with securitization – the *ex-ante* inefficiency, i.e., the bank's moral hazard problem; and the *ex-post* inefficiency, i.e., inefficient ex-post allocation due to the adverse selection problem.

We have shown that even with the help of the originators' reputational incentives, the ex-ante efficiency and ex-post efficiency are in general at odds. Hence, our results suggest that the effectiveness of various proposals should be judged based on their effects on the ex-ante inefficiency, the ex-post inefficiency, and – most importantly – the tension between them. The regulation discussed in this paper resolves the tension between the ex-ante inefficiency and the ex-post inefficiency by enhancing the role of reputation. Therefore, the ex-ante efficiency can be improved without undermining the ex-post efficiency.

In light of these findings, there are a few natural extensions that could be profitably be studied. First, because we have assumed that the investment is perfectly persistent, the moral hazard problem is a one-time problem.<sup>15)</sup> An important step would be to analyze what happens when the investment is not perfectly persistent, that is, the bank faces a moral hazard problem at each moment with a positive probability. Even in such an environment, we believe that our main result will still hold: namely, that the tension between the ex-ante efficiency and the ex-post efficiency leads to no efficiency gain from securitization under a positive reputational equilibrium when the cost of

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<sup>15)</sup>Since the unobservable interim type in our model has persistency, our model is related to the model in Board and Meyer-ter Vehn (2010) namely, a model of firm reputation where product quality is persistent and depends stochastically on the firm's past investments. In their model, as in our model, reputation is modeled directly as market belief about quality.

investment is large enough.

We also believe that a regulation which uses the observed prices in the market as verifiable information to reward the good performance can ease the tension between the ex-ante and ex-post inefficiency. An infinite-period model whose analysis becomes more intricate due to the multiplicity of equilibria seems to be more suitable to analyze a scenario in which the moral hazard problem is not a one-time problem.

Another natural extension would be to study the possibility of voluntary provision of assets that play a role similar to that of the regulation discussed in our paper. Recall that the regulation we have discussed in this paper is budget-balancing. Therefore, a long-lived third-party (with a deep pocket) may be willing to sell a “reverse” credit default swap (CDS), for which the bank pays  $\tau$  as a premium, and which pays  $s$  if the loan does not default at the end of  $t = 1$ . If the bank can credibly commit to the purchase of such reverse CDSs, then that purchase works as a signaling device. This suggests the possibility that what is achievable under a contract (which is not renegotiation-proof) could be achieved in the spot market. Investigating when such assets working as a signaling device are voluntarily provided into the market would be an interesting future topic.

Finally, the improved ex-post efficiency may create another type of inefficiency which our model does not capture: the entry of “inefficient” banks that should never originate a loan. How to deter the entry and facilitate the exit of such inefficient banks is therefore also an important question for future study.

## 8 Appendix

### Proof of Claim 2

We first show (i). Note that a buyer never offers  $p > p(\mu)$ . If a buyer is offering  $p < p(\mu)$ , then the other buyer can yield a positive payoff by offering  $p + \varepsilon$  for some  $\varepsilon < p(\mu) - p$ , a contradiction. The similar logic leads to (iii).

Next, we show (ii). Observe that a buyer never offers  $p \in (\pi_b, \gamma\pi_g)$ . Indeed, a good bank never sells at such  $p$  because it is below the reservation value of holding a good loan. Also, note that when a buyer is offering  $\pi_b$ , the other buyer’s payoff from offering  $\gamma\pi_g$  and  $\pi_b$  are both zero when a good bank accepts  $\gamma\pi_g$ . Q.E.D.

### Proof of Lemma 1

Suppose there is an equilibrium in which the bank invests with probability  $\mu > \bar{\mu}$ . Then, by Claim 2, the highest offers in  $t = 1$  and  $t = 2$  are both  $p(\mu)$ . In this case, the payoff of a bad bank exceeds the payoff of a good bank by  $c$ . Therefore, the bank chooses not to invest, a contradiction.

Next, suppose there is an equilibrium in which the bank invests with probability  $\mu < \bar{\mu}$ . Then by Claim 2, the highest offers in  $t = 1$  and  $t = 2$  are both  $\pi_b$ . By Assumption 1, however, a good bank's payoff  $2\gamma\pi_g - c$  is higher than a bad bank's payoff  $2\pi_b$ . Hence, the bank chooses to invest, a contradiction.

Hence, if there is an equilibrium, then the bank invests with probability  $\bar{\mu}$ . Now we show there is an equilibrium in which the bank invests with probability  $\bar{\mu}$ . When the bank invests with probability  $\bar{\mu}$ , buyers are indifferent between offering  $\gamma\pi_g$  and  $\pi_b$  by Claim 2.

Let  $\alpha_t$  be the ex-ante probability that the highest offer in  $t$  is  $\gamma\pi_g$ . Then, the payoffs of being a good bank and a bad bank are, respectively,  $2\gamma\pi_g - c$ , and

$$u_B(\alpha_1, \alpha_2) \equiv (\alpha_1 + \alpha_2)\gamma\pi_g + ((1 - \alpha_1) + (1 - \alpha_2))\pi_b. \quad (8.1)$$

Since there is a pair  $(\alpha_1, \alpha_2) \in [0, 1] \times [0, 1]$  such that  $u_B(\alpha_1, \alpha_2) = 2\gamma\pi_g - c$ , we have established the existence of an equilibrium.

Note that for any pair  $(\alpha_1, \alpha_2) \in [0, 1] \times [0, 1]$  that satisfies  $u_B(\alpha_1, \alpha_2) = 2\gamma\pi_g - c$ , we have  $\max\{\alpha_1, \alpha_2\} > 0$  and  $\alpha_1 \times \alpha_2 < 1$ . Hence the required results follows. Q.E.D.

### Proof of Claim 3

To facilitate the discussion, we introduce a few notations. By  $\mu_2^{sh}, \mu_2^{sl}$ , and  $\mu_2^{hl}$ , we denote the posterior beliefs on the bank being good when (i) it sold a loan and yielded the high return; (ii) it sold a loan and yielded the low return; and (iii) it held its loan, respectively.

Now, suppose that the buyers in  $t = 2$  believe that the loan sold in  $t = 1$  is good with probability  $\mu_1$ . Then they update their beliefs by the Bayes' rule. Hence,

$$\mu_2^{sh} = \mu_2^{sh}(\mu_1) \equiv \frac{\mu_1\pi_g}{\mu_1\pi_g + (1 - \mu_1)\pi_b}$$

$$\text{and } \mu_2^{sl} = \mu_2^{sl}(\mu_1) \equiv \frac{\mu_1(1 - \pi_g)}{\mu_1(1 - \pi_g) + (1 - \mu_1)(1 - \pi_b)}.$$

To prove Claim 3, suppose the bank is good with some exogenous probability  $\mu$ . Moreover, assume that there is an equilibrium in which buyers offer

$p(\mu)$  in  $t = 1$ , which a good bank accepts. Then payoff of a good bank  $u_G(\mu)$  is

$$u_G(\mu) \equiv \begin{cases} p(\mu) + (\pi_g p(\mu_2^{sh}(\mu)) + (1 - \pi_g) \gamma \pi_g) - c & \text{if } \mu \leq \bar{\mu}^P \\ p(\mu) + (\pi_g p(\mu_2^{sh}(\mu)) + (1 - \pi_g) p(\mu_2^{sl}(\mu))) - c & \text{if } \mu > \bar{\mu}^P \end{cases},$$

where  $\bar{\mu}^P$  is the solution of  $\mu_2^{sl}(\mu) = \bar{\mu}$ . Note that  $u_G(\mu)$  is the highest payoff a good bank can obtain in an equilibrium in which a good loan is traded with a positive probability in  $t = 1$ . Then, it is straightforward to see that a good bank deviates and hold if  $\mu < \underline{\mu}$ , where  $u_G(\underline{\mu}) = 2\gamma\pi_g - c$ . In contrast, if  $\mu \geq \underline{\mu}$ , then we can indeed construct the system of beliefs which induces a good bank to accept  $p(\mu)$ . Q.E.D.

### Proof of Lemma 2

We prove the following two claims first: (a) there is no negative reputational equilibrium with  $\mu < \bar{\mu}$ ; (b) if the bank is good with some exogenous probability  $\mu \in [\bar{\mu}, 1)$ , then there is an equilibrium in which a good bank accepts  $p_1$  if and only if  $p_1 \geq \pi_g$ ; and (c) the probability that a loan is traded in  $t = 1$  is zero in any negative reputational equilibrium with  $\mu \in [\bar{\mu}, 1)$ , if there is any.

We start with (a). Suppose there is a negative reputational equilibrium with  $\mu < \bar{\mu}$ . Then a bad bank accepts  $\pi_b$  with a positive probability on the equilibrium path. If not, then on any equilibrium path, the highest offer in  $t = 2$  is  $\pi_b$ . Therefore, a bad bank's payoff is  $\gamma\pi_b + \delta\pi_b$ . On the other hand, the payoff of accepting  $\pi_b$  in  $t = 1$  is  $2\pi_b > \gamma\pi_b + \pi_b$ , a contradiction. This in turn implies the ex-ante equilibrium payoff of a bad bank is at most  $2\pi_b$ , which is lower than the payoff from investing and holding. This is a contradiction.

Next, we show (b), i.e., we show that if the bank is good with some exogenous probability  $\mu \in [\bar{\mu}, 1)$ , then there is an equilibrium in which

1. a good bank accepts  $p_1$  if and only if  $p_1 \geq \pi_g$  in  $t = 1$ ;
2. a bad bank
  - (a) accepts  $p_1$  if  $p_1 \geq \bar{p}_1$  in  $t = 1$ , where  $\bar{p}_1 + \pi_b = \gamma\pi_b + \pi_g$ ;
  - (b) rejects  $p_1 \in [\bar{p}'_1, \bar{p}_1)$  with probability  $r(p_1)$  in  $t = 1$ , where  $r(p_1)$  satisfies the following

$$\underbrace{p_1 + \pi_b}_{\text{payoff of accepting } p_1} = \underbrace{\gamma\pi_b + p\left(\frac{\mu}{\mu + (1 - \mu)r(p_1)}\right)}_{\text{payoff of rejecting } p_1},$$

and  $\bar{p}'_1 = p(\mu) - (1 - \gamma)\pi_b$ ; and

(c) rejects  $p_1 < \tilde{p}'_1$  with probability 1.

To see this, consider the following system of beliefs,

$$\left( \mu_2^{sh}, \mu_2^{sl}, \mu_2^{hl} \right) = \begin{cases} \left( \mu_2^{sh}(\mu), \mu_2^{sl}(\mu), 1 \right) & \text{if } p_1 \geq \pi_g \\ (0, 0, 1) & \text{if } p_1 \in [\tilde{p}_1, \pi_g) \\ \left( 0, 0, \frac{\mu}{\mu + (1-\mu)r(p_1)} \right) & \text{if } p_1 \in [\tilde{p}'_1, \tilde{p}_1) \\ (0, 0, \mu) & \text{if } p_1 < \tilde{p}'_1 \end{cases} .$$

If  $p_1 \geq \pi_g$ , then both a good bank and a bad bank accept  $p_1$ , irrespective of posterior beliefs of buyers in  $t = 2$ . If  $p_1 \in [\tilde{p}_1, \pi_g)$ , then by construction of  $\tilde{p}_1$ , a bad bank prefers to accept  $p_1$ . However, a good bank prefers to reject  $p_1$ . Indeed, for a good bank, while the payoff of accepting  $p_1$  is  $p_1 + \gamma\pi_g$ , the payoff of rejecting  $p_1$  is  $\gamma\pi_g + \pi_g > p_1 + \gamma\pi_g$ . If  $p_1 \in (\tilde{p}'_1, \tilde{p}_1)$ , then by construction of  $r(p_1)$ , a bad bank is indifferent between accepting and rejecting  $p_1$ . In contrast, a good bank strictly prefers to hold. If  $p_1 < \tilde{p}'_1$ , then both a good and a bad bank strictly prefer to hold. Indeed, if a bad bank accepts  $p_1$ ,

Now we show (c). Suppose there is a negative reputational equilibrium with  $\mu \in [\bar{\mu}, 1)$ . Note that the buyers in  $t = 1$  never make an offer a good bank accepts. Therefore, the highest offer in  $t = 1$  is at most  $\pi_b$  in any negative reputational equilibrium. If there is a negative reputational equilibrium in which a bad bank accepts  $\pi_b$  with a positive probability, then it means its equilibrium payoff is  $2\pi_b$ . Then the bank prefers to invest and hold, which contradicts that  $\mu < 1$ .

To prove lemma, let  $\bar{c}^N \equiv \gamma(\pi_g - \pi_b)$ . First, suppose  $c > \bar{c}^N$ . Then (a) there is no negative reputational equilibrium with  $\mu \in (\bar{\mu}, 1)$ ; and (b) there is a negative reputational equilibrium with  $\mu = \bar{\mu}$ . To see (a), suppose there is a negative reputational equilibrium with  $\mu \in (\bar{\mu}, 1)$ . Then, the payoffs of a bad bank and a good bank are  $\gamma\pi_b + p(\mu)$  and  $\gamma\pi_g + p(\mu) - c$ , respectively. Therefore, the bank strictly prefers not to invest, a contradiction. To see (b), note that in a negative reputational equilibrium with  $\mu = \bar{\mu}$ , the highest offer in  $t = 2$  when the bank has held its loan in  $t = 1$  is either  $\gamma\pi_g$  and  $\pi_b$ . This follows from the same argument as Claim 2. Let  $\alpha$  be the probability that highest offer is  $\gamma\pi_g$ . Then, the payoffs of a bad bank and a good bank are, respectively,  $\gamma\pi_b + (\alpha\gamma\pi_g + (1-\alpha)\pi_b)$  and  $\gamma\pi_g + \gamma\pi_g - c$ . Note that  $\gamma\pi_b + \gamma\pi_g > \gamma\pi_g + \pi_g - c$  and  $\gamma\pi_b + \pi_b < \gamma\pi_g + \gamma\pi_g - c$ . Therefore, for any  $c > \bar{c}^N$ , there exists a unique  $\alpha \in (0, 1)$  that solves

$$\gamma\pi_b + (\alpha\gamma\pi_g + (1-\alpha)\pi_b) = \gamma\pi_g + \gamma\pi_g - c.$$

This concludes that there is a negative reputational equilibrium with  $\bar{\mu}$ .

Now, suppose that  $c = \bar{c}^N$ . Then for  $\mu \geq \bar{\mu}$ ,  $\gamma\pi_b + p(\mu) = \gamma\pi_g + p(\mu) - c$ . Therefore, the bank is indifferent between investing and not investing when buyers in  $t = 2$  offer  $p(\mu)$ . Therefore, there is a negative reputational equilibrium with  $\mu = \bar{\mu}$  for any  $\bar{\mu} \in [\bar{\mu}, 1)$ .

Lastly, suppose  $c < \bar{c}^N$ . If there is a negative reputational equilibrium, then the bank strictly prefers to invest, i.e.,  $\mu = 1$ , which is a contradiction. Q.E.D.

### Proof of Lemma 3

Let  $\bar{\mu}^P$  be the solution of  $\mu_2^{sl}(\mu) = \bar{\mu}$ . Recall that by the proof of Claim 3, and Assumption 2, we know that  $\mu \in [\underline{\mu}, \bar{\mu}^P]$  in any positive reputational equilibrium.

First, we note that the highest offer in  $t = 1$  in a positive reputational equilibrium is  $p(\mu)$  when  $\mu \in (\underline{\mu}, \bar{\mu}^P]$ ; and is either  $p(\underline{\mu})$  or  $\pi_b$  when  $\mu = \underline{\mu}$ .

Therefore, the payoff of a bad bank in a positive reputational equilibrium with  $\mu \in (\underline{\mu}, \bar{\mu}^P]$  is

$$u_B(\mu; \alpha) \equiv \begin{cases} p(\mu) + \begin{pmatrix} \pi_b p(\mu_2^{sh}(\mu)) \\ + (1 - \pi_b)(\alpha\gamma\pi_g + (1 - \alpha)\pi_b) \end{pmatrix} & \text{if } \mu = \bar{\mu}^P \\ p(\mu) + (\pi_b p(\mu_2^{sh}(\mu)) + (1 - \pi_b)\pi_b) & \text{if } \mu \in (\underline{\mu}, \bar{\mu}^P) \end{cases},$$

where  $\alpha$  is the probability that the highest offer is  $\gamma\pi_g$  in  $t = 2$  when the loans sold in  $t = 1$  yields the high return. Also the payoff of a good bank is

$$u_G(\mu_1, c) \equiv p(\mu) + (\pi_g p(\mu_2^{sh}(\mu)) + (1 - \pi_g)\gamma) - c.$$

Now, define  $\underline{c}^P$  and  $\bar{c}^P$  as follows:  $\underline{c}^P$  solves  $\lim_{\mu \rightarrow \underline{\mu}} u_B(\mu) = \lim_{\mu \rightarrow \underline{\mu}} u_G(\mu, \underline{c}^P)$ , and  $\bar{c}^P$  solves  $u_B(\bar{\mu}^P; 0) = u_G(\bar{\mu}^P, \bar{c}^P)$ . We show that  $\underline{c}^P$  and  $\bar{c}^P$  are well defined, and  $\underline{c}^P < \bar{c}^P$ . To see this, note that for  $\mu < \bar{\mu}^P$ ,  $\frac{\partial u_G(\mu, c)}{\partial \mu} > \frac{\partial u_B(\mu, c)}{\partial \mu}$ . Hence,  $u_B(\mu; \alpha) - u_G(\mu, c)$  is decreasing in  $\mu$ ; and increasing in  $c$ .<sup>16)</sup>

If  $c < \underline{c}^P$ , then  $u_B(\mu; \alpha) < u_G(\mu, c)$  for any  $\mu < \bar{\mu}^P$ . Therefore, if there exists a positive reputational equilibrium, then it has to satisfy  $\mu = \bar{\mu}^P$ . Note that by Assumptions 1 and 2, there exists a unique  $\alpha$  such that  $u_B(\bar{\mu}^P; \alpha) = u_G(\bar{\mu}^P, c)$ . This implies that there is a positive reputational equilibrium in which  $\gamma\pi_g$  is offered with probability  $\alpha$  when the loan sold in  $t = 1$  yields the low return.

If  $c > \bar{c}^P$ , then for any  $\alpha \in [0, 1]$  and  $\mu_1 \in (\underline{\mu}, \bar{\mu}^P]$ , we have  $u_B(\mu; \alpha) > u_G(\mu, c)$ . Therefore, if there exists a positive reputational equilibrium, then it has to

<sup>16)</sup>See Figure 1.



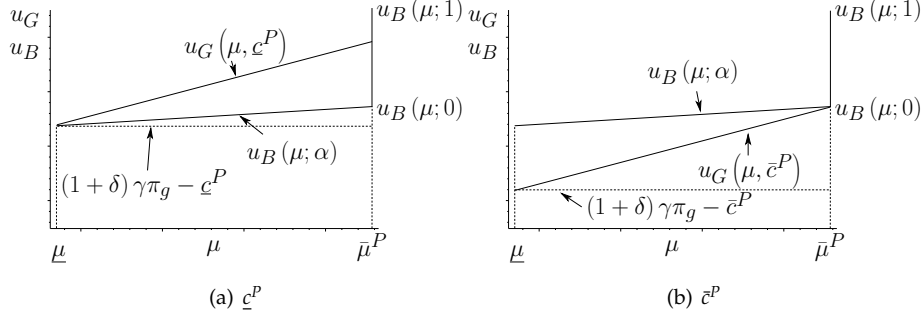


Figure 1:  $\underline{c}^P$  and  $\bar{c}^P$

satisfy  $\mu = \underline{\mu}$ . Suppose that  $p(\underline{\mu})$  and  $\pi_b$  are offered with probabilities  $\beta$  and  $1 - \beta$ , respectively, where

$$\beta \lim_{\mu \rightarrow \underline{\mu}} u_B(\underline{\mu}) + (1 - \beta) \times 2\pi_b = 2\gamma\pi_g - c.$$

Then, the bank is indifferent between investing and not investing. Therefore, there is a positive reputational equilibrium with  $\mu = \underline{\mu}$ .

Lastly, suppose that  $c \in [\underline{c}^P, \bar{c}^P]$ . Then the same argument as above implies that there are positive reputational equilibrium with  $\mu = \underline{\mu}$  and  $\mu = \bar{\mu}^P$ . Also, there is a unique  $\tilde{\mu}$  that solves  $u_B(\tilde{\mu}; \alpha) = u_G(\tilde{\mu}, c)$ . Hence, there is a positive reputational equilibrium with such a  $\tilde{\mu}$ . Q.E.D.

#### Proof of Lemma 4

We show that the following strategy constitutes an equilibrium: (i) the bank invests with probability  $\mu$  which is close to one; (ii) the buyers in  $t = 1$  offer  $p(\mu)$ ; (iii) the buyers in  $t = 2$  offer  $p(\mu^{sh}(\mu))$  and  $p(\mu^{sl}(\mu))$  when the loan sold in  $t = 1$  yielded the high return, and the low return, respectively; and (iv) the government imposes tax

$$\tau(\mu) \equiv p(\mu) \left( \frac{c}{(\pi_g - \pi_b)} - (p(\mu^{sh}(\mu)) - p(\mu^{sl}(\mu))) \right),$$

and subsidy  $s = \frac{\tau}{\mu\pi_g + (1-\mu)\pi_b}$ .

It is straightforward to see that a good bank accepts  $p(\mu)$  in  $t = 1$ . Hence, the optimality of buyers' strategy immediately follows. Since the regulator's expected revenue is

$$\tau - (\mu\pi_g + (1 - \mu)\pi_b)s = 0,$$

we have  $s = \frac{\tau}{\mu\pi_g + (1-\mu)\pi_b}$ .

What remains to be shown is that the bank is indifferent between investing and not investing. To see this, note that the difference in the payoffs of investing and not investing is

$$\Delta = (\pi_g - \pi_b) \left( p(\mu^{sh}(\mu)) - p(\mu^{sl}(\mu)) + s \right) - c. \quad (8.2)$$

Using  $s = \frac{\tau}{\mu\pi_g + (1-\mu)\pi_b}$ , we can show that  $\Delta = 0$  zero if and only if

$$\tau(\mu) \equiv p(\mu) \left( \frac{c}{(\pi_g - \pi_b)} - \left( p(\mu^{sh}(\mu)) - p(\mu^{sl}(\mu)) \right) \right)$$

Q.E.D.

#### Proof of Proposition 4

Fix  $\mu$  close to one, and suppose that the regulator uses  $\tau$  and  $s$  specified in the proof of Lemma 4. Then, the bank's payoff is

$$U(\mu) \equiv 2\pi_g - \frac{(1-\mu)(\pi_g - \pi_b)}{(\mu\pi_g + (1-\mu)\pi_b)} \tau(\mu) - c.$$

Note that  $U(\mu)$  is continuous in  $\mu$  around  $\mu = 1$ , and  $\lim_{\mu \rightarrow 1} U(\mu) = 2\pi_g - c$ . Therefore, we obtain the required result. Q.E.D.

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