Buyer Resistance to Cartel Conduct*

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Abstract

A common feature of most procurements is that if the bids are viewed as too high a buyer may make no award and re-auction the project at a later date. In practice, the highest bid that a buyer accepts is set based on the buyer’s own estimate of the project’s cost which may or may not be publicly released prior to bidding. I analyze the role of information disclosure in a two-period procurement model with the following information structure. The sellers’ costs have both private and common components. The common component is known to all the sellers, but the buyer privately observes only the realization of a noisy signal which is correlated with the sellers’ common cost component. In addition, the buyer is uncertain as to whether he faces a cartel or noncooperative sellers. I show that in this model the buyer can increase his expected payoff by following a policy of concealing the signal in the initial round of bidding. Intuitively, if the buyer’s signal is sufficiently correlated with the true costs, then through a policy of concealing his signal, the buyer can limit the incentives of a low cost cartel to represent itself as high cost. It is also shown that the disclosure of the signal is irrelevant for the buyer when collusion is not a possibility. Thus, nondisclosure of the buyer’s signal may be viewed as a tool to combat collusion.

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1 Introduction

Public and private buyers often rely on a competitive bidding process to purchase required goods or services. Government agencies are usually required by law to purchase all goods, works and services through competitive procurement processes, and billions of dollars are awarded annually.

Competitive bidding can allow buyers to obtain substantial surplus from their purchase. The competitive process only generates this surplus, however, when sellers act independently. When sellers collude, competition is suppressed and prices are artificially inflated.

When collusion is a concern, buyers may react strategically to counter the potential collusion by sellers. There are a number of actions that the buyer can take in a procurement context to oppose collusive bidding. For example, the buyer may refuse to purchase if the bids are viewed as too high. Almost all procurement rules are designed so that the buyer has the discretion to make no award and reject all bids if the bids appear to be unacceptably high. In practice, it is often the case that the buyer refuses to award the project when the lowest received bid is in excess of what is considered a reasonable market price by the buyer. Unawarded projects are typically re-procured at a later date.\(^1\) In most real-world procurements, it is common for a buyer to use its own estimate of the project’s cost to judge the acceptability of the bids. Interestingly, it is also common that the buyer’s cost estimate is not announced prior to bidding, but is only revealed after the initial bidding has ended.

For non-collusive bidders, it is a well-known result in auction theory that the public announcement of all information known to the auctioneer/procurer about the item for sale/procurement may benefit the auctioneer/procurer. In an environment with common value uncertainty, the release of information has two effects on bidding behavior. First, it can undermine the winning bidder’s private information and, as a result, reduce his information rents and second, it can lead to more aggressive bidding by reducing the “winner’s curse.” While much of the existing literature on information disclosure maintains the assumption of competitive bidding, the question of whether it is in the auctioneer’s interest to reveal his information to potentially collusive bidders is less explored.

In this paper, I study the problem of information disclosure in a procurement context where sellers may be colluding. The setting for my analysis is a procurement market in which the buyer is relatively uninformed about the project to be procured.

There are many procurement environments in which sellers have superior information that is relevant to the project. Sellers typically have better information about various aspects of

\(^1\)Re-auctioning of the unsold items is common in auctions as well. See e.g. Cassady (1967), Ashenfelter (1989), and Porter (1995).
their production costs than a buyer. They are usually more familiar with demand and input market conditions as well. As a result, the quality of information held by the buyer is often scarce compared to that of the sellers. For example, highway construction companies may be more accurate in their estimation of the cost of a particular road construction project than the government transportation agencies that are generally the buyers in this industry. Likewise, in the auction setting, timber mills may be more accurate in assessing the quality of timber in a particular area than the Forest Service. In such environments, collusion may enable bidders to extract a potentially substantial rent for their information that would be transferred to the buyer if their bidding were competitive.²

Motivated by common procurement practices, I envision a strategic buyer who is relatively uninformed about the cost of a project but may resist the collusive price increases by rejecting “unreasonable” bids and re-auctioning the project. Re-auctioning is costly to the buyer, but enables the buyer to have a more reliable estimate of the sellers’ costs by the second procurement. Sellers, who may or may not be collusive, need to take account of such resistance by the buyer in order to achieve high profits. In the setting I consider, the disclosure of the buyer’s information has nothing to add to what the sellers believe about their costs. Although, by manipulating the information that is revealed to the sellers, the buyer may influence the sellers’ perceptions about buyer resistance, i.e., as to whether the bids get accepted or rejected. The main finding of this paper is that nondisclosure of the buyer’s information combined with re-auctioning can be useful for combating collusion in an environment where sellers are informationally advantaged and the buyer’s signal is sufficiently informative regarding the project’s cost.

I construct a two-stage procurement model in which a single buyer faces two potential sellers. The sellers’ costs have both common and private components. The common component is known to all sellers but not to the buyer, and the private component is privately observed by each seller. The buyer privately observes the realization of a noisy signal that is correlated with the sellers’ common cost component. After the initial round of bidding, the buyer can purchase from one of the sellers or reject all bids and re-conduct the procurement. If the buyer makes no purchase in the initial round, the buyer may suspend the procurement to acquire a more accurate signal for the second round. Acquiring a more accurate signal causes a delay in purchasing. To prevent the delay, the buyer can re-conduct the procurement.

²The Government buyers may be more susceptible to collusion by informationally advantaged sellers. “It is also difficult for agency personnel to identify collusive patterns or inflated bidding if they are not familiar with the products used in the contracts they are to procure. This problem is exacerbated by the dwindling government acquisition workforce. With fewer contracting offices, the current contract officers are handling a larger number of contracts for a variety of different products and services. As a result, the contracting officers may be less familiar with the products or services on which contractors are bidding.” (“Procurement of Construction and Design Contracts”, Michael T. Callahan, 2005, pg. 1110)
without acquiring a new signal.

The buyer is strategic in this model. In each round, the buyer sets a minimum acceptable bid according to the pre-announced award rule, which is a mapping from the buyer’s signal and the observed bids to a decision whether to award the project. I consider two information disclosure policies, one under which the buyer publicly reveals his private signal in the initial round of bidding and another under which the buyer reveals no information at all. The buyer commits to either a revealing or concealing policy before observing the signal and prior to soliciting bids.

I analyze the buyer’s expected payoff under a revealing and a concealing policy in two circumstances: when there is a cartel of all sellers and when individual sellers act non-cooperatively. Collusion is modeled as an exogenous event. In the event of collusion, only the seller with the lowest cost submits a “serious” bid, and this is common knowledge.

I show that in this model the revealing and the concealing policies are payoff-equivalent for the buyer when sellers act non-cooperatively. However, when the sellers are functioning as a cartel, the buyer can increase his payoff by following a policy of concealing the signal if the buyer’s signal is sufficiently correlated with the sellers’ costs. When the buyer publicly reveals his signal, both low and high cost cartels bid the highest amount that the buyer is willing to accept, conditional on the information that is revealed to the cartel. Unlike the public disclosure policy, when the buyer follows a concealing policy, the cartel remains uncertain about how high to bid without causing rejection of the bid by the buyer. Since the sellers’ costs are correlated with the buyer’s signal, the low (high) cost cartel attaches a greater probability to the buyer’s signal and hence the highest acceptable bid being low (high). Therefore, given that the low signal buyer is more likely to re-auction for a given bid, the highest bid the low-cost cartel believes it can submit without generating rejection is less than the highest bid the high-cost cartel expects it can get away with. If correlation between the buyer’s signal and the sellers’ costs is strong, the low-cost cartel has incentives to bid closer to its actual cost. Thus, the buyer can limit the incentives of a low-cost cartel to represent itself as high cost through a concealing policy. As a result, the concealing policy yields a higher expected payoff for the buyer.

While the objective of this paper is to study the impact of a disclosure of relatively “uninformative” signals on the cartel’s ability to elevate prices, most previous studies examine the effect of the release of information from the auctioneer in a competitive bidding environment where the auctioneer and all bidders receive signals that are affiliated in the Milgrom and Weber sense.\(^3\)

\(^3\)Goeree and Offerman (2000) study the effect of the public release of information in a first-price auction model in which bidders receive both private and common value signals.
Closer to this paper is the literature on secret reserve prices. Several authors have argued that the use of a secret reserve price may be useful for deterring collusive bidding (see, e.g., Ashenfelter (1989), Porter (1993), McAfee and McMillan (1992)). The intuition of this argument is that a secret reserve price introduces additional noise into the auction process and, hence, makes collusion more difficult; however, it is not apparent why the auctioneer would prefer a secret reserve price to a more aggressive public reserve price which would also reduce the rents available to the cartel (as in Graham and Marshall (1987)). Alternative justifications of the use of secret reserve prices include increasing bidder participation in common value auctions (Vincent (1995)) or risk aversion on the part of bidders (Li and Tan (2000), Brisset and Naegelen (2006)).

There is also literature on multi-round auctions that is relevant to this paper. Most related is the paper by Horstmann and LaCasse (1997), where the authors show that the joint use of re-auctioning and secret reserve prices can be profitable for the auctioneer.\footnote{See Ji and Li (2008) for an empirical analysis of multi-round procurement auctions with secret reserve prices.} In contrast to my paper, they consider a pure common value environment in which the auctioneer knows the true value of the object and may re-auction the item to signal his private information that can not be directly transmitted to the bidders.\footnote{Other studies of multi-round auctions include Brusco, Lopomo and Marx (2010), McAfee and Vincent (1997), Skreta (2004), among others.}

The paper proceeds as follows. In Section 2, I review the common procurement practices adopted by the US government purchasing agencies and extract some essential features. In Section 3, I describe a two-stage procurement model that closely parallels the procurement settings described in Section 2. In Section 4, I present results of my model which are consistent with the findings from Section 2. In Section 5, I provide the numerical results. I conclude in Section 6.

## 2 Background

In this section, I describe the public procurement practices currently adopted by the US government agencies.\footnote{See Appendix C for examples.} I focus on some common features of public procurements that were implemented by the government purchasers to police collusive behavior.

In the United States most government procurements are held as first-price sealed-bid auctions and proceed as follows. Before solicitation of bids, usually prequalification is used to identify and invite potential bidders. For a firm to become prequalified, its work experience,
resources and financial assets are typically assessed to determine its capability of undertaking the project. In most procurements, it is common for a buyer to prepare independent cost estimates of the project before soliciting bids. This is called the engineer’s cost estimate and is used as a benchmark for evaluating proposed bids. Bidding is then open to all eligible bidders.

After the bid submission deadline, the bids are evaluated. The purpose of bid evaluation is to determine whether the received bids are acceptable to the buyer. This is a key component of any procurement process. After evaluating the bids, the buyer either awards the contract to the lowest responsible bidder or makes no award and rejects all bids. Almost all procurement rules give the buyer the right to reject any or all bids if the bids appear to be unacceptably high.

 Buyers often void the initial procurement when the low bid exceeds the engineer’s estimate by a substantial margin. If great differences between bid prices and the engineer’s estimate are found, the proposed bids are often viewed as unreasonably high. The procurement agency then rejects all bids and re-auctions the project at a later date.

Since the engineer’s estimate is used to establish a reserve value for a project, its reliability is considered essential. Under-estimating the project value causes delay while overestimating causes inefficient use of funds. There are three main methods for estimating the project costs: cost-based, historical, and a combination of the two. Regardless of the method used to prepare the estimates, buyers generally do not have access to the detailed information held by the contractors. As a result, the cost estimates prepared by the buyer usually are not as accurate as those prepared by contractors for bidding purposes.

Although it is realized that the engineer’s estimate may not always accurately reflect the market value of the project, most government procurement rules explicitly prohibit making an award to the lowest bidder if the lowest bid is not within a certain percent, usually 10%-15%, of the engineer’s estimate. These rules are typically pre-announced. When the lowest

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7Preparation of the engineer’s estimate is optional for private buyers, but typically required by law when a purchaser is a public agency.

8See Federal Acquisition Regulations, Section 14.404 Rejection of bids: “Invitations may be cancelled and all bids rejected before award but after opening when ... (6) All otherwise acceptable bids received are at unreasonable prices, or only one bid is received and the contracting officer cannot determine the reasonableness of the bid price; (7) The bids were not independently arrived at in open competition, were collusive, or were submitted in bad faith.” (Available on-line at http://www.acquisition.gov/Far/reissue/FARvol1ForPaperOnly.pdf)

9Rejection of all bids and re-bidding can be initiated due to other reasons as well, e.g., bids are found to be substantially non-responsive, bid documents are defective and/or incomplete, or there is evidence of inadequate competition.

bid is above the maximum tolerance of the engineer’s estimate, usually a more thorough review of the project costs is required to investigate causes for the excessive bids. An additional investigation is especially important when limited competition is anticipated by the purchasing authorities. In those circumstances where the level of competition for the project is considered as adequate, even an apparently excessive bid may be accepted, but a proper justification is required to ensure that the bids are not collusive.\textsuperscript{11}

An interesting feature of procurements that varies across U.S. states is the information provided to bidders regarding the engineer’s estimate. Some states release this information prior to bidding, while others withhold the engineer’s estimate and use it as a secret reserve price. For example, the evidence from the U.S. transportation industry suggests that more than 60% of state departments of transportation do not release the engineer’s estimate prior to bidding.\textsuperscript{12}

There are different views and justifications, both pro and con, for keeping the engineer’s estimate secret. In some states, public procurement rules require publication of the engineer’s estimate to ensure that contracts are fairly awarded.\textsuperscript{13}

In the absence of legal constraints, publication of the engineer’s estimate is justified for efficiency reasons. The argument for publishing the engineer’s estimate is that disclosure of the estimate helps prospective bidders understand the scope of the project better and makes the project value more predictable to them.

The major concern with respect to releasing the engineer’s estimate is that it may facilitate bid rigging. The U.S. Department of Transportation advises (but does not require) buyers to keep the engineer’s estimate confidential. The Department’s view concerning the confidentiality of the engineer’s estimate is that releasing the estimate creates an enhanced opportunity for collusion among bidders.\textsuperscript{14} The rationale of this argument is that publishing

\textsuperscript{11}The evidence on cartel behavior in the US construction industry prior to 1980’s suggests that it was in deed a normal practice for cartels to achieve excess profits by manipulating the engineer’s estimate. For example, such practice was common in the case of a New York State highway bid-rigging cartel: “the evidence was ample to permit the jury to find that the bid-rigging conspiracies resulted in the low bidders’ submitting bids that were well in excess of what they would have been in the absence of bid rigging, and that the State and the County therefore were in fact injured by the conspiracies... In some instances, the State simply revised its estimate upward on the theory that ‘the competitive bids [were] a better reflection of costs in the current market.’ The State accepted at face value the bidders’ noncollusion representations and operated on the assumption that the bids received reflected the operation of free market forces without any collusion among bidders.” (State of New York v. Hendrickson Brothers Inc., et al, 840 F.2d 1065 (United States Court of Appeals, 2nd Circuit, 1988)) See also Feinstein, et al (1985) for more examples of cartels that manipulated engineer’s estimates on highway construction projects let by NCDOT.

\textsuperscript{12}See Appendix C, Table C.1 for a summary of the EE disclosure policy for individual state DOT’s.

\textsuperscript{13}Publication of the engineer’s estimate leads to a more transparent procedure for the award of public contracts as it eliminates the possibility from state employees to secretly release the estimate to only one or some of the bidders.

\textsuperscript{14}This argument is discussed in “Guidelines on Preparing Engineer’s Estimate, Bid Reviews and Evaluating...
the engineer’s estimate invites bidders to submit bids very close to the estimated amount, even if the cost of the work is less.

According to the Federal Highway Administration guidelines, the state engineer’s estimate is a useful piece of information for firms desiring to rig bids. If bidders know what the state thinks a job is worth, the rigged price may then exceed the engineer’s estimate but not by so much that the bids will be rejected. However, when the engineer’s estimate is kept secret, bidders are uncertain about how high to bid without jeopardizing the award of the project and could be inclined to bid close to competitive prices.

The DOT and DOJ also address this issue in their joint 1983 guidance, “Suggestions for the Detection and Prevention of Construction Contract Bid Rigging,” where the primary reason for maintaining confidentiality of the estimate would be to reduce the possibility of collusion by preventing bidders from knowing the maximum amount that the purchasing agency is willing to pay for the project. The suggestions also note that: “it is not necessary to help them [bidders] estimate the cost of materials, since bidders are intimately familiar with these costs,” concluding that the “bidding process would not be impaired if the state engineer’s estimates were withheld from prospective bidders.”

It is noteworthy that the government policies regarding disclosure of the engineer’s estimate went through remarkable changes in the 1980s, in response to the bid rigging problems that were prevalent during the preceding decades. As a result of the bid rigging scandal in early 1980s and the subsequent changes in the government policy, many states adopted the policy of keeping the engineer’s estimate secret for the purpose of combating bidder collusion.

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16 “Suggestions for the Detection and Prevention of Construction Bid Rigging” (1983), Section 2, pg. 7. “Two major antitrust scandals in the construction industry highlighted the pervasiveness of the practice. ... In 1979, federal prosecutors undertook a major investigation into bid rigging on federally aided highway and airport runway projects in Illinois ... The expanded investigation ultimately covered 24 states and resulted in 340 criminal suits against 183 companies and 210 individuals. Fines totaling $65.5 million and aggregate jail sentences of more than 61 years were imposed. Other construction-related industries ... have been the focus of federal and state investigations.” (“Procurement of Construction and Design Contracts”, Michael T. Callahan, 2005, pg. 1110)
17 The view that collusion is more effective against buyers who reveal the engineer’s estimate to potential bidders was largely influenced by the empirical evidence of cartel behavior prior to the 1980s. The following example of a cartel that was convicted for rigging bids on Oklahoma highway construction projects shows that cartel members were indeed using the engineer’s estimate in determining their collusive bids: “The mechanics of the conspiracy remained essentially the same during its operation: After the ODOT advertised its intent to award certain highway contracts, interested contractors would request a bidding proposal, which included specifications for the job and an estimate of costs prepared by the state’s engineers. ... Once the winning contractor finished his bid, he would tell the other contractors who had promised to submit ‘complimentary
Revealing versus concealing the engineer’s estimate appears to be an important issue outside the United States as well. For instance, the World Bank advises procurement authorities to keep their engineers’ estimates confidential in World Bank-financed road construction projects. The Bank’s view on this problem is that publishing the estimates can produce lower bids when the market is competitive. However, publication of the estimate can facilitate collusion as the cost estimate might serve as a target for potential conspirators.\textsuperscript{19} In support of this view, the World Bank provides a comparison of the estimated price and the winning bids on 46 Bank-financed road construction and repair projects let during 2009 and 2010 in Eastern European countries. It was concluded that “this degree of correspondence is unimaginable in the absence of collusion.”\textsuperscript{20}

The above description of procurement examples establishes several important facts that are relevant to my model: (1) Buyers usually use their own estimate of the project’s cost to judge the reasonableness of bids. (2) Most public procurements have rigid award criteria related to the buyer’s estimate (i.e. the lowest bid should be within a certain percent of the estimate to be recommended for award). (3) If the initial bids are viewed as reasonable, then the buyer makes an award to the lowest bidder. (4) If the initial bids are viewed as unreasonably high, the bids are subject to a more detailed review and investigation before re-letting the project.

\textsuperscript{19}In some cases bidding cartels were discovered after the procurement officials became suspicious because all the received bids were very close to the engineer’s estimate. For example, the "World Bank staff became suspicious when only three bids were submitted for one of the first contracts on the Bali Urban Infrastructure Project. Suspicions were heightened when, despite wide variations in labor and materials prices on the bidders’ bills of quantity, the prices submitted by all three were within 0.02 percent of the engineer’s estimate...Additional investigation confirmed the existence of a bid-rigging cartel". (Aguilar, Gill and Pivio (2000): “Preventing fraud and corruption in World Bank projects. A guide for staff,” p. 22).

\textsuperscript{20}“Preventing fraud and corruption in World Bank projects. A guide for staff,” p. 21.
3 Model

In this section, I present a stylized two-stage procurement model in which the buyer is relatively uninformed.

3.1 The Environment

There is one buyer $B$ who wants to purchase a single good. The buyer’s valuation for the good is equal to $v_B$.

There are two sellers $S_1$ and $S_2$ who can provide the good. The cost of providing the good for seller $i$ is $c_i$, which consists of a common component $\theta$ and a private component $\sigma_i$. Common and private components are additive: $c_i = \theta + \sigma_i$. The common component $\theta$ is low (equal to $\theta_l$) with probability $p_l$ and high (equal to $\theta_h$) with probability $p_h$. Each seller $i$ independently draws a private cost component $\sigma_i$ from a distribution $F(\sigma)$ that has a support $[\underline{\sigma}, \overline{\sigma}]$. Thus, conditional on $\theta$, $c_i$’s are drawn independently from $F[\theta + \sigma, \theta + \overline{\sigma}]$. I let $G$ denote the cdf for the minimum of two random variables drawn from the distribution $F$. I assume that the cdf’s $F(\cdot)$ and $G(\cdot)$ admit continuously differentiable density functions $f(\sigma)$ and $g(\sigma)$ that are bounded away from zero on $[\underline{\sigma}, \overline{\sigma}]$ and have decreasing reverse hazard rates.\footnote{This is equivalent to the assumption of an increasing hazard rate in auctions.} I also assume that the following conditions hold: $\sigma \geq 0$, $\theta_l \geq 0$, $\theta_l + \sigma \leq \theta_h + \sigma$ and $v_B \geq \theta_h + \overline{\sigma}$. All the distribution functions and $v_B$ are commonly known by both sellers and the buyer.

The buyer solicits bids via a first-price sealed-bid procurement. There are potentially two rounds of bidding. After the initial round, the buyer either purchases the good from one of the bidders or rejects the initial bids and re-solicits new bids in the second round.

The common cost component $\theta$ is drawn in round 1 and remains in place for round 2. The private cost components $\sigma_i$’s are drawn from $F[\theta + \sigma, \theta + \overline{\sigma}]$ in each round, independently of each other and of all past realizations. I assume that $\theta$ is commonly observed by $S_1$ and $S_2$. Realization of $\sigma_i$ is privately observed by $S_i$ at the beginning of each round. Neither $\theta$, nor $\sigma_i$’s are observed by the buyer.

The buyer observes a noisy signal of $\theta$ at the beginning of round 1. The signal is labelled as $\widehat{\theta}$. Conditional on $\theta$, $\widehat{\theta}$ takes on two values, high or low: $\widehat{\theta} \in \{\widehat{\theta}_l, \widehat{\theta}_h\}$. The joint distribution of $(\theta, \widehat{\theta})$ is given by $\Pr(\theta_l, \widehat{\theta}_l) = p_{l|l}$, $\Pr(\theta_l, \widehat{\theta}_h) = p_{l|h}$, $\Pr(\theta_h, \widehat{\theta}_l) = p_{h|l}$ and $\Pr(\theta_h, \widehat{\theta}_h) = p_{h|h}$. The conditional probabilities are denoted as follows: $\Pr(\widehat{\theta}|\theta) = p_{\theta|\widehat{\theta}}$ and $\Pr(\theta|\widehat{\theta}) = p_{\theta|\widehat{\theta}}$. I assume that $p_{l|l} > p_{h|l}$ and $p_{h|h} > p_{l|h}$. Thus, the signal $\widehat{\theta}_l$ is more likely to be realized when the sellers’ costs are low, and $\widehat{\theta}_h$ is more likely to be realized the higher are the sellers’ costs.
Realization of $\hat{\theta}$ is $B$’s private information. If the buyer makes no purchase in the initial round, he may suspend the procurement to acquire an accurate signal $\hat{\theta}^{acc}$ that is perfectly informative about $\theta$, i.e., $\hat{\theta}^{acc} = \theta$. Acquiring an accurate signal causes a delay in purchasing. To prevent the delay, the buyer can re-conduct the procurement immediately after the first round, without acquiring a new signal.

Finally, I assume that the buyer is vulnerable to collusion. The environment is such that with probability $\lambda$ the market is competitive, i.e., $S_1$ and $S_2$ are noncooperative bidders. With probability $1 - \lambda$, the environment is collusive, i.e., the sellers form a cartel. I assume that $\lambda$ is independent of $\theta$ and $\sigma_i$’s. If $S_1$ and $S_2$ are in a cartel, the seller with the lowest cost submits a serious bid at the procurement and the second seller submits a complementary bid and this is common knowledge. Both sellers observe if the cartel was formed, but the buyer does not.

All players are risk-neutral. If a purchase is made, the buyer’s payoff is his valuation minus his payment and the seller’s payoff is the payment he gets from the buyer minus his cost. If the buyer suspends the initial procurement to draw an accurate signal and makes a purchase in the second round, then in round 1 all players discount their payoffs at a rate $\delta$. If the buyer holds the second round immediately after the first round without drawing an accurate signal, then the second round payoffs are not discounted. If the buyer does not purchase the good, both sellers and the buyer get zero payoffs.

### 3.2 Timing and Notation

Consistent with the timing as it occurs in practice, I assume that at the beginning of round 1, prior to bidding and before observing the signal, the buyer announces the first round award rule $R_1(\hat{\theta}, b_1^1, b_2^2)$. Conditional on the signal value $\hat{\theta}$ and the received bids $b_1^1, b_2^2$, this announcement commits the buyer to accept no bid greater than $R_1(\hat{\theta}, b_1^1, b_2^2)$ and accept the lowest bid if it is less than or equal to $R_1(\hat{\theta}, b_1^1, b_2^2)$. There are two signal disclosure policies available to the buyer: revealing and concealing. The buyer commits to either a revealing or concealing policy before observing the signal and prior to soliciting the bids.

If the buyer follows a revealing policy, he discloses the signal $\hat{\theta}$ to the sellers before soliciting the first round bids. The sellers then make their bids. If the lowest bid is below $R_1(\hat{\theta}, b_1^1, b_2^1)$, the lowest bidder wins the project and pays his bid. If both bids are above the reserve price, the buyer voids the initial bids and announces re-bidding in the second round.\footnote{I assume that each seller must bid in each round. In other words, if a seller wants to withdraw from the procurement, he may submit an arbitrarily high bid, but he may not refrain from bidding.}
If the buyer follows a concealing policy, he withholds $\hat{\theta}$ from the sellers. After receipt of the first round bids, $\hat{\theta}$ is publicly displayed and if $R_1(\hat{\theta}, b_1^1, b_2^1)$ exceeds the lowest received bid, then the project is awarded to that seller. Otherwise, the buyer rejects both first round bids. I assume that the buyer can credibly commit to not modifying the award rule after observing the bids.\textsuperscript{23} If the first round bids are rejected, the buyer may suspend the procurement to acquire a completely informative signal $\tilde{\theta}^{acc}$ before re-soliciting bids in the second round.\textsuperscript{24} If the buyer holds the second round of bidding immediately after the first round, then $B$ keeps the initial signal $\hat{\theta}$. I assume that when $\lambda < 1$, i.e., when there is a positive probability of collusion among sellers, the buyer must suspend the procurement and obtain an accurate signal $\tilde{\theta}^{acc}$ for the second round if the lowest first round bid is greater than $\max\{R_1, \theta_h + \sigma\}$.\textsuperscript{25}

In the second round, which is the final stage in the model, the buyer publicly reveals his signal and the first round bids prior to soliciting the second round bids. Based upon his inferences about $\theta$ from the initial bids $b_1^1, b_2^1$ and the signal, the buyer commits to the second round award rule $R_2(b_1^2, b_2^2)$ prior to bidding. If the lowest second round bid is below $R_2$, the buyer awards the project to the lowest bidder, otherwise he makes no award. If the buyer does not purchase in the second round, then he cancels the project.\textsuperscript{26}

The timing in the model is as follows:

**Initial procurement**

1. $B$ commits to the signal disclosure policy: revealing or concealing.
2. $S_1$ and $S_2$ observe $\theta$ and $\lambda$. Each $S_i$ observes $\sigma_i^1$.
3. $B$ announces an award rule $R_1$ prior to bidding.
4. $B$ observes $\hat{\theta}$.
5. If $B$ follows revealing policy, then $B$ publicly discloses $\hat{\theta}$. If the buyer follows concealing policy, $B$ keeps $\hat{\theta}$ secret.

\textsuperscript{23}This assumption is quite realistic in most governemnt procurements. Clearly, though, there may be situations where this assumption is less realistic.

\textsuperscript{24}I assume for simplicity that the buyer can draw a perfectly informative signal, but the second signal can be made noisy. The crucial assumption is that the second signal is more informative than the first round bids. Without the latter assumption, finding an optimal award rule is a much more complex problem.

\textsuperscript{25}Note that such requirements are very common in most government procurements. Procurement authorities are usually not allowed to revise their estimates upward without conducting a thorough investigation of costs. Such investigations usually take time.

\textsuperscript{26}In certain circumstances the buyer cannot credibly commit to cancel the project if the project is of a critical importance to the buyer. This scenario is a special case of my model when the second round reserve price is non-binding.
6. $S_1$ and $S_2$ submit their bids $b_1^1$ and $b_1^2$.

7. If $\min \{b_1^1, b_1^2\} \leq R_1$, the buyer awards the project to the lowest bidder. Otherwise, $B$ voids the first round bids and re-procures the project in the second round.

**Potential delay**

If the initial procurement is voided, $B$ decides either to hold the re-procurement stage immediately after the initial procurement or to suspend the procurement. If $B$ suspends the procurement, then there is a delay before the re-procurement stage and $B$ draws $\hat{\theta}^{\text{acc}}$. If $B$ does not suspend the procurement, then $B$ keeps the initial signal $\hat{\theta}$.

**Re-procurement**

1. Each $S_i$ gets a new draw $\sigma_i^2$. $\theta$ remains in place.

2. $B$ announces an award rule $R_2$ prior to bidding.

3. $S_1$ and $S_2$ submit their bids $b_1^2$ and $b_2^2$.

4. If $\min \{b_1^2, b_2^2\} \leq R_2$, then $B$ awards the project to the lowest bidder. Otherwise, $B$ voids the second round bids and cancels the project.

The players’ strategies and payoffs under the revealing and concealing policies are defined as follows.

If the second round is reached, each noncooperative seller $i$ bids $\beta^2_{\text{nc}}(\theta, \sigma_i)$, where for all $i \in \{1, 2\}$, the second round noncooperative bid function $\beta^2_{\text{nc}}(\theta, \cdot)$ is defined by

$$\beta^2_{\text{nc}}(\theta, \sigma_i) \in \arg\max_b \left\{ E_{\sigma_{-i}} \left[ \min \{b - c_i, 1_{b \leq \min \{\beta^2_{\text{nc}}(\theta, \sigma_{-i}), R_2\}} \right] \right\}.$$

I consider only symmetric equilibria of the noncooperative game, so I omit the subscript $i$ throughout.

If the market is collusive, the cartel member with the lowest cost bids $\beta^2_c(\theta, \sigma_m)$, where $\sigma_m \equiv \min \{\sigma_1, \sigma_2\}$ and the second member provides a complementary bid. The cartel bid function $\beta^2_c(\theta, \cdot)$ is defined by

$$\beta^2_c(\theta, \sigma_m) \in \arg\max_b \left\{ (b - c_m)1_{b \leq R_2} \right\}.$$

The second round award rule is defined by

$$R_2(b_1^2, b_2^2) \in \arg\max_{R_2} \left\{ \rho E_\theta \left[ U_{\text{nc}}^2(\theta) \mid \hat{\theta}, b_1^1, b_1^2 \right] + (1 - \rho) E_\theta \left[ U_c^2(\theta) \mid \hat{\theta}, b_1^1, b_2^1 \right] \right\},$$
where \( \rho \equiv \Pr(\text{noncooperative sellers}|\theta, b^1_1, b^1_2) \) is the buyer’s posterior probability of noncooperative sellers.

\[
U^2_{nc}(\theta) \equiv E_{\sigma_1,\sigma_2} \left[ (v_B - \min \left\{ \beta^2_{nc}(\theta, \sigma_1), \beta^2_{nc}(\theta, \sigma_2) \right\}) 1_{\min \left\{ \beta^2_{nc}(\theta, \sigma_1), \beta^2_{nc}(\theta, \sigma_2) \right\} \leq R_2 | \theta} \right]
\]

and

\[
U^2_c(\theta) \equiv E_{\sigma_1,\sigma_2} \left[ (v_B - \beta^2_c(\theta, \sigma_m)) 1_{\beta^2_c(\theta, \sigma_m) \leq R_2 | \theta} \right].
\]

In round one, under a revealing policy, the buyer announces an award rule \( R^P_1(\widehat{\theta}, b^1_1, b^1_2) \) and publicly reveals the signal \( \widehat{\theta} \) prior to bidding. Bidder \( i \)'s problem is then to bid below the highest acceptable price or to wait for the second round. Waiting strategy for bidder \( i \) means bidding above the highest price that the buyer accepts in the first round. The second round is reached only if both bidders choose to wait in the first round. Seller \( i \)'s incentives to bid an acceptable price in the first round or wait depends upon the payoff he expects to get in the second round.

In round 1, the expected second round payoff of a noncooperative seller \( i \) is

\[
\pi^2_{nc}(\theta) \equiv E_{\sigma_1,\sigma_{-i}} \left[ (\beta^2_{nc}(\theta, \sigma_i) - c_i) 1_{\beta^2_{nc}(\theta, \sigma_i) \leq \min \{ \beta^2_{nc}(\theta, \sigma_{-i}); R_2 \}} \right].
\]

The expected second round payoffs to the cartel is

\[
\pi^2_c(\theta) \equiv E_{\sigma_m} \left[ (\beta^2_c(\theta, \sigma_m) - c_m) 1_{\beta^2_c(\theta, \sigma_m) \leq R_2} \right].
\]

Each noncooperative bidder \( i \) bids \( \beta^{1,P}_{nc}(\theta, \sigma_i) \), where for all \( i \in \{1, 2\} \), the first round bid function \( \beta^{1,P}_{nc}(\theta, \cdot) \) under a revealing policy is defined by

\[
\beta^{1,P}_{nc}(\theta, \sigma_i) \in \operatorname{arg \ max}_b E_{\sigma_{-i}} \left[ \frac{(b - c_i) 1_{b \leq \min \{ \beta^{1,P}_{nc}(\theta, \sigma_{-i}), R^P_1 \}}}{\delta \pi^2_{nc}(\theta) 1_{R^P_1 \leq \min \{ b, \beta^{1,P}_{nc}(\theta, \sigma_{-i}) \}}} \right].
\]

The cartel bids \( \beta^{1,P}_c(\theta, \sigma_m) \), where the first round cartel bid function \( \beta^{1,P}_c(\theta, \cdot) \) is defined by

\[
\beta^{1,P}_c(\theta, \sigma_m) \in \operatorname{arg \ max}_b \left[ (b - c_m) 1_{b \leq R^P_1} + \delta \pi^2_c(\theta) 1_{b > R^P_1} \right].
\]

The first round award rule under a revealing policy is defined by

\[
R^P_1(\widehat{\theta}, b^1_1, b^1_2) \in \operatorname{arg \ max}_{R^P_1} \left\{ \lambda E_{\theta} \left[ U^{1,P}_{nc}(\theta) \right] + (1 - \lambda) E_{\theta} \left[ U^{1,P}_c(\theta) \right] \right\}
\]
where

\[
U_{nc}^1(\theta) = E_{\sigma_1, \sigma_2}[(v_B - \min\{\beta_{nc}^{1,P}(\theta, \sigma_1), \beta_{nc}^{1,P}(\theta, \sigma_2)\})1_{\min\{\beta_{nc}^{1,P}(\theta, \sigma_1), \beta_{nc}^{1,P}(\theta, \sigma_2)\} \leq R_1^P} + \delta U_{nc}^2(\theta)1_{R_1^P < \min\{\beta_{nc}^{1,P}(\theta, \sigma_1), \beta_{nc}^{1,P}(\theta, \sigma_2)\}}] \]

and

\[
U_c^1(\theta) = E_{\sigma_1, \sigma_2}[(v_B - \beta_{c}^{1,P}(\theta, \sigma_m))1_{\beta_{c}^{1,P}(\theta, \sigma_m) \leq R_1^P} + \delta U_{c}^2(\theta)1_{\beta_{c}^{1,P}(\theta, \sigma_m) > R_1^P}] \]

Under a concealing policy, the buyer announces the first round award rule \(R_{1}^{NP}(\hat{\theta}, b_1, b_2)\) prior to bidding, but does not reveal \(\hat{\theta}\) to the bidders. Each noncooperative bidder \(i\) bids \(\beta_{nc}^{1,NP}(\theta, \sigma_i)\), where for all \(i \in \{1, 2\}\), the first round bid function \(\beta_{nc}^{1,NP}(\theta, \cdot)\) is defined by

\[
\beta_{nc}^{1,NP}(\theta, \sigma_i) \in \arg\max_b E_{\sigma_i, \hat{\theta}} \left[ (b - c_i)1_{b \leq \min\{\beta_{nc}^{1,NP}(\theta, \sigma_i), R_{1}^{NP}\}} + \delta \pi_{nc}^2(\theta)1_{R_{1}^{NP} < \min\{b, \beta_{nc}^{1,NP}(\theta, \sigma_i)\}} \right].
\]

The cartel bids \(\beta_c^{1,NP}(\theta, \sigma_m)\), where the first round cartel bid function \(\beta_c^{1,NP}(\theta, \cdot)\) is defined by

\[
\beta_c^{1,NP}(\theta, \sigma_m) \in \arg\max_b E_{\hat{\theta}} \left[ (b - c_m)1_{b \leq R_{1}^{NP}} + \delta \pi_c^2(\theta)1_{b > R_{1}^{NP}} \right].
\]

The first round award rule under a concealing policy is defined by

\[
R_{1}^{NP}(\hat{\theta}, b_1, b_2) \in \arg\max_{R_{1}^{NP}} \left\{ \lambda E_{\theta} [U_{nc}^{1,NP}(\theta)] + (1 - \lambda)E_{\theta} [U_c^{1,NP}(\theta)] \right\},
\]

where

\[
U_{nc}^{1,NP}(\theta) = E_{\sigma_1, \sigma_2}[(v_B - \min\{\beta_{nc}^{1,NP}(\theta, \sigma_1), \beta_{nc}^{1,NP}(\theta, \sigma_2)\})1_{\min\{\beta_{nc}^{1,NP}(\theta, \sigma_1), \beta_{nc}^{1,NP}(\theta, \sigma_2)\} \leq R_{1}^{NP} + \delta U_{nc}^2(\theta)1_{R_{1}^{NP} < \min\{\beta_{nc}^{1,NP}(\theta, \sigma_1), \beta_{nc}^{1,NP}(\theta, \sigma_2)\}}] \]

and

\[
U_c^{1,NP}(\theta) = E_{\sigma_1, \sigma_2}[(v_B - \beta_{c}^{1,NP}(\theta, \sigma_m))1_{\beta_{c}^{1,NP}(\theta, \sigma_m) \leq R_{1}^{NP} + \delta U_{c}^2(\theta)1_{\beta_{c}^{1,NP}(\theta, \sigma_m) > R_{1}^{NP}]} \]

Note that, in this model, a concealing policy by the buyer is equivalent to a revealing policy when the signal \(\hat{\theta}\) is completely informative.\(^{27}\) This is so because the buyer receives no private information when \(\hat{\theta} = \theta\). When buyer is uncertain about the underlying distribution, i.e., when \(\hat{\theta}\) is noisy, the buyer has private knowledge of his beliefs about \(\theta\). Therefore,

\(^{27}\)The same is true when the signal is completely uninformative.
Disclosure policies of the buyer’s signal have different implications regarding what sellers know before participating in the procurement, which leads to differences in equilibrium behavior and payoffs.

4 Results

To analyze the buyer’s expected payoff under revealing and concealing policies, I first focus on the case of noncooperative bidders. In Section 4.1, I show that revealing and concealing policies are payoff-equivalent for the buyer, when the buyer is certain that the market is competitive. In section 4.2, I consider the case of collusive bidders. I establish in Proposition 4 that when \( \theta \) is sufficiently correlated with \( \theta \), the concealing policy yields strictly higher payoff for the buyer. A similar result holds when the buyer believes with probability \( \lambda \) that the sellers are noncooperative and attaches probability \( 1 - \lambda \) to the existence of a cartel.

The main message of this analysis is that the disclosure of the buyer’s private signal is irrelevant for a buyer who faces informationally advantaged noncooperative sellers. However, if the buyer anticipates (with some probability) that the sellers are functioning as a cartel, he can increase his payoff and reduce cartel rents by concealing the signal.

4.1 Noncooperative Sellers

In this section, I assume that sellers \( S_1 \) and \( S_2 \) are noncooperative with probability one. To analyze the buyer’s expected payoff, consider first the best scenario for the buyer, i.e., when \( \hat{\theta} = \theta \). As noted earlier, revealing and concealing policies are identical under the assumption of a completely informative signal. In addition, when the buyer knows with certainty the distributional source of the seller’s costs, no additional information can be inferred from the received bids. Therefore, the optimal award rule of the buyer is a simple cutoff rule that is not a function of bids (i.e. a reserve price).

Let \( R_{nc}^1(\theta)^* \) and \( R_{nc}^2(\theta)^* \) denote the optimal first and second round award rules when the sellers are noncooperative and the buyer observes the realized value of \( \theta \) at \( t = 1 \).

Define \( U_{nc}^1(\theta)^* \) and \( U_{nc}^2(\theta)^* \) to be the buyer’s maximum expected payoffs in rounds one and two, respectively, when \( \hat{\theta} \) is completely informative.

I argue in Proposition 1 below that when sellers are noncooperative and the buyer receives a noisy signal of \( \theta \), the buyer can achieve the maximum ex ante expected payoff \( p_t U_{nc}^1(\theta_t)^* + \)

\(^{28}\)When \( \theta \) is known by the buyer, the optimal award rule in each round is the standard reserve price as in Myerson (1981).
\((1 - p_l)U_{nc}^1(\theta_h)^*\), as if \(\theta\) were known to the buyer both under a revealing and a concealing policy.

**Proposition 1** Suppose that sellers are noncooperative. Under both revealing and concealing policies, there exists equilibrium award rule that gives the buyer ex ante expected payoff equal to \(p_l U_{nc}^1(\theta_1)^* + (1 - p_l) U_{nc}^1(\theta_h)^*\).

**Proof.** The proof is by construction. I first characterize the equilibrium award rule and bid strategies when \(\theta\) is common knowledge. Then I show that both under revealing and concealing policies, there exists an equilibrium in which \(B\) commits to the following award rule

\[
R^P_{1}(\tilde{\theta}, b_1^1, b_2^1) = \begin{cases} 
R_{nc}^1(\theta_1)^*, & \text{if } \min\{b_1^1, b_2^1\} \leq \theta_l + \sigma \\
R_{nc}^1(\theta_h)^*, & \text{if } \min\{b_1^1, b_2^1\} \geq \theta_h + \sigma
\end{cases}
\]

and \(S_i\)’s bid as if \(\theta\) were known to the buyer. (see Appendix A for the proof) ■

When the sellers are noncooperative the buyer can rely on the observed bids to learn the true \(\theta\), whether high or low, and set the highest acceptable price accordingly. Since the common cost component \(\theta\) is perfectly correlated across sellers, the buyer can use correlation to fully extract the noncooperative bidders’ knowledge about \(\theta\) irrespective of the signal disclosure policy. Although in the procurement mechanism the buyer can not directly ask the sellers to make reports about \(\theta\), he uses the fact that the equilibrium bid functions are disjoint for \(\theta_l\) and \(\theta_h\), to fully extract information about \(\theta\). Because the buyer can learn \(\theta\) through the first round bids, he conditions the award rule on the observed bids and disregards his noisy signal. Thus, a concealing policy by the buyer is equivalent to a revealing policy when the sellers are noncooperative bidders.

The intuition of this result is that even when there is a significant asymmetry in production information between the buyer and his suppliers, the buyer can rely on supplier rivalry to mitigate the sellers’ informational advantage if the market is competitive.

### 4.2 Collusive Sellers

In this section, I consider the case of collusive sellers. First, I assume that the buyer faces a known cartel, i.e., the buyer is certain that \(S_1\) and \(S_2\) are functioning as a cartel.

As it was shown in the previous section, when sellers are noncooperative, they are unable to capture positive rents for their common knowledge of \(\theta\). When sellers are collusive, however, they can command rents for their superior information by limiting the information that is revealed to the buyer through bidding. The buyer can resist collusive prices by rejecting bids above the highest acceptable price and suspending the procurement to obtain
a more accurate estimate of the sellers’ costs before re-procuring the project. There are two
conflicting factors that govern the buyer’s optimal choice of the award rule when the buyer
faces a cartel. An award rule that commits the buyer to accept a price higher than the actual
costs may result in a substantial payment by the buyer, while an award rule that commits
the buyer to set the highest acceptable price below the actual costs leads to a delay in the
award. I call these two factors No overpay and No delay effects.

As I argued in the previous section, when the buyer knows the true $\theta$, it is optimal for
the buyer to commit to an award rule that does not depend on bids, as there is no useful
information contained in the observed bids. Let $R^1_c(\theta)^*$ and $R^2_c(\theta)^*$ denote the optimal first
and second round award rules when sellers are collusive and the buyer observes $\theta$ at $t = 1$.
Define $U^1_c(\theta)^*$ and $U^2_c(\theta)^*$ to be the buyer’s maximum expected payoffs in rounds one and
two, respectively, and $\pi^2_c(\theta)$ be the cartel’s maximum expected payoff when it is a common
knowledge that $\widehat{\theta}$ is completely informative.

I begin by considering the case when the buyer commits to a policy of publicly revealing
his signal to the bidders. When the buyer reveals the signal $\widehat{\theta}$, the buyer moves first, in
the sense that he fully reveals the first round award rule $R^P_1(\widehat{\theta}, b^1_{ctl})$ prior to bidding. Given
the award rule, the cartel’s problem is to choose between winning the project in the first
round, or waiting for the second round, depending upon the expected payoff that the cartel
expects to get if bidding proceeds to the second round. In equilibrium, the cartel bids either
the maximum price that the buyer accepts in the first round or submits a bid that will be
rejected by the buyer. Because the signal that the buyer uses to design the award rule is
noisy, the buyer always makes a mistake with positive probability in his choice of the award
rule. In other words, the equilibrium award rule that is revealed to the sellers is always
suboptimal compared to when $\theta$ is known by the buyer.

The following proposition characterizes the buyer’s expected payoff under a revealing
policy when $\widehat{\theta}$ is sufficiently accurate.

**Proposition 2** Suppose that sellers are collusive. Under the revealing policy, if $p_{l,\widehat{\theta}}$ is suf-

ficiently high, then in any equilibrium involving non-weakely dominated strategies, the buyer
has ex ante expected payoff strictly less than $p_l U^1_c(\theta_l)^* + p_h (p_{\widehat{\theta}|h} \delta U^2_c(\theta_h)^* + p_{\widehat{\theta}|h} U^1_c(\theta_h)^*)$

**Proof.** I show in Appendix A that in any PBNE, there exists $\tilde{p} \in (0, 1)$ such that in the
first round the buyer with signal $\widehat{\theta}$ adopts the following award rule:

if $p_{l,\widehat{\theta}} \geq \tilde{p}$,

$$R^P_1(\widehat{\theta}, b^1_{ctl}) = R^1_c(\theta_l)^*, \text{ for all } b^1_{ctl}$$
and if $p_{t, \hat{\theta}} < \hat{p}$,

$$R_1^P(\hat{\theta}, b_{ctl}^1) = \begin{cases} R_c^1(\theta_t)^*, & \text{if } b_{ctl}^1 \leq \theta_t + \sigma \\ r(p_{t, \hat{\theta}}), & \text{if } b_{ctl}^1 > \theta_t + \sigma \end{cases}$$

where $r(\cdot) : [0, 1] \rightarrow (\theta_h + \sigma + \delta \pi_c^2(\theta_h), R_c^1(\theta_h)^*)$. In the first round, the cartel either bids the highest price that the buyer accepts or waits for the second round. The equilibrium award rule and bid functions are then used to construct an upper bound on the buyer’s ex ante expected payoff.

Proposition 2 shows that under the public disclosure policy, the buyer either incurs unnecessary costs of delay or pays an inflated price with some probability, regardless of the accuracy of the buyer’s signal. When $p_{t, \hat{\theta}}$ is sufficiently high, the buyer with a signal $\hat{\theta}$ chooses the first round award rule as if $\theta = \theta_t$ were the case, because the No overpay effect dominates the No delay effect. If the true cost environment appears low, then the buyer achieves the maximum possible payoff, however if the true cost environment appears high, the buyer incurs the costs of delaying the procurement. When $p_{t, \hat{\theta}}$ is sufficiently low, the optimal award rule is such that the highest acceptable bid in the first round is greater than $\theta_h + \sigma + \delta \pi_c^2(\theta_h)$, because the No delay effect dominates the No overpay effect. If the true cost environment is high, the buyer incurs the costs of investigation with lower probability, but if the true costs are low, the buyer pays a price substantially higher than the actual costs.

Now I analyze the equilibrium of the game under a concealing policy. Unlike the revealing policy, when the buyer follows a concealing policy, the cartel moves first, i.e., submits a bid without learning the first round award rule $R_1^{NP}(\hat{\theta}, b_{ctl}^1)$. Let $R_c^{NP}(\hat{\theta}_t)$ and $R_c^{NP}(\hat{\theta}_h)$ be the highest prices that the buyer accepts in the first round if the signals are $\hat{\theta}_t$ and $\hat{\theta}_h$, respectively. (Suppose w.l.o.g. that $R_c^{NP}(\hat{\theta}_t) \leq R_c^{NP}(\hat{\theta}_h)$). Then in the first round the cartel believes that the highest acceptable price is equal to $R_c^{NP}(\hat{\theta}_t)$ with probability $\Pr(\hat{\theta}_t | \theta)$ and equal to $R_c^{NP}(\hat{\theta}_h)$ with probability $\Pr(\hat{\theta}_h | \theta)$. Given these beliefs, the cartel’s problem is to choose between bidding $R_c^{NP}(\hat{\theta}_t)$ and $R_c^{NP}(\hat{\theta}_h)$ or waiting for the second round by submitting a bid that will be rejected by the buyer with probability one.\(^{29}\)

Bidding under uncertainty about the award rule involves the following trade-off for the cartel. If the cartel submits a bid substantially greater than the true cost, the cartel might achieve a high payoff. However, if such a high bid gets rejected and the buyer suspends the procurement to draw a more accurate signal, the cartel’s payoff is substantially discounted. Proposition 3 below shows that when $\hat{\theta}$ is sufficiently accurate, the buyer can use this disincentive of the cartel to bid too high, to structure the award rule in a way that induces the

\(^{29}\)Clearly, the cartel never bids below $R_c^{NP}(\hat{\theta}_t)$, or in the interval $(R_c^{NP}(\hat{\theta}_t), R_c^{NP}(\hat{\theta}_h))$ in equilibrium.
Proposition 3 Suppose that sellers are collusive. Under the concealing policy, if $p_{l,h}$ is sufficiently high, then there exists an equilibrium in which the buyer has ex ante expected payoff equal to $p_{l}U_{c}^{1}(\theta_{l})^{*} + p_{h}(p_{l,h}\delta U_{c}^{2}(\theta_{h})^{*} + p_{h,h}U_{c}^{1}(\theta_{h})^{*})$.

Proof. I show in Appendix A that there exists $\bar{p}^{NP} \in (0,1)$ such that if $p_{l,h} \geq \bar{p}^{NP}$, then there is a PBNE in which the buyer adopts the following award rule:

$$R_{c}^{1,NP}(\widehat{\theta}_{l},b_{c}^{1}) = R_{c}^{1}(\theta_{l})^{*}, \text{ for all } b_{c}^{1}$$

$$R_{c}^{1,NP}(\widehat{\theta}_{h},b_{c}^{1}) = \begin{cases} R_{c}^{1}(\theta_{l})^{*}, & \text{if } b_{c}^{1} \leq \theta_{l} + \sigma \\ R_{c}^{1}(\theta_{h})^{*}, & \text{if } b_{c}^{1} > \theta_{l} + \sigma \end{cases}$$

If $\theta = \theta_{l}$, the cartel bids $R_{c}^{1}(\theta_{l})^{*}$ or $\theta_{l} + \sigma$ and if $\theta = \theta_{h}$, the cartel bids $R_{c}^{1}(\theta_{h})^{*}$ or $\theta_{h} + \sigma$. The equilibrium award rule and bid strategies are then used to construct the buyer’s ex ante expected payoff. ■

To obtain the payoff as in Proposition 3, the buyer needs to design the award rule so that the cartel bids as if the true $\theta$ were known to the buyer. The intuition of why this can be accomplished through a concealing policy is as follows. The buyer with the signal $\widehat{\theta}_{l}$ penalizes the cartel by delaying the procurement when he gets bids greater than $\theta_{l} + \sigma$ and rewards the cartel by holding the second round immediately after the first round when bids lower than $\theta_{l} + \sigma$ are received. When $\widehat{\theta}_{l}$ is sufficiently correlated with $\theta$, the cartel’s incentives are such that it never bids above $\theta_{l} + \sigma$ if the true $\theta$ is equal to $\theta_{l}$ and it bids above $\theta_{l} + \sigma$ if the true $\theta$ is equal to $\theta_{h}$. Thus, by concealing the signal, the buyer can induce the cartel to reveal the true $\theta$ through its bid in the first round.

We can now state the main result of this section.

Proposition 4 Suppose that sellers are collusive. For any prior distribution of $\theta$, if $p_{l,h}$ is sufficiently high, then there exists an equilibrium under the concealing policy in which the buyer has a strictly higher ex ante expected payoff than in any equilibrium under the revealing policy.

Proof. The proof directly follows from Propositions 2 and 3. ■

The intuition of why the concealing policy yields a higher expected payoff for the buyer is quite simple. While both high and low cost cartels bid the highest acceptable price when the signal is revealed prior to bidding, under the concealing policy, the buyer can use correlation
between his signal and the true costs to induce different incentives for the high and low cost cartels. In particular, the buyer can limit the incentives of the low-cost cartel to represent itself as high cost. As a result, the concealing policy yields a higher payoff for the buyer.

The basic result that the buyer prefers the concealing policy over the revealing policy for sufficiently accurate signals continues to hold when the buyer is uncertain as to whether sellers are operating as a cartel or not.

**Proposition 5** Suppose that sellers are noncooperative with probability \( \lambda \) and collusive with probability \( 1 - \lambda \). For any \( \lambda \in (0, 1) \) and any prior distribution of \( \theta \), if \( p_l \hat{\theta} \) is sufficiently high, then there exists an equilibrium under the concealing policy in which the buyer has a strictly higher ex ante expected payoff than in any equilibrium under the revealing policy.

**Proof.** See Appendix A for the proof. ■

Similar to the case of a known cartel, the concealing policy enables the buyer to discourage the cartel from misrepresenting the true \( \theta \) if the buyer’s signal is sufficiently accurate. In addition to delaying the procurement, when the buyer believes with positive probability that the sellers are noncooperative, the buyer further penalizes the cartel for misrepresenting the cost environment by announcing a more aggressive award rule in the second round if the initial bids get rejected and the buyer acquires an accurate signal for the second round.

5 Numerical Examples

In this section, I illustrate by way of numerical examples the conditions under which a concealing policy may not be preferable for the buyer. In the examples presented below, I assume that the buyer knows that sellers are in a cartel. In addition, I assume the following parameterization: \( v_B = 10 \), \( F(\cdot) \) is uniformly distributed on \([\underline{\theta}, \bar{\theta}]\) where \( \underline{\theta} = 0 \) and \( \bar{\theta} = 5 \); \( \theta_l = 0 \), \( \theta_h = 5 \), \( p_l = p_h = 0.5 \) and \( p_l \hat{\theta} = p_l \bar{\theta} = p_h \hat{\theta} = p_h \bar{\theta} \equiv p \).

Given these parametrization, one can solve for the cartel’s equilibrium bid function and the buyer’s award rule for different values of \( p \) and \( \delta \).

Figure 1 below illustrates a comparison between the buyer’s equilibrium ex ante expected payoffs under revealing and concealing policies for a variety of parameters \( p \) and \( \delta \). Figure 1 below illustrates a comparison between the buyer’s equilibrium ex ante expected payoffs under revealing and concealing policies for a variety of parameters \( p \) and \( \delta \).

Consistent with the findings from the previous section, these examples suggest that the buyer’s expected payoff under a concealing policy is higher than that under a revealing policy when \( \hat{\theta} \) is sufficiently correlated with \( \theta \). However, as one can see from Figure 1, if the

\[\text{See Appendix B for a summary of the numerical results.}\]
correlation between $\hat{\theta}$ and $\theta$ is not high, the revealing policy might yield a higher payoff for the buyer.

![Figure 1: Buyer's expected payoff under concealing and revealing policies](image)

As it was argued above, under a concealing policy, the cartel either bids the highest price that the low signal buyer accepts or the highest price that the high signal buyer accepts, given that the cartel prefers to win the project in the first round. Such conduct by the cartel works to the advantage of the buyer if $p$ is sufficiently high, but if $p$ is not high the buyer may end up paying more to the cartel when he conceals the signal relative to when the signal is publicly revealed. Intuitively, when the buyer’s signal is sufficiently correlated with $\theta$, the concealing policy provides incentives for the low-cost cartel to bid below $\theta_l + \sigma$ in order to avoid the rejection of the bid by the low signal buyer. But, when the correlation between $\hat{\theta}$ and $\theta$ is weak, the extent to which a concealing policy by the buyer drives down the bid by the low-cost cartel is limited, but the extent to which a low signal publicly revealed by the buyer forces the low-cost cartel to lower its bid is not limited. As a result, the buyer may achieve a greater payoff by revealing a less accurate signal than by concealing it.

There are two additional observations regarding these examples. First, a threshold value of $p$ above which a concealing policy does better for the buyer and below which it does worse than a revealing policy, is lower for smaller values of the discount factor $\delta$. When there is a large gap in time between the first and second rounds of bidding, the punishment for the
cartel for misrepresenting the true cost environment may be substantial, should the buyer reject the first round bids and suspend the procurement to draw an accurate signal for the second round. However, the delay in the award is less of a concern for the cartel when \( \delta \) is high and a higher degree of accuracy of the buyer’s signal is required to ensure that the buyer achieves a greater expected payoff by concealing the signal.

Second, when the degree of correlation between \( \hat{\theta} \) and \( \theta \) is sufficiently low, the buyer has the same expected payoff both under revealing and concealing policies. This occurs because, when \( \hat{\theta} \) is very inaccurate, the cost of paying an excessive cartel price outweighs the cost of delaying the award for the buyer and, thus, the buyer adopts the first round award rule that is optimal against the low cost cartel both under a revealing and concealing policy.

In summary, the preceding analysis demonstrates that the buyer’s expected payoff is greater under a concealing policy when the buyer’s signal is sufficiently informative regarding the sellers’ costs and delay in the award due to re-procurement is substantial.

6 Conclusion

This paper shows that nondisclosure of the buyer’s information, relative to disclosure, can increase the buyer’s payoff when sellers collude. In a two-stage procurement model that closely parallels the informational environment present in many real-world procurements, I analyze the effect of information disclosure by the buyer on bidders’ behavior and buyer’s expected payoff. The model predicts that the disclosure of the buyer’s information is irrelevant when sellers act non-cooperatively. When sellers act collusively, the buyer’s payoff is higher under a concealing policy if the buyer’s signal is sufficiently correlated with the sellers’ costs. When the buyer follows a concealing policy, the correlated signals reduce the cartel’s expected rent from misrepresenting its true cost and thus helps the buyer to moderate the cartel overcharges. Lastly, I show through numerical analysis that when the degree of correlation is weak, the buyer can be better-off by publicly revealing the signal than by concealing it. These results provide insights into the common procurement policies concerning the confidentiality of the engineer’s cost estimate.

A potentially important question for future research is to study how the disclosure of the buyer’s signal affects the sellers’ ability to collude. For example, it seems reasonable that sellers who have the best information on their costs would be able to come to a collusive agreement relatively easily if the buyer overestimates the cost of a project and reveals the estimate prior to bidding. Thus, the buyer may create stronger incentives for collusion by revealing his imprecise knowledge about the sellers’ costs. Then the question is whether the buyer can reduce incentives for collusion by concealing the noisy information that is available
to him regarding the project’s costs.
A Appendix: Proofs

Proof of Proposition 1. We first prove the following two lemmas.

Lemma A.1 Suppose it is a common knowledge $\hat{\theta}$ is completely informative, i.e. $\hat{\theta} = \theta$. In equilibrium each noncooperative bidder $i$ bids according to the following bid function

$$
\beta_{nc}^1(\theta, \sigma_i)^* = \begin{cases} 
\tilde{\beta}_{nc}(\theta, \sigma_i), & \text{if } c_i \leq \tilde{c}(\theta) \\
\sigma + \beta_{nc}(\theta, \sigma_i), & \text{if } c_i > \tilde{c}(\theta)
\end{cases}
$$

and

$$
\beta_{nc}^2(\theta, \sigma_i)^* = \begin{cases} 
\tilde{\beta}_{nc}(\theta, \sigma_i), & \text{if } c_i \leq R_{nc}^2(\theta)^* \\
\sigma + \beta_{nc}(\theta, \sigma_i), & \text{if } c_i > R_{nc}^2(\theta)^*
\end{cases}
$$

where $\tilde{c}(\theta) = R_{nc}^1(\theta)^* - \pi_{nc}^2(\theta)$

Proof of Lemma 1. To see that it is a best response for noncooperative bidders to bid according to $\beta_{nc}^1(\theta, \sigma_i)^*$ and $\beta_{nc}^2(\theta, \sigma_i)^*$, note that the second round is a standard IPV first price procurement with two bidders and a reserve price $R_{nc}^2(\theta)^*$. Therefore, if $c_i \leq R_{nc}^2(\theta)^*$, the optimal bid is given by $\tilde{\beta}_{nc}^2(\theta, \sigma_i) = \frac{R_{nc}^2(\theta)^*[1 - F_{\theta}(R_{nc}^2(\theta)^*)]}{1 - F_{\theta}(\sigma_i)} + \frac{R_{nc}^2(\theta)^*}{1 - F_{\theta}(\sigma_i)} \int_{\sigma_i}^{\tilde{c}(\theta)} udF_{\theta}(u)$. Since the buyer knows $\theta$, he does not suspend the procurement, so the second round payoffs are not discounted. In the first round, a noncooperative bidder with the cost draw equal to $\tilde{c}(\theta)$ bids the first round reserve price $R_{nc}^1(\theta)^*$, as he is indifferent between winning the first round at the reserve price and waiting for the second round. Therefore, if $c_i \leq \tilde{c}(\theta)$, the optimal bid is given by $\tilde{\beta}_{nc}^1(\theta, \sigma_i) = \frac{R_{nc}^1(\theta)^*[1 - F_{\theta}(R_{nc}^1(\theta)^*)]}{1 - F_{\theta}(\sigma_i)} + \frac{\tilde{c}(\theta)}{1 - F_{\theta}(\sigma_i)} \int_{\sigma_i}^{\tilde{c}(\theta)} udF_{\theta}(u)$. If $c_i > \tilde{c}(\theta)$, bidder $i$ prefers to wait for the second round, i.e. the optimal bid is above the reserve price (the upper support). The optimal reserve price in each round is the standard reserve price as in Myerson (1981). Q. E. D.

Lemma A.2 Suppose $\hat{\theta}$ is noisy. If the buyer follows a revealing policy, then the following is PBNE: In the first round the buyer commits to the award rule

$$
R_1^P(\hat{\theta}, b_1^1, b_1^2) = \begin{cases} 
R_{nc}(\theta_1)^*, & \text{if } \min\{b_1^1, b_1^2\} \leq \theta_1 + \sigma \\
R_{nc}(\theta_h)^*, & \text{if } \min\{b_1^1, b_1^2\} \geq \theta_h + \sigma
\end{cases}
$$

In the first round each noncooperative bidder bids according to the bid function $\beta_{nc}^1(\theta, \sigma_i)^*$ defined in Lemma 1. If the buyer observes at least one bid less than $\theta_1 + \sigma$ in the first round, the buyer believes that $\theta$ is equal to $\theta_1$ with probability one. If the buyer observes both bids
greater than $\theta_h + \sigma$ in the first round, the buyer believes that $\theta$ is equal to $\theta_h$ with probability one. If the buyer observes at least one bid less than $\theta_1 + \sigma$ in the first round, then in the second round he commits to the award rule

$$R_2(b_1^2, b_2^2) = R_{nc}^2(\theta_1)^* \text{ for all } \{b_1^2, b_2^2\}$$

and if the buyer observes both bids greater than $\theta_h + \sigma$ in the first round, then in the second round he commits to the award rule

$$R_2(b_1^2, b_2^2) = R_{nc}^2(\theta_h)^* \text{ for all } \{b_1^2, b_2^2\}$$

In the second round each noncooperative bidder bids according to the bid function $\beta_{nc}^2(\theta, \sigma_i)^*$ defined in Lemma 1.

Proof of Lemma 2. The buyer’s beliefs are consistent with the Bayes’ rule given the bid strategies. To see that it is a best reply for a noncooperative bidder $i$ to bid according to the bid functions $\beta_{nc}^1(\theta, \sigma_i)^*$ and $\beta_{nc}^2(\theta, \sigma_i)^*$, note that if $\theta = \theta_1$, in the first round bidder $-i$ bids according to the bid function $\beta_{nc}^0(\theta_1, \sigma_i)^*$ which is bounded above by $\theta_1 + \sigma$. Since bidder $-i$ always bids below $\theta_1 + \sigma$ in the first round, the buyer rejects the lowest first round bid if it is greater than $R_{nc}^1(\theta_1)^*$ and announces the reserve price equal to $R_{nc}^2(\theta_1)^*$ in the second round. Then, by the definition of $R_{nc}^2(\theta_1)^*$, it is a best reply for bidder $i$ to bid $b_i^2 \in [\theta_1, \sigma_i)$ in the second round and for costs greater than $R_{nc}^2(\theta_1)^*$, it is a best reply to bid $\theta_1 + \sigma$. In the first round, by the definition of $\beta_{nc}^1(\theta_1, \sigma_i)$, for the costs less than or equal to $R_{nc}^1(\theta_1)^* - \pi_{nc}^2(\theta_1)$ it is a best reply for bidder $i$ to bid $\beta_{nc}^1(\theta_1, \sigma_i)$ and for costs greater than $R_{nc}^1(\theta_1)^* - \pi_{nc}^2(\theta_1)$, it is a best reply to bid $\theta_1 + \sigma$.

If $\theta = \theta_h$, in the first round bidder $-i$ bids according to the bid function $\beta_{nc}^1(\theta_h, \sigma_i)^*$ which is bounded below by $\theta_h + \sigma$. If bidder $i$ bids less than $R_{nc}^1(\theta_1)^*$, then the buyer accepts $i$’s bid and $i$ gets a negative payoff. Bidder $i$’s payoff from a bid greater than $R_{nc}^1(\theta_1)^*$, but less than $\theta_1 + \sigma$ is equal to zero as the buyer rejects both first period bids and announces a reserve price equal to $R_{nc}^2(\theta_1)^*$ in the second round. If bidder $i$ bids above $\theta_h + \sigma$, the buyer rejects the lowest first round bid if it is greater than $R_{nc}^1(\theta_h)^*$ and announces the reserve price equal to $R_{nc}^2(\theta_h)^*$ in the second round. Then, by the definition of $\beta_{nc}^2(\theta_h, \sigma_i)$, for costs less than or equal to $R_{nc}^2(\theta_h)^* - \pi_{nc}^2(\theta_h)$, it is a best reply for bidder $i$ to bid $\beta_{nc}^2(\theta_h, \sigma_i)$ in the second round, and for costs greater than $R_{nc}^2(\theta_h)^*$ it is a best reply to bid $\theta_h + \sigma$. In the first round, by the definition of $\beta_{nc}^1(\theta_h, \sigma_i)$, for the costs less than or equal to $R_{nc}^1(\theta_h)^* - \pi_{nc}^2(\theta_h)$ it is a best reply for bidder $i$ to bid $\beta_{nc}^1(\theta_h, \sigma_i)$ and for costs greater than $R_{nc}^1(\theta_h)^* - \pi_{nc}^2(\theta_h)$, it is a best reply to bid $\theta_h + \sigma$. It follows then from the definition of $\beta_{nc}^1(\theta, \sigma_i)^*$ and $\beta_{nc}^2(\theta, \sigma_i)^*$ that
it is a best reply for bidder $i$ to bid according to the bid functions $\beta_{nc}^1(\theta, \sigma_i)^*$ and $\beta_{nc}^2(\theta, \sigma_i)^*$.

Given the buyer’s beliefs, by the definition of $R_{nc}^2(\theta)^*$, it is a best reply for the buyer to set the second round reserve price equal to $R_{nc}^2(\theta_i)^*$ if the lowest first round bid is less than $\theta_l + \sigma$ and set the second round reserve price equal to $R_{nc}^2(\theta_h)^*$ if the lowest first round bid is greater than $\theta_h + \sigma$.

To show that it is indeed optimal for the buyer to commit to the first round award rule as defined in Lemma 2, note that the buyer’s ex ante expected payoff is $p_l U_{nc}^1(\theta_l) + p_h U_{nc}^1(\theta_h)^*$. The buyer can not get a higher payoff by choosing any other award rule $R_{P}^1(b_1, b_2) = \cdot$, as the optimal reserve price is $R_{c}^1(\theta_l)^*$, conditional on $\theta = \theta_l$ and the optimal reserve price is $R_{c}^1(\theta_h)^*$, conditional on $\theta = \theta_h$.

With exactly the same logic we can argue that the reserve price and noncooperative bid functions defined in Lemma 2 are mutual best responses when the buyer follows a concealing policy as well. In equilibrium the buyer adopts the award rule that does not depend on the value of $\hat{\theta}$, so the disclosure policy of the buyer’s signal does not affect the noncooperative bid functions. Q. E. D.

**Continuation of the Proof of Proposition 1.** Proposition 1 follows then by combining Lemmas 1 and 2. Q.E.D.

**Proof of Proposition 2.** We first prove the following two lemmas.

**Lemma A.3** Suppose it is a common knowledge that $\hat{\theta}$ is completely informative, i.e. $\hat{\theta} = \theta$. In equilibrium the buyer adopts a constant award rule $R_{c}^1(\hat{\theta}, b_1, b_2) = R_{c}^1(\theta)^*$ in the first round, and a constant award rule $R_{c}^2(b_2 | b_1) = R_{c}^2(\theta)^*$ in the second round. The cartel bids according to the following bid functions

\[
\beta_{c}^1(\theta, \sigma_m)^* = \begin{cases} 
R_{c}^1(\theta)^*, & \text{if } c_m \leq R_{c}^1(\theta)^* - \pi_{c}^2(\theta) \\
\theta + \sigma, & \text{if } c_m > R_{c}^1(\theta)^* - \pi_{c}^2(\theta)
\end{cases}
\]

\[
\beta_{c}^2(\theta, \sigma_m)^* = \begin{cases} 
R_{c}^2(\theta)^*, & \text{if } c_m \leq R_{c}^2(\theta)^* \\
\theta + \sigma, & \text{if } c_m > R_{c}^2(\theta)^*
\end{cases}
\]

**Proof of Lemma 3.** Note that when $\theta$ is a common knowledge, the second round is a standard IPV first price procurement with one bidder. Therefore, the optimal award rule in the second round is a constant reserve price (that does not depend on bids).

Given $\theta$, let $R_{c}^2(\theta)^*$ be the optimal second round reserve price. Then, for the costs less than $R_{c}^2(\theta)^*$, it is a best reply for the cartel to bid the reserve price and for the costs greater than $R_{c}^2(\theta)^*$, it is a best reply to bid above the reserve price.
$R^2_c(\theta)^*$ is given by

$$R^2_c(\theta)^* = \arg\max_R \{(v_B - R) G_\theta(R)\}$$

By assumption, $G_\theta(R)$ has a decreasing reverse hazard rate, so $R^2_c(\theta)^*$ is unique and $R^2_c(\theta)^* \in [\theta + \sigma, \theta + \overline{\sigma}]$.

Let $U^2_c(\theta)^*$ and $\pi^2_c(\theta)$ be the maximum ex ante expected second round payoffs to the buyer and to the cartel.

$$U^2_c(\theta)^* \equiv (v_B - R^2_c(\theta)^*) G_\theta(R^2_c(\theta)^*)$$

$$\pi^2_c(\theta) \equiv E_{\sigma_m} [R^2_c(\theta)^* - (\theta + \sigma_m)]$$

The first round is also a standard first price procurement with only one bidder where the buyer has an outside option equal to $U^2_c(\theta)^*$ and the cartel has an outside option equal to $\pi^2_c(\theta)$. Therefore, the optimal award rule in the first round is a constant reserve price (that does not depend on bids). Let $R^1_c(\theta)^*$ be the optimal first round reserve price. Then, in the first round, the cartel’s payoff from a bid equal to the reserve price is $R^1_c(\theta)^* - c_m$ and the cartel’s payoff from a bid greater than the reserve price is $\pi^2_c(\theta)$. So, for the costs less than $R^1_c(\theta)^* - \pi^2_c(\theta)$, it is a best reply for the cartel to bid $R^1_c(\theta)^*$ and for the costs greater than $R^1_c(\theta)^* - \pi^2_c(\theta)$, it is a best reply for the cartel to bid $\theta + \overline{\sigma}$.

$R^1_c(\theta_l)^*$ and $R^1_c(\theta_h)^*$ are given by

$$R^1_c(\theta_l)^* \in \arg\max_R \Psi_l(R)$$

$$R^1_c(\theta_h)^* \in \arg\max_R \Psi_h(R)$$

where

$$\Psi_l(R) = (v_B - R) G_l(R - \pi^2_c(\theta_l)) + U^2_c(\theta_l)^* (1 - G_l(R - \pi^2_c(\theta_l)))$$

$$\Psi_h(R) = (v_B - R) G_h(R - \theta_l) + \delta U^2_c(\theta_h)^* (1 - G_h(R - \delta \pi^2_c(\theta_h)))$$

By assumption, $G_\theta(R)$ has a decreasing reverse hazard rate, so $R^1_c(\theta_l)^*$ is unique and $R^1_c(\theta_l)^* \in [\theta_l + \sigma + \pi^2_c(\theta_l), \theta_l + \overline{\sigma}]$ and $R^1_c(\theta_h)^*$ is unique and $R^1_c(\theta_h)^* \in [\theta_h + \sigma + \delta \pi^2_c(\theta_h), \theta_h + \overline{\sigma}]$.

Q.E.D.

Let $U^1_c(\theta_l)^*$ and $U^1_c(\theta_h)^*$ be the maximum ex ante expected first round payoffs to the buyer, when $\theta = \theta_l$ and $\theta = \theta_h$, respectively.

$$U^1_c(\theta_l)^* \equiv (v_B - R^1_c(\theta_l)^*) G_l(R^1_c(\theta_l)^* - \pi^2_c(\theta_l)) + U^2_c(\theta_l)^* (1 - G_l(R^1_c(\theta_l)^* - \pi^2_c(\theta_l)))$$
\[ U^1_c(\theta_h) = (v_B - R^1_c(\theta_h))G_h(R^1_c(\theta_h) - \delta \pi^2_c(\theta_h)) + \delta U^2_c(\theta_h) \left(1 - G_h(R^1_c(\theta_h) - \delta \pi^2_c(\theta_h)) \right) \]

Q.E.D.

**Lemma A.4** Suppose \( \hat{\theta} \) is noisy. If the buyer follows a revealing policy, then in every PBNE, there exists \( \tilde{p} \in (0, 1) \) such that in the first round the buyer adopts the following award rule: if \( p_{l, \hat{\theta}} \geq \tilde{p} \),

\[ R^1_{l}(\hat{\theta}, b_c^l) = R^1_c(\theta_l)^*, \text{ for all } b_c^l \]

and if \( p_{l, \hat{\theta}} < \tilde{p} \),

\[ R^1_{l}(\hat{\theta}, b_c^l) = \begin{cases} 
R^1_c(\theta_l)^*, & \text{If } b_c^l \leq \theta_l + \sigma \\
r(p_{l, \hat{\theta}}), & \text{If } b_c^l > \theta_l + \sigma
\end{cases} \]

where \( r(\cdot) : [0, 1] \rightarrow (\theta_h + \sigma + \delta \pi^2_c(\theta_h), R^1_c(\theta_h)^*] \). If \( \theta = \theta_l \) and the buyer reveals \( \hat{\theta} \) such that \( p_{l, \hat{\theta}} \geq \tilde{p} \), the cartel bids according to the bid function

\[ \beta^1_{l,P}(\theta_l, \sigma_m) = \begin{cases} 
R^1_c(\theta_l)^*, & \text{if } c_m \leq R^1_c(\theta_l)^* - \pi^2_c(\theta_l) \\
\theta_l + \sigma, & \text{if } c_m > R^1_c(\theta_l)^* - \pi^2_c(\theta)
\end{cases} \]

If \( \theta = \theta_l \) and the buyer reveals \( \hat{\theta} \) such that \( p_{l, \hat{\theta}} < \tilde{p} \), the cartel bids according to the bid function

\[ \beta^1_{l,P}(\theta_l, \sigma_m) = \begin{cases} 
r(p_{l, \hat{\theta}}), & \text{if } c_m \leq r(p_{l, \hat{\theta}}) - \pi^2_c(\theta_l) \\
\theta_l + \sigma, & \text{if } c_m > r(p_{l, \hat{\theta}}) - \pi^2_c(\theta)
\end{cases} \]

If \( \theta = \theta_h \) and the buyer reveals \( \hat{\theta} \) such that \( p_{l, \hat{\theta}} \geq \tilde{p} \), the cartel bids \( \theta_h + \sigma \). If \( \theta = \theta_h \) and the buyer reveals \( \hat{\theta} \) such that \( p_{l, \hat{\theta}} < \tilde{p} \), the cartel bids according to the bid function

\[ \beta^1_{l,P}(\theta_h, \sigma_m) = \begin{cases} 
r(p_{l, \hat{\theta}}), & \text{if } c_m \leq r(p_{l, \hat{\theta}}) - \delta \pi^2_c(\theta_h) \\
\theta_h + \sigma, & \text{if } c_m > r(p_{l, \hat{\theta}}) - \delta \pi^2_c(\theta)
\end{cases} \]

If in the first round the cartel bid is less than or equal to \( \theta_l + \sigma \), the buyer believes that \( \theta = \theta_l \) with probability one. If the first round bid is less than or equal to \( \theta_l + \sigma \) and is rejected by the buyer, the buyer holds the second round of bidding immediately after the first round. If in the first round the cartel bid is greater than \( \theta_h + \sigma \) and is rejected by the buyer, the buyer suspends the procurement and draws an accurate signal \( \hat{\theta}_{acc} \). In the second round, the buyer
announces the award rule
\[ R_2(b_c^1) = R_c^2(\theta)^* \text{ for all } b_c^1 \]
when \( \hat{\theta}^{acc} = \theta \). The cartel bids according to the following bid function
\[
\beta_c^2(\theta, \sigma_m) = \begin{cases} 
R_c^2(\theta)^*, & \text{if } c_m \leq R_c^2(\theta)^* \\
\theta + \sigma, & \text{if } c_m > R_c^2(\theta)^*
\end{cases}
\]

Proof of Lemma 4. If the second round is reached, \( B \) learns \( \theta \) with certainty. (Either because the procurement was suspended and \( B \) obtained a completely informative signal, or because the first round bid was less than \( \theta_t + \sigma \), which reveals to the buyer that \( \theta = \theta_t \), because in any equilibrium involving non-weakly dominated strategies, the cartel does not bid above \( \theta_h + \sigma \) when \( \theta = \theta_h \). Therefore, the second round is a standard first-price procurement with only one bidder. Then, by the definition of \( R_c^2(\theta_t)^* \) and \( R_c^2(\theta_h)^* \), it is a best response for the buyer to set the second round reserve price equal to \( R_c^2(\theta_t)^* \) if \( \theta = \theta_t \) and set the reserve equal to \( R_c^2(\theta_h)^* \) if \( \theta = \theta_h \), respectively. The ex ante expected second round payoffs to the buyer are given by \( U_c^2(\theta_t)^* \) and \( U_c^2(\theta_h)^* \) and to the cartel - by \( \pi_c^2(\theta_t) \) and \( \pi_c^2(\theta_h) \).

Let \( R_c^{1,P}(\hat{\theta}) \) denote the maximum price that the buyer accepts in the first round when he observes a signal \( \hat{\theta} \), i.e. \( R_c^{1,P}(\hat{\theta}) \equiv \max_{b_1} \{ R_c^{1,P}(\hat{\theta}, b_1^1) \} \).

Given \( R_c^{1,P}(\hat{\theta}) \), the cartel’s expected payoff from submitting a bid equal to \( R_c^{1,P}(\hat{\theta}) \) is \( R_c^{1,P}(\hat{\theta}) - c_m \). If \( \theta = \theta_t \), the cartel’s payoff from submitting a bid greater than \( R_c^{1,P}(\hat{\theta}) \) and less than \( \theta_t + \sigma \) is \( \pi_c^2(\theta_t) \). If \( \theta = \theta_h \), the cartel’s payoff from submitting a bid greater than \( R_c^{1,P}(\hat{\theta}) \) and less than \( \theta_t + \sigma \) is zero (because the buyer rejects the bid and chooses the reserve price equal to \( R_c^2(\theta_h)^* \) in the second round). If \( \theta = \theta_t \), the cartel’s payoff from submitting a bid greater than \( \max \{ R_c^{1,P}(\hat{\theta}), \theta_t + \sigma \} \) is \( \delta \pi_c^2(\theta_t) \). If \( \theta = \theta_h \), the cartel’s payoff from submitting a bid greater than \( \max \{ R_c^{1,P}(\hat{\theta}), \theta_h + \sigma \} \) is \( \delta \pi_c^2(\theta_h) \) (because the buyer must suspend the procurement and draw a completely informative signal when the first round bid is greater than \( \max \{ R_c^{1,P}(\hat{\theta}), \theta_h + \sigma \} \)).

Thus, if \( \theta = \theta_t \), it is a best reply for the cartel to bid in the first round according to the bid function
\[
\beta_c^{1,P}(\theta_t, \sigma_m) = \begin{cases} 
R_c^{1,P}(\hat{\theta}), & \text{if } c_m \leq R_c^{1,P}(\hat{\theta}) - \pi_c^2(\theta_t) \\
\theta_t + \sigma, & \text{if } c_m > R_c^{1,P}(\hat{\theta}) - \pi_c^2(\theta_t)
\end{cases}
\]

That is, when \( \theta = \theta_t \), the cartel bids either the maximum price that the buyer accepts \( R_c^{1,P}(\hat{\theta}) \), or submits a bid less than or equal to \( \theta_t + \sigma \) to avoid the delay before the second round.

\[31\] Clearly, the cartel does not bid below \( R_c^{1,P}(\hat{\theta}) \) in equilibrium.
If $\theta = \theta_k$, for all costs $c_m$, it is a best reply for the cartel to bid $\theta_h + \bar{\sigma}$ if $R_{c}^{1,P}(\hat{\theta})$ is less than $\theta_h + \bar{\sigma} + \delta \pi^2_c(\theta_h)$. If $R_{c}^{1,P}(\hat{\theta})$ is greater than $\theta_h + \bar{\sigma} + \delta \pi^2_c(\theta_h)$, it is a best reply for the cartel to bid according to the bid function

$$
\beta_{c}^{1,P}(\theta_h, \sigma_m) = \begin{cases} 
R_{c}^{1,P}(\hat{\theta}), & \text{if } c_m \leq R_{c}^{1,P}(\hat{\theta}) - \delta \pi^2_c(\theta_h) \\
\theta_h + \bar{\sigma}, & \text{if } c_m > R_{c}^{1,P}(\hat{\theta}) - \delta \pi^2_c(\theta_h)
\end{cases}
$$

Given the cartel’s equilibrium bid function, the buyer’s belief that $\theta = \theta_t$ with probability one when the first round bid is less than or equal to $\theta_t + \bar{\sigma}$ is consistent with the Bayes’ rule.

To determine the optimal first round award rule, note that, given the cartel’s equilibrium bid function, the buyer’s ex ante expected payoff in the first round is given by

$$
\begin{align*}
&\Psi_t(R) \equiv (v_B - R_{c}^{1,P}(\hat{\theta}))G_t(R_{c}^{1,P}(\hat{\theta}) - \pi^2_c(\theta_t)) + \\
&\quad U_{c}^{2}(\theta_t)^*(1 - G_t(R_{c}^{1,P}(\hat{\theta}) - \pi^2_c(\theta_t))) \quad 1_{R_{c}^{1,P}(\hat{\theta}) \leq \theta_t + \bar{\sigma} + \delta \pi^2_c(\theta_h)} + \\
&\quad (v_B - R_{c}^{1,P}(\hat{\theta}))1_{R_{c}^{1,P}(\hat{\theta}) > \theta_t + \bar{\sigma} + \delta \pi^2_c(\theta_h)}
\end{align*}
$$

$$
\begin{align*}
&\Psi_h(R) \equiv (v_B - R_{c}^{1,P}(\hat{\theta}))G_h(R_{c}^{1,P}(\hat{\theta}) - \delta \pi^2_c(\theta_h)) + \\
&\quad \delta U_{c}^{2}(\theta_h)^*(1 - G_h(R_{c}^{1,P}(\hat{\theta}) - \delta \pi^2_c(\theta_h))) \quad 1_{R_{c}^{1,P}(\hat{\theta}) > \theta_t + \bar{\sigma} + \delta \pi^2_c(\theta_h)} + \\
&\quad (v_B - R_{c}^{1,P}(\hat{\theta}))1_{R_{c}^{1,P}(\hat{\theta}) > \theta_t + \bar{\sigma} + \delta \pi^2_c(\theta_h)}
\end{align*}
$$

Introduce the following notation

$$
\Psi_t(R) \equiv (v_B - R)G_t(R - \pi^2_c(\theta_t)) + U_{c}^{2}(\theta_t)^*(1 - G_t(R - \pi^2_c(\theta_t)))
$$

$$
\Psi_h(R) \equiv (v_B - R)G_h(R - \delta \pi^2_c(\theta_h)) + \delta U_{c}^{2}(\theta_h)^*(1 - G_h(R - \delta \pi^2_c(\theta_h)))
$$

By assumption, $G_{\theta}(R)$ has a decreasing reverse hazard rate which implies that $\Psi_t(R)$ and $\Psi_h(R)$ are st. concave on $[\theta_t + \bar{\sigma} + \pi^2_c(\theta_t), \theta_t + \bar{\sigma}]$ and $[\theta_h + \bar{\sigma} + \delta \pi^2_c(\theta_h), \theta_h + \bar{\sigma}]$, respectively, so the following holds:

$$
\left. \frac{d\Psi_t(R)}{dR} \right|_{R=R_{c}^{1,P}(\theta_t)} = 0 \quad \text{and} \quad \left. \frac{d\Psi_h(R)}{dR} \right|_{R=R_{c}^{1,P}(\theta_h)} = 0
$$
Consider the following three cases.

Case 1: \( R^{1,P}_{c}(\hat{\theta}) \leq \theta_h + \sigma + \delta \pi^2_c(\theta_h) \)
If in equilibrium the buyer adopts an award rule \( R^{1,P}_{c}(\hat{\theta}, b^1_c) \) such that \( R^{1,P}_{c}(\hat{\theta}) \leq \theta_h + \sigma + \delta \pi^2_c(\theta_h) \), then \( R^{1,P}_{c}(\hat{\theta}) \) must solve the following maximization problem

\[
R^{1,P}_{c}(\hat{\theta}) \in \arg \max_R \left[ (v_B - R)G_t(R - \pi^2_c(\theta_l)) + U^2_c(\theta_l)(1 - G_t(R - \pi^2_c(\theta_l))) \right]
\]

By the definition of \( R^{1}_c(\theta_l) \), \( R^{1,P}_{c}(\hat{\theta}) = R^{1}_c(\theta_l) \) in this case.

Case 2: \( R^{1,P}_{c}(\hat{\theta}) \in (\theta_h + \sigma + \delta \pi^2_c(\theta_h), \theta_h + \sigma + \pi^2_c(\theta_l)) \)
If in equilibrium the buyer adopts an award rule \( R^{1,P}_{c}(\hat{\theta}, b^1_c) \) such that \( R^{1,P}_{c}(\hat{\theta}) \in (\theta_h + \sigma + \delta \pi^2_c(\theta_h), \theta_h + \sigma + \pi^2_c(\theta_l)) \), then \( R^{1,P}_{c}(\hat{\theta}) \) must solve the following maximization problem

\[
R^{1,P}_{c}(\hat{\theta}) \in \arg \max_R \left\{ p_{l,\hat{\theta}} \Psi_l(R) + (1 - p_{l,\hat{\theta}}) \Psi_h(R) \right\}
\]

As \( R^{1,P}_{c}(\hat{\theta}) \) is interior, it should solve the F.O.C.

\[
p_{l,\hat{\theta}} \frac{d \Psi_l(R)}{dR} + (1 - p_{l,\hat{\theta}}) \frac{d \Psi_h(R)}{dR} \equiv 0
\]

For given \( p_{l,\hat{\theta}} \), let us denote such interior solution, if it exists, by \( m(\ p_{l,\hat{\theta}}) \)

Case 3: \( R^{1,P}_{c}(\hat{\theta}) \geq \theta_h + \sigma + \pi^2_c(\theta_l) \)
If the buyer adopts an award rule \( R^{1,P}_{c}(\hat{\theta}, b^1_c) \) such that \( R^{1,P}_{c}(\hat{\theta}) \geq \theta_h + \sigma + \pi^2_c(\theta_l) \), then \( R^{1,P}_{c}(\hat{\theta}) \) must solve the following maximization problem

\[
R^{1,P}_{c}(\hat{\theta}) \in \arg \max_R \left\{ p_{l,\hat{\theta}}(v_B - R) + (1 - p_{l,\hat{\theta}}) \Psi_h(R) \right\}
\]

If \( R^{1,P}_{c}(\hat{\theta}) \) is not interior, then \( R^{1,P}_{c}(\hat{\theta}) \) is equal to \( \theta_h + \sigma + \pi^2_c(\theta_l) \) if

\[
- p_{l,\hat{\theta}} + (1 - p_{l,\hat{\theta}}) \left( \frac{d \Psi_h(R)}{dR} \right)_{\theta_h + \sigma + \pi^2_c(\theta_l)} < 0
\]

Since \( \frac{d \Psi_h(R)}{dR} \bigg|_{\theta_h + \sigma + \pi^2_c(\theta_l)} > 0 \), there exists \( p \in (0, 1) \) such that

\[
- p_{l,\hat{\theta}} + (1 - p_{l,\hat{\theta}}) \left( \frac{d \Psi_h(R)}{dR} \right)_{\theta_h + \sigma + \pi^2_c(\theta_l)} < 0, \text{ if } p_{l,\hat{\theta}} > \bar{p}
\]
and

\[- p_{t,\hat{b}} + (1 \, - \, p_{t,\hat{b}}) \frac{d\Psi_h(R)}{dR} \bigg|_{\theta_h + \pi_c^2(\theta_i)} > 0, \text{ if } p_{t,\hat{b}} < \bar{p}\]

If \( R^{1, P}_c(\hat{\theta}) \) is interior, then \( R^{1, P}_c(\hat{\theta}) \) solves the F.O.C.

\[- p_{t,\hat{b}} + (1 \, - \, p_{t,\hat{b}}) \frac{d\Psi_h(R)}{dR} \equiv 0\]

Note that for \( R^{1, P}_c(\hat{\theta}) \) to satisfy the necessary F.O.C. it must lie in the interval \((\theta_h + \sigma + \pi^2_c(\theta_i), R^{1, \theta}_c(\theta_h)^*)\). For given \( p_{t,\hat{b}} \), let us denote such interior solution, if it exists, by \( z(p_{t,\hat{b}}) \).

Thus, the equilibrium award rule is such that \( R^{1, P}_c(\hat{\theta}) = R^{1, \theta}_c(\theta_i)^* \) if the following inequalities are satisfied

\[ \left[ p_{t,\hat{b}} \Psi_t(R) + (1 \, - \, p_{t,\hat{b}}) \Psi_h(R) \right]_{m(p_{t,\hat{b}})} > \left[ p_{t,\hat{b}} \left[ U^{1}_c(\theta_t)^* \right] + (1 \, - \, p_{t,\hat{b}}) \delta U^{2}_c(\theta_h)^* \right] \]

\[ \left[ p_{t,\hat{b}} (v_B - R) + (1 \, - \, p_{t,\hat{b}}) \Psi_h(R) \right]_{\theta_h + \pi_c^2(\theta_i)} > \left[ p_{t,\hat{b}} \left[ U^{1}_c(\theta_t)^* \right] + (1 \, - \, p_{t,\hat{b}}) \delta U^{2}_c(\theta_h)^* \right] \]

\[ \left[ p_{t,\hat{b}} (v_B - R) + (1 \, - \, p_{t,\hat{b}}) \Psi_h(R) \right]_{z(p_{t,\hat{b}})} > \left[ p_{t,\hat{b}} \left[ U^{1}_c(\theta_t)^* \right] + (1 \, - \, p_{t,\hat{b}}) \delta U^{2}_c(\theta_h)^* \right] \]

In the above inequalities the R.H.S. is a linear, st. increasing function of \( p_{t,\hat{b}} \) and the L.H.S. is continuous and non-decreasing in \( p_{t,\hat{b}} \) (by the Envelope Theorem). Therefore, it must be true that the L.H.S. and the R.H.S. cross only once (these functions cross because L.H.S.<R.H.S. when \( p_{t,\hat{b}} \rightarrow 1 \) and L.H.S.>R.H.S. when \( p_{t,\hat{b}} \rightarrow 0 \)). So, there exists \( \tilde{p} \in (0, 1) \) such that the above inequalities are satisfied if \( p_{t,\hat{b}} > \tilde{p} \) and at least one of the inequalities is not satisfied if \( p_{t,\hat{b}} < \tilde{p} \).

We can conclude that if \( p_{t,\hat{b}} \geq \tilde{p} \), the optimal first round award rule is given by

\( R^{1, P}_c(\hat{\theta}, b^1_c) = R^{1, \theta}_c(\theta_i)^* \) for all \( b^1_c \) and if \( p_{t,\hat{b}} < \tilde{p} \), the optimal first round award rule is given by

\[ R^{1, P}_c(\hat{\theta}, b^1_c) = \begin{cases} 
R^{1, \theta}_c(\theta_i)^*, & \text{if } b^1_c \leq \theta_i + \sigma \\
r(p_{t,\hat{b}})^*, & \text{if } b^1_c > \theta_i + \sigma 
\end{cases} \]

where \( r(p_{t,\hat{b}}) : [0, 1] \rightarrow (\theta_h + \sigma + \delta \pi^2_c(\theta_h), \, R^{1, \theta}_c(\theta_h)^*) \).

Q.E.D.

Continuation of the Proof of Proposition 2. It follows from Lemma 3 and Lemma 4 that if \( p_{t,\hat{b}} \geq \tilde{p} \), the buyer’s ex ante expected payoff is strictly below \( p_tU^{1, \theta}_c(\theta_i)^* + p_h(p_{t,\hat{b}} \delta U^{2}_c(\theta_h)^* + p_{h,\hat{b}}U^{1}_c(\theta_h)^*) \). Q.E.D.

Proof of Proposition 3. We first prove the following lemma.
Lemma A.5 Suppose $\hat{\theta}$ is noisy. If the buyer follows a concealing policy, then there exists $\hat{p}^{NP} \in (0,1)$ such that if $p_{\hat{t}} \geq \hat{p}^{NP}$, the following is PBNE. In the first round the buyer adopts the following award rule:

$$R^{1,NP}_{c}(\hat{\theta}_t, b^1_c) = R^1_c(\theta_t)^*, \text{ for all } b^1_c$$

$$R^{1,NP}_{c}(\hat{\theta}_h, b^1_c) = \begin{cases} R^1_c(\theta_t)^*, & \text{if } b^1_c \leq \theta_t + \sigma \\ R^1_c(\theta_h)^*, & \text{if } b^1_c > \theta_t + \sigma \end{cases}$$

If $\theta = \theta_t$, the cartel believes with probability $p_{\hat{t}|\theta}$ that the award rule is $R^{1,NP}_{c}(\hat{\theta}_t, b^1_c)$ and believes with probability $p_{\hat{t}|\theta}^\prime$ that the award rule is $R^{1,NP}_{c}(\hat{\theta}_t, b^1_c)$. Given the cartel’s beliefs, the cartel bids according to the bid function

$$\beta^{1,NP}_{c}(\theta_t, \sigma_m) = \begin{cases} R^1_c(\theta_t)^*, & \text{if } c_m \leq R^1_c(\theta_t)^* - \pi^2_c(\theta_t) \\ \theta_t + \sigma, & \text{if } c_m > R^1_c(\theta_t)^* - \pi^2_c(\theta_t) \end{cases}$$

If $\theta = \theta_h$, the cartel believes with probability $p_{\hat{t}|\theta}$ that the award rule is $R^{1,NP}_{c}(\hat{\theta}_t, b^1_c)$ and believes with probability $p_{\hat{t}|\theta}^\prime$ that the award rule is $R^{1,NP}_{c}(\hat{\theta}_t, b^1_c)$. Given the cartel’s beliefs, the cartel bids according to the bid function

$$\beta^{1,NP}_{c}(\theta_h, \sigma_m) = \begin{cases} R^1_c(\theta_h)^*, & \text{if } c_m \leq R^1_c(\theta_h)^* - \delta \pi^2_c(\theta_h) \\ \theta_h + \sigma, & \text{if } c_m > R^1_c(\theta_h)^* - \delta \pi^2_c(\theta_h) \end{cases}$$

If in the first round the cartel bid is less than or equal to $\theta_t + \sigma$, the buyer believes that $\theta = \theta_t$ with probability one. If the first round bid is rejected and is less than $\theta_t + \sigma$, the buyer holds the second round of bidding immediately after the first round. If in the first round the cartel bid is rejected and is greater than $\theta_h + \sigma$, the buyer suspends the procurement and draws an accurate signal $\hat{\theta}^{acc}$. In the second round, the buyer announces the award rule

$$R_{2}(b^1_c) = R^2_c(\theta)^* \text{ for all } b^1_c$$

when $\hat{\theta}^{acc} = \theta$. The cartel bids according to the following bid function

$$\beta^2_{c}(\theta, \sigma_m) = \begin{cases} R^2_c(\theta)^*, & \text{if } c_m \leq R^2_c(\theta)^* \\ \theta + \sigma, & \text{if } c_m > R^2_c(\theta)^* \end{cases}$$

Proof of Lemma 5. The outcome of the second round is the same as that under a revealing policy.
Let $R_{c}^{1,NP}({\hat{\theta}})$ denote the maximum price that the buyer accepts in the first round when he observes a signal ${\hat{\theta}}$, i.e. $R_{c}^{1,NP}({\hat{\theta}}) \equiv \max_{b_{1}}\{R_{c}^{1,NP}({\hat{\theta}}, b_{1})\}$. Without loss of generality, assume that $R_{c}^{1,NP}({\hat{\theta}}) < R_{c}^{1,NP}({\hat{\theta}})_{h}$.

Given the true $\theta$, the cartel’s beliefs are: $R_{c}^{1,NP}({\bar{\theta}}) = R_{c}^{1,NP}({\hat{\theta}})$ with probability $p_{\bar{\theta}}$ and $R_{c}^{1,NP}({\hat{\theta}}) = R_{c}^{1,NP}({\hat{\theta}})_{h}$ with probability $1 - p_{\bar{\theta}}$. If the cartel bids $R_{c}^{1,NP}({\hat{\theta}})$, then the cartel believes that the bid will be accepted by the buyer with probability one. If the cartel bids $R_{c}^{1,NP}({\hat{\theta}})_{h}$, then the cartel believes that the bid will be rejected with probability $p_{\bar{\theta}}$ and accepted with probability $(1 - p_{\bar{\theta}})$.

Consider first the cartel’s problem when $\theta = \theta_{l}$. If $R_{c}^{1,NP}({\hat{\theta}})_{h}$ is less than $\theta_{l} + \sigma$, then the cartel never bids above $\theta_{l} + \sigma$ in the first round to avoid the delay. If $\pi_{c}^{2}(\theta_{l}) > R_{c}^{1,NP}({\hat{\theta}})_{h} - c_{m}$, then the cartel prefers to wait for the second round, so it bids the upper support $\theta_{l} + \sigma$. If $\pi_{c}^{2}(\theta_{l}) \leq R_{c}^{1,NP}({\hat{\theta}})_{h} - c_{m}$, then the cartel prefers to participate in the first round and bids $R_{c}^{1,NP}({\hat{\theta}})_{l}$ if $R_{c}^{1,NP}({\hat{\theta}})_{l} - c_{m} \leq p_{\bar{\theta}}\pi_{c}^{2}(\theta_{l}) + (1 - p_{\bar{\theta}})(R_{c}^{1,NP}({\hat{\theta}})_{h} - c_{m})$ and bids $R_{c}^{1,NP}({\hat{\theta}})_{h}$ if $R_{c}^{1,NP}({\hat{\theta}})_{l} - c_{m} < p_{\bar{\theta}}\pi_{c}^{2}(\theta_{l}) + (1 - p_{\bar{\theta}})(R_{c}^{1,NP}({\hat{\theta}})_{h} - c_{m})$.

If $R_{c}^{1,NP}({\hat{\theta}})_{h}$ is greater than $\theta_{l} + \sigma$, then if the cartel bids $R_{c}^{1,NP}({\hat{\theta}})_{h}$, the buyer will reject the cartel’s bid and suspend the procurement if the buyer’s signal is $\hat{\theta}_{l}$. If $\pi_{c}^{2}(\theta_{l}) > \max\{R_{c}^{1,NP}({\hat{\theta}})_{l} - c_{m}, p_{\bar{\theta}}\pi_{c}^{2}(\theta_{l}) + (1 - p_{\bar{\theta}})(R_{c}^{1,NP}({\hat{\theta}})_{h} - c_{m})\}$, then the cartel prefers to wait for the second round, so it bids $\theta_{l} + \sigma$ to avoid the delay. The cartel prefers to bid $R_{c}^{1,NP}({\hat{\theta}})_{l}$ if $R_{c}^{1,NP}({\hat{\theta}})_{l} - c_{m} \geq \max\{\pi_{c}^{2}(\theta_{l}), p_{\bar{\theta}}\pi_{c}^{2}(\theta_{l}) + (1 - p_{\bar{\theta}})(R_{c}^{1,NP}({\hat{\theta}})_{h} - c_{m})\}$ and bids $R_{c}^{1,NP}({\hat{\theta}})_{h}$ if $p_{\bar{\theta}}\pi_{c}^{2}(\theta_{l}) + (1 - p_{\bar{\theta}})(R_{c}^{1,NP}({\hat{\theta}})_{h} - c_{m}) > \max\{\pi_{c}^{2}(\theta_{l}), R_{c}^{1,NP}({\hat{\theta}})_{l} - c_{m}\}$.

Next consider the cartel’s problem when $\theta = \theta_{h}$. If $R_{c}^{1,NP}({\hat{\theta}})_{h}$ is less than $\theta_{h} + \sigma + \delta\pi_{c}^{2}(\theta_{h})$, the cartel prefers to wait for the second round for all $c_{m}$, so it bids $\theta_{h} + \sigma$. If $R_{c}^{1,NP}({\hat{\theta}})_{h}$ is greater than $\theta_{h} + \sigma + \delta\pi_{c}^{2}(\theta_{h})$, then the cartel prefers to wait for the second round if $\delta\pi_{c}^{2}(\theta_{h}) > R_{c}^{1,NP}({\hat{\theta}})_{h} - c_{m}$, so it bids $\theta_{h} + \sigma$. If $\delta\pi_{c}^{2}(\theta_{h}) \leq R_{c}^{1,NP}({\hat{\theta}})_{h} - c_{m}$, then the cartel prefers to bid $R_{c}^{1,NP}({\hat{\theta}})_{l}$ if $R_{c}^{1,NP}({\hat{\theta}})_{l} - c_{m} \geq p_{\bar{\theta}}\pi_{c}^{2}(\theta_{h}) + (1 - p_{\bar{\theta}})(R_{c}^{1,NP}({\hat{\theta}})_{h} - c_{m})$ and bids $R_{c}^{1,NP}({\hat{\theta}})_{h}$ if $p_{\bar{\theta}}\pi_{c}^{2}(\theta_{h}) + (1 - p_{\bar{\theta}})(R_{c}^{1,NP}({\hat{\theta}})_{h} - c_{m}) > \max\{\pi_{c}^{2}(\theta_{l}), R_{c}^{1,NP}({\hat{\theta}})_{l} - c_{m}\}$.

Let us define the set $\Sigma$ as follows

$$\Sigma \equiv \{(p_{\bar{\theta}}, \delta) \mid p_{\bar{\theta}} \in [0, 1], \delta \in (0, 1) \text{ and } p_{\bar{\theta}}\delta\pi_{c}^{2}(\theta_{l}) + (1 - p_{\bar{\theta}})(R_{c}^{1}(\theta_{h})^{*} - c_{m}) \leq \max\{\pi_{c}^{2}(\theta_{l}), R_{c}^{1}(\theta_{l})^{*} - c_{m}\} \text{ for all } c_{m} \in [\theta_{l} + \sigma, \theta_{l} + \sigma + \delta]\}$$

If the parameters $(p_{\bar{\theta}}, \delta)$ belong to $\Sigma$, then if $R_{c}^{1,NP}({\hat{\theta}})_{h}$ is equal to $R_{c}^{1}(\theta_{l})^{*}$ and $R_{c}^{1,NP}({\hat{\theta}})_{h}$ is equal to $R_{c}^{1}(\theta_{h})^{*}$, the low-cost cartel bids below $\theta_{l} + \sigma$ for all realizations of $\sigma_{m}$. Note that $\Sigma$ is non-empty for all $\delta \in (0, 1)$ if $p_{\bar{\theta}}$ sufficiently close to 1.

We can now conclude that, given the buyer’s award rule in Lemma 5, it is a best reply for the low cost cartel to bids $R_{c}^{1}(\theta_{l})^{*}$, if $c_{m} \leq R_{c}^{1}(\theta_{l})^{*} - \pi_{c}^{2}(\theta_{l})$ , and bid $\theta_{l} + \sigma$ if $c_{m} >$
$R_c^1(\theta_i)^* - \pi_c^2(\theta_i)$. This follows from the definition of $\Sigma$. It is easy to see that for the high cost cartel it is a best reply to bid $R_c^1(\theta_h)^*$ if $c_m \leq R_c^1(\theta_h)^* - \delta \pi_c^2(\theta_h)$ and bid $\theta_h + \sigma$ if $c_m > R_c^1(\theta_h)^* - \delta \pi_c^2(\theta_h)$.

Given the cartel’s equilibrium bid function, the buyer’s belief that $\theta = \theta_l$ with probability one when the first round bid is less than or equal to $\theta_l + \sigma$ is consistent with the Bayes’ rule.

Finally, we show that it is indeed optimal for the buyer to choose the first round award rule as defined in Lemma 5.

If the buyer commits to the award rule described in Lemma 5, then the buyer’s ex ante expected payoff is $p_l U_c^1(\theta_l)^* + p_h (p_{il} | h) U_c^2(\theta_h)^* + p_{ih} | h U_c^1(\theta_h)^*)$.

Consider possible deviations for the buyer. Let us denote by $U_c^1(\theta, R_c^{NP}(\theta, b_c^l))$ the buyer’s expected payoff in the first round, conditional on $\theta$ and $\hat{\theta}$, when the buyer’s first round award rule is $R_c^{NP}(\theta, b_c^l)$. If the buyer adopts any award rule $R_c^{NP}(\hat{\theta}, b_c^l)$ that is different from the award rule described in Lemma 5, then the buyer’s ex ante expected payoff is

$$p_{l|\theta} U_c^1(\theta_l, R_l(\hat{\theta}, b_c^l)) + p_{h|\theta} U_c^1(\theta_h, R_l(\hat{\theta}, b_c^l)) + p_{h|\theta} U_c^1(\theta_h, R_l(\hat{\theta}, b_c^l))$$

Because the optimal reserve price is $R_c^1(\theta_l)^*$, conditional on $\theta = \theta_l$ and the optimal reserve price is $R_c^1(\theta_h)^*$, conditional on $\theta = \theta_h$, we have $U_c^1(\theta_l, R_l(\hat{\theta}, b_c^l)) < U_c^1(\theta_l)^*$, $U_c^1(\theta_h, R_l(\hat{\theta}, b_c^l)) < U_c^1(\theta_h)^*$, and $U_c^1(\theta_h, R_l(\hat{\theta}, b_c^l)) < U_c^1(\theta_h)^*$ for all $R_l(\hat{\theta}, b_c^l)$.

The supremum of $U_c^1(\theta_h, R_l(\hat{\theta}, b_c^l))$ is $U_c^1(\theta_h)^*$ for all possible $R_l(\hat{\theta}, b_c^l)$.

Then, there exists a value $\tilde{p}_{h,\hat{\theta}}$ such that for all $p_{h,\hat{\theta}} < \tilde{p}_{h,\hat{\theta}}$, the following inequality holds

$$\begin{align*}
p_{l|\theta} U_c^1(\theta_l, R_l(\hat{\theta}, b_c^l)) + p_{h|\theta} U_c^1(\theta_l, R_l(\hat{\theta}, b_c^l)) + p_{h|\theta} U_c^1(\theta_h, R_l(\hat{\theta}, b_c^l)) < \\
p_{l|\theta} U_c^1(\theta_l)^* + p_{h|\theta} U_c^1(\theta_h, R_l(\hat{\theta}, b_c^l)) + p_{h|\theta} U_c^1(\theta_h)^* \end{align*}$$

Let, $\tilde{p}_{l,\hat{\theta}} \equiv 1 - \tilde{p}_{h,\hat{\theta}}$. Let use define $\tilde{p}^{NP}$ to be the value in $(0, 1)$ such that if $p_{l,\hat{\theta}} \geq \tilde{p}^{NP}$, then $p_{l,\hat{\theta}} \geq \tilde{p}_{l,\hat{\theta}}$ and $(p_{l|\hat{\theta}}, \delta) \in \Sigma$. We can conclude that if $p_{l,\hat{\theta}} \geq \tilde{p}^{NP}$, it is optimal for the buyer to commit to the award rule as described in Lemma 5. Q.E.D.

**Continuation of the Proof of Proposition 3.** Proposition 3 follows from Lemma 5. Q.E.D.

**Proof of Proposition 5.** Let us define $R_c^1(\theta, b_1^l, b_2^l)^*$ and $R_2^2(b_2^l, b_2^2)^*$ to be the buyer’s optimal award rules when it is a common knowledge that $\hat{\theta} = \theta$ and the buyer believes with probability $\lambda$ that sellers are noncooperative and believes with probability $1 - \lambda$ that sellers are collusive. Using similar arguments as in the proofs of Propositions 2 and 3, one can show that when the buyer follows a concealing policy, for any $\lambda < 1$, there exists
\( \bar{p}_{\lambda}^{NP} \in (0, 1) \) such that if \( p_{t, \lambda} \geq \bar{p}_{\lambda}^{NP} \), the low signal buyer chooses the first round award rule \( R_{\lambda}^{1, NP}(\bar{\theta}_l, b_1^1, b_2^1) \) such that \( R_{\lambda}^{1, NP}(\bar{\theta}_l, b_1^1, b_2^1) = R_{\lambda}^1(\theta_l, b_1^1, b_2^1)^* \) and the high signal buyer chooses the award rule \( R_{\lambda}^{1, NP}(\bar{\theta}_h, b_1^1, b_2^1) \) such that

\[
R_{\lambda}^{1, NP}(\bar{\theta}_h, b_1^1, b_2^1) = \begin{cases} 
R_{\lambda}^1(\theta_l, b_1^1, b_2^1)^*, & \text{if } \min\{b_1^1, b_2^1\} \leq \theta_l + \sigma \\
R_{\lambda}^1(\theta_h, b_1^1, b_2^1)^*, & \text{if } \min\{b_1^1, b_2^1\} > \theta_l + \sigma 
\end{cases}
\]

and both noncooperative and collusive sellers bid as if \( \theta \) were known to the buyer and the award rules were given by \( R_{\lambda}^1(\theta, b_1^1, b_2^1)^* \) and \( R_{\lambda}^2(b_1^2, b_2^2)^* \). When the buyer follows a revealing policy, one can show that for any \( \lambda < 1 \), there exists \( \bar{p}_{\lambda} \in (0, 1) \) such that if \( p_{t, \lambda} \geq \bar{p}_{\lambda} \), the low signal buyer chooses the first round award rule \( R_{\lambda}^{1, P}(\bar{\theta}_l, b_1^1, b_2^1) = R_{\lambda}^1(\theta_l, b_1^1, b_2^1)^* \) and the high signal buyer chooses the award rule \( R_{\lambda}^{1, P}(\bar{\theta}_h, b_1^1, b_2^1) \) such that \( R_{\lambda}^{1, P}(\bar{\theta}_h, b_1^1, b_2^1) \neq R_{\lambda}^{1, NP}(\bar{\theta}_h, b_1^1, b_2^1) \). We can conclude that for a given \( \lambda \), if \( p_{t, \lambda} \geq \max\{\bar{p}_{\lambda}, \bar{p}_{\lambda}^{NP}\} \), then the buyer’s ex ante expected payoff is higher under the concealing policy. \( Q.E.D. \)

## B Appendix: Numerical Examples

Table B.1 reports the percent differences between the buyer’s expected payoff under the concealing policy and the buyer’s expected payoff under the revealing policy, normalized by the difference between the buyer’s expected payoff when \( \bar{\theta} \) is completely informative and the buyer’s expected payoff when \( \bar{\theta} \) is completely informative. (A positive/negative entry indicates that the buyer’s payoff under the concealing policy is higher/lower).

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<th>( \delta = 0.6 )</th>
<th>( \delta = 0.7 )</th>
<th>( \delta = 0.8 )</th>
<th>( \delta = 0.9 )</th>
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<td>19.3%</td>
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<td>( p=0.90 )</td>
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<td>( p=0.75 )</td>
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<tr>
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<td>0.0%</td>
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<tr>
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</tr>
<tr>
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<td>0.0%</td>
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</tr>
<tr>
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<td>0.0%</td>
</tr>
</tbody>
</table>
Table B. 2 reports the buyer’s expected payoff under the revealing and concealing policies.

**Table B.2: Buyer’s expected payoff under concealing and revealing policies**

<table>
<thead>
<tr>
<th></th>
<th>Concealing</th>
<th>Revealing</th>
<th>Concealing</th>
<th>Revealing</th>
<th>Concealing</th>
<th>Revealing</th>
<th>Concealing</th>
<th>Revealing</th>
<th>Concealing</th>
<th>Revealing</th>
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</thead>
<tbody>
<tr>
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<td>4.1371</td>
<td>4.1553</td>
<td>4.1553</td>
<td>4.1775</td>
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<td>4.1577</td>
<td>4.081</td>
<td>4.1233</td>
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<td>3.6475</td>
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<td>3.7106</td>
<td>3.7807</td>
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</tbody>
</table>

Table B. 3 reports the highest acceptable price by the low/high signal buyer in the initial round of bidding when the buyer follows a revealing policy.

**Table B.3: Highest acceptable price under revealing policy**

<table>
<thead>
<tr>
<th></th>
<th>δ=0.6</th>
<th>δ=0.7</th>
<th>δ=0.8</th>
<th>δ=0.9</th>
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<tbody>
<tr>
<td>R_{L,1}^{P}(\theta_{l}^*)</td>
<td>2.97</td>
<td>6.92</td>
<td>2.97</td>
<td>6.89</td>
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<tr>
<td>R_{H,1}^{P}(\theta_{h}^*)</td>
<td>2.97</td>
<td>6.83</td>
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<td>R_{L,1}^{P}(\theta_{l}^*)</td>
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<td>6.74</td>
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<td>R_{H,1}^{P}(\theta_{h}^*)</td>
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<td>6.63</td>
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<tr>
<td>R_{L,1}^{P}(\theta_{l}^*)</td>
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<td>6.54</td>
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<tr>
<td>R_{H,1}^{P}(\theta_{h}^*)</td>
<td>2.97</td>
<td>2.97</td>
<td>2.97</td>
<td>2.97</td>
</tr>
<tr>
<td>R_{L,1}^{P}(\theta_{l}^*)</td>
<td>2.97</td>
<td>2.97</td>
<td>2.97</td>
<td>2.97</td>
</tr>
<tr>
<td>R_{H,1}^{P}(\theta_{h}^*)</td>
<td>2.97</td>
<td>2.97</td>
<td>2.97</td>
<td>2.97</td>
</tr>
<tr>
<td>R_{L,1}^{P}(\theta_{l}^*)</td>
<td>2.97</td>
<td>2.97</td>
<td>2.97</td>
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</tr>
<tr>
<td>R_{H,1}^{P}(\theta_{h}^*)</td>
<td>2.97</td>
<td>2.97</td>
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</tr>
<tr>
<td>R_{L,1}^{P}(\theta_{l}^*)</td>
<td>2.97</td>
<td>2.97</td>
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</tr>
<tr>
<td>R_{H,1}^{P}(\theta_{h}^*)</td>
<td>2.97</td>
<td>2.97</td>
<td>2.97</td>
<td>2.97</td>
</tr>
<tr>
<td>R_{L,1}^{P}(\theta_{l}^*)</td>
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<td>2.97</td>
<td>2.97</td>
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<td>R_{H,1}^{P}(\theta_{h}^*)</td>
<td>2.97</td>
<td>2.97</td>
<td>2.97</td>
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</tr>
</tbody>
</table>

Table B. 4 reports the highest acceptable price by the low/high signal buyer in the initial
round of bidding when the buyer follows a concealing policy.

Table B.4: Highest acceptable price under concealing policy

<table>
<thead>
<tr>
<th>p</th>
<th>δ=0.6</th>
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<th>δ=0.8</th>
<th>δ=0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R_{1, NP}^{1,NF}(\theta_l^*)</td>
<td>R_{1, NP}^{1,NF}(\theta_h^*)</td>
<td>R_{1, NP}^{1,NF}(\theta_l^*)</td>
<td>R_{1, NP}^{1,NF}(\theta_h^*)</td>
</tr>
<tr>
<td>0.95</td>
<td>2.97</td>
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<tr>
<td>0.90</td>
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<tr>
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<tr>
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<tr>
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<td>2.97</td>
<td>2.97</td>
<td>2.97</td>
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</tbody>
</table>

C Appendix: Bid rejections and re-bidding in practice

Table B.1 summarizes the current disclosure policy of the engineer’s estimate for the US Departments of Transportation.\(^{32}\)

Table B.1: State DOT’s Policies on EE release

<table>
<thead>
<tr>
<th>EE Policy</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>No release before the bid letting - EE release after</td>
<td>AK, AZ, CO, DE, GA, ID, IN, KY, ME, MN, NM, OH, SC, TN, WV</td>
</tr>
<tr>
<td>No release before or after the bid letting</td>
<td>AR, IL, IA, KS, MD, NE, VT, VA</td>
</tr>
<tr>
<td>Release of a range of values before no release after</td>
<td>AL, NJ, MO, WI</td>
</tr>
<tr>
<td>Release of a range of values before EE release after</td>
<td>CT, HI, MS, MT, NY, ND, OR, WA, WY, PA</td>
</tr>
<tr>
<td>Release of a budgeted estimate before EE release after</td>
<td>CA, FL, SD, NC</td>
</tr>
<tr>
<td>Release of a budgeted estimate before no EE release after</td>
<td>RI</td>
</tr>
<tr>
<td>EE release before</td>
<td>LA, MA, MI, NV, OK, TX, UT, NH</td>
</tr>
</tbody>
</table>

\(^{32}\)Table B.1 is from De silva, et al (2005).
Table B.2 summarizes twenty recent examples of procurements in which all initial bids were rejected by the relevant government decision maker because the lowest responsive bid was unacceptably high for the buyer.\(^{33}\)

Table B.2: Bid rejections and re-bidding\(^{34}\)

<table>
<thead>
<tr>
<th>City</th>
<th>Project</th>
<th>Industry</th>
<th>Number of Bidders</th>
<th>Date</th>
<th>Reason for Rejection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belmont</td>
<td>Overhaul and upgrade Sewer and Pump Station pumps, holding tanks, and consultants</td>
<td>Construction / Renovation</td>
<td>4</td>
<td>01.09.07</td>
<td>Not sufficient funding in project budget to award to low bidder</td>
</tr>
<tr>
<td>Belmont-2</td>
<td>Sanitary Sewer Rehabilitation Ralston Avenue Pipe Bursting and Pipelining</td>
<td>Construction / Renovation</td>
<td>2</td>
<td>09.14.04</td>
<td>Two received bids exceed the anticipates costs. The City will redesign and re-advertise the project</td>
</tr>
<tr>
<td>Clinton</td>
<td>Install water and sewer infrastructure for Sampson Square Apartments</td>
<td>Construction</td>
<td>3</td>
<td>02.16.10</td>
<td>Lowest bid greater than grant funding</td>
</tr>
<tr>
<td>Des Moines</td>
<td>Golf Course Repairs – damaged from erosion and slope failure</td>
<td>Construction</td>
<td>2</td>
<td>10.11.10</td>
<td>Lowest bid was 53% over project estimate and exceeded project budget</td>
</tr>
<tr>
<td>Folsom</td>
<td>Revitalization Project</td>
<td>Construction</td>
<td>2</td>
<td>07.20.09</td>
<td>Low bid exceeded engineer's estimate</td>
</tr>
<tr>
<td>Fresno</td>
<td>Delivery of Ortho Poly Phosphate Blend to the Surface Water Treatment Facility</td>
<td>Ortho Poly Phosphate Blend Delivery</td>
<td>1</td>
<td>05.01.07</td>
<td>Want to obtain greater bidder participation and lower pricing</td>
</tr>
<tr>
<td>Fresno-2</td>
<td>Landscaping around City Hall and Santa Fe Depot</td>
<td>Landscaping</td>
<td>4</td>
<td>10.02.07</td>
<td>There is a reasonable expectation that additional bids will be received through a future rebid, thereby, reducing the cost of this item</td>
</tr>
<tr>
<td>Lacey</td>
<td>Construct a treatment facility and booster station at reservoir site</td>
<td>Construction</td>
<td>5</td>
<td>05.24.07</td>
<td>Low bidder withdrew because of data errors and next apparent low bidder's value higher than engineer's estimate</td>
</tr>
<tr>
<td>Missoula</td>
<td>Stripping and stockpiling topsoil, and large rocks, rough grading, earth moving, landscape contouring and removal of excess granular materials</td>
<td>Construction</td>
<td>2</td>
<td>6.3.09</td>
<td>Both bids were above the anticipated budget for this project</td>
</tr>
<tr>
<td>Piedmont</td>
<td>Build children's play area</td>
<td>Construction</td>
<td>3</td>
<td>07.19.04</td>
<td>Large discrepancy between architect's estimate for the base bid work versus the low bid</td>
</tr>
</tbody>
</table>

\(^{33}\)Table B.2 is from Kumar, Marshall, Marx and Samkharadze (2011).

\(^{34}\)The procurements are labelled by the name of the city. The full citations are provided at the end of this
<table>
<thead>
<tr>
<th>City</th>
<th>Project</th>
<th>Industry</th>
<th>Number of Bidders</th>
<th>Date</th>
<th>Reason for Rejection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pinole</td>
<td>Information Network Technology Support Services</td>
<td>IT Support</td>
<td>2</td>
<td>06.15.10</td>
<td>Both responses were for more than double the budgeted amount</td>
</tr>
<tr>
<td>Plant City</td>
<td>Furnishing and Installing a 12,000 Gallon Diesel Tank</td>
<td>Fueling</td>
<td>13</td>
<td>8.24.09</td>
<td>Lowest bid was above City's budget for project</td>
</tr>
<tr>
<td>San Rafael</td>
<td>Tennis and Basketball Court Renovation</td>
<td>Construction</td>
<td>4</td>
<td>08.02.10</td>
<td>Lowest bid exceeded Engineer's Estimate</td>
</tr>
<tr>
<td>Shasta Lake</td>
<td>Build Native American Cultural Resource Center</td>
<td>Construction</td>
<td>7</td>
<td>09.08.10</td>
<td>Low bid exceeds available funding</td>
</tr>
<tr>
<td>Silver City</td>
<td>Re-roof library and replace HVAC units in library</td>
<td>Construction/Roofing</td>
<td>4</td>
<td>11.10.09</td>
<td>Town issued bid up to $185,000 from fund but all bids exceeded this amount</td>
</tr>
<tr>
<td>Suisun City</td>
<td>Landscaping along Bikeway</td>
<td>Landscaping</td>
<td>7</td>
<td>09.07.10</td>
<td>Lowest bid exceeded engineer's estimate</td>
</tr>
<tr>
<td>Tracy</td>
<td>Fire Department wants to purchase Triple Combination Fire Pumper</td>
<td>Fire Apparatus Manufacturers</td>
<td>6</td>
<td>08.05.08</td>
<td>The low bid with tax was higher than the authorized budgeted amount</td>
</tr>
<tr>
<td>Villa Park</td>
<td>Mesa Drive Widening &amp; Guard Rail Project</td>
<td>Construction</td>
<td>9</td>
<td>12.16.08</td>
<td>The lowest qualified bid was approximately 44% higher than the engineer’s estimate of the project</td>
</tr>
<tr>
<td>Woodinville</td>
<td>Build bridge</td>
<td>Construction</td>
<td>2</td>
<td>06.13.05</td>
<td>The lowest bid exceeded engineer’s estimate by approximately 30%</td>
</tr>
<tr>
<td>Woodinville-2</td>
<td>Install Fire Detection and Alarm System at City Hall Annex Building</td>
<td>Maintenance</td>
<td>2</td>
<td>07.02.01</td>
<td>The lowest bid was higher than the project funding.</td>
</tr>
</tbody>
</table>

References for Appendix B


References


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