

Product Switching in a Model of Learning

Olga A. Timoshenko*
Yale University

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Abstract

New exporters add and drop products with much greater frequency than old exporters. This paper rationalizes this behavior with a model of demand learning. In the model, an exporter's profitability on the demand side is determined by a time-invariant firm-destination appeal index, and transient firm-destination-year preference shocks. New exporters must learn about their appeal index in the presence of these shocks, and respond to fluctuations in demand by adding and dropping products more frequently than older exporters because they have less information about their attractiveness to consumers. Calibrated to match cross-section distribution of sales and scope, the model quantitatively accounts for the contribution of the extensive margins to aggregate Brazilian exports. The model predicts that in response to a decline in trade costs existing exporters add new products and new exporters enter a destination. Counterfactual implies that the contribution of product adding to export growth resulting from trade liberalization is three times larger than the contribution of exporter entry.

Keywords: Learning, product switching, firm dynamics, exports, Brazil

JEL Classification: F12, F14, L11, L25

1 Introduction

Product adding and dropping (product switching) is a substantial channel of microeconomic and macroeconomic adjustments in the economy, as first argued in Bernard, Redding, and Schott (2010). A large fraction of firms is engaged into product switching.¹ Those firms

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¹For example, see Bernard, Redding, and Schott (2010); Iacovone and Javorcik (2010), with the exception of Goldberg, Khandelwal, Pavcnik, and Topalova (2010b) where wide-spread product adding and infrequent product dropping among Indian firms were driven by industrial regulations (Goldberg, Khandelwal, Pavcnik, and Topalova, 2010a).

adjust by adding and dropping products to changes in competition, regulations, trade costs, and exchange rates. Such changes have been shown to translate into substantial productivity adjustments at the firm as well as the aggregate level, and to account for a large part of changes in traded volumes.² While product switching behavior is widespread and economically important, relatively little is known about how firms make product switching decisions and how these decisions are affected by firms' experience. This paper rationalizes product switching behavior with a model of demand learning, and demonstrates the model provides a quantitatively appealing framework for policy analysis.

Using an extensive data set of Brazilian exporters, I find that new exporters engage in substantially more product adding and dropping than old ones. The proportion of recently introduced products and the share of exports arising from those new products decline with the duration of export experience. An experienced exporter with a given value of export sales derives a smaller fraction of sales from new products and adds fewer products compared to a young exporter with the same value of export sales. As I argue in this paper, this age dependence of product switching naturally arises from a model of demand learning.

In the model, a firm's profitability in a product in a given year depends on firm, firm-product and firm-destination characteristics. On the supply side, a particular product's productivity is a function of firm "ability" and firm-product efficiency. On the demand side, product's attractiveness in a particular export destination is determined by the firm-destination "appeal" of the firms' brand as well as transient firm-destination-year preference shocks. While productivity parameters are known to the firm upon entry, new firms must learn about the appeal of their brand in the presence of these shocks. A key insight of the model is that this learning leads to differential product switching behavior among new and old firms: new firms respond to fluctuations in demand by adding and dropping products more frequently than older firms because they have less information about their attractiveness to consumers.

The model of demand learning is quantitatively consistent with the margins of export sales. Calibrated to match cross-sectional statistics on the export sales and export scope distributions of Brazilian exporters, the model predicts that exporter turnover and product switching are equally important in explaining aggregate Brazilian exports, as implied by the data. Thus, a model of demand learning provides not only a solid rationalization for product switching behavior, but also a quantitatively appealing framework for policy analysis, in particular trade liberalizations.

The model yields striking quantitative implications regarding the role of the product switching and exporter turnover margins in explaining the growth in exports resulting from a decline in trade costs. In response to a decline in trade costs, the model predicts that existing exporters adjust by adding new products and new exporters enter a destination. In a counterfactual simulation of a bilateral trade liberalization, the calibrated model predicts that the product adding contributes three times more to the growth in export sales compared to the exporter entry.

The modeling of the learning on the demand side, as opposed to the cost side, is motivated by the data on the evolution of plant's physical productivity versus plant's demand shock

²Bernard, Redding, and Schott (2010, 2011); Goldberg, Khandelwal, Pavcnik, and Topalova (2010a,b); Gopinath and Neiman (2011)

analyzed by Foster, Haltiwanger, and Syverson (2008). On the one hand, the authors find that there is no substantial difference in physical productivity (measured as output per unit of input) across plants of different ages. In my model a plant is equivalent to a firm. Thus, I assume that firm’s productivity level, referred to as firm’s “ability”, stays constant through the firm’s life-cycle. On the other hand, the authors find that plant’s demand shock is lower for younger plants compared to the old ones. An increase in the demand component through firm’s life-cycle motivates the assumption of learning on the demand side.

Demand learning is not the only way to generate product switching among exporters, or firms in general. Such behavior can be generated by a model with random productivity and demand shocks, as shown in the model of Bernard, Redding, and Schott (2010). In the Bernard, Redding, and Schott (2010) model, product specific demand shocks yield individual product adding and dropping, while productivity specific shocks yield expansion or contraction of the range of products produced by a firm. A framework with random shocks, however, cannot deliver the age dependence of product switching conditional on size, which is the focus of this paper.³

The paper complements the line of research analyzing the role of firm’s age in the growth and expansion of firms. The age dependence of the growth rate of firms conditional on size is a well documented fact (Evans, 1987). This relationship can partially be accounted by the decline in the frequency of product switching as firms age in the market, as suggested by the empirical finding of this paper. While Arkolakis and Papageorgiou (2010) argue that a model with demand learning can predict the negative relation between the growth rate of a firm and its age conditional on size, I apply a similar notion of demand learning to analyze the age dependence of product switching behavior.

An important contribution of this paper lies in demonstrating that a model with demand learning performs well quantitatively. Previous papers have focused on testing qualitative implications of an assumed learning mechanism (Albornoz, Pardo, Corcos, and Ornelas, 2011; Timoshenko, 2010). Some reduced form quantitative evidence has been suggested in the work by Ruhl and Willis (2008), where the authors assume a deterministic learning process, and demonstrate that such assumption is helpful in predicting an increasing survival rate of exporters. In contrast to this earlier work, I incorporate an endogenous demand learning mechanism into a general equilibrium setup, and perform a quantitative evaluation of the learning model.

The rest of the paper is organized as follows. In Section 2 I briefly present the data and document the patterns that motivate the theoretical analysis. In Section 3 I develop a model of demand learning and product choice, and study the model’s implications for product switching behavior. In Section 4 I perform a quantitative evaluation of the model. Section 5 presents the conclusions.

³In the context of a model where a firm is associated with a single product, Arkolakis (2010) shows that a model with random productivity evolution cannot generate age dependence of the growth rate of firms conditional on size.

2 Empirical Trends

In the context of Brazilian export data, this section provides evidence on the importance of product switching behavior, and how this behavior is affected by firm's experience. First, I show that the contribution of product switching margin is at least as large as the exporter turnover margin in predicting the level and growth of Brazilian exports. Second, I show that, conditional on size, the fraction of added products and the fraction of export sales from added products decline in exporter's experience in a market.

2.1 Data

I use data on export sales that comes from the customs declarations for merchandise exports by Brazilian exporters collected at SECEX (*Secretaria de Comercio Exterior*)⁴. It is a four-dimensional panel data set by firm, product, destination, and year between 1990 and 2001. The data are reported in current USA dollars. Where appropriate, I convert exports into real terms (2001 USA dollars) using annual USA consumer price index (from the International Monetary Fund).

A product corresponds to a 6-digit Harmonized Tariff System (HS) code.⁵ I restrict the sample to include solely exports in manufacturing products which results in the total of 4,637 products exported by Brazil between 1990 and 2001. Exports in manufacturing products comprise on average 91% of total Brazilian exports in a given year; on average 92% of exporters in a given year export at least one manufacturing product. Over the period 1990-2001 Brazilian manufacturing products have been exported to 240 destinations.

2.2 Extensive Margins of Brazilian Exports

The contribution of sales from new products by incumbent exporters is at least as large as the contribution of sales from new exporters in explaining the level of Brazilian exports, as can be seen from Table 1. Four point four percent of total Brazilian manufacturing exports in a given year arise from products introduced by incumbent exporters, firms that exported in the previous and current year (third column of Table 1). The new exporter margin comprises a half of that magnitude. Only 2.6 percent of total Brazilian manufacturing exports are accounted by new entrants, firms that began exporting in the current period.

Similar pattern holds for the level of exports to a particular destination (columns one and two of Table 1). For example, 4.5 percent of Brazilian manufacturing exports to the US, Brazil's top export destination, are generated by incumbent exporters introducing new products to the US. The new exporter margin is not as small as for the total Brazilian manufacturing exports, yet it remains below the new product margin.

⁴For detailed description of the data set refer to Molinaz and Muendler (2009)

⁵The original data is reported at the 8-digit level of which the first six digits correspond to the first six digits of the HS classification. To make the results comparable to other data sets, I aggregate the data to the 6-digit HS level: 6-digit HS codes are standardized across countries.

2.3 Margins of Brazilian Exports Growth

The role of product switching margin versus exporter turnover margin is even more pronounced in explaining the growth in Brazilian exports than the level of exports. Nearly 40 percent of the annual growth in Brazilian manufacturing exports arises from the product switching margin, as can be seen from the third column of Table 2. The exporter turnover margin accounts for less than 15 percent of the annual growth in exports.

When considering Brazilian exports to individual destinations, Argentina and the US, both the product switching margin and exporter turnover margin are equally important in explaining the growth in export sales. Each margin accounts for 15 to 20 percent of the growth in annual export sales, as can be inferred from the first and second columns of Table 2.

2.4 Fraction of Sales from New Products

The sample mean of the intra-firm extensive margin is 20.0 percent, as shown in the third column of Table 3. An average surviving exporter derives 20.0 percent of its total export sales from newly added products. The magnitude of the margin is similar when computed for individual export destinations. For example, firms that export to the US derive 18.4 percent of exports to the US from the products they introduced into the US in a given year (second column of Table 3).

The magnitude of the intra-firm extensive margin varies across firms with different export age. The export age is defined as the number of consecutive years a firm is observed exporting positive amount of at least one good. While firms with two years of exporting derive 27.1 percent of sales from new goods, firms with 5 years of experience derive 14.3 percent of sales from new goods (third column in Table 4). Similar pattern holds within individual destinations. Firms that exported to the US for two year derive 24.6 percent of sale to the US from new products in the US, while firms that exported for 5 years derive 12.7 percent of sales from new products. Overall, the magnitude of intra-firm extensive margin gradually declines with firm's export age suggesting that exporters are engaged into product switching less actively as they export for a longer period of time.

Results from the ordinary least squares (OLS) regression of the intra-firm extensive margin on the export age and the logarithm of export sales confirm that the observed relation between the margin and the age is not a statistical artifact. One might explain the results in Table 4 by growing sales of surviving firms. If a firm on average introduces the same amount of products per year, sales from new products will account for less and less of total export sales as total firm's sales grow. In such case the decline in the intra-firm extensive margin is not an indicator of less intense product switching. Thus, in order to claim that the intensity of product switching declines with export experience, it is important to control for firm's total export sales. This is achieved in the aforementioned ordinary least squares regression by inclusion of the logarithm of firm's sales as a control variable. Controlling for firm's total sales, the effect of export age on the intra-firm extensive margin remains negative and statistically significant (column (2) in Table 5). The result is preserved both at the aggregate (column (2)) and destination (column (4)) levels, verifying the conjecture that, conditional on sales, the longer a firm exports the smaller is the fraction of export sales

from new products.

2.5 Fraction of new products

The fact that intensity of product switching declines with export experience can alternatively be seen in the behavior of the fraction of new/added products, the percent of currently exported products a firm added between two consecutive periods. On average, close to a third of products exported by a Brazilian firm to the world are newly added products (third column of Table 6).

The fraction of newly added products varies with the duration of firm's export experience. A firm that became an exporter a year ago exhibit 36.1 percent of current products to be newly added, while a firm with 5 years of export experience exhibits 26.3 percent of current product to be new (third column in Table 7). The statistic seems to decline with export age for the young and old exporters, while appears to be constant in the sample mid-range of export age. The statistics exhibit much stronger declining pattern in age when a subsample of only those exporters that add a positive amount of products in a given year is considered (sixth column in Table 7). Similar pattern holds withing individual destinations.

Results from ordinary least squares regression of the fraction of new products on the export age and the logarithm of export sales (see Table 8) confirm that the observed relation between the fraction and the age is not a statistical artifact.

In the next section I describe a model of product switching behavior that is consistent with the observed patterns.

3 The Model

This section describes a model of learning with product switching and discusses model's implications for firm's quantity, scope, and market participation decisions. The model is built on two key assumptions: Jovanovic (1982) assumption of demand learning and Arkolakis and Muendler (2010) assumption of product ordering. The two assumptions are incorporated into Melitz (2003) monopolistic competition framework with heterogeneous firms.

The world consists of $N + 1$ countries. Time is discrete, and is denoted by t . Firms decide whether to enter or exit a particular market, how many products to sell in that market (scope decisions), and in which quantities to sell those products (scale decision). Firm's products are faced with a demand shock in a given market, and a firm must make quantity and scope decisions before observing the shock. After the decisions are made, prices adjust to clear the markets, and a firm infers its demand shock from the market clearing price. A firm makes quantity and scope decisions based on the expectations about current demand shock. The expectations are formed based on the mean and number of the previously observed demand shocks. Products are imperfect substitutes.

3.1 Preferences and Demand

Each country j is populated with a measure L_j of infinitely lived identical consumers. The preferences of a representative consumer in country j are given by the Dixit and Stiglitz

(1977) constant elasticity of substitution (CES) utility function over the consumptions of the composite goods C_{jt}

$$U_j = E \sum_{t=0}^{+\infty} \beta^t \ln(C_{jt}),$$

where β is the discount factor and ρ is the inter-temporal elasticity of substitution. The consumption of the composite good C_{jt} takes the CES form

$$C_{jt} = \left(\sum_{i=1}^{N+1} \int_{\Omega_{ijt}} (e^{a_{jt}(\omega)})^{\frac{1}{\sigma}} c_{jt}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}},$$

where Ω_{ijt} is the mass of products (both imported and domestic) available for consumption in country j imported from country i in period t ; $c_{jt}(\omega)$ is the consumption of a product $\omega \in \Omega_{ijt}$ in country j . σ is the elasticity of substitution between products. $a_{jt}(\omega)$ is a demand shock for product ω in country j .

Every product ω is offered in $G_{ijt}(\omega)$ differentiated varieties, thus $c_{jt}(\omega)$ is a product composite index which also takes the CES form

$$c_{jt}(\omega) = \left(\sum_{g=1}^{G_{ijt}(\omega)} c_{jgt}(\omega)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.$$

g indexes varieties within product ω ; $c_{jgt}(\omega)$ is the consumption of variety g of product ω in country j .

Aggregate price index P_{jt} associated with the consumption of the composite good C_{jt} is given by

$$P_{jt} = \left(\sum_{i=1}^N \int_{\Omega_{ijt}} e^{a_{jt}(\omega)} \sum_{g=1}^{G_{ijt}(\omega)} p_{jgt}(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}},$$

where $p_{jgt}(\omega)$ is the price of variety g of product ω in country j .

A consumer is endowed with one unit of labor that he inelastically supplies to the market. A consumer receives wage w_{jt} per unit of labor and owns a share of domestic firms. Given prices $p_{jgt}(\omega)$ and income, the level of $c_{jgt}(\omega)$ is chosen to minimize the cost of acquiring C_{jt} yielding demand equation

$$q_{jgt}(\omega) = e^{a_{jt}(\omega)} \frac{p_{jgt}(\omega)^{-\sigma}}{P_{jt}^{1-\sigma}} Y_{jt},$$

where Y_{jt} is the aggregate spending level defined as the sum of total labor income, $w_{jt}L_j$, and profits of domestic firms, Π_{jt} : $Y_{jt} = w_{jt}L_j + \Pi_{jt}$.

3.2 Production

At any point in time t there exists a continuum of firms. Upon entry, a firm is associated with a brand name, product ω (hereafter called brand), that becomes globally unique. Within own brand, a firm can produce differentiated varieties of its product (hereafter called just products).

Firms differ in their ability level φ to produce products within their brand. All else equal, a firm with a greater ability is able to produce each of its products at a lower cost compared to a less able firm.

There are two additional components that influence profitability of firm's products. First, a component that is product specific but the same across destinations and time, a product specific productivity φ_g . Second, a component common across products within a firm but different across destinations and time, a destination specific demand shock a_{jt} .

The destination demand shock is given by the sum of two components: time invariant brand appeal index θ_j and an inter-temporal preference shock ϵ_{jt} . The brand appeal index in a given destination is drawn from a normal distribution with mean $\bar{\theta}_j$ and variance σ_θ^2 . The inter-temporal preference shock is drawn from a normal distribution with zero mean and variance σ_ϵ^2 . The shocks are independently and identically distributed over time.

The brand appeal index is not known to the firm. Before entering a market, a firm does not know how well perceived its product going to be in that market. As the firm continues to supply the market, the appeal index is subject to inter-temporal preference shocks. Thus, a firm never observes own appeal index, but must learn about it upon observing demand shocks.

One of the simplifying assumptions I introduce in my analysis is the independence of the draws of the firm's appeal index across destinations. An alternative formulation that allows for correlated across destinations draws of the appeal index is proposed in Defever, Heidz, and Larchx (2010), Alborno, Pardo, Corcos, and Ornelas (2011), and Nguyen (2011). This modeling approach is essential for the analysis of the sequential market entry patterns by exporters pursued in the aforementioned papers. Since the goal of this paper is to characterize product switching in a given destination conditional on entry, the assumption of correlated draws is not crucial to my analysis.

The level of current demand shock a_{jt} is not known to the firm at the time of making production decisions. Thus, the firm bases its decisions on the belief regarding the appeal index and, subsequently, the demand shock.

Before entry into a market a firm does not have any information regarding the appeal index and thus its prior about the appeal index is given by the original distribution from which θ_j is drawn.

Upon observing n demand shocks that yield mean \bar{a}_j , the posterior distribution regarding the appeal index is given by the normal distribution with mean μ_{nj} and variance v_n^2 (DeGroot, 2004) where

$$\begin{aligned}\mu_{nj} &= \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + n\sigma_\theta^2}\bar{\theta}_j + \frac{n\sigma_\theta^2}{\sigma_\epsilon^2 + n\sigma_\theta^2}\bar{a}_j, \\ v_n^2 &= \frac{\sigma_\epsilon^2\sigma_\theta^2}{\sigma_\epsilon^2 + n\sigma_\theta^2}.\end{aligned}$$

Firms from country i face fixed costs of supplying market j , $F_{ij}(G)$, that reflect advertising and marketing costs, costs associated with complying to regulatory standards, and market research costs. The fixed costs are increasing in the number of products produced G (scope), yet there might be either economies or dis-economies of scope. Bernard, Redding, and Schott (2010) assume the fixed costs to be linear in scope, yet Arkolakis and Muendler (2010) estimate the incremental fixed cost to be increasing in scope. I am going to allow a fixed cost structure that can potentially encompass all possible cases. Let $F_{ij}(G)$ be given by

$$F_{ij}(G) = f_{ij} \sum_{g=1}^G g^\gamma.$$

Since the incremental fixed costs are not constant the decisions to export any given product are not independent across each other (as is true in Bernard, Redding, and Schott (2010) model, for example). As such a firm will order products from the most to the least productive and will continue to add products until variable profit from a marginal product is at least as large as the incremental fixed cost.

It is convenient to relabel products a firm can produce by their efficiencies in relation to each other. Let the subscript g on the product's productivity φ_g refer to the productivity rank of a product within the firm. $g = 1$ refers to a product a firm produces with highest efficiency. As we move to lower ranks the efficiency declines.

Further, The ability of the firm φ influences φ_g to the extent that φ_g is increasing in φ reflecting the fact that more able firms can produce all products with greater efficiency. Thus, productivities φ_g have to satisfy two properties: be increasing in φ , and be decreasing in g . Following Arkolakis and Muendler (2010), I am going to assume that φ_g takes the following functional form $\varphi_g = \frac{\varphi}{g^\alpha}$, where $\alpha > 0$.

All products are subject to the iceberg transportation cost τ_{ij} . τ_{ij} units of a good must be shipped from country i to j in order for one unit to arrive at destination j . For $i \neq j$ $\tau_{ij} > 1$ reflecting the fact that it is costly to ship products abroad. I further assume that in the home country firms do not face additional transportation costs beyond variable and fixed costs of production, $\tau_{ii} = 1$.

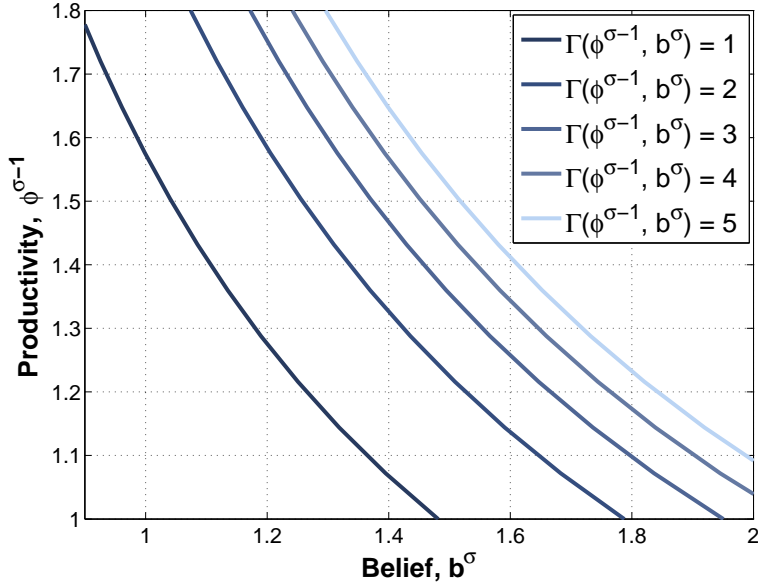
Conditional on entry into a market, firms choose quantity of every good and the number of goods supplied to that market to maximize per period expected profit given posterior distribution about the appeal index. The resulting optimal quantity (scale) of product g supplied to market j from i is given by

$$q_{ijgt}(\varphi, \bar{a}_j, n) = \left(\frac{\sigma - 1}{\sigma} \right)^\sigma \left(\frac{\varphi}{g^\alpha} \right)^\sigma b_j^\sigma \frac{P_{jt}^{\sigma-1} Y_{jt}}{(\tau_{ij} w_{it})^\sigma}, \quad (1)$$

where b_j^σ captures the effect of product appeal learning (demand learning) on optimal quantity. The complete characterization of the maximization problem is described in Appendix A.

b_j is defined as $E_{a_{jt}|\bar{a}_j, n} \left[e^{\frac{a_{jt}}{\sigma}} \right]$. It is the expected value of the exponent of current demand shock (normalized by the elasticity of substitution for convenience). The expectation is taken with respect to the current demand shock a_{jt} given the posterior distribution regarding the

Figure 1: **Scope Indifference Curves**



appeal index, which is completely described by \bar{a}_j and n . I will refer to b_j^σ as the expected demand level.

Optimal scale decision yields expected variable profit from a product that is increasing in firm's ability and expected demand level, and decreasing in product's rank g :

$$\pi_{ijgt}(\varphi, \bar{a}_j, n) = \frac{(\sigma - 1)^{\sigma-1}}{\sigma^\sigma} \left(\frac{\varphi}{g^\alpha} \right)^{\sigma-1} b_j^\sigma \frac{P_{jt}^{\sigma-1} Y_{jt}}{(\tau_{ij} w_{it})^{\sigma-1}}.$$

In determining optimal number of products a firm compares the expected variables profit for a given product to the incremental fixed costs and keeps adding products as long as the profit is greater than or equal to the costs.

Expected variable profit from a given product declines in product's rank, while ex-ante I do not impose any assumptions on the behavior of the incremental fixed costs. To ensure a well defined solution to the scope problem the incremental fixed cost must decline in product's rank no faster than the variables profit yielding the following constrain on the parameters of the cost function: $\gamma > -\alpha(\sigma - 1)$.

The optimal scope decision is then characterized by a set of scope indifference curves $\Gamma(\varphi^{\sigma-1}, b_j^\sigma) = g$ for $g \geq 1$ implicitly determined by equation (2) below and schematically depicted in Figure 1.

$$\varphi^{\sigma-1} b_j^\sigma = \frac{\sigma^\sigma}{(\sigma - 1)^{\sigma-1}} f_{ij} g^{\gamma + \alpha(\sigma-1)} \frac{(\tau_{ij} w_{it})^{\sigma-1}}{P_{jt}^{\sigma-1} Y_{jt}}. \quad (2)$$

A firm with $\varphi^{\sigma-1}, b_j^\sigma$ such that $\Gamma(\varphi^{\sigma-1}, b_j^\sigma) = g$, makes zero net profit on the marginal product g , and thus sells g products. A firm with $\varphi^{\sigma-1}, b_j^\sigma$ such that $g < \Gamma(\varphi^{\sigma-1}, b_j^\sigma) < g + 1$ makes positive net profit on the g th product, and considers adding another $g + 1$ st product.

Yet $g + 1$ st product cannot generate enough variable profit to cover the incremental fixed cost. Firms in such range, thus, produce g products.

Notice that firm's ability φ stays constant through firm's life cycle, while expected demand $b_j^\sigma(\bar{a}_j, n)$ adjusts as the firm observes new demand shocks.

The range of values $\varphi^{\sigma-1}$ and b_j^σ such that $\Gamma(\varphi^{\sigma-1}, b_j^\sigma) < 1$ is of special interest. Firms in the range of values below the scope indifference curve $\Gamma(\varphi^{\sigma-1}, b_j^\sigma) = 1$ make negative net profit on the first product they can potentially produce. In the static environment such firms would immediately exit the market. In the dynamic learning environment considered in this paper, it would be optimal for some of these firms to incur one period losses in exchange for observing current demand shock which has some information value. The following section describes the value of the firm and firm's optimal exit and entry decisions.

3.3 Firm Exit and Entry

Denote by $V_{ijt}(\varphi, \bar{a}_j, n)$ the continuation value of firm in country i of being an exporter to market j .

An incumbent firm enters period t with the knowledge of its own ability φ , mean of the previously observed demand shocks \bar{a}_j , and the number of the observed shocks n . The firm must decide whether to participate in a given market in the current period or exit. If the firm decides to participate in market j it receives expected profit $\Pi_{ijt}(\varphi, \bar{a}_j, n)$ and expected discounted continuation value of begin an exporter $\beta(1 - \delta)E_{\bar{a}'_j|\bar{a}_j, n}V_{ijt}(\varphi, \bar{a}'_j, n + 1)$, where δ is an exogenous probability of a shock that forces a firm to exit, and β is the rate at which a firm discounts future profits. If the firm decides to exit it receives the option value of non-exporting.

In the context of this model the option value of non-exporting is zero. A firm must export at least one product in a given period in order to observe current demand shock and be able to update posterior regarding the appeal index. Once the firm exits market j , it will no longer receive any new information regarding its appeal index in market j . Thus, if it is optimal for a firm to exit in a given period, it will be optimal to stay out of the market for all subsequent periods, yielding the value of exit to be zero.

The optimal market participation decision for incumbent firms is thus a policy function associated with the following Bellman Equation

$$V_{ijt}(\varphi, \bar{a}_j, n) = \max\{\Pi_{ijt}(\varphi, \bar{a}_j, n) + \beta(1 - \delta)E_{\bar{a}'_j|\bar{a}_j, n}V_{ijt}(\varphi, \bar{a}'_j, n + 1), 0\}. \quad (3)$$

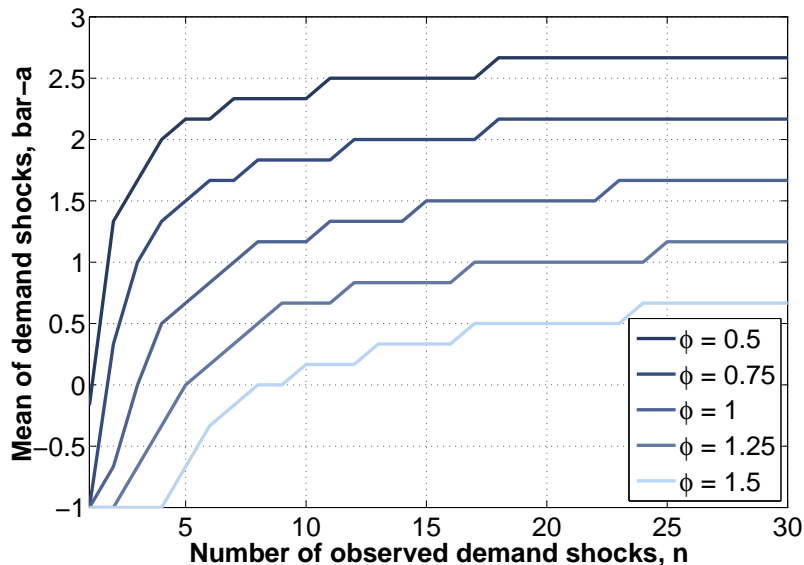
Proposition 1 *The optimal market participation decision is given by a set of market participation thresholds $\bar{a}_{ij}^*(\varphi, n)$ such that a firm from destination i continues to export to destination j when $\bar{a}_j \geq \bar{a}_{ij}^*(\varphi, n)$ and exits export market when $\bar{a}_j < \bar{a}_{ij}^*(\varphi, n)$. Policy function $\bar{a}_{ij}^*(\varphi, n)$ satisfies the following properties: (a) $\frac{\partial \bar{a}_{ij}^*(\varphi, n)}{\partial \varphi} < 0$, (b) $\bar{a}_{ij}^*(\varphi, n + 1) > \bar{a}_{ij}^*(\varphi, n)$, and (c) $\lim_{n \rightarrow +\infty} \bar{a}_{ij}^*(\varphi, n) = \bar{a}_{ij}^*(\varphi)$.*

Proof. See Appendix B.

■

The behavior of the market participation thresholds, drawn in Figure 2, reflects the role of information and learning in the economy.

Figure 2: Market Participation Thresholds



For a given ability level φ the short run threshold, $\bar{a}_{ij}^*(\varphi, n)$ when n is small, approaches the long run value, $\bar{a}_{ij}^*(\varphi, \infty)$ when n approaches infinity, from below thereby giving the firms time to learn about their true appeal index. I explain the exact meaning of this notion in the discussion below.

Consider market participation decision of an exporter in the limit when $n \rightarrow \infty$ (in the long run). The behavior in the long run can be view as the behavior of firms should they have had perfect information about own appeal index. As n goes to $+\infty$, the mean of the observed demand signals converges to the true appeal index θ_j :

$$\lim_{n \rightarrow +\infty} \frac{\sum_{k=1}^n a_{j(t+k)}}{n} = \lim_{n \rightarrow +\infty} \frac{\sum_{k=1}^n \theta_j + \epsilon_{j(t+k)}}{n} = \theta_j + \lim_{n \rightarrow +\infty} \frac{\sum_{k=1}^n \epsilon_{j(t+k)}}{n} = \theta_j + E(\epsilon_{jt}) = \theta_j \quad (4)$$

Thus, in the long run the mean of the demand signal can be viewed as the true appeal index of the firm, and the threshold level $\bar{a}_{ij}^*(\varphi)$ can be viewed as the exit threshold for the appeal index. Thus, had the firm had perfect information regarding the appeal index, they would have exited the export market immediately if $\theta_j < \bar{a}_{ij}^*(\varphi)$.

In the short run the estimate of the appeal index, the mean of the observed demand shocks, might be above or below the true value of the appeal index depending on random preference shocks. If the short run value of the market participation threshold stayed at the long run level, a slight negative preference shock would force many firms out of the market immediately, while from the profitability perspective it would have been optimal for those firms to export. To accommodate the effect of occasional negative preference shocks on the estimate of the appeal index, the short run value of the market exit threshold for a firm with productivity φ must be below it long run equivalent. As a firm receives more and more demand signals, its estimate of own appeal index becomes more accurate. Thus, a firm is less sensitive to occasional negative preference shock. As a result the threshold level gradually increases to the long run value.

Having described the continuation value of being an exporter and optimal market participation by incumbents, I turn to the description of the entry of new firms.

Following Chaney (2008), I assume that in every country j there exists an exogenous mass of prospective entrants J . Upon entry every firm draws its ability level φ from a Pareto distribution with the scale parameter φ_{min} and the shape parameter ξ .⁶ After observing its ability level a firm in market i decides in which market j to participate by comparing the expected value of entry $V_{ij}^E(\varphi)$ to the cost of entry. Since I do not assume the presence of the sunk market entry cost, the entry cost is zero. Furthermore, since there is no sunk entry cost, the decisions to enter destinations are independent of each other.

A firm enters a particular market whenever the expected value of entry is greater than zero. The expected value of entry into a market is given by the sum of the initial expected profit in that market and the expected continuation value of being an exporter

$$V_{ij}^E(\varphi) = \Pi_{ijt}(\varphi, 0, 0) + \beta(1 - \delta)E_{\bar{a}'_j|0,0}V_{ijt}(\varphi, \bar{a}'_j, 1).$$

Since prior to entry a firm has not observed any demand shocks the value of the mean demand shocks is normalized to zero, and the number of the observed demand shocks is zero. The expectation of firm's initial profits is computed based on the firm's prior regarding the demand parameter given by the distribution from which the appeal index is drawn. Notice that since $\Pi_{ijt}(\varphi, 0, 0)$ is increasing in φ , so is the expected value of entry. Thus productivity entry threshold φ_{ij}^* is determined by equating the expected value of entry to zero:

$$V_{ij}^E(\varphi_{ij}^*) = 0.$$

A firm with ability level above φ_{ij}^* will enter destination j and begin serving that market. A firm with ability level below φ_{ij}^* for all destinations j will exit the economy forever.

3.4 Determination of Equilibrium

I am going to solve for the stationary general equilibrium in case of the symmetric countries. Thus, I assume that all countries are of equal size $L_j = L$; they have the same number of entrants $J_j = J$; they are separated by equal trade cost $\tau_{ij} = \tau, > 1$ for $i \neq j$ and $\tau_{ii} = 1$. Firms in every country face the same fixed cost structure $f_{ij} = f$. The fundamental distribution of the appeal index is also the same across countries, $\hat{\theta}_j = \hat{\theta}$. Wage w is normalized to 1.

For a given set of parameters of the model, a symmetric stationary equilibrium is given by ability entry thresholds into domestic and export markets, φ_d^* and φ_x^* , mean demand shock exit thresholds from the domestic and export market, $\bar{a}_d^*(\varphi, n)$ and $\bar{a}_x^*(\varphi, n)$, scope indifference curves in the domestic and export market, $\Gamma_d(\varphi, b^\sigma)$ and $\Gamma_x(\varphi, b^\sigma)$, aggregate price level and expenditure level P and Y , masses of domestic and exporting firms, M_d

⁶For simplicity I assume that parameters of Pareto distribution are the same across countries. An alternative approach is taken by Eaton, Kortum, and Kramarz (2011), where differences in scale parameters across countries reflect technological advancements of some countries over the others. While such assumption is more realistic, it is not crucial for the analysis pursued in this paper.

and M_x , and a measure function in the domestic and export market, $m_d(\varphi, \bar{a}, n, a)$ and $m_x(\varphi, \bar{a}, n, a)$. In what follows I describe how all equilibrium values are determined.

For a given value of P and Y all equilibrium thresholds and scope indifference curves are given by the solution to the firm's maximization problem. Given equilibrium thresholds and firms' beliefs about the distribution of demand shock, Proposition 2 below describes the equilibrium measure function of firms, $m_i(\varphi, \bar{a}, n, a)$ for $i \in \{d, x\}$.

Proposition 2 *Equilibrium measure function of firms $m_i(\varphi, \bar{a}, n, a)$, $i \in \{d, x\}$, is given by*

$$m_i(\varphi, \bar{a}, n, a) = \frac{1}{\sqrt{v_n^2 + \sigma_\epsilon^2}} \phi \left(\frac{a - \mu_n}{\sqrt{v_n^2 + \sigma_\epsilon^2}} \right) m_i(\varphi, \bar{a}, n),$$

where $m_i(\varphi, \bar{a}, n)$ is such that

$$m_i(\varphi, 0, 0) = J \left(\frac{\varphi_{min}}{\varphi_i^*} \right)^\xi \frac{\xi(\varphi_i^*)^\xi}{\varphi_i^{\xi+1}},$$

and

$$m_i(\varphi, \bar{a}, n+1) = \int_{\bar{a}_i^*(\varphi, n)}^{+\infty} m_i(\varphi, \bar{a}', n) \frac{1}{\sqrt{v_n^2 + \sigma_\epsilon^2}} \phi \left(\frac{\bar{a}(n+1) - \bar{a}'n - \mu_n}{\sqrt{v_n^2 + \sigma_\epsilon^2}} \right) d\bar{a}'.$$

Proof. See Appendix E. ■

Using the equilibrium measure function, the equilibrium mass of firms can be written as (see Appendix C)

$$M_i(\varphi_i^*) = J \left(\frac{\varphi_{min}}{\varphi_i^*} \right)^\xi \int_{\varphi_i^*}^{+\infty} \frac{\xi(\varphi_i^*)^\xi}{\varphi_i^{\xi+1}} H_i(\varphi) d\varphi, \quad (5)$$

where

$$H_i(\varphi) = \sum_{n=0}^{+\infty} \int_{\bar{a}_i^*(\varphi, n)}^{+\infty} \dots \int_{\bar{a}_i^*(\varphi, 1)}^{+\infty} p(\bar{a}|\bar{a}_{n-1})p(\bar{a}_{n-1}|\bar{a}_{n-2})p(\bar{a}_1)d\bar{a}_1\dots d\bar{a}_{n-1}d\bar{a}$$

with $p(\bar{a}_k|\bar{a}_{k-1})$ being transitional densities of the form $p(\bar{a}_k|\bar{a}_{k-1}) = \frac{1}{\sqrt{v_{k-1}^2 + \sigma_\epsilon^2}} \phi \left(\frac{\bar{a}_k k - \bar{a}_{k-1}(k-1) - \mu_{k-1}}{\sqrt{v_{k-1}^2 + \sigma_\epsilon^2}} \right)$.

Since normal densities take positive values, $H_i(\varphi)$ is positive for all φ .

From equation (5) observe that an increase in φ_i^* increases the lower bound of integration. Since integrand is a positive function, the value of the integral declines. Thus, $M_i(\varphi_i^*)$ declines in φ_i^* .

Define by M the total mass of varieties available in an economy. M is given by

$$M(\varphi_d^*, \varphi_x^*) = M_d(\varphi_d^*) + NM_x(\varphi_x^*). \quad (6)$$

In solving for equilibrium values of P , Y , M_d , and M_x I will use results in Proposition 3 below.

Proposition 3 *There exist $u_d^* = \varphi_d^* PY^{\frac{1}{\sigma-1}}$ and $u_x^* = \frac{\varphi_x^* PY^{\frac{1}{\sigma-1}}}{\tau}$ such that*

- (a) $u_d^* = u_x^* = u^*$;
- (b) u^* is completely determined by the exogenous parameters of the model;
- (c) Equilibrium mean profit level of firms is equal across destinations and is completely determined by u^* ;
- (d) Equilibrium mean revenue level of firms is equal across destinations and is completely determined by u^* .

Proof. See Appendix F. ■

Proposition 3 states that equilibrium values of P , Y , φ_d^* , and φ_x^* can be normalized in such a way as to yield a single variable u^* . This variable is completely determined by exogenous parameters of the model, and is useful in characterizing mean profit and revenue level of firms in a destination as a function of exogenous parameters only.

Notice that part (a) of Proposition 3 implies $\varphi_x^* = \tau\varphi_d^*$. Thus, equilibrium mass of varieties in equation (6) can be written as a function of a single threshold φ_d^* :

$$M(\varphi_d^*) = M_d(\varphi_d^*) + NM_x(\tau\varphi_d^*). \quad (7)$$

Using results from parts (c) and (d) of Proposition 3, and rational expectations of firms about the aggregate price level, the goods market clearing condition can be written as (see Appendix D)

$$L + M\tilde{\pi} = M\tilde{r}, \quad (8)$$

where $\tilde{\pi}$ is the mean profit level across firms in a given destination, $\tilde{\pi} = E_{m_d}(\pi_d(\varphi, \bar{a}, n, a)) = E_{m_x}(\pi_x(\varphi, \bar{a}, n, a))$. \tilde{r} is the mean revenue level across firms in a given destination, $\tilde{r} = E_{m_d}(r_d(\varphi, \bar{a}, n, a)) = E_{m_x}(r_x(\varphi, \bar{a}, n, a))$.

Equation (8) determines equilibrium mass of varieties as a function of exogenous parameters only

$$M = \frac{L}{\tilde{r} - \tilde{\pi}}.$$

The equilibrium threshold φ_d^* is then a solution to $M = M_d(\varphi_d^*) + NM_x(\tau\varphi_d^*)$.

Given φ_d^* , $\varphi_x^* = \tau\varphi_d^*$, the mass domestic and exporting firms is determined by equation (5), $Y = L + M\tilde{\pi}$; $P = \frac{u^*}{\varphi_d^* Y^{\frac{1}{\sigma-1}}}$. This completes characterization of equilibrium.

3.5 Properties of the Solution

I will demonstrate properties of the optimal solution for the stationary symmetric equilibrium described in the previous section.

In demonstrating how learning affects optimal scale and scope decision of the firms, it is instructive to compute how scale will respond to an additional demand shock signal.

Proposition 4 *In the steady state there exists a threshold level of mean demand shock \hat{a} given by $\bar{\theta} + \frac{\sigma_{\hat{a}}^2}{2\sigma}$, such that scale adjustment in response to an additional demand shock, defined as $\frac{q_g(\varphi, \bar{a}_j, n+1)}{q_g(\varphi, \bar{a}_j, n)}$, is greater than 1 if $\bar{a} > \hat{a}$, $\frac{q_g(\varphi, \bar{a}, n+1)}{q_g(\varphi, \bar{a}, n)} < 1$ if $\bar{a} < \hat{a}$, and $\frac{q_g(\varphi, \bar{a}, n+1)}{q_g(\varphi, \bar{a}, n)} = 1$ if $\bar{a} = \hat{a}$.*

Proof. See Appendix G. ■

Proposition 4 states that firms with sufficiently high realized value of the mean demand shock are prompted to expand the scale of products in response to an additional demand shock, while firms with sufficiently low realized value of the mean demand shocks are prompted to contract the scale of products in response to an additional shock. The intuition for the obtained result is as follows.

Upon entry, firm's expectation about appeal index are given by the mean of the distribution from which the parameter is drawn, $\bar{\theta}$. After some demand socks have been observed, a firm learns of whether its true appeal index is higher or lower compared to the baseline value, $\bar{\theta}$. Suppose the realized mean of the demand shock \bar{a} is higher than the originally thought value ($\bar{a} > \hat{a}$ to be exact, as stated in Proposition 4). Conditional on ability and \bar{a} , an additional demand shock reassures the firm that its appeal index is higher than originally thought (through the channel of reducing the variance of the posterior belief v_n^2 , which declines in n). As a result, the firm expands the scale of produced products. On the contrary, if the realized mean of the demand shock \bar{a} is lower than the originally thought value, the firm adjusts to the new information by contracting the scale of every product.

Results of Proposition 4 are helpful in understanding the mechanism by which demand learning induces firms to add and drop products. As the scale of a given product expands or contracts in response to an additional demand shock, so does the variable profit generated by that product. Since product incremental fixed cost remains unchanged, changes in product's profitability translate into changes in the range of products sold by a firm.

Consider a firm with $\varphi^{\sigma-1}$ and b^σ such that $g < \Gamma(\varphi^{\sigma-1}, b^\sigma) < g + 1$. As discussed before, such firm will produce g products and will generate positive net profit from g th product. The firm does not yet sell $g + 1$ st product since the variable profit is not sufficient to cover the incremental fixed costs. As the firm receives an additional demand signal, the expected demand adjusts appropriately. On the one hand, for the firms with $\bar{a} > \bar{\theta} + \frac{\sigma_{\bar{\theta}}^2}{2\sigma}$ expected demand increases in n conditional on \bar{a} . [find appropriate place to discuss this] Thus, an additional demand signal will increase the profitability of the marginal $g + 1$ st product without affecting its marginal variable cost. For a sufficiently large increase in b^σ the firm will add an additional product. On the other hand, for the firms with $\bar{a} < \bar{\theta} + \frac{\sigma_{\bar{\theta}}^2}{2\sigma}$ expected demand declines in n conditional on \bar{a} . Thus, an additional demand signal will reduce the profitability of the marginal product g without affecting its marginal variable cost. For a sufficiently large decline in b^σ the firm drop its marginal product g .

Proposition 4 describes how the process of learning affects firm's decision to adjust scale of produced products: As firms learn, they either expand or contract the scale of their products. Proposition 5 below sates that, conditional on size, scale adjustment decision is age dependent, where in the context of this paper age corresponds to the number of the observed demand shocks in a given market.

Proposition 5 *In the steady state there exist a threshold level of expected demand \hat{b}^σ given by $\bar{\theta} + \frac{\sigma_{\bar{\theta}}^2 + \sigma_\epsilon^2}{2\sigma}$, such that conditional on b^σ , $\frac{q_g(\varphi, \bar{a}_j(b^\sigma), n+1)}{q_g(\varphi, \bar{a}(b^\sigma), n)}$ declines in n for $b^\sigma > \hat{b}^\sigma$, increases in n for $b^\sigma < \hat{b}^\sigma$, and stays constant for $b^\sigma = \hat{b}^\sigma$.*

Proof. See Appendix H. ■

In analyzing the age dependence of the scale adjustment decision we are interested in controlling for the initial scale of the firm. Notice from equation (1), that the optimal scale is completely determined by firms ability φ and expected demand b^σ . Firms with the same expected demand, however, vary in their mean demand shock level and age (the number of observed demand signals). Proposition 5 states that scale adjustment varies with age, conditional on the same initial expected demand value. Younger firms with high expected demand expand at higher rate compared to older firms with the same initial expected demand. Younger firms with low expected demand contract at smaller rate compared to older firms with the same initial expected demand.

The expected demand threshold $\hat{b}^\sigma = \bar{\theta} + \frac{\sigma_{\hat{\theta}}^2 + \sigma_{\epsilon}^2}{2\sigma}$ defined in Proposition 5 has an intimate relation to the mean demand shock threshold $\hat{a} = \bar{\theta} + \frac{\sigma_{\hat{\theta}}^2}{2\sigma}$ defined in Proposition 4: When $\bar{a} = \hat{a}$, $b^\sigma = \hat{b}^\sigma$. Thus, firms with expected demand above the threshold demand are exactly those firms with the realized mean demand shock above the base line value. Such firm can be thought of as the firms which have underestimated their true appeal index. As soon as they discover that they are more appealing to consumers than originally believed, they rapidly expand. An older firm with the same expected demand is more convinced of its appeal index, and thus does not adjust the scale by as much as a younger firm who just discovered its high appeal index and is eager to expand.

Slightly opposite story holds for the firms that overestimated their demand level: firms with the realized expected demand $b^\sigma < \hat{b}^\sigma$ or the realized mean of demand shocks $\bar{a} < \hat{a}$. As they discover mean demand shock below the baseline value, they certainly respond by contracting. Older firms, however, contract by more since they are more confident, after receiving a greater amount of negative demand shocks, that they are of the low appeal to consumers. Younger firms are not as strongly convinced that they are not liked by consumers as much as they originally believed, and thus gradually adjust product scale downward as they discover with greater confidence they the are of the low appeal type.

The age dependence of scale decision translates into the age dependence of product adding and dropping decision. Conditional on firm's ability and expected demand level, younger firms adjust scale by a greater amount compared to older firms. As a result, all products of younger firms will face a larger increase in product's profitability compared to older firms. Thus, younger firms will be prompted to add more products compared to older firms. Since initial scope of the firms was the same, younger firms will exhibit greater fraction of added products withing the range of exported products.

The model yields predictions regarding the response of firm's intensive and extensive margins to a change in trade costs. Proposition 6 below establishes the effect of a change in trade costs on individual product scale and sales.

Proposition 6 *In a symmetric stationary equilibrium $\frac{dq_{gx}}{d\tau} < 0$ and $\frac{dr_{gx}}{d\tau} < 0$, $\frac{dr_{gd}}{d\tau} > 0$, $\frac{dq_{gd}}{d\tau} > 0$ if $\frac{N}{\tau\xi+N} > \frac{1}{\sigma-1}$ and $\frac{dq_{gd}}{d\tau} < 0$ otherwise.*

Proof. See Appendix I ■

Proposition 6 describes how the withing firm intensive margin adjusts in response to changes in trade costs. First, in response to a decline in trade costs firm adjust the scale at with they produce their products: the quantity of exported goods unambiguously increases,

while the response of the quantity of domestically supplied goods depends on the number of trading partner, original trade costs, and elasticity of substitution. Second, the changes to the scale of product sold unambiguously translate into change to per product revenue. The sales from exported goods increase, while the sales from domestically supplied goods unambiguously decline in response to a decline in trade costs.

Consider the case of an exporter facing a decline in trade cost. Since sales (and as a result variable profit) of every potential product increase without affecting the marginal fixed market entry cost, it might become optimal for a firm to expand the range of produced products. Similarly, a firm selling to the domestic market will experience a decline in variable profit and might find it optimal to reduce the number of products supplied.

There are two potential channels that affect the scope of supplied products (extensive margin) due to a change in trade costs. First, as discussed above, an increase or decrease in product's profitability might induce a marginal product to surpass the scope indifference curve threshold resulting in product adding or dropping. Second, the scope thresholds are themselves affected by a change in trade costs. For an export market the scope indifference curves move toward the origin in response to a decline in trade costs, inducing firms to add products. For a domestic market the scope indifference curves move away from the origin in response to a decline in trade costs, inducing firms to drop products (see Appendix J).

The relative importance of the intensive versus extensive margins of firm adjustment in predicting changes in trade flows are evaluated numerically in the following section. I calibrate the symmetric two country stationary equilibrium to match Brazilian world exports. I use the calibrated model to evaluate the model's ability to predict margins of trade flows, and I perform a counterfactual simulation of a decline in trade costs to study the relative importance of the margins as predicted by the model.

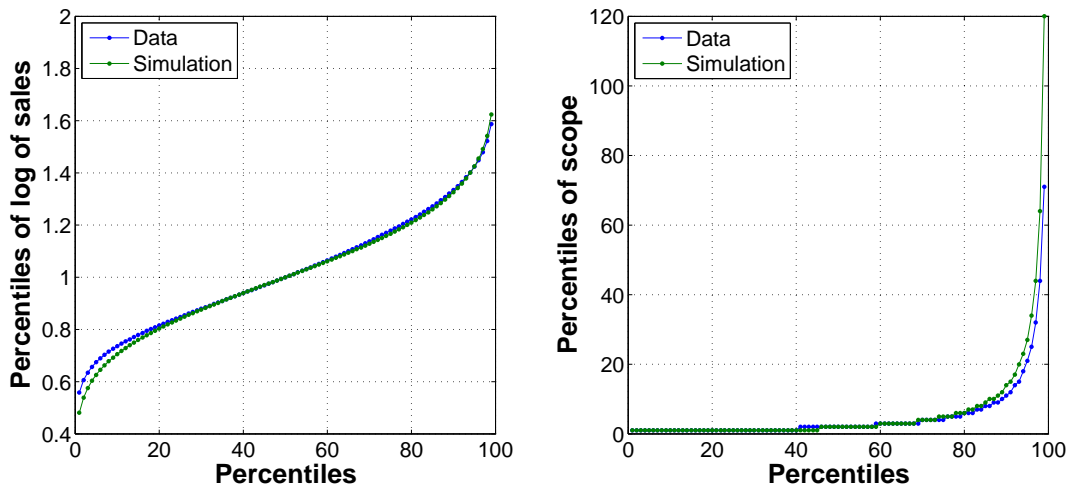
4 Quantitative Analysis

I perform quantitative analysis of the model to answer the following two questions. First, can a model with demand learning, parametrized to match cross section distribution of sales and scope, account for the extensive and intensive margins of aggregate export flows? Second, what is the relative importance of extensive versus intensive margins in generating export sales growth due to trade liberalization?

I use the indirect inference method of Gourieroux and Monfort (1996) to estimate parameters of the model that govern the cost structure and the distribution of shocks: f , α , γ , σ_θ^2 , and σ_ϵ^2 . The method involves solving firm's maximization problem for each parameter guess. Based on the optimal solution to the firm's problem, firm level data is simulated. From the simulated data a number of moments is computed and compared to the identical moments from the data. The sum of squares of the percent deviation of the simulated moments from the data moments is used as the criterion function. Parameters are then chosen to minimize the criterion.

The following five moments are chosen from the data, and help identify the five parameters mentioned above: the mean and standard deviation of the logarithm of firm's total export sales, the relative log-sales size of newcomers to incumbents, the fraction of multi-product exporters, and the fraction of product switching exporters. The moments values are

Figure 3: Simulated and Actual Distributions



reported in the first column of Table 9.

To solve the firm’s maximization problem I calibrate three parameters of the model: β , σ , and δ . Firm’s discount factor is given by β and is set to 0.9606, which is consistent with the period length of a year assumed in the simulations. The elasticity of substitution across varieties is given by σ and is set to 7.49. This value is reported in Broda and Weinstein (2006) as the mean elasticity of substitution across 5 digit SITC product categories. The exogenous death rate of firms is given by δ and is set to 0.03. 3 percent corresponds to the annual exit rate among Brazilian exporters in the top 5 percent of export sales distribution. Further, the following parameters are normalized to the corresponding values: $\bar{\theta} = 0$, $\xi = 7.4$, $\varphi_{min} = 1$.

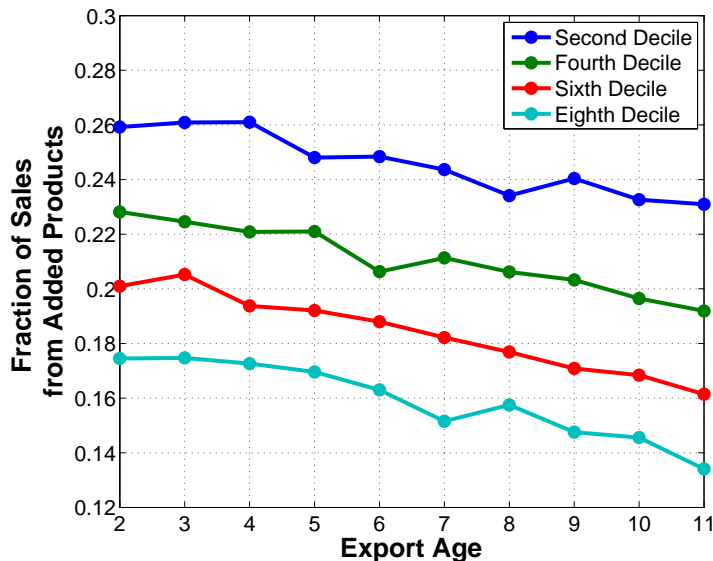
4.1 Estimation Results

The estimation yields parameter values reported in Table 10. The simulated data moments are reported in the second column of Table 9. The model slightly under predicts the fraction of product switching exporters due to a trade off between firm size and frequency of product switching. Lower value of $\alpha(\sigma - 1) + \gamma$ moves scope indifference curves closer together, as can be inferred by looking at the indifference curve equation (2). Thus, for a given dynamics in demand beliefs, a firm would be more likely to add and drop products the lower is the coefficient $\alpha(\sigma - 1) + \gamma$. At the same time lower value of $\alpha(\sigma - 1) + \gamma$ moves indifference closer to the origin. Thus, for a given ability level and demand belief, a firm will export greater amount of products that would yield higher sales.

Parametrized as described above, the model matches well the cross section distribution of scope and logarithm of sales as depicted in Figure 3. The thicker tail in the scope distribution is a result of size and product switching trade off described above. The model slightly over predicts the proportion of large firms in the economy at the benefit of matching the frequency of product switching behavior better.

Parametrized to match cross section moments of sales and scope distribution, the model accurately predicts the role of margins in explaining trade flows, and the role of export age

Figure 4: Age Dependence of Products Switching: Simulations



in explaining product switching behavior. The model predicts equal importance of exporter turnover and product switching margin in accounting for total Brazilian exports as described in Table 11. The first column replicates the first column of Table 1, while the second column reports the results from the simulated data. Although the model slightly under predicts the magnitudes of both of the extensive margins, the model does well in predicting equal importance of both extensive margins.

The model predicts age dependence of product switching behavior as depicted in Figure 4. Each curve in the Figure plots the relationship between fraction of sales from added products and firm’s export age for the firms belonging to a given export sales decile. For example, a firm with export sale in the sixth decile will derive 20 percent of sales from new products in the second year of being an exporter, while the same firm will derive only 16 percent of sales from new products when it has been exporting for 11 consecutive years.

4.2 Counter Factual Analysis

In this section I use parametrized model to perform a counter factual analysis of trade liberalization reform that yields 4.8 percent growth in export sales. Counter factual simulations imply a substantial adjustment on the product switching margin. 15 percent of the growth in total exports is attributed to the firms’ expansion of the range of exported products, while only 6 percent can be attributed to exporter turnover margin. (See Table 12).

The relative importance of the product switching margin in relation to the exporter turnover margin predicted by the simulations is comparable to the magnitudes found in the literature. For example, in the study of Indian trade liberalization in the 1990’s, Goldberg, Khandelwal, Pavcnik, and Topalova (2010a) find that product switching margin accounted for nearly 25 percent of total Indian manufacturing output growth in the 1990’s.

5 Conclusion

This paper identifies new patterns in the product switching behavior of exporters and develops a model of demand learning to explain them, both qualitatively and quantitatively. Specifically, I incorporate a model of demand learning into a general equilibrium setup and perform a quantitative assessment of the learning model in terms of its ability to predict the data.

Using customs record data on the universe of Brazilian exporters, I begin by documenting the importance of product switching over exporter turnover margin in export sales. I contribute to the literature by documenting this pattern in the context of export data as opposed to domestic or import data. I then document a novel pattern in the product switching behavior of exporters: conditional on size, younger exporters more eager to engage in product switching than exporters with more experience. The ‘eagerness’ is defined in terms of two statistics: the fraction of sales from added products, and the fraction of added products in the current product mix. Conditional on size, both variables decline with the age of exporting.

The demand learning model provides a natural explanation for the age dependence of product switching behavior. Products of individual firms are characterized by their appeal index, which is not known to the firms *ex ante* and is subject to inter-temporal preference shocks. Upon observing the sum of their appeal index and preference shocks, the demand shocks, firms learn their true appeal index. Variations in inter-temporal preference shocks translate into variations in the observed mean of the demand shocks, which further translate into product adding and dropping. The convergence of the mean of the observed demand shocks to the true appeal index generates age dependence of product switching.

Finally, the paper shows that the learning model, calibrated to match cross-sectional statistics for the sales and scope distributions of exporters, quantitatively predicts the product switching and exporter turnover margins in export trade flows, as well as the age dependence of product switching conditional on size. These findings contribute to a fast growing literature that focuses on demand learning to study various aspects of firm growth and survival.

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Appendices

A Firm's Maximization Problem

Profit maximization problem of a firm based in country i with ability φ , mean of observed demand shocks \bar{a}_j , and the number of observed shocks n , in destination j takes the following form

$$\pi_{ijt}(\varphi, \bar{a}_j, n) \equiv \max_{q_{ijgt}, G_{ijt} \geq 1} E_{a_{jt}|\bar{a}_j, n} \left[\sum_{g=1}^{G_{ijt}} \left(q_{ijgt}^{\frac{\sigma-1}{\sigma}} e^{\frac{a_{jt}}{\sigma}} P_{jt}^{\frac{\sigma-1}{\sigma}} Y_{jt}^{\frac{1}{\sigma}} - \frac{\tau w_{it} g^\alpha}{\varphi} q_{ijgt} \right) - f_{ij} \sum_{g=1}^{G_{ijt}} g^\gamma \right].$$

Note that the expectation operator in the objective function refers to the random variable $e^{\frac{a_{jt}}{\sigma}}$. Since in relation to $e^{\frac{a_{jt}}{\sigma}}$ other variables are treated as non-random, and as such the objective is linear in $e^{\frac{a_{jt}}{\sigma}}$, we can apply expectation operator directly to the variable. Denote by $b_j = E_{a_{jt}|\bar{a}_j, n} \left[e^{\frac{a_{jt}}{\sigma}} \right]$

The maximization can be split into a two step procedure. First, given the number of products G_{ijt} , solve for the optimal quantity of a given product, q_{ijgt} . Second, given optimal quantity, solve for the optimal number of products.

Taking first order conditions of the objective function with respect to q_{ijgt} , keeping G_{ijt} fixed, yields optimal quantity decision

$$q_{ijgt}(\varphi, \bar{a}_j, n) = \left(\frac{\sigma-1}{\sigma} \right)^\sigma \left(\frac{\varphi}{g^\alpha} \right)^\sigma b_j^\sigma \frac{P_{jt}^{\sigma-1} Y_{jt}}{(\tau_{ij} w_{it})^\sigma}.$$

Substitute optimal quantity decision back into the objective function to obtain

$$\pi_{ijt}(\varphi, \bar{a}_j, n) = \max_{G_{ijt} \geq 1} \sum_{g=1}^{G_{ijt}} \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} \left(\frac{\varphi}{g^\alpha} \right)^{\sigma-1} b_j^\sigma \frac{P_{jt}^{\sigma-1} Y_{jt}}{(\tau_{ij} w_{it})^{\sigma-1}} - f_{ij} \sum_{g=1}^{G_{ijt}} g^\gamma.$$

Compare variable profit from a given product to the incremental fixed cost. For a firm to profitably produce a product, it must be that the variable profit is greater than or equal to the marginal fixed cost of supplying that product to a market

$$\frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} \left(\frac{\varphi}{g^\alpha} \right)^{\sigma-1} b_j^\sigma \frac{P_{jt}^{\sigma-1} Y_{jt}}{(\tau_{ij} w_{it})^{\sigma-1}} > f_{ij} g^\gamma. \quad (9)$$

Rearrange equation (9) is the following way

$$\varphi^{\sigma-1} b_j^\sigma > \frac{\sigma^\sigma}{(\sigma-1)^{\sigma-1}} f_{ij} g^{\gamma+\alpha(\sigma-1)} \frac{(\tau_{ij} w_{it})^{\sigma-1}}{P_{jt}^{\sigma-1} Y_{jt}}. \quad (10)$$

At the point of making a scope decision the left hand side of inequality (10) is constant and positive. The right hand side uniformly increases from zero to $+\infty$ if $\gamma + \alpha(\sigma-1) > 0$, or declines from $+\infty$ to zero if $\gamma + \alpha(\sigma-1) < 0$. In the latter case the maximization problem

does not have a well defined and economically meaningful solution for all possible firm's ability levels. For example, for a firm with φ high enough such that for $g = 1$ inequality (10) is satisfied, it would be optimal to produce infinite amount of products. Thus, for the maximization problem to have a well defined solution I assume $\gamma + \alpha(\sigma - 1) > 0$. In looking at inequality (9) the condition implies that the incremental fixed cost must decline slower than the variable profit, or must increase in rank g .

With $\gamma + \alpha(\sigma - 1) > 0$ the right hand side of inequality (10) increases from zero to $+\infty$, and a firm will keep adding products as long as inequality (10) is satisfied.

Notice that when $\varphi^{\sigma-1}b_j^\sigma$ are such that inequality (10) holds with equality for some g , a firm will make zero net profit on the last (marginal) product g . For a firm with $\varphi^{\sigma-1}b_j^\sigma$ slightly below such level the inequality will not hold and the production of g will not be profitable. Thus the firm will produce $g - 1$ products. On the contrary, for a firm with $\varphi^{\sigma-1}b_j^\sigma$ slightly above such level the inequality will be satisfied implying that production of g th product will bring positive net profit. The firm will produce g products until $\varphi^{\sigma-1}b_j^\sigma$ increases sufficiently enough as to make zero net profit on the next $g + 1$ st product. Thus,

$$\varphi^{\sigma-1}b_j^\sigma = \frac{\sigma^\sigma}{(\sigma - 1)^{\sigma-1}} f_{ij} g^{\gamma+\alpha(\sigma-1)} \frac{(\tau_{ij} w_{it})^{\sigma-1}}{P_{jt}^{\sigma-1} Y_{jt}}.$$

implicitly defines scope indifference curves in the $(\varphi^{\sigma-1}, b_j^\sigma)$ space for each scope level g . Denote this indifference curves by $\Gamma(\varphi^{\sigma-1}, b_j^\sigma) = g$. A firm with $\varphi^{\sigma-1}, b_j^\sigma$ such that $g \leq \Gamma(\varphi^{\sigma-1}, b_j^\sigma) < g + 1$ will produce exactly g products.

B Firm's Market Participation Problem

[To be described]

C Equilibrium Mass of Firms

Using the measure function $m_i(\varphi, \bar{a}, n)$ the equilibrium mass of firms is defined as

$$M_i = \sum_{n=0}^{+\infty} \int_{\varphi_i^*}^{+\infty} \int_{\bar{a}_i^*(\varphi, n)}^{+\infty} m_i(\varphi, \bar{a}, n) d\varphi d\bar{a}. \quad (11)$$

By recursive substitution using equations (16) and (17) $m_i(\varphi, \bar{a}, n)$ can be expressed as a function of demand exit thresholds and posterior distributions in the following way

$$m_i(\varphi, \bar{a}, n) = J \left(\frac{\varphi_{min}}{\varphi_i^*} \right)^\xi \frac{\xi(\varphi_i^*)^\xi}{\varphi^{\xi+1}} \int_{\bar{a}_i^*(\varphi, n-1)}^{+\infty} \dots \int_{\bar{a}_i^*(\varphi, 1)}^{+\infty} p(\bar{a}|\bar{a}_{n-1})p(\bar{a}_{n-1}|\bar{a}_{n-2})p(\bar{a}_1)d\bar{a}_1 d\dots \bar{a}_{n-1}, \quad (12)$$

where $p(\bar{a}_k|\bar{a}_{k-1}) = \frac{1}{\sqrt{v_{k-1}^2 + \sigma_\epsilon^2}} \phi \left(\frac{\bar{a}_k k - \bar{a}_{k-1}(k-1) - \mu_{k-1}}{\sqrt{v_{k-1}^2 + \sigma_\epsilon^2}} \right)$ is the transitional density from state $(\bar{a}_{k-1}, k - 1)$ to state (\bar{a}_k, k) , and $p(\bar{a}_1) = \frac{1}{\sqrt{\sigma_\theta^2 + \sigma_\epsilon^2}} \phi \left(\frac{\bar{a}_1 - \bar{\theta}_i}{\sqrt{\sigma_\theta^2 + \sigma_\epsilon^2}} \right)$.

Substitute equation (12) in equation (11) to obtain

$$\begin{aligned}
M_i &= \sum_{n=0}^{+\infty} \int_{\varphi_i^*}^{+\infty} \int_{\bar{a}_i^*(\varphi,n)}^{+\infty} J \left(\frac{\varphi_{min}}{\varphi_i^*} \right)^\xi \frac{\xi(\varphi_i^*)^\xi}{\varphi^{\xi+1}} \int_{\bar{a}_i^*(\varphi,n-1)}^{+\infty} \dots \int_{\bar{a}_i^*(\varphi,1)}^{+\infty} p(\bar{a}|\bar{a}_{n-1})p(\bar{a}_{n-1}|\bar{a}_{n-2})p(\bar{a}_1)d\bar{a}_1\dots d\bar{a}_{n-1}d\bar{a}d\varphi \\
&= J \left(\frac{\varphi_{min}}{\varphi_i^*} \right)^\xi \int_{\varphi_i^*}^{+\infty} \frac{\xi(\varphi_i^*)^\xi}{\varphi^{\xi+1}} \sum_{n=0}^{+\infty} \int_{\bar{a}_i^*(\varphi,n)}^{+\infty} \dots \int_{\bar{a}_i^*(\varphi,1)}^{+\infty} p(\bar{a}|\bar{a}_{n-1})p(\bar{a}_{n-1}|\bar{a}_{n-2})p(\bar{a}_1)d\bar{a}_1\dots d\bar{a}_{n-1}d\bar{a}d\varphi
\end{aligned}$$

Denote

$$H_i(\varphi) = \sum_{n=0}^{+\infty} \int_{\bar{a}_i^*(\varphi,n)}^{+\infty} \dots \int_{\bar{a}_i^*(\varphi,1)}^{+\infty} p(\bar{a}|\bar{a}_{n-1})p(\bar{a}_{n-1}|\bar{a}_{n-2})p(\bar{a}_1)d\bar{a}_1\dots d\bar{a}_{n-1}d\bar{a},$$

yielding

$$M_i(\varphi_i^*) = J \left(\frac{\varphi_{min}}{\varphi_i^*} \right)^\xi \int_{\varphi_i^*}^{+\infty} \frac{\xi(\varphi_i^*)^\xi}{\varphi^{\xi+1}} H_i(\varphi) d\varphi.$$

D Goods Market Clearing Condition

Aggregate expenditure level Y is defines as the sum of payment to labor and dividends from national firms

$$Y = L + \Pi_d + N\Pi_x, \quad (13)$$

where Π_d denotes profit from firms selling into domestic market, and Π_x denotes profit from exports to a given destination. Total profits from a destination can further be expressed as the product of the mass of firms selling to that destination and average profit level of those firms as shown below.

$$\begin{aligned}
\Pi_i &= \sum_{n=0}^{+\infty} \int_{\varphi^*}^{+\infty} \int_{\bar{a}_i^*(\varphi,n)}^{+\infty} \int_{-\infty}^{+\infty} \pi_i(\varphi, \bar{a}, n, a) m_i(\varphi, \bar{a}, n, a) dad\bar{a}d\varphi = \\
&= M_i \sum_{n=0}^{+\infty} \int_{\varphi^*}^{+\infty} \int_{\bar{a}_i^*(\varphi,n)}^{+\infty} \int_{-\infty}^{+\infty} \pi_i(\varphi, \bar{a}, n, a) \varsigma_i(\varphi, \bar{a}, n, a) dad\bar{a}d\varphi = \\
&= M_i \tilde{\pi}_i,
\end{aligned}$$

where M_i is the mass of firms selling to destination i , and $\varsigma_i(\varphi, \bar{a}, n, a)$ is the density associated with measure $m_i(\varphi, \bar{a}, n, a)$ and defined as $\frac{m_i(\varphi, \bar{a}, n, a)}{\sum_{n=0}^{+\infty} \int_{\varphi^*}^{+\infty} \int_{\bar{a}_i^*(\varphi,n)}^{+\infty} \int_{-\infty}^{+\infty} m_i(\varphi, \bar{a}, n, a) dad\bar{a}d\varphi} = \frac{m_i(\varphi, \bar{a}, n, a)}{M_i}$. $\tilde{\pi}_i$ is the mean profit level As stated in part (c) of Proposition 3, $\tilde{\pi}_i$ s are equal across destinations. Thus, equation (13) can we written as

$$Y = L + (M_d + NM_x)\tilde{\pi}, \quad (14)$$

yielding left hand side of equation (8) in the text.

I will show that Y also equals the total revenue of firms using goods market clearing conditions and rational expectations of the firms.

Rational expectation of the firms imply that the price index that firms take as given P , results from individual prices resulting in equilibrium. Consider the definition of the price index

$$P^{1-\sigma} = \sum_{n=0}^{+\infty} \int_{\varphi_d^*}^{+\infty} \int_{\bar{a}_d^*(\varphi,n)}^{+\infty} \int_{-\infty}^{+\infty} e^a \sum_{g=1}^{G(\varphi,\bar{a},n,a)} p_{dg}^{1-\sigma} m_d(\varphi, \bar{a}, n, a) dad\bar{a}d\varphi + \\ + N \sum_{n=0}^{+\infty} \int_{\varphi_x^*}^{+\infty} \int_{\bar{a}_x^*(\varphi,n)}^{+\infty} \int_{-\infty}^{+\infty} e^a \sum_{g=1}^{G(\varphi,\bar{a},n,a)} p_{xg}^{1-\sigma} m_x(\varphi, \bar{a}, n, a) dad\bar{a}d\varphi$$

Market clearing implies optimal price level given by

$$p_{ig} = \frac{\sigma}{\sigma-1} \tau b^{-1} \frac{g^\alpha}{\varphi} e^{\frac{a}{\sigma}}.$$

Substitute p_{ig} into the price index to obtain

$$P^{1-\sigma} = \sum_{n=0}^{+\infty} \int_{\varphi_d^*}^{+\infty} \int_{\bar{a}_d^*(\varphi,n)}^{+\infty} \int_{-\infty}^{+\infty} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \varphi^{\sigma-1} b^{\sigma-1} e^{\frac{a}{\sigma}} \sum_{g=1}^{G(\varphi,\bar{a},n)} g^{\alpha(1-\sigma)} M_d \varsigma_d(\varphi, \bar{a}, n, a) dad\bar{a}d\varphi + \\ + N \sum_{n=0}^{+\infty} \int_{\varphi_x^*}^{+\infty} \int_{\bar{a}_x^*(\varphi,n)}^{+\infty} \int_{-\infty}^{+\infty} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \left(\frac{\varphi}{\tau}\right)^{\sigma-1} b^{\sigma-1} e^{\frac{a}{\sigma}} \sum_{g=1}^{G(\varphi,\bar{a},n)} g^{\alpha(1-\sigma)} M_x \varsigma_x(\varphi, \bar{a}, n, a) dad\bar{a}d\varphi = \\ = \sum_{n=0}^{+\infty} \int_{\varphi_d^*}^{+\infty} \int_{\bar{a}_d^*(\varphi,n)}^{+\infty} \int_{-\infty}^{+\infty} \frac{r_d(\varphi, \bar{a}, n, a)}{P^{\sigma-1} Y} M_d \varsigma_d(\varphi, \bar{a}, n, a) dad\bar{a}d\varphi + \\ + N \sum_{n=0}^{+\infty} \int_{\varphi_x^*}^{+\infty} \int_{\bar{a}_x^*(\varphi,n)}^{+\infty} \int_{-\infty}^{+\infty} \frac{r_x(\varphi, \bar{a}, n, a)}{P^{\sigma-1} Y} M_x \varsigma_x(\varphi, \bar{a}, n, a) dad\bar{a}d\varphi = \\ = \frac{M_d \tilde{r}_d + N M_x \tilde{r}_x}{P^{\sigma-1} Y}.$$

Together with the result of part (d) of Proposition 3 the last equality yields

$$Y = (M_d + N M_x) \tilde{r} \tag{15}$$

Combining equation (14) and (24), and using $M = M_d + N M_x$ we obtain

$$L + M \tilde{\pi} = M \tilde{r}.$$

E Proof of Proposition 2

To describe the measure function $m_i(\varphi, \bar{a}, n, a)$ for $i \in \{d, x\}$, notice that it can be written as

$$m_i(\varphi, \bar{a}, n, a) = \varsigma(a|\varphi, \bar{a}, n) m_i(\varphi, \bar{a}, n),$$

where $\varsigma(a|\varphi, \bar{a}, n)$ is the probability of current demand shock to be a for a firm of type (φ, \bar{a}, n) . The distribution of demand shock for a firm of type (φ, \bar{a}, n) is given by the firm's posterior distribution: $N(\mu_n, v_n^2 + \sigma_\epsilon^2)$. Thus, $\varsigma(a|\varphi, \bar{a}, n)$ is given by $\frac{1}{\sqrt{v_n^2 + \sigma_\epsilon^2}} \phi\left(\frac{a - \mu_n}{\sqrt{v_n^2 + \sigma_\epsilon^2}}\right)$.

$m_i(\varphi, \bar{a}, n)$ is the measure of firms of type (φ, \bar{a}, n) . All firms enter with $\bar{a} = 0$ and $n = 0$, the mass of entrants is J out of which only firms with $\varphi > \varphi_j^*$ drawn from a Pareto distribution enter. Thus, the mass of entrants is $J \left(\frac{\varphi_{min}}{\varphi_i^*}\right)^\xi$. Conditional on entry, the ability level is distributed Pareto with support $[\varphi_j^*, +\infty)$. Thus, the initial condition is given by

$$m_i(\varphi, 0, 0) = J \left(\frac{\varphi_{min}}{\varphi_i^*}\right)^\xi \frac{\xi(\varphi_i^*)^\xi}{\varphi^{\xi+1}} \quad (16)$$

for $\varphi > \varphi_i^*$ and $m_i(\varphi, 0, 0) = 0$ otherwise.

Next, I describe the transition rule to state $(\varphi, \bar{a}, n + 1)$. A firm in state (φ, \bar{a}', n) can transfer to state $(\varphi, \bar{a}, n + 1)$ with the appropriate demand shock a given by $a = \bar{a}(n + 1) - \bar{a}'n$. Conditional on (φ, \bar{a}', n) , the demand shock is distributed $N(\mu_n, v_n^2 + \sigma_\epsilon^2)$. Thus, by the law of large number, the measure of firms in state (φ, \bar{a}', n) that transitions to state $(\varphi, \bar{a}, n + 1)$ is given by $m_i(\varphi, \bar{a}', n) \frac{1}{\sqrt{v_n^2 + \sigma_\epsilon^2}} \phi\left(\frac{\bar{a}(n + 1) - \bar{a}'n - \mu_n}{\sqrt{v_n^2 + \sigma_\epsilon^2}}\right)$, implying the transition rule given by

$$m_i(\varphi, \bar{a}, n + 1) = \int_{\bar{a}_i^*(\varphi, n)}^{+\infty} m_i(\varphi, \bar{a}', n) \frac{1}{\sqrt{v_n^2 + \sigma_\epsilon^2}} \phi\left(\frac{\bar{a}(n + 1) - \bar{a}'n - \mu_n}{\sqrt{v_n^2 + \sigma_\epsilon^2}}\right) d\bar{a}'. \quad (17)$$

F Proof of Proposition 3

F.1 Parts (a) and (b)

Consider per period firm's profit described in Appendix A

$$\pi_{ijt}(\varphi, \bar{a}_j, n) = \sum_{g=1}^{G_{ijt}(\varphi^{\sigma-1} b_j^\sigma)} \frac{(\sigma - 1)^{\sigma-1}}{\sigma^\sigma} \left(\frac{\varphi}{g^\alpha}\right)^{\sigma-1} b_j^\sigma \frac{P_{jt}^{\sigma-1} Y_{jt}}{(\tau_{ij} w_{it})^{\sigma-1}} - f_{ij} \sum_{g=1}^{G_{ijt}(\varphi^{\sigma-1} b_j^\sigma)} g^\gamma.$$

In the symmetric stationary equilibrium the profit can be written as

$$\pi(\varphi, \bar{a}, n) = \sum_{g=1}^{G(\varphi^{\sigma-1} b^\sigma)} \frac{(\sigma - 1)^{\sigma-1}}{\sigma^\sigma} \left(\frac{\varphi}{g^\alpha}\right)^{\sigma-1} b^\sigma \frac{P^{\sigma-1} Y}{(\tau)^{\sigma-1}} - f \sum_{g=1}^{G(\varphi^{\sigma-1} b^\sigma)} g^\gamma,$$

where $\tau > 1$ for an export market, and $\tau = 1$ for domestic market.

Firm's dynamic behavior is determined by the evolution of beliefs about the demand parameter described by \bar{a} , and n . Ability φ , and aggregate variables P and Y stay constant through firm's life cycle. Thus, to characterize firm's dynamic behavior consider the following normalization $u = \frac{\varphi P Y^{\frac{1}{\sigma-1}}}{\tau}$. Using normalization u firm's profit can be written as

$$\pi(u, \bar{a}, n) = \sum_{g=1}^{G(u, b^\sigma)} \frac{(\sigma - 1)^{\sigma-1}}{\sigma^\sigma} g^{\alpha(1-\sigma)} b^\sigma u^{\sigma-1} - f \sum_{g=1}^{G(u, b^\sigma)} g^\gamma.$$

The convenience of this representation is that given u , there are no more endogenous equilibrium objects determining firm's profit. Also notice that conditional on u , the profit from domestic and foreign market look identical: the effect of transportation costs τ is taken into account in defining u .

Using normalization u firm's market participation problem can be written as

$$V(u, \bar{a}, n) = \max \left\{ \sum_{g=1}^{G(u, b^\sigma)} \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} g^{\alpha(1-\sigma)} b^\sigma u^{\sigma-1} - f \sum_{g=1}^{G(u, b^\sigma)} g^\gamma + \beta(1-\delta) E_{\bar{a}'|\bar{a}, n} V(u, \bar{a}', n+1), 0 \right\}.$$

Observe that conditional on u , the problem for domestic and export market are identical.

The solution to this problem is characterized by a set of market exit thresholds $\bar{a}^*(u, n)$, which are, as expected, do not depend on a specific destination. Conditional on u the thresholds are completely determined by exogenous parameters of the model.

The uniqueness of the market exit thresholds implies $\bar{a}^*(u, n) = \bar{a}_i^*(\varphi, n)$. A firm in state (u, n) exits the market when $\bar{a} < \bar{a}^*(u, n) = \bar{a}^* \left(\frac{\varphi PY^{\frac{1}{\sigma-1}}}{\tau}, n \right)$. Thus, a firm with ability $\varphi = \frac{\tau u}{PY^{\frac{1}{\sigma-1}}}$ exits the market when $\bar{a} < \bar{a}^* \left(\frac{\varphi PY^{\frac{1}{\sigma-1}}}{\tau}, n \right)$. At the same time the exit threshold is given by $\bar{a}_i^*(\varphi, n)$. Uniqueness of the exit threshold implies $\bar{a}^* \left(\frac{\varphi PY^{\frac{1}{\sigma-1}}}{\tau}, n \right) = \bar{a}_i^*(\varphi, n)$, or $\bar{a}^*(u, n) = \bar{a}_i^*(\varphi, n)$.

Firm's value of entry into either domestic or export market can be written as

$$V^E(u) = \Pi(u, 0, 0) + \beta(1-\delta) E_{\bar{a}'|0,0} V(u, \bar{a}', 1). \quad (18)$$

The entry threshold u^* is determined by $V^E(u^*) = 0$. Notice that conditional on u equation (18) does not involve any endogenous variables. Thus, u^* is completely determined by exogenous parameters of the model. Furthermore, it is equal across destinations.

A firm will enter a given destination whenever $u \geq u^*$, or equivalently $\varphi \geq \frac{\tau u^*}{PY^{\frac{1}{\sigma-1}}}$. Since the entry thresholds φ_d^* and φ_x^* are unique, it must be true that $\varphi_d^* = \frac{u^*}{PY^{\frac{1}{\sigma-1}}}$ and $\varphi_x^* = \frac{\tau u^*}{PY^{\frac{1}{\sigma-1}}}$.

F.2 Parts (c) and (d)

Denote by $r_i(\varphi, \bar{a}, n, a)$ revenue of firms in state (φ, \bar{a}, n, a) in market i . The mean revenue level of firms in destination i is defined as

$$\tilde{r}_i = \sum_{n=0}^{+\infty} \int_{\varphi_i^*}^{+\infty} \int_{\bar{a}_i^*(\varphi, n)}^{+\infty} \int_{-\infty}^{+\infty} r_i(\varphi, \bar{a}, n, a) \frac{m_i(\varphi, \bar{a}, n, a)}{M_i} da d\bar{a} d\varphi,$$

where $\frac{m_i(\varphi, \bar{a}, n, a)}{M_i}$ defines the density associated with measure $m_i(\varphi, \bar{a}, n, a)$. Substitute equilibrium revenue, substitute the measure function defined in Proposition 2, integrate out a to obtain

$$\tilde{r}_i = \sum_{n=0}^{+\infty} \int_{\varphi_i^*}^{+\infty} \int_{\bar{a}_i^*(\varphi, n)}^{+\infty} \left(\frac{\sigma-1}{\sigma} \right)^{\sigma-1} \frac{\varphi^{\sigma-1} P^{\sigma-1} Y}{\tau^{\sigma-1}} b^\sigma \sum_{g=1}^G g^{\alpha(1-\sigma)} \frac{m_i(\varphi, \bar{a}, n)}{M_i} d\bar{a} d\varphi.$$

Using recursive definition of $m_i(\varphi, \bar{a}, n)$ described in Appendix C, mean revenue level can be further written as

$$\tilde{r}_i = J \left(\frac{\varphi_{min}}{\varphi_i^*} \right)^\xi \int_{\varphi_i^*}^{+\infty} \frac{\xi(\varphi_i^*)^\xi}{\varphi^{\xi+1}} \left(\frac{\sigma-1}{\sigma} \right)^{\sigma-1} \frac{\varphi^{\sigma-1} P^{\sigma-1} Y}{\tau^{\sigma-1}} \sum_{n=0}^{+\infty} \int_{\bar{a}_i^*(\varphi, n)}^{+\infty} b^\sigma \sum_{g=1}^G g^{\alpha(1-\sigma)} \frac{h_i(\varphi, \bar{a}, n)}{M_i} d\bar{a} d\varphi,$$

where $h_i(\varphi, \bar{a}, n) = \int_{\bar{a}_i^*(\varphi, n-1)}^{+\infty} \dots \int_{\bar{a}_i^*(\varphi, 1)}^{+\infty} p(\bar{a}|\bar{a}_{n-1})p(\bar{a}_{n-1}|\bar{a}_{n-2})p(\bar{a}_1)d\bar{a}_1d\dots\bar{a}_{n-1}$ (individual components are defined in Appendix C).

Consider change of variables $u = \frac{\varphi P Y^{\frac{1}{\sigma-1}}}{\tau}$. $h_i(\varphi, \bar{a}, n)$ depends on φ through thresholds $\bar{a}_i^*(\varphi, n)$ which are equal to $\bar{a}^*(u, n)$. Thus $h_i(\varphi, \bar{a}, n) = h(u, \bar{a}, n)$.

Further

$$\frac{d\varphi}{\varphi^{\xi+1}} = \frac{\tau du \left(P Y^{\frac{1}{\sigma-1}} \right)^{\xi+1}}{P Y^{\frac{1}{\sigma-1}} (\tau u)^{\xi+1}}.$$

Apply the transformation first to the mass of firms M_i defined in Appendix C

$$M_i(\varphi_i^*) = J \left(\frac{\varphi_{min}}{\varphi_i^*} \right)^\xi \int_{\varphi_i^*}^{+\infty} \frac{\xi(\varphi_i^*)^\xi}{\varphi^{\xi+1}} H_i(\varphi) d\varphi = \quad (19)$$

$$= J(\varphi_{min})^\xi \xi \left(\frac{P Y^{\frac{1}{\sigma-1}}}{\tau} \right)^\xi \int_{u^*}^{+\infty} \frac{1}{u^{\xi+1}} H(u) du \quad (20)$$

Apply transformations to the mean revenue equation which now can be written as

$$\tilde{r}_i = \frac{\int_{u^*}^{+\infty} \frac{1}{u^{\xi+1}} \left(\frac{\sigma-1}{\sigma} \right)^{\sigma-1} u^{\sigma-1} \sum_{n=0}^{+\infty} \int_{\bar{a}^*(u, n)}^{+\infty} b^\sigma \sum_{g=1}^G g^{\alpha(1-\sigma)} h(u, \bar{a}, n) d\bar{a} du}{\int_{u^*}^{+\infty} \frac{1}{u^{\xi+1}} H(u) du}. \quad (21)$$

Equation (21) completely determines mean revenue in destination i as a function of exogenous parameters only. Notice that the right hand side of equation (21) is independent of i implying the mean revenue is the same across destinations. Similar calculations apply to the mean profit level.

G Proof of Proposition 4

Recall that $q_{ijg}(\varphi, \bar{a}_j, n)$ is given by $q_{ijg}(\varphi, \bar{a}_j, n) = \left(\frac{\sigma-1}{\sigma} \right)^\sigma \left(\frac{\varphi}{g^\alpha} \right)^\sigma b_j^\sigma \frac{P_j^{\sigma-1} Y_j}{(\tau_{ij} w_i)^\sigma}$, where $b_j = E_{a_{jt}|\bar{a}_j, n} \left[e^{\frac{a_{jt}}{\sigma}} \right]$. Using posterior distribution to compute the expectation $b_j^\sigma = \exp \left[\mu_{jn} + \frac{1}{2} \left(\frac{v_n^2 + \sigma_\epsilon^2}{\sigma} \right) \right]$. Thus,

$$\begin{aligned} \frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)} &= \frac{\left(\frac{\sigma-1}{\sigma} \right)^\sigma \left(\frac{\varphi}{g^\alpha} \right)^\sigma \exp \left[\mu_{j, n+1} + \frac{1}{2} \left(\frac{v_{n+1}^2 + \sigma_\epsilon^2}{\sigma} \right) \right] \frac{P_j^{\sigma-1} Y_j}{(\tau_{ij} w_i)^\sigma}}{\left(\frac{\sigma-1}{\sigma} \right)^\sigma \left(\frac{\varphi}{g^\alpha} \right)^\sigma \exp \left[\mu_{jn} + \frac{1}{2} \left(\frac{v_n^2 + \sigma_\epsilon^2}{\sigma} \right) \right] \frac{P_j^{\sigma-1} Y_j}{(\tau_{ij} w_i)^\sigma}} = \\ &= \exp \left(\mu_{j, n+1} - \mu_{jn} + \frac{v_{n+1}^2 - v_n^2}{2\sigma} \right). \end{aligned}$$

Substitute for μ_{jn} , $\mu_{j,n+1}$, v_n^2 , v_{n+1}^2 , rearrange terms to obtain

$$\frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)} = \exp\left(\frac{\lambda(2\sigma(\bar{a}_j - \bar{\theta}_j) - \sigma_\theta^2)}{2\sigma(1+(n+1)\lambda)(1+n\lambda)}\right), \quad (22)$$

where $\lambda = \frac{\sigma_\theta^2}{\sigma_\epsilon^2}$. The relation of $\frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)}$ to 1 depends on the sign of $\frac{\lambda(2\sigma(\bar{a}_j - \bar{\theta}_j) - \sigma_\theta^2)}{2\sigma(1+(n+1)\lambda)(1+n\lambda)}$. Notice that the denominator is always positive. The numerator is linearly increasing in \bar{a}_j and equals zero when $\bar{a}_j = \bar{\theta}_j + \frac{\sigma_\theta^2}{2\sigma}$. Thus, the threshold level $\hat{a}_j = \bar{\theta}_j + \frac{\sigma_\theta^2}{2\sigma}$.

$$\text{When } \bar{a}_j > \hat{a}_j, \frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)} > 0 \Rightarrow \exp\left(\frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)}\right) > 1 \Rightarrow \frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)} > 1.$$

$$\text{When } \bar{a}_j < \hat{a}_j, \frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)} < 0 \Rightarrow \exp\left(\frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)}\right) < 1 \Rightarrow \frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)} < 1.$$

$$\text{When } \bar{a}_j = \hat{a}_j, \frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)} = 0 \Rightarrow \exp\left(\frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)}\right) = 1 \Rightarrow \frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)} = 1.$$

H Proof of Proposition 5

Recall that b_j^σ is expressed in terms of \bar{a}_j and n as

$$b_j^\sigma = \exp\left[\mu_{jn} + \frac{1}{2}\left(\frac{v_n^2 + \sigma_\epsilon^2}{\sigma}\right)\right].$$

Manipulating the equation yields

$$\bar{a}_j - \bar{\theta}_j = \frac{\ln b_j^\sigma(1 + \lambda n)}{\lambda n} - \frac{\bar{\theta}_j(1 + \lambda n)}{\lambda n} - \frac{\sigma_\theta^2}{2\sigma\lambda n} - \frac{\sigma_\epsilon^2(1 + \lambda n)}{2\sigma\lambda n} \quad (23)$$

Substitute equation (23) into equation (22) to obtain

$$\frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)} = \exp\left(\frac{\ln b_j^\sigma - \bar{\theta}_j - \frac{\sigma_\theta^2 + \sigma_\epsilon^2}{2\sigma}}{(1 + (1 + n)\lambda)}\right)$$

For $b_j^\sigma = \bar{\theta}_j + \frac{\sigma_\theta^2 + \sigma_\epsilon^2}{2\sigma}$ (which corresponds to $\bar{a}_j = \bar{\theta}_j + \frac{\sigma_\theta^2}{2\sigma}$) the exponent equals 1 and thus scale does not change with n . For $b_j^\sigma > \hat{b}_j^\sigma$ the numerator in the exponent is positive, thus the ratio declines in n . For $b_j^\sigma < \hat{b}_j^\sigma$ the numerator in the exponent is negative, thus the ratio increases in n .

I Proof of Proposition 6

In proving the result I am going to use the following three equations

$$u^* = \varphi_d^* P Y^{\frac{1}{\sigma-1}} \quad (24)$$

$$u^* = \varphi_d^* P Y^{\frac{1}{\sigma-1}} \frac{1}{\tau} \quad (25)$$

$$K(u^*) = \left(P Y^{\frac{1}{\sigma-1}}\right)^\xi + N \frac{\left(P Y^{\frac{1}{\sigma-1}}\right)^\xi}{\tau^\xi}. \quad (26)$$

Equation (26) is obtained by manipulating $M = M_d(\varphi_d^*) + NM_x(\varphi_x^*)$, where expression of $M_i(\varphi_i^*)$ in terms of u^* is used (equation (20)). $K(u^*) = \frac{M}{J(\varphi_{min})^\xi \int_{u^*}^{+\infty} \frac{1}{u^{\xi+1}} H(\varphi) du}$

An important thing to note is that u^* is completely determined by exogenous parameters of the model(excluding τ) by Proposition 3, and as such does not respond to variation in τ . Denote by $X = PY^{\frac{1}{\sigma-1}}$.

The optimal product sales are given by

$$r_{gi}(\varphi, \bar{a}, n, a) = \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma-1} \left(\frac{\varphi}{g^\alpha} \right)^{\sigma-1} b^{\sigma-1} \frac{P^{\sigma-1} Y}{\tau^{\sigma-1}} e^{\frac{a}{\sigma}}$$

Using equations (24) or (25) they can be written as

$$r_{gi}(\varphi, \bar{a}, n, a) = \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma-1} \left(\frac{\varphi}{g^\alpha} \right)^{\sigma-1} b^{\sigma-1} \left(\frac{u^*}{\varphi_i^*} \right)^{\sigma-1} e^{\frac{a}{\sigma}}$$

Thus, the sign of the derivative of $r_{gi}(\varphi, \bar{a}, n, a)$ with respect to τ is the same as the sign of the derivative of φ_i^* with respect to τ , which I compute below.

Total differentiation of equation (26) yields

$$\frac{dX}{X} \frac{\tau}{d\tau} = \frac{N}{\tau^\xi + N}.$$

Thus, $0 < \frac{dX}{X} \frac{\tau}{d\tau} < 1$. Total differentiation of equation (24) yields

$$\frac{d\varphi_d^*}{\varphi_d^*} \frac{\tau}{d\tau} = -\frac{dX}{X} \frac{\tau}{d\tau}.$$

Thus, $\frac{d\varphi_d^*}{d\tau} < 0$ implying $\frac{dr_{gd}}{d\tau} > 0$.

Total differentiation of equation (25) yields

$$\frac{d\varphi_x^*}{\varphi_x^*} \frac{\tau}{d\tau} = 1 - \frac{dX}{X} \frac{\tau}{d\tau}.$$

Since $0 < \frac{dX}{X} \frac{\tau}{d\tau} < 1$, $\frac{d\varphi_x^*}{d\tau} > 0$ implying $\frac{dr_{gx}}{d\tau} < 0$.

Consider characterizing the response of a product's sale which is given by

$$q_{gi}(\varphi, \bar{a}, n) = \left(\frac{\sigma - 1}{\sigma} \right)^\sigma \left(\frac{\varphi}{g^\alpha} \right)^\sigma b^\sigma \frac{P^{\sigma-1} Y}{\tau^\sigma},$$

and using equations (24) or (25) can be written as

$$q_{gi}(\varphi, \bar{a}, n) = \left(\frac{\sigma - 1}{\sigma} \right)^\sigma \left(\frac{\varphi}{g^\alpha} \right)^\sigma b^\sigma \left(\frac{u^*}{\varphi_i^*} \right)^{\sigma-1} \frac{1}{\tau}. \quad (27)$$

Differentiate equation (27) with respect to τ to obtain

$$\frac{dq_{gi}}{d\tau} = \left(\frac{\sigma - 1}{\sigma} \right)^\sigma \left(\frac{\varphi}{g^\alpha} \right)^\sigma b^\sigma \frac{u^{*(\sigma-1)}}{\varphi_i^{*(\sigma-1)} \tau^2} \left[-(\sigma - 1) \frac{d\varphi_i^*}{\varphi_i^*} \frac{\tau}{d\tau} - 1 \right].$$

Since $\frac{d\varphi_x^*}{d\tau} > 0$, $\frac{dq_{gx}}{d\tau} < 0$.

For the domestic market $-(\sigma - 1) \frac{d\varphi_i^*}{\varphi_i^*} \frac{\tau}{d\tau} - 1 = (\sigma - 1) \frac{N}{\tau^\xi + N} - 1$. Thus, $\frac{dq_{gd}}{d\tau} > 0$ when $\frac{N}{\tau^\xi + N} > \frac{1}{\sigma-1}$, and negative otherwise.

J Scope Indifference Curves and Trade Costs

Consider equation (2) describing the scope indifference curves. The curves are parametrized by the endogenous variable $B = \frac{\tau^{\sigma-1}}{P^\sigma Y}$. A higher value of B induces higher value of the product $\varphi^{\sigma-1}b^\sigma$ to yield the same scope level g . Thus in response to an increase in B the level curves will move away from the origin. As a result firm's with in the state (φ, b) will find itself lying on a lower scope indifference curve and will be forced to drop marginal products. In a similar way a decline in the value of B will move level curves $\Gamma(\varphi^{\sigma-1}b^\sigma) = g$ toward the origin, inducing firms to add marginal products.

I will show below that $\frac{dB_x}{d\tau} > 0$ and $\frac{dB_d}{d\tau} < 0$.

$$B_x = \frac{\tau^{\sigma-1}}{P^\sigma Y} = \left(\frac{\varphi_x^*}{u^*} \right)^{\sigma-1}.$$

Since, as shown in Appendix 6, $\frac{d\varphi_x^*}{d\tau} > 0 \Rightarrow \frac{dB_x}{d\tau} > 0$. In response to a decline in trade costs, B_x declines, shifting scope indifference curves toward the origin, and inducing firms to add products.

$$B_d = \frac{1}{P^\sigma Y} = \left(\frac{\varphi_d^*}{u^*} \right)^{\sigma-1}.$$

Since, as shown in Appendix 6, $\frac{d\varphi_d^*}{d\tau} < 0 \Rightarrow \frac{dB_d}{d\tau} < 0$. In response to a decline in trade costs, B_d increases, shifting scope indifference curves away from the origin, and inducing firms to drop products.

Table 1: Sources of Brazilian Manufacturing Exports to Argentina, the US, and the World in an Average Year (percent of export sales)

Contribution of	Argentina	US	World
New Exporters	6.4	4.2	2.6
New Products by Incumbent Exporters	7.5	4.5	4.4
Incumbent Products by Incumbent Exporters	86.2	91.5	93

Note: The table reports the share of Brazilian exports in manufacturing products in a given year that arises either from new exporters, or from the sales of previously not exported products by continuing exporters, or from the sales of previously exported products by continuing exporters. New product is defined at the exporter level. Mean across annual observations.

Table 2: Decomposition of Brazilian Manufacturing Export Sales Growth to Argentina, the US, and the World in an Average Year (percent)

Margin	Argentina	US	World
Exporter Turnover	5.25	0.61	0.42
Product Switching	5.19	0.58	1.12
Intensive	13.30	2.68	1.38
Total	23.73	3.87	2.87

Note: The table reports the decomposition of the growth rate of real exports in manufacturing products by margins of trade. Mean across annual observations.

Table 3: Mean of the Fraction of Export Sales from New Products among Firms Exporting to Argentina, the US, and the World

	Argentina	US	World
Intra-Firm Extensive Margin	17.9	18.4	20.0

Note: The table reports the percent of exporter's sales derived from products introduced between two consecutive periods (intra-firm extensive margin). Sample of all surviving exporters is considered.

Table 4: Mean of the Fraction of Export Sales from New Products among Firms Exporting to Argentina, the US, and the World: Decomposition by Export Age

Export age	Argentina	US	World
2	24.2	24.6	27.1
3	17.5	17.6	19.8
4	15.2	14.4	16.0
5	13.4	12.7	14.3
6	13.7	10.3	14.1
7	13.7	11.8	12.9
8	10.2	9.7	12.7
10	9.0	9.6	9.8
11	8.2	6.6	7.6

Note: The table reports the intra-firm extensive margin computed by export age categories. Sample of all surviving exporters with defined export age is considered.

Table 5: OLS Regression. Dependent Variable Fraction of Sales From New Products

	(1)	(2)	(3)	(4)
Export Age	-0.025 ^{***} (0.001)	-0.018 ^{***} (0.001)	-0.016 ^{***} (0.0003)	-0.013 ^{***} (0.0003)
Log Exports		-0.026 ^{***} (0.001)		-0.018 ^{***} (0.0003)
Constant	0.294 ^{***} (0.003)	0.561 ^{***} (0.007)	0.217 ^{***} (0.001)	0.402 ^{***} (0.004)
R^2	0.02	0.05	0.01	0.02
Number of Obs.	56,638	56,638	225,569	225,569
Level of Obs.	Firm-Year	Firm-Year	Firm-Dest.-Year	Firm-Dest.-Year

Note: The table reports the results of OLS regression. The dependent variables is the fraction of exporter's sales derived from products introduced between two consecutive periods. A sample of surviving exporters is considered.

^{***} Statistically significant at the 1% level.

Table 6: Mean of the Fraction of Newly Added Products among Firms Exporting to Argentina, the US, and the World

	Argentina	US	World
Percent of new products	28.3	23.1	29.4

Note: The table reports the percent of currently exported products an exporter added between two consecutive periods. Sample of all surviving firms is considered.

Table 7: Mean of the Fraction of Newly Added Products among Firms Exporting to Argentina, the US, and the World: Decomposition by Export Age

Export age	All Incumbent Exporters			Incumbent Exporters that Added Products		
	Argentina	US	World	Argentina	US	World
2	33.3	31.0	36.1	66.0	69.0	67.5
3	27.8	25.8	30.6	57.4	62.4	59.1
4	25.8	22.8	27.5	53.0	58.5	54.6
5	25.0	21.6	26.3	49.6	54.3	51.6
6	27.0	20.0	27.5	50.9	53.1	51.5
7	27.5	22.4	26.9	49.1	53.0	49.8
8	25.5	19.5	27.2	46.1	50.4	47.9
9	23.5	20.7	26.9	42.6	48.6	47.9
10	23.0	21.6	25.4	40.9	50.2	46.5
11	22.1	21.0	24.6	40.3	45.4	43.2

Note: The table reports the percent of currently exported products an exporter added between two consecutive periods computed by export age categories. A sample of surviving exporters with defined export age is considered.

Table 8: OLS Regression. Dependent Variable Fraction of Added Products in Exporter's Product Mix

	(1)	(2)	(3)	(4)
Export Age	-0.016 ^{***} (0.001)	-0.015 ^{***} (0.001)	-0.009 ^{***} (0.0003)	-0.008 ^{***} (0.0004)
Log Exports		-0.001 ^{***} (0.001)		-0.004 ^{***} (0.0003)
Constant	0.370 ^{***} (0.003)	0.388 ^{***} (0.008)	0.270 ^{***} (0.001)	0.314 ^{***} (0.004)
R^2	0.01	0.01	0.003	0.004
Number of Obs.	56,638	56,638	225,569	225,569
Level of Obs.	Firm-Year	Firm-Year	Firm-Dest.-Year	Firm-Dest.-Year

Note: The table reports the results of OLS regression. The dependent variables is the fraction of added products in exporter's product mix. A sample of surviving exporters is considered.

^{***} Statistically significant at the 1% level.

Table 9: Data Moments

Moment	Data	Simulation
1. Mean of log-sales	11.57	11.60
2. Standard deviation of log-sales	2.63	2.80
3. Mean log-sales of entrants to mean log-sales of all firms	0.86	0.93
4. Fraction of multi-product exporters	0.59	0.54
5. Share of product switching exporters	0.50	0.36
Criterion		0.095

Note: Statistics are computed at the level of total firm's exports (aggregated across destinations). For the first three moments, the mean is taken across firm-year observations. For the last two moments the mean is taken across annual observations.

Table 10: Parameter values

Parameter	α	γ	σ_θ	σ_ϵ	f
Value	0.15	0.02	2.19	16.66	39,734.38

Table 11: Decomposition of Brazilian exports (percent of export sales)

Margin	Data	Simulation
Exporter Turnover	2.6	1.1
Product Switching	4.4	1.6
Intensive	93	97

Table 12: Margins of Export Sales Growth: Simulations

Growth of exports	Exporter Turnover Margin	Product Switching Margin	Intensive Margin
4.8%	0.3%	0.8%	3.7%