Endogenous Property Rights*

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Abstract

It is often argued that additional checks and balances provide economic agents with better protection from expropriation of their wealth or productive capital. We demonstrate that in a dynamic political economy model this intuition may be flawed. Surprisingly, increasing the number veto players or the majority requirement for redistribution may reduce property right protection on the equilibrium path. The reason is that there are two complementary mechanisms of property rights protection. One is the individual's veto power, his ability to prevent any redistribution which is not in his favor. The other is the mutual protection of non-veto players with similar wealth in equilibrium. Thus, the model explains why property rights of individuals who do not possess a lot of political power are nevertheless often respected. Non-veto players anticipate that the expropriation of one of them will ultimately hurt others, and thus combine their influence to prevent redistributions; the flip-side of this is that individual investment efforts might require coordination. The property rights of non-veto players may suffer if the environment changes, even if the number of veto players or the supermajority requirement increases. The model also predicts that distribution of wealth in societies with weaker formal institutions (smaller supermajority requirements) among the non-veto players will tend to be more homogenous.

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Preliminary. Comments are very welcome

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1 Introduction

The protection of property rights is widely viewed as a cornerstone of efficiency and economic growth (e.g., Coase, 1960, Alchian, 1965, Demsetz 1967, Barro 1998).[1] But whether property rights are effectively protected depends on the political economy of the respective society and its institutions. Among political institutions, checks and balances have long been viewed as an essential tool to limit the power of government to expropriate its subjects. The ideas date back at least to the Roman republic (Polybius [2010], Macchiavelli [1984]) and, in modern times, to Montesquieu's *Spirit of the Laws* (1748 [1989]) and the Federalist papers, the intellectual foundation of the United States Constitution.[2] In Federalist papers #51, James Madison argued for the need to contrive the government "as that its several constituent parts may, by their mutual relations, be the means of keeping each other in their proper places." Riker (1987) concurs: "For those who believe, with Madison, that freedom depends on countering ambition with ambition, this constancy of federal conflict is a fundamental protection of freedom."

In modern political economy checks and balances have been associated with various beneficial consequences.[3] North and Weingast (1989) argued that the checks and balances established by the Glorious Revolution in 1688 provided "the credible commitment by the government to honour its financial agreement [that] was part of a larger commitment to secure private rights".[4] Similarly, Root (1989) has argued that checks and balances on British monarchs led to lower borrowing costs compared to the French Kings. Persson, Roland, and Tabellini (1997, 2000) model checks and balances as the separation of taxing and spending decisions within budgetary decision-makings. They argue that properly designed checks and balances improve the accountability of elected officials and thereby voter utility. They may also limit rent-seeking by politicians. Keefer (2004) argues that "The absence of multiple veto players in countries often means that some groups in society are less represented than they otherwise would be. [...] That is, if the number of veto players tracks the extent to which all citizens are represented, government officials are more likely to grant special interest favors where there are few veto players." [5]

[1] Alchian (2008) defines private property as "the exclusive authority to determine how a resource is used."

[2] Many political theorists refer to the broader notion of "liberty" that encompasses property rights, but may also include certain political rights such as freedom of expression etc.

[3] Recent research in political economy, however, has also pointed out that in some domains check and balances may lead to inefficiencies. Diermeier and Myerson (1999) have shown that bicameralism may lead to inefficiencies due to incentives to create restrictive internal procedures as a consequence of inter-chamber bargaining.

[4] Much of the literature on the political economy of checks and balances has focused on incentives for expropriation by a monarch and the associated commitment problem (e.g. North and Weingast 1989, Root 1989). Our approach also allows us to consider the issue of democratic expropriation, e.g., the expropriation of a minority by a majority. This was one of the central concerns of, e.g., Montesquieu and Madison, that led to various power sharing arrangements from multi-cameralism to Federal political structures.

[5] In a recent paper Acemoglu, Robinson, and Torvik (2011) have argued that this phenomenon can be used to

In this paper, we study political mechanisms that ensure protection of individuals' property. Checks and balances come in different forms such as the separation of powers between the legislative, executive, and judicial branches of government, multi-cameralism, or Federalism. However, at its core are institutional limits on majority rule, usually in the form of veto rights for individuals or groups. Such limits allow individuals or collectives to block any redistribution without their consent. In our model these serve as abstract representations of checks and balances.[6] Those include a president with veto powers, a supreme court that can strike down a law as unconstitutional, or the Spartan *Gerousia*, the Council of Elders, that could veto motions passed by the *Apella*, the citizens' assembly as described in Plutarch's "Life of Lycurgus". However, if we interpret property rights as institutions that sustain allocations unless changed by the legislature, we can formally investigate the effect of checks and balances on the allocation of property rights and prerogatives.[7]

But a simple mapping from political prerogatives to economic rights would be inappropriate. In a political economy approach property rights regimes are to be understood as equilibrium outcomes of interacting economic agents. The property rights of an individual may be respected in equilibrium not because he is strong enough to protect them on his own, i.e. has veto power, but because he is a member of a larger group, formed in equilibrium, the members of which oppose the expropriation of each other because they know that once a member of the group is expropriated, others will be next in the line for expropriation. Such groups serve as *endogenous* veto groups, i.e. collections of agents that *in equilibrium* can block any path that makes any of the members worse off. That is, an agent's "commitment" to the protection of a particular property rights system hinges on the rational foresight of what might happen once the redistributive process is initiated. This means that property rights systems can be stable even in the absence of explicit veto powers.[8] Paradoxically, adding additional exogenous protection (e.g. by increasing the number of veto players) may lead to the break-down of an equilibrium with stable property rights, as the newly protected agent (e.g., one who was granted or has acquired

explain why voters may sometimes dismantle checks and balances. The idea is that under checks and balances politicians receive less rents throug the political process which makes them more susceptible to be bribed by elites. Removing checks and balances leads to more rents but better policy outcomes for the majority.

[6] Some scholars have argued that veto player arrangements are the most important feature of political governance structures (e.g. Tsebelis 2002).

[7] While we use the language of "voting" and "vetos" which suggests a legislative process, our model can also be interpreted as power struggle between individuals and coalitions. Here a "veto" simply means that an individual has the military or economic means to block any redistribution without his consent.

[8] Similar effects can be observed in models of legislative bargaining in dynamic policy environments with a persistent proposer (Diermeier and Fong 2011). In that model it is the fact that the persistent agenda setter can always reconsider a proposal that induces a group of voters to "protect" each other's allocation, as, e.g. in the context of entitlement program . That is, self-interested voters may block any policy proposal when some other voters are substantially worse off. In the Diermeier and Fong model the proposer effectively serves as the sole veto player.

on his own veto power) is no longer interested in protecting others. Thus, property rights need not necessarily become better protected with an increase of the number of exogenous veto players or by additional supermajority requirements. In other words, even if formal institutions are weak, property rights may nevertheless arise as an equilibrium phenomenon. Strengthening formal institutions, however, may worsen property rights protection.

Modeling property rights, we need to consider dynamic environments. For example, the right to own a parcel of land means that the owner benefits from the income stream generated from the asset. In other words, a status quo allocation stays in place for the next period unless it is changed by the political decision mechanism in which case the newly chosen policy becomes the status quo for the next period. There is a growing literature in political economy that has investigated dynamic policy environments (Baron, 1996; Kalandrakis, 2004, 2007; Duggan and Kalandrakis 2011; Bowen and Zahran, 2012; Battaglini and Coate, 2007, 2008; Diermeier and Fong, 2011). Examples of continuing policies include entitlements such as social security, monetary policy, but also allocations of land, property and privileges, and many others. In such environments, the policy or allocation remains in effect unless it is changed by the legislature. Considering dynamic environments can have important consequences.

Example 1 *Five legislators decide how to split* 15 *units of wealth, with the status quo being* $(4, 3, 2, 1; 5)$*. Legislator* #5 *is the sole veto player, re-allocation requires a majority of votes, and, for the sake of simplicity, we assume that when agents are indifferent, they support the proposer. In a standard legislative bargaining model in the tradition of Baron and Ferejohn (1989), a proposer, say legislator* #5*, would simply build a coalition to expropriate two agents, say agents* #1 *and* #2*, and capture the surplus resulting in, e.g.,* $(0, 0, 2, 1; 12)$ *allocation. But this logic does not hold in a model with assets that might be reallocated further. Indeed, starting with* $(0, 0, 2, 1; 12)$*, agent* #5 *might propose to expropriate agents* #3 *and* #4 *by proposing a* $(0, 0, 0, 0; 15)$ *division, which is accepted in equilibrium. Anticipating this, agents* #3 *and* #4 *should not agree to the first expropriation, thus becoming the effective guarantors of property rights of agents* #1 *and* #2.[9]

To demonstrate which allocations are stable when a further reallocation is possible, consider the simplest possible case. Allocation $(0, 0, 0, 0; 15)$ *is stable as agent* #5 *has veto power, while allocations where one agent among* $\#1 - 4$ *has some wealth and the rest has zero (e.g.,* $(0, 1, 0, 0; 14)$*,* $(3, 0, 0, 0; 12)$*, or* $(1, 1, 0, 0; 13)$*) are unstable as the veto player can always use two of the remaining agents to redistribute all wealth to himself. However, allocation* $(1, 1, 1, 0; 12)$ *is stable: in order to improve his position, the veto player must either move to* $(0, 0, 0, 0; 15)$ *or to an*

[9] We will see the later that the status quo is not stable either, and will lead to the allocation $(3, 3, 3, 0; 6)$.

unstable allocation (which ultimately leads to the same result). For any such transition, the veto player will not get enough votes, so $(1,1,1,0;12)$ *is stable. Notice, however, that* $(1,1,1,1;11)$ *is unstable: the veto player can expropriate one of the non-veto ones and gain consent of the remaining three as they know that there will be no further reallocations after the initial expropriation. One can prove that allocation* $(x_1, x_2, x_3, x_4; x_5)$ *is stable if and only if among the four non-veto agents, three have the same wealth (positive or zero), and the remaining agent has zero.*

This simple example points out to an important phenomenon, which is much more general. Protection of property rights does not emerge from formal set of rules, but from equilibrium behavior of economic agents, whose commitment to protection of property rights hinges on rational foresight.

The logic of endogenous veto power explains how a small change in formal political rules might lead to a dramatic reallocation. (This might help to explain why those in power often oppose even small concessions to political opposition. E.g., many monarchs from Charles I of England to Nicholas II of Russia would not concede a part of their formidable formal power until the very end.) Our next example demonstrates the workings of this logic formally. A small change in political rights, an addition of a veto player, drastically change the equilibrium: the process of redistribution will involve the majority of agents (with about one half of agents being expropriated) and the most part of the society's wealth.

Example 2 *Now, let us modify Example 1. There are again five agents, and three votes are required to make a change, but now there are* two veto *players instead of one, agents #4 and #5. If only these two players may make a proposal, all allocations of the type* $(0,0,0;x_4,x_5)$ *are stable, and those of the type* $(x_1,x_2,x_3;x_4,x_5)$*, where one or two out of* x_1, x_2, x_3 *are zeroes, are unstable (the two veto players will get the vote of one agent with zero and redistribute the assets of the remaining two agents). But then any allocation such that* $x_1 = x_2 = x_3 = 1$ *is stable, because, within the realm of stable states, it is impossible to improve the welfare of the veto players without hurting all of non-veto players. Proceeding inductively, one can prove that an allocation is stable if and only if* $x_1 = x_2 = x_3$*. Thus, the allocation* $(1,1,0;1,12)$ *with two veto players, players #4 and #5, is unstable as* $x_1 = x_2 \neq x_3$*. The addition of a new veto player (agent #4) leads to expropriation of agents #1 and #2: if there is an offer to move to* $(0,0,0;3,12)$*,* $(0,0,0;2,13)$*, or* $(0,0,0;1,14)$*, there will be the majority of three, including both veto players, in support of this motion.*[10]

We see here an interesting phenomenon. The naive intuition would suggest that giving one

[10] We ignore for the moment the possibility that agent #5 might block moving to $(0,0,0;2,13)$ in hope that the eventual outcome is $(0,0,0;1,14)$ which player #5 prefers.

extra player (agent #4 in this example) veto power would make it more difficult for agent #5 to expropriate the rest of the group. Our analysis, however, shows that this is not the case. On the contrary, the introduction of a new veto player breaks the stable coalition of non-veto players, and makes #5, the agenda-setter, more powerful. Beforehand, non-veto players sustained an equal allocation, precisely because they were individually more vulnerable. With only one veto player and an equal allocation for agents #2, #3, and #4, the three non-veto players were an endogenous veto group, which blocked any transition that hurts the group as a whole (even one of them). Here, an extra veto player makes expropriation more, not less, likely.

There is another important observation. With one veto player, #5,, allocation $(1,1,1,0;2)$ is stable. Adding one veto player (let it be agent #3) makes allocation $(1,1,0;1,2)$ unstable as agent #3 (note that the order of individual amounts listed in the allocation vector changed) would now be willing to join $\#4-5$ in expropriating $\#1-2$. The critical role is played by #4 (formerly, agent #3) : previously, he was opposing expropriation of $\#1-2$ as he would have been vulnerable in the next round. Once he is safe as a veto player, he drops opposition to expropriation of $\#1-2$. A resulting stable allocation is $(0,0,0;x,y)$, where $x \geq 1$, $y \geq 2$, and $x+y=5$. Both the amount of wealth being redistributed (3 units out of the total 5) and the number of agents affected by expropriation is significant. The number of agents who stand to lose is two, which close to half of agents, and the amount of wealth redistributed by voting exceeds three quarters of the total wealth.

Example 3 *As above, consider an economy with five agents, which makes redistributive decisions by majority, and one of which (#5) has veto power. With one veto player, #5, allocation* $(1,1,1,0;5)$ *is stable. Now, instead of a change in the number of veto agents, consider a change in supermajority requirement. If a new rule requires four votes, rather than three, the status quo allocation becomes unstable. Instead,* $(1,1,0,0;3)$ *becomes stable, and this move is supported by coalition of 4 agents out of 5. (The veto-player, #5, benefits from the move, #4 is indifferent as he gets* 0 *in both allocations, and* $\#1-2$ *support this move as they realize that with the new supermajority requirement they form a group which is sufficient to protect its members against any expropriation.) Thus, an increase in supermajority results in expropriation and redistribution.*

In the above example the results are again somewhat counterintuitive. The naive logic suggests that such measure as raising the supermajority requirement should strengthen property rights. As Example 3 demonstrates, this might produce the opposite: a number of agents are expropriated. Proposition 5 below establishes that this phenomenon, as well the one discussed in Example 2, is generic: adding a veto plater or raising the supermajority requirement almost always leads to a wave of redistribution.

To provide a general characterization of politically stable allocations of wealth, we use the technique developed in Acemoglu, Egorov, and Sonin (2012) and Acemoglu, Egorov, and Sonin (2009). In particular, we demonstrate that the (Markov perfect) political equilibrium is a unique outcome of a non-cooperative voting game, either with simultaneous, or sequential voting. We show that all the major results hold in the cases where there are one or more veto players. Intuitively, the existence of a stable equilibrium and a recursive algorithm to calculate it corresponds to the von Neumann stability concept.[11]

Our results also provide a new perspective on the role of veto players on policy stability. The existing literature has focused on the role of veto players in a static environment. One approach has conceptualized veto players as constraints on majority rule in a social choice theoretic environment (e.g. Tsebelis 2002). Another approach has adapted the sequential bargaining approach developed by Baron and Ferejohn (1989). Examples include Diermeier and Myerson (1994), McCarthy (2000), Cameron and McCarthy (2004), and Persson, Roland, and Tabellini (1997, 2000). The policy environments in these models, however, static: once a decision is made it is not revised and the chosen policy does not influence future decisions.

Tsebelis (2002) summarizes the main insights of the standard (static) veto players theory on policy consequences as follows: "The veto players theory expects policy stability (impossibility of significant change of the status quo) to be caused by many veto players, by big ideological distances among them, by high qualified majority thresholds (or equivalents) in any collective veto player."[12] Keefer (2004) asserts that "as the number of veto players increases, independent of their preferences, their incentives to offer favors to special interests diminishes," and corroborates this with empirical evidence.

Our approach shows that a dynamic perspective may lead to a more subtle understanding of veto players. One the one hand, endogenous veto groups may protect each other in equilibrium even in the absence of formal veto rights. One the other hand, adding more veto players may lead to more instability and policy change if such additions upset dynamic equilibria where agents mutually protect each other.

The remainder of the paper is organized as follows. Section 2 introduces our general model.

[11] Our analysis contains the results of Diermeier and Fong (2011) and Diermeier and Fong (2012) as a special case. Diermeier and Fong considered a single persistent proposer, who also, by definition, was the single veto player. We separate proposer and veto power, which allows us to study the effects of changing the number of veto players and super-majority rules paper hold if we relax the assumption that there is a single proposer.

[12] Spiller and Tomassi (2008, in Handbook of NIE) argue that "The main deterrent to stable policies is that government controls both the administrative and the legislative processes. Thus, political changes that bring about a change in government can also bring about legislative changes. By having few institutional checks and balances, such systems have an inherent instability that raises questions about their ability to provide regulatory commitment."Making a more general point, Allston and Mueller (2008, in Handbook of NIE) state: "A set of universally shared beliefs in a system of checks and balances is what separates populist democracies from democracies with respect for the rule of law."

In Section 3, we establish the existence of (pure-strategy Markov perfect) equilibrium in a non-cooperative game and provide full characterization of stable wealth allocations. Section 4 focuses on the impact of changes in the number of veto players or supermajority requirements. Section contains an extension, in which legislative decision over wealth distribution is preceded by an investment stage.

2 Setup

Consider a set N of $n = |N|$ political agents who have to choose an outcome out of a finite set $\mathbb{A}$ of feasible allocations. We assume that $\mathbb{A} \subset \mathbb{R}^n$, and we will use lower indices (x_i) to denote the amount that individual i gets under allocation $x \in \mathbb{A}$. Throughout, we focus on the case where agents redistribute b indivisible objects, so

$$\mathbb{A} = \left\{ x \in (\mathbb{N} \cup \{0\})^n : \sum\nolimits_{i=1}^{n} x_i = b \right\}.$$

Time is discrete and indexed by $t > 0$, and in each period there is a status quo $x^t \in \mathbb{A}$. The initial status quo x^0 is given exogenously, while x^t for $t \geq 1$ is determined through a voting procedure in period t. More precisely, in period t an alternative x defeats x^{t-1} and becomes x^t if it gains the support of a winning coalition of agents.[13] In each period t, each agent i gets instantaneous utility x_i^t and acts as to maximize his continuation utility

$$U_i^t = x_i^t + \sum_{j=1}^{\infty} \beta^j x_i^{t+j},$$

where $\beta \in (0, 1)$ is a common discount factor (we would be interested in cases where β is close to 1, meaning that individuals are sufficiently forward-looking).

To define which coalitions are powerful enough to redistribute, we use the language of winning coalitions. Let $V \subset N$ be a non-empty set of veto players (denote $v = |V|$; without loss of generality, let us assume that V corresponds to the last v agents $n - k + 1, \dots n$), and let $k \in [v, n]$ be a positive integer. A coalition W is winning if and only if (a) $V \subset W$ and (b) $|W| \geq k$; the set of winning coalitions is denoted by $\mathcal{W}$:

$$\mathcal{W} = \left\{ X \in 2^N \setminus \varnothing : |X| \geq k \text{ and } V \subset X \right\}.$$

In this case, we say that the society is governed by a k-rule with veto players V, meaning that a transition is successful if it is supported by at least k players and no veto player opposes it.

[13] Acemoglu, Egorov and Sonin (2012) consider an environment where political power may depend on the state. The characterization results in this paper are made possible by keeping power allocation (i.e. the set of all possible winning coalitions in each state) constant, which is natural in legislative bargaining context.

We will compare the results for different k and V. We maintain the assumption that there is at least one veto player—that V is non-empty—throughout the paper; this helps us capture various political institutions, e.g. a supreme court, but it is also helpful in ruling out cycles. We do no require that $k > n/2$, so we allow for minority rules. For example, 1-rule with the set of veto players $\{i\}$ is a dictatorship of legislator i.

The timing of the game below uses the notion of a protocol (see, e.g., Acemoglu, Egorov, and Sonin, 2012, or Ray and Vohra, 2013). A protocol is a sequence (permutation) of legislators who are able to make proposals. We assume that only veto players are able to make proposals,[14] so the sequence is described by the order of veto players, $\pi_1, \ldots, \pi_v$ (e.g., according to seniority). The protocol is the same for all states and defines both the sequence in which legislators make proposals and vote (both assumptions are non-consequential for the results). More precisely, the timing of the game in period $t \geq 1$ is the following.[15]

1. For $j = 1$, legislator π_j is recognized as an agenda-setter as described by the protocol $(\pi_1, \ldots, \pi_v)$, and either proposes an alternative $z^j \neq x^{t-1}$ or passes (in which case we write $z^j = \varnothing$).
2. If legislator j passed and $j < v$, the game moves to stage 1 with j increased by 1, and if $j = n$, then $x^t = x^{t-1}$ and the game moves to stage 5.
3. If $z_j \neq \varnothing$, then all legislators vote, in a sequence given by protocol π, *yes* or *no*.
4. If the set of those who voted *yes*, $Y^j \in \mathcal{W}$, then the new allocation is $x^t = z^j$, otherwise $x^t = x^{t-1}$.
5. Each legislator i gets instantaneous payoff x_i^t.

The equilibrium concept we use in this game of full information is Markov Perfect equilibrium in pure strategies, with the additional requirement: we impose that when a legislator is indifferent (between continuation payoffs given by the two subgames), he votes *yes*, and when he is indifferent between making a proposal and not, he does not.[16] One can interpret it as that proposing a redistribution, as well as resisting a redistribution, is slightly costly, but passively approving it is not. We do not model cost of redistribution explicitly to simplify exposition and to save on notation.

[14] This assumption simplifies the analysis and formal statements considerably, but it is not necessary for the main results in this paper.

[15] There are many game forms that would yield to identical results. For example, we could have each agenda-setter nominate an alternative and then proceed to choosing one that will be put for a vote against the status-quo. To simplify the exposition and proofs, we opted for a simpler game.

[16] This assumption is technical and allows for a simpler characterization of stable allocations. Without it, the main insights on stability and the effects of majority requirements would still hold, but would be less transparent.

3 Analysis

Our strategy is as follows. We start by proving some basic results about equilibria of the non-cooperative game described above. Then, we characterize stable allocations, i.e. allocations with no expropriations, and demonstrate that the stable allocations correspond to equilibria of the non-cooperative game. We then proceed to studying comparative statics with respect to the number of veto players, different voting rules (majority requirements), and equilibrium paths that follow an exogenous shock to some agent's wealth.

3.1 Non-cooperative Characterization

Every pure-strategy Markov Perfect Equilibrium (MPE) σ of the non-cooperative game gives rise to a transition mapping $\phi = \phi_\sigma$ (we will skip σ whenever it leads to no ambiguity). This mapping is defined as follows: If x^t is the allocation in period t, then the allocation in period $t+1$ determined by the profile of equilibrium strategies σ is $x^{t+1} = \phi\left(x^t\right)$. The Markovian property ensures that this transition mapping is well-defined. The usefulness of mapping ϕ is twofold: first, it allows us to capture equilibrium paths in terms of allocations and transitions rather than individuals' agenda-setting and voting strategies (i.e., more concisely), and second, it allows continuation utilities to be written in a very simple form. If allocation x^t is chosen in period t, then continuation utility of legislator i from period t on is

$$U_i^t\left(x^t\right) = x^t + \sum_{j=1}^{\infty} \beta^j \left[\phi^j\left(x^t\right)\right]_i .$$

Iterating the mapping ϕ gives a sequence of mappings $\phi, \phi^2, \phi^3, \ldots$, which must converge if ϕ is acyclic. (Mapping ϕ is *acyclic* if $x \neq \phi(x)$ implies $x \neq \phi^\tau(x)$ for any $\tau > 1$; we will show that every MPE satisfies this property.) Denote this limit by ϕ^∞, which is simply ϕ^{t_0} for some t_0 as the set $\mathbb{A}$ is finite. We say that mapping ϕ is *one-step* if $\phi = \phi^\infty$ (this is equivalent to $\phi = \phi^2$), and we call MPE σ *simple* if ϕ_σ is one-step. Given an MPE σ, we call allocation x *stable* if $\phi_\sigma(x) = x$. Naturally, ϕ_σ^∞ maps any allocation into a stable allocation.

Our first result deals with existence of an equilibrium. The following theorem shows that an MPE exists for any protocol when the discount factor β is sufficiently high. More importantly, it demonstrates formally that our approach is essentially protocol-free and justifies our focus on simple MPE: for any two protocols, the fixed point of an equilibrium transition mapping under one protocol is a reshuffling of a fixed point of the equilibrium transition mapping of the other protocol.

Proposition 1 *There exists $\beta_0 < 1$ such that for any discount factor $\beta \in (\beta_0, 1)$:*

1. *For any protocol π, there exists a Markov Perfect Equilibrium σ. Moreover, there is a simple MPE.*

2. *Every MPE σ is acyclic. If a transition mapping χ is the limit of transition iterations under some MPE σ (i.e., $\chi = \phi_{\sigma}^{\infty}$ for some σ), then there is a simple MPE σ' such that $\chi = \phi_{\sigma'}$.*

Apart from existence, this result implies that we can, without any loss of generality, focus on simple equilibria. Indeed, they always exist (part 1) and they result in the same ultimate allocations that an MPE can support (part 2); moreover, we show in the Appendix that the set of mappings that may be supported by a simple MVE does not depend on the protocol σ. The proof is technically cumbersome and is relegated to the Appendix.

Proposition 1 shows existence of an equilibrium. The following Example 4 demonstrates that the equilibrium is not necessarily unique: there may be multiple equilibria (which are the same for any protocol).

Example 4 *Suppose there are $b = 3$ units of good available, four agents, the required number of votes is $k = 3$, and the set of veto players is $V = \{\#4\}$. In this case, there is a simple equilibrium with transition mapping ϕ, under which allocations $(3, 0, 0, 0)$, $(1, 1, 0, 1)$, $(1, 0, 1, 1)$ and $(1, 1, 1, 0)$ are stable, any allocation $\phi(0, 1, 0, 2) = \phi(0, 2, 1, 0) = \phi(0, 1, 2, 0) = \phi(1, 1, 1, 1) = (1, 1, 1, 0)$ and any allocation with $x_1 = 2$ has $\phi(x) = (3, 0, 0, 0)$. However, another mapping ϕ' coinciding with ϕ except that $\phi'((0, 1, 0, 2)) = (2, 0, 1, 0)$ may also be supported in equilibrium.*

3.2 Stable Allocations

Our next goal is to get a more precise characterization of equilibrium mappings and stable allocations. Let us define a binary relation $\rhd$ (interpreted as "dominance" relation) on $\mathbb{A}$ as follows:

$$x \rhd y \Longleftrightarrow \{i \in N : x_i \geq y_i\} \in \mathcal{W} \text{ and } x_j > y_j \text{ for some } j \in V.$$

Intuitively, allocation x dominates allocation y if some agenda-setter strictly prefers x to y so as to be willing to make this motion, and there is a winning coalition that (weakly) prefers x to y. Note that this does not imply that x will be proposed or supported in an actual voting against y because of further changes this move may lead to. Following the standard definition (von Neumann and Morgenstern, 1947, Greenberg, 1990), we call a set of states $\mathbf{S} \subset \mathbb{A}$ *von Neumann-Morgenstern- (vNM-)stable* if the following two conditions hold: (i) For no states $x, y \in \mathbf{S}$, it holds that $y \rhd x$ (*internal stability*); and (ii) For each $x \notin \mathbf{S}$, there exists $y \in \mathbf{S}$ such that $y \rhd x$ (*external stability*).

Below, we prove that in our case, the vNM-stable set is unique, and corresponds to fixed points of transition mappings of non-cooperative equilibria described in Proposition 1. Let us denote $q = k - v$, the number of non-veto players that is required in any winning coalition, $d = m - q + 1 = n - k + 1$, the size of a minimal blocking coalition of non-veto players, and, finally, $r = \lfloor m/d \rfloor$, the maximum number of pairwise disjoint blocking coalition that non-veto players may be split into.

Proposition 2 *1. For the binary relation $\rhd$, a vNM-stable set exists and is unique.*

2. Each element x of this set $\mathbf{S}$ has the following structure: the set of non-veto players $M = N \setminus V$ may be split into a disjoint union of r groups $G_1, \ldots, G_r$ of size d and one (perhaps empty) group G_0 of size $m - rd$, such that inside each group, the distribution of wealth is equal: $x_i = x_j = x_{G_k}$ whenever $i, j \in G_k$ for some $k \geq 1$, and $x_i = 0$ for any $i \in G_0$.[17] In other words, up to a permutation of members in M:

$$x \in \mathbf{S} \Leftrightarrow x = \left(\underbrace{\lambda_1, \ldots, \lambda_1}_{d \text{ times}}, \underbrace{\lambda_2, \ldots, \lambda_2}_{d \text{ times}}, \ldots, \underbrace{\lambda_r, \ldots, \lambda_r}_{d \text{ times}}, \underbrace{0, \ldots, 0}_{m-rd \text{ times}}, \underbrace{x_{m+1}, \ldots, x_n}_{\text{veto players}} \right).$$

3. There exists $\beta_0 < 1$ such that for any discount factor $\beta \in (\beta_0, 1)$ and any MPE σ, $\phi_\sigma(x) = x \Leftrightarrow x \in \mathbf{S}$.

The proof of Parts 1 and 2 is important for understanding the structure of endogenous veto groups, and we prove it in the text (the proof of Part 3 is technical, and we relegate it to the Appendix). We show that starting from any wealth allocation $x \in \mathbf{S}$, it is impossible to redistribute the units between agents without making at least d agents worse off, and thus no redistribution would gain support from a winning coalition. In contrast, starting from any allocation $x \notin \mathbf{S}$, such redistribution is possible. Furthemore, our proof will show that in any transition, the number of individuals who are worse-off as a result is limited to the $d - 1$ richest non-veto players.

Proof of Proposition 2, Parts 1 and 2. We will prove that set $\mathbf{S}$, as defined in Part 2, is vNM-stable, thus ensuring existence. (For a finite To show internal stability, suppose that $x, y \in \mathbf{S}$ and $y \rhd x$, and let the r groups be $G_1, \ldots, G_r$ and $H_1, \ldots, H_r$, respectively. Without loss of generality, we can assume that each set of groups is ordered so that x_{G_j} and y_{H_j} are nonincreasing in j for $1 \leq j \leq r$. Let us prove, by induction, that $x_{G_j} \leq y_{H_j}$ for all j.

[17] It is permissible that two groups have equal allocations, $x_{G_j} = x_{G_k}$, or that members of some or all groups get zero. In particular, any allocation x where $x_i = 0$ for all $i \in M$ is in $\mathbf{S}$. Notice that if non-veto players get the same under two allocations x and y, so $x|_M = y|_M$, then $x \in \mathbf{S} \Leftrightarrow y \in \mathbf{S}$; moreover, this is true if $x_i = y_{\pi(i)}$ for all $i \in M$ and some permutation i on M.

The induction base is as follows. Suppose that the statement is false and $x_{G_1} > y_{H_1}$; then $x_{G_1} > y_s$ for all $s \in M$. This yields that for all agents $i \in G_1$, we have $x_i > y_i$. Since the total number of agents in G_1 is d, G_1 is a blocking coalition, and therefore it cannot be true that $y_j \geq x_j$ for a winning coalition, contradicting that $y \rhd x$.

For the induction step, suppose that $x_{G_l} \leq y_{H_l}$ for $1 \leq l < j$, and also assume, to obtain a contradiction, that $x_{G_j} > y_{H_j}$. Given the ordering of groups, this means that for any l, s such that $1 \leq l \leq j$ and $j \leq s \leq r$, $x_{G_l} > y_{H_s}$. Consequently, for agent $i \in \bigcup_{l=1}^{j} G_l$ to have $y_i \geq x_i$, he must belong to $\bigcup_{s=1}^{j-1} H_s$. This implies that for at least $jd - (j-1)\, d = d$ agents in $\bigcup_{l=1}^{j} G_l \subset M$, it cannot hold that $y_i \geq x_i$, which contradicts the assumption that $y \rhd x$. This establishes that $x_{G_j} \leq y_{H_j}$ for all j, and therefore $\sum_{i \in M} x_i \leq \sum_{i \in M} y_i$. But $y \rhd x$ would require that $x_i \leq y_i$ for all $i \in V$ with at least one inequality strict, which implies $b = \sum_{i \in N} x_i < \sum_{i \in N} y_i = b$, a contradiction. This proves internal stability of set $\mathbf{S}$.

Let us now show that the external stability condition holds. To do this, we take any $x \notin \mathbf{S}$ and will show that there is $y \in \mathbf{S}$ such that $y \rhd x$. Without loss of generality, we can assume that x_i is nonincreasing for $1 \leq i \leq m$ (i.e., non-veto players are ordered from richest to poorest). Let us denote $G_j = \{(j-1)\, d + 1, \ldots, jd\}$ for $1 \leq j \leq r$ and $G_0 = M \setminus \left(\bigcup_{j=1}^{r} G_j \right)$. Since $x \notin \mathbf{S}$, it must be that either for some G_j, $1 \leq j \leq r$, the agents in G_j do not get the same allocation, or they do, but some individual $i \in G_0$ has $x_i > 0$. In the latter case, we define y by

$$y_i = \begin{cases} x_i & \text{if } i \leq dr \text{ or } i > m+1; \\ 0 & \text{if } dr < i \leq m; \\ x_i + \sum_{j \in G_0} x_j & \text{if } i = m+1 \end{cases}$$

(in other words, we take everything possessed by individuals in G_0 and distribute it among veto players, for example, giving everything to one of them). Obviously, $y \in \mathbf{S}$ and $y \rhd x$.

If there exists a group G_j such that not all of it members have the same amount of wealth, let j be the smallest such number. For $i \in G_l$ with $l < j$, we let $y_i = x_i$. Take the first $d-1$ members of group G_j, $Z = \{(j-1)\, d + 1, \ldots, jd - 1\}$. Together, they possess $z = \sum_{i=(j-1)d+1}^{jd-1} x_i > (d-1)\, x_{jd}$ (the inequality is strict precisely because not all x_i in G_j are equal). Let us now take these z units and redistribute it among all the agents (perhaps including those in Z) in the following way. For each $s : j < s < r$, we let $y_{(s-1)d} = y_{(s-1)d+1} = \cdots = y_{sd-1} = x_{(s-1)d}$; this makes these d agents having the same amount of wealth and being weakly better off as the agent with number $(s-1)\, d$ was the richest among them.

Now, observe that in each group s, we spend at most $(d-1) \left(x_{(s-1)d} - x_{sd-1} \right) \leq (d-1) \left(x_{(s-1)d} - x_{sd} \right)$. For $s = r$, we take d agents as follows: $D = \{(r-1)\, d, \ldots, m\} \cup Z'$, where $Z' \subset Z$ is a subset of the first $d - (m - (r-1)\, d + 1) = rd - m - 1$ agents needed to make D a collection of exactly d agents (notice that $Z' = \varnothing$ if $|G_0| = d-1$ and $Z' = Z$ if $G_0 = \varnothing$). For all

$i \in D$, we let $y_i = x_{(r-1)d}$ (thus making all members of G_0 weakly better off and spending at most $(d-1)\, x_{(r-1)d}$ units) and we let $y_i = 0$ for each $i \in Z \setminus Z'$. We have thus defined y_i for all $i \in M$ and distributed $c \leq (d-1)\left(x_{jd} - x_{(j+1)d} + \cdots + x_{(r-2)d} - x_{(r-1)d} + x_{(r-1)d}\right) = (d-1)\, x_{jd}$, thus having $z - c > 0$ remaining in our disposal. As before, we let $y_{m+1} = x_{m+1} + z - c$ and $y_i = x_i$ for $i > m+1$. We have thus constructed $y \in \mathbf{S}$ such that $y_{m+1} > x_{m+1}$ and $\{i \in N : y_i < x_i\} \subset Z$. The latter, given $|Z| \leq d-1$, implies $\{i \in N : y_i \geq x_i\} \in \mathcal{W}$, which means $y \rhd x$. This completes the proof of external stability, and thus $\mathbf{S}$ is vNM-stable.

Let us now show that $\mathbf{S}$ is a unique stable set defined by $\rhd$. Suppose not, so there is $\mathbf{S}'$ that is also vNM-stable. Let us prove that $x \in \mathbf{S} \Leftrightarrow x \in \mathbf{S}'$ by induction on $\sum_{i \in M} x_i$. The induction base is trivial: if $x_i = 0$ for all $i \in M$, then $x \in \mathbf{S}$ by definition of $\mathbf{S}$. If $x \notin \mathbf{S}'$, then there must be some y such that $y \rhd x$. But for such y, $b = \sum_{i \in N} y_i \geq \sum_{i \in V} y_i > \sum_{i \in V} x_i = \sum_{i \in N} x_i = b$, a contradiction.

The induction step is as follows. Suppose that for some x with $\sum_{i \in M} x_i = j > 0$, $x \in \mathbf{S}$ but $x \notin \mathbf{S}'$ (the vice-versa case is treated similarly). By external stability of $\mathbf{S}'$, $x \notin \mathbf{S}'$ implies that for some $y \in \mathbf{S}'$, $y \rhd x$, which in turn yields that $\sum_{i \in V} y_i > \sum_{i \in V} x_i$. We have $\sum_{i \in M} y_i = b - \sum_{i \in V} y_i < b - \sum_{i \in V} x_i = \sum_{i \in M} x_i = j$. For y such that $\sum_{i \in M} y_i < j$ induction yields that $y \in \mathbf{S} \Leftrightarrow y \in \mathbf{S}'$, and thus $y \in \mathbf{S}$. Consequently, there is $y \in \mathbf{S}$ such that $y \rhd x$, but this contradicts $x \in \mathbf{S}$. This contradiction establishes uniqueness of the stable set.■

The characterization of stable allocations obtained in Proposition 1 gives several important insights. First, the set of stable allocations (fixed points of any transition mapping under any MPE) does not depend on the mapping; it maps into itself when either veto players V or non-veto players $N \setminus V$ are reshuffled in any way. Second, the allocation of wealth among veto players does not have any effect on stability of allocations. Third, each stable allocation has a well-defined "class" structure: every non-veto player with a positive allocation is part of a group of size d (or a multiple of d) of equally-endowed individuals who have incentives to protect each other's interests.

To demonstrate how this protection works, consider the following example.

Example 5 *There are $b = 15$ units. Take $n = 5$ individuals with one veto agent (#5) and a supermajority requirement $k = 4$ (i.e., a supermajority of four-fifths is needed for a transition). Then $d = 2$ and $r = 2$, so stable allocations have two groups of size two. In particular, allocation $x = (4, 4, 2, 2; 3)$ is stable. Let ϕ be a transition mapping for some simple MPE σ.*

Suppose that we, outside of the game, remove a unit from agent #2 and give it to the veto player; i.e., consider $y = (4, 3, 2, 2; 4)$. This resulting allocation is unstable, and agent #1 will necessarily be expropriated. The way redistribution may take place is not unique; for example,

$\phi(y) = (3,3,2,2;5)$ is natural, but $\phi(y) = (2,3,3,2;5)$ or $\phi(y) = (2,3,2,3;5)$ are, in fact, natural as well. Note that the allocation structure is still unique up to a re-indexing of agents.

Now suppose that one of the agents possessing two units, say agent 3, was expropriated, i.e., take $z = (4,4,1,2;4)$. Then it is possible that the other member, agent 4, would be expropriated as well: $\phi(y'') = (4,4,1,1;5)$. But it is also possible that one of the richer agents may be expropriated instead: e.g., a transition to $\phi(z) = (4,1,1,4;5)$ would be supported by all agents except agent #2.

Example 5 demonstrates that equilibrium protection that agents provide to each other extends beyond members of the same wealth group. In the latter case, agent #2 would oppose a move from $(4,4,2,2;3)$ to $(4,4,1,2;4)$ if the subgame the next move is to $(4,1,1,4;5)$.

We see that in general, an exogenous shock may lead to expropriation, on the subsequent equilibrium path, of agents belonging to different wealth groups; the particular path depends on the equilibrium mapping, which is not unique. However, if we apply the refinement that only equilibria with a "minimal" (in terms of the number of units that need to be transferred) redistribution along the equilibrium path are allowed, then only the agents with exactly the same wealth endowment would suffer from the redistribution that follows a shock.

Also, Example 5 demonstrates that if a non-veto player becomes poorer, at least $d-1$ agents other agents would suffer in the subsequent redistribution. This is makes them willing to oppose any redistribution from any of their members. Their number, if we include the initial expropriation target himself, is d, which is sufficient to block a transition. The next proposition generalizes Example 5 to establish formally this mutual protection result. It also highlights that protection of a non-veto player is more likely to come from equally endowed or richer individuals than from poorer ones. Proposition 3 focuses on equilibrium consequences of a negative shock to some agent's wealth, starting with a stable allocation.

Proposition 3 *Consider any MPE σ and let $\phi = \phi_\sigma$. Suppose that the voting rule is not unanimity ($k < n$), so $d > 1$. Take any stable allocation $x \in \mathbf{S}$, some non-veto plater $i \in M$, and let $y \in \mathbf{A}$ be such $y|_{M\setminus\{i\}} = x|_{M\setminus\{i\}}$ and $y_i < x_i$. Then:*

1. *Player i will never be as well off as before the shock, but he will not get any worse off: $y_i \leq [\phi(y)]_i < x_i$. Furthermore, the number of agents that suffer as a result of a redistribution on the equilibrium path defined by σ is limited:*

$$\left|\left\{j \in M \setminus \{i\} : [\phi(y)]_j < y_j\right\}\right| = d-1;$$

2. *Suppose, in addition, that for any $k \in M$ with $x_k < x_i$, $x_k \leq y_i$ (i.e., the shock did not make agent i poorer than the agents in the next wealth group; e.g., this would always be*

the case if $y_i = x_i - 1$). Then $[\phi(y)]_j < y_j$ implies $x_j \geq x_i$; i.e., in this case, member of poorer wealth groups do not suffer from redistribution.

The essence of Proposition 3 is that following a negative shock to an agent's wealth, at least $d-1$ other agents are expropriated, and agent i never fully recovers. If the shock is relatively minor so the ranking of agent i relative to other wealth groups did not change (weak inequalities are preserved), then it must be equally endowed or richer people who suffer from subsequent redistribution and who therefore have incentives to protect i in the initial stable allocation. This result may be generalized to the case when a negative shock affects more than one (but less than d) non-veto players. It is straightforward when all the affected agents belong to the same wealth group, however this is not necessary for the result to be true. If expropriated agents belong to different groups, the the lower bound of the resulting wealth after redistribution is the amount of wealth that the poorest (after shock) agent possesses, and the number of agents who suffer as a result of the redistribution following the shock is still limited by $d-1$.

Our next step is comparative statics with respect to different voting rules (k and v).

4 Comparing Voting Rules

Suppose that we vary the supermajority requirement, k, and the number of veto players, v. The following result easily follows from the characterisation in Proposition 2.

Proposition 4 *Fix the number of individuals n. The size of each group G_j, $j \geq 1$, is decreasing as the majority requirement k increases. In particular, for $k = v+1$, $d = n-v = m$, and thus all the non-veto players form a single group; for $k = n$ (unanimity rule), $d = 1$, and so each agent can veto any change. The number of groups is weakly increasing in k, from 1 for $k = v+1$ (and 0 for lower k) to m for $k = n$. The size of these groups does not depend on the number of veto players, but as v increases, the number of groups weakly decreases, reaching zero for $v > n-d$.*

The last result is most remarkable. The size of groups does not depend on the number of veto players, but only on the majority requirement, as it determines the minimal size of blocking coalitions. As the majority requirement increases, groups become smaller. This has a very simple intuition: as redistribution becomes harder (it is necessary to get approval of more agents), it takes fewer non-veto players to defend themselves; as such, smaller groups are sufficient. Conversely, the largest group (all non-veto players together) is formed when a single vote from a non-veto player is sufficient for veto players to accept a redistribution; in this case, non-veto players can only keep a positive payoff by holding equal amounts.

Example 6 *Suppose that $k = v + 1$; as before, $d = n - v$. In this case, an allocation x is stable if $x_i = x_j$ for all non-veto players i and j, i.e., if all non-veto players hold the same amount. This is consistent with Proposition 1 of Diermeier and Fong (2009), which shows this result for the special case $n = 3$, $v = 1$. More generally, a single group of non-veto players with positive amount of wealth may be formed if and only if $k - v \equiv q \leq (m + 1)/2$ (in this case, some $n - k + 1$ non-veto players belong to the group and get the same amount, and the rest get zero).*

One important implication of Proposition 4 is that k and v affect the equilibrium heterogeneity of the society, at least for the non-veto players. Indeed, take n large and v small (so that m is large enough to be interesting) and start with the smallest possible value of $k = v + 1$. Then all the non-veto players possess the same allocation in any equilibrium. In other words, all agents, except perhaps those explicitly endowed with veto power, must be equal. If we increase k, then two groups will form, one of which may possess a positive amount, while the rest possesses zero, which is clearly more heterogenous than for $k = v + 1$. If we increase k further beyond $v + (m + 1)/2$, three groups will form, etc. In other words, as k increases, so does the number of groups, which implies that the society becomes less and less homogenous and can support more and more groups of smaller size. We see that in this model, heterogeneity of the society is directly linked to difficulty of expropriation, measured by the degree of majority needed for expropriation or, equivalently, by the minimal size of a coalition that is able to resist attempts to expropriate. If we interpret the equally-endowed groups as economic classes, then we have the following result: the more politically difficult it is to expropriate, the larger is the number of classes in the society that can exist without expropriation.

Proposition 4 dealt with comparing stable allocations for different k and v. We now study whether an allocation that was stable under some rules k and v remains stable if these rules change. For example, suppose that we make an extra individual a veto player (increase v), or increase the majority rule requirement (increase k). A naive intuition would say that in both these cases, individuals would not be worse off from better property rights protection. As the next proposition shows, in general, the opposite is likely to be true.

Proposition 5 *Suppose that allocation x is stable for k ($k < n$) and V ($x \in \mathbf{S}_{k,v}$). Then:*

1. *If we increase the number of veto players by granting an individual $i \notin V$ veto power so that $V' = V \cup \{i\}$, then allocation x remains stable if and only if $x_i = 0$;*

2. *Suppose $k + 1 < n$ and all groups G_j, $j \geq 0$, had different amounts of wealth under x: $x_{G_j} \neq x_{G_{j'}}$ for $j' \neq j$ (and $x|_M \neq 0$). If we increase the majority requirement from k to $k' = k + 1$, and $k' < n$, then x becomes unstable.*

The first part of this proposition suggests that adding a veto player makes an allocation unstable, and therefore will lead to a redistribution hurting some individual. There is only one exception to this rule: if the new veto player had nothing in the allocation, then the allocation will remain stable. On the other hand, if the new veto player had a positive amount of the good, then, while he will be weakly better off from becoming a veto player, there will be at least one other non-veto player who will be worse off. Indeed, removing a member of one of the groups G_j without changing the required sizes of the groups *must* lead to redistribution. This logic would not apply if $V' = N$, so that all agents are now veto players; however, then i would have to be the last non-veto player, and under $k < n$ he would have to get $x_i = 0$ in a stable allocation x. Interestingly, removing a veto player i (making him non-veto) will also make x unstable as long as $x_i > 0$; this is of course less surprising, as this individual may be expected to be worse off.

The second part says that if all groups got different allocations (which is a generic situation), then an increase in k would decrease the required group sizes, leading to redistribution. When some groups have equal amounts of wealth in a stable allocation, then allocation x may, in principle, remain stable. This is trivially true when all non-veto players get zero ($x_i = 0$ for all $i \notin V$), but, as the following Example 7 demonstrates, this is possible in other cases as well.

Example 7 *Suppose $n = 7$, $V = \{\#7\}$, $b = 6$ and the supermajority requirement is $k = 5$. Then $x = (1, 1, 1, 1, 1, 1; 0)$ is a stable allocation, because $d = 3$ and the non-veto players form two groups of size three. If we increase k to $k' = 6$, then x would remain stable, as then $d' = 2$ and x has three groups of size two.*

5 Veto Power and Incentives to Invest

Our analysis so far has focused on the game of pure redistribution. This allowed us to identify two distinctive sources of property rights: legal rights (modeled as exogenous veto power) and equilibrium property rights. In Section 3, we showed that the consequences of these rights might be different: in particular, in response to a shock, agents with equilibrium property rights may suffer, and this makes them to protect each other on the equilibrim path. A natural question is whether equilibrium property rights are different from legal ones when it comes to important economic activities such as production, investment and, ultimately the implications for economic growth. While a comprehensive analysis of these issues is, obviously, beyond the scope of the paper, we argue below that the distinction between two types of property rights has very real implications. To show why, we consider a very simple extension to the game of Section 2. Consider first the following example.

Example 8 *Take four agents* $n = 4$ *and assume that one of them is veto;* $V = \{\#4\}$*; let* $b = 12$*. The supermajority requirement is* $k = 2$*, i.e., it takes the veto player and one non-veto player to redistribute successfully. In this case, the size of the minimal blocking coalition of non-veto players is* $d = 3$*. Start with a stable allocation* $x = (3, 3, 3; 3)$*.*

Suppose that before the redistribution game is played, in period 0*, some agents (a subset* L*) get an opportunity to work and produce: if an individual* i *works, he gets a disutility* $c < 1$ *from labor, but creates* 1 *unit of wealth that initially goes to him:* $x_i^0 = x_i + 1$*. Assume also that agents know who has a production opportunity and who does not.*

The incentives of the veto player #4 are clear: he produces whenever he is given an opportunity to, because he will necessarily keep the product of his labor. Now, consider agent #1. He is only going to keep the product of his labor if the other two non-veto players produce successfully, so that each of them gets 4 *units. Therefore, he will not invest unless* $L \subset \{1, 2, 3\}$*, and even in that case, multiple equilibria are possible: with the three non-veto players investing and with the three of them not investing. The reason for this multiplicity is that under the threat of redistribution, production is a coordination game: a agent can retain what he produces only if all the agents in his group get richer; otherwise he gets expropriated. For example, if #1 is the only one who invested, the allocation is* $x_i^0 = (4, 3, 3; 3)$*, which will lead, after redistrubution, to* $x_i^t = (3, 3, 3; 4)$ *from the next period on. This means that if the opportunity to produce comes to all the agents with equal probability and independently, a veto player is ex ante strictly more likely to invest than a non-veto player – even though they may be equal in terms of the initial endowment.*

Now let us modify the example by increasing the supermajority requirement to $k = 3$*. In that case,* $d = 2$*; let us start with a stable allocation* $\tilde{x} = (4, 4, 0; 4)$*. Again, the veto player makes the efficient decision to produce: he does so whenever he has an opportunity. Agents #1 and #2 produce only if both of them have an opportunity to do so: indeed, if only one agent produces, he is going to be expropriated: e.g., if* $\tilde{x}^0 = (5, 4, 0; 4)$*, then redistribution may lead to* $(4, 4, 0; 5)$ *or* $(0, 4, 4; 5)$*, but in any case agent #1 would not keep the extra unit that he produces. In other words, agents #1 and #2 are again less likely to produce than the veto player; however, in this case, they do not need to have agent #3 producing in order to coordinate on production. This implies that production by non-veto players is a more likely event when property rights are better defended: when it is harder to expropriate, coordination between fewer players is needed for a production decision to be undertaken.*

The intuition developed in Example 8 holds more generally. To study the consequences of different types of property rights, we develop a simple growth model below.

For simplicity and tractability, let us focus on the case where $q \leq (m+1)/2$, so that in any stable allocation, there is only one group of non-veto agents with positive allocation (see Example 6); in addition, assume that there is only one veto player. In this case, the condition $q \leq (m+1)/2$ may then be rewritten as $k \leq n/2+1$. These assumptions ensure that the distribution of wealth is characterized by only two variables: the amount that the veto player possesses and the amount that non-veto players possess.

Assume that in the beginning every period (which starts with allocation x^{t-1} inherited from the previous period) each agent gets an opportunity to produce with probability p, independently across agents and periods; production requires effort $c > 0$. Let L denote the set of agents who have the opportunity to produce. All agents observe who belongs to L and make production decisions ($e_i^t \in \{0,1\}$). After producing and observing the intermediate outcome $\tilde{x}^t$ given by

$$\tilde{x}_i^t = x_i^{t-1} + e_i,$$

they play a within-period redistribution game as in Section 2, with the additional assumption that voting against a proposal costs a agent $\varepsilon \in (0,1)$ (before, agents weakly preferred to agree with proposals, now we assume that they strictly prefer to do so). We are interested in Markov Perfect equilibria of the game.

To obtain a precise characterization, let us furthermore assume that the initial allocation, x^0, is stable in the corresponding game without production, i.e., $x^0 \in \mathbf{S}$, and the group structure among non-veto players is already established, namely, $x_i^0 > 1$ for some $i \in n \setminus V$ (let the set of such non-veto players be G). Finally, let us focus on MPEs with minimal redistribution on the equilibrium path: for every x, the difference $\|\phi_\sigma(x) - x\|$ achieves its minimum among all possible σ.

Example 9 *Suppose there are four agents, one of them (#4) has veto power, with $k=3$ and $b=3$. Then equilibrium with $\phi((2,1,0;0)) = (1,1,0;1)$ (and similar mappings for other allocations of $2,1,0$ among non-veto players) is minimal. On the other hand, ϕ' such that $\phi'((2,1,0;0)) = (0,1,1;1)$ is not minimal.*

Focusing on minimal equilibria appears to be a reasonable equilibrium selection criterion as the actual process of transfering units of wealth might involve some costs. Under the assumptions discussed above, we get the following results.

Proposition 6 *Suppose that the status-quo x^0 is a stable allocation, and let G be the subset of non-veto players with $x_i^0 > 1$. There exists $\beta_0 > 0$ such that for any discount factor $\beta \in (\beta_0, 1)$, there exists threshold $p_0 > 0$ such that for any $p \in (0, p_0)$:*

(i) the veto player (#n) produces whenever he has the opportunity ($i \in L$);

(ii) there always exists a minimal MPE where non-veto players never produce, and in any minimal MPE, non-veto players with zero endowment produce nothing: for $i \notin V$, $x_i^{t-1} = 0$ implies $e_i = 0$.

(iii) in the equilibrium maximizing social welfare, non-veto players from group G produce whenever all of them have this opportunity, i.e., $G \subset L$ implies $e_i = 1$ for $i \in G$.

In other words, veto players make an efficient decision to produce. As for non-veto players, there always exists an equilibrium where they do not invest. They may only invest if all members of their group have an opportunity to do so; only then may they coordinate on investment.

Proposition 6 yields an important conclusion: equilibrium property rights that non-veto players possess create less incentives to invest, and ultimately to inefficient outcomes. For non-veto players whose property rights are just an equilibrium phenomenon, production effectively becomes a game of coordination. Producing alone does not make sense, as the result would be expropriated along the equilibrium path. It is only worth the effort if there are multiple agents that will produce and will be able and willing to protect each other (and obviously, if the result of production was uncertain, the decision to produce would be even harder to coordinate on). As a result, non-veto players produce only if others produce, and therefore they do so rarer than veto players do. It is straightforward to show that in the absence of redistribution, they will be more investment by non-veto players.

Let us now study comparative statics, focusing on the following variable of interest: $g = \max_\sigma \mathbb{E} \sum_{i \in N} e_i$, where the maximum is taken over all minimal MPE (and this maximum does not depend on the period). This g has a natural interpretation as (average) economic growth.

Proposition 7 *Under the assumptions of Proposition 6, in the minimal MPE that maximizes social welfare:*

1. *the growth rate g is increasing in p, the rate at which production opportunities arrive;*

2. *the growth rate g is increasing in k, the degree of property rights protection.*

These comparative statics results are intuitive: the more often individuals have a chance to produce, the faster wealth accumulates; the smaller is the group of agents that need to coordinate, the more likely they are to succeed in that. Yet they have important implications: the harder it is to expropriate, the more likely production or investment decisions are, and the faster the economic growth is. Remarkably, this effect does not result from the actions of agents whose property rights are protected (veto players), but rather from those who only enjoy equilibrium property rights.

We therefore have important takeaways from this analysis. First, property rights have different implications for growth: the individuals who have legal rights are more likely to produce and to accumulate wealth than other agents. Non-veto players who possess property rights in equilibrium also can invest, but do so rarer and only if they are able to coordinate. In other words, while the equilibrium effects we study in this paper are able to maintain a status quo, they do a poorer job promoting growth and investment, especially if expropriation is relatively easy. Second, production decisions by individuals or groups of individuals may be different even if their observed wealth is the same: these decisions depend on whether their property rights are protected by law or are an equilibrium phenomenon maintained by others' fear of being expropriated. These insights may be helpful for understanding allocation of wealth and means of production, economic growth, and structure of society in a historical perspective.

6 Conclusion

The modern political economy literature on checks and balances has mainly concentrated on public spending with less attention being paid to the original motivation of protecting of property rights. The reason may lie in the intuition that the relationship between additional checks and balances and better protection of rights seems so fundamental that there is little left to explain. Yet, from a political economy perspective property rights systems are to be understood as equilibrium outcomes rather than exogenous fixed constraints. Legislators cannot commit to entitlements, prerogatives, and rights. Rather, any allocation must be maintained in equilibrium. By varying the characteristic of the political institutions (checks and balances) one can assess the consequences for economic institutions (property rights).

It is often argued that additional checks and balances provide economic subjects with protection from expropriation of their wealth of productive capital. In this paper, we demonstrate that this is not necessarily true in a setting, where decisions to redistribute wealth are made by the (super) majority of votes in a legislature, with some agents having veto power over the collective decision. Surprisingly, providing some of the members of the legislature with additional veto power might result in expropriation of other agents. The driving force of this result is that the protection of property rights might be supported by equilibrium behavior of non-veto players, whose opposition to expropriation of others is caused by the fear that those who were expropriated might support the next round expropriation that would affect the current set of decision-makes.

Our results point to the importance of looking beyond formally defined property rights, and more, generally, beyond formal institutions. While formal instiutions provide better incentives for investment and production, the incentives provided by informal (equilibrium) institutions are substantial as well. Thus, a change in formal institutions might strengthen protection of property rights of designated agents, yet have negative consequences for protection of property rights of the others, and, as a result, a negative overall effect.

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Appendix [Incomplete]

Proofs

Our strategy of proof is the following. First, we prove three auxiliary lemmas; then, we finish the proof of Proposition 2, and after that, we prove Proposition 1.

Let $\beta_0 = \left(\frac{|\mathbf{A}|}{|\mathbf{A}|+1}\right)^{\frac{1}{|\mathbf{A}|}}$. We show that for $\beta \in (\beta_0, 1)$, and any acyclic mapping ϕ, if an individual prefers the ultimate allocation $\phi^\infty(x)$ to $\phi^\infty(y)$, then he prefers the continuation utility starting from x to one starting from y.

Lemma 1 *Suppose $\beta \in (\beta_0, 1)$. Then for any acyclic mapping ϕ, any agent $i \in N$ and any allocations $x, y \in \mathbf{A}$, $[\phi^\infty(x)]_i > [\phi^\infty(y)]_i$ implies $U_i(x) > U_i(y)$.*

Proof of Lemma 1. Simple algebra shows that β satisfies $\left(1-\beta^{|\mathbf{A}|}\right)|\mathbf{A}| > \beta^{|\mathbf{A}|}$. Now, since ϕ is acyclic, we have $\phi^\infty = \phi^t$ for $t \geq |\mathbf{A}|$. Using these two results, we get

$$\begin{aligned}
U_i(x) &= \left(x_i + [\phi(x)]_i + \cdots + \left[\phi^{|\mathbf{A}|-1}(x)\right]_i\right) + \frac{\beta^{|\mathbf{A}|}}{1-\beta}[\phi^\infty(x)]_i \\
&\geq \frac{\beta^{|\mathbf{A}|}}{1-\beta}[\phi^\infty(x)]_i > \frac{1-\beta^{|\mathbf{A}|}}{1-\beta}|\mathbf{A}| + \frac{\beta^{|\mathbf{A}|}}{1-\beta}\left([\phi^\infty(x)]_i - 1\right) \\
&\geq \frac{1-\beta^{|\mathbf{A}|}}{1-\beta}|\mathbf{A}| + \frac{\beta^{|\mathbf{A}|}}{1-\beta}[\phi^\infty(y)]_i \\
&\geq \left(y_i + [\phi(y)]_i + \cdots + \left[\phi^{|\mathbf{A}|-1}(y)\right]_i\right) + \frac{\beta^{|\mathbf{A}|}}{1-\beta}[\phi^\infty(y)]_i = U_i(y).
\end{aligned}$$

■

To formulate the next lemma, we introduce the following notation. Take any (not necessarily simple) MPE σ; then for each status quo x there is a set of stages $J_x \subset \{1, \ldots, v\}$ such that if stage $j \in J_x$ is reached, agent $\pi_{x,j}$ proposes an alternative $z_{x,j} \neq \varnothing$, and it is accepted in subsequent voting.

Lemma 2 *Suppose $\beta > \beta_0 = \left(\frac{|\mathbf{A}|}{|\mathbf{A}|+1}\right)^{\frac{1}{|\mathbf{A}|}}$. For any (acyclic) MPE σ and any $x \in \mathbf{A}$:*

1. *if proposal y is accepted at some stage j, if made, then the set of agents $\{i \in N : [\phi_\sigma^\infty(y)]_i \geq [\phi_\sigma^\infty(x)]_i\} \in \mathcal{W}$;*

2. *if proposal y is accepted at some stage j, if made, then for any $i \in V$, $U_i(y) \geq U_i(x)$;*

3. *the set of agents $\{i \in N : U_i(\phi_\sigma(x)) \geq U_i(x)\} \in \mathcal{W}$;*

4. *the set of agents $\{i \in N : [\phi_\sigma^\infty(x)]_i \geq x_i\} \in \mathcal{W}$;*

5. *either $\phi_\sigma^\infty(x) = x$ or $\phi_\sigma^\infty(x) \rhd x$.*

Proof of Lemma 2. Part 1. Suppose not; then take any $i \in N$ for whom the opposite holds: $[\phi_\sigma^\infty(y)]_i < [\phi_\sigma^\infty(x)]_i$. By Lemma 1, this must imply $U_i(y) < U_i(x)$. Hence, the set $\{i \in N : U_i(y) \geq U_i(x)\} \notin \mathcal{W}$. But this implies that y cannot be the outcome of the voting at that stage, which contradicts the assumption.

Part 2. This is trivial: if $U_i(y) < U_i(x)$ and y is accepted in equilibrium, it means that agent i votes Yes, but it would be a best response to vote No and thus veto the proposal, which cannot be true in an equilibrium.

Part 3. This statement is trivial if $\phi_\sigma(x) = x$. Otherwise, $J_x \neq \varnothing$ and at the stage $j = \min J_x$, the proposal $z_{x,j} = \phi_\sigma(x)$ is made and is accepted. Each agent i gets utility $U_i(\phi_\sigma(x))$ if it is accepted and utility $U_i(x)$ if it is not. Assuming, to obtain a contradiction, that $\{i \in N : U_i(\phi_\sigma(x)) \geq U_i(x)\} \notin \mathcal{W}$, would imply that the proposal cannot be accepted in equilibrium; this proves the result.

Part 4. The statement is trivial if $\phi_\sigma(x) = x$, so consider the case $\phi_\sigma(x) \neq x$. Consider a mapping $\phi_0 : \mathbf{A} \to \mathbf{A}$ given by $\phi_0(y) = \phi(y)$ if $y \neq x$ and $\phi_0(x) = x$; since ϕ is acyclic, ϕ_0 is acyclic, too. Consider an individual $i \in N$ such that $[\phi_\sigma^\infty(x)]_i < x_i$. Since $\phi_0^\infty(\phi_\sigma(x)) = \phi_\sigma^\infty(x)$ and $\phi_0^\infty(x) = x$, then applying Lemma 1 to mapping ϕ_0 yields $U_i(\phi_\sigma(x)) < x_i/(1-\beta)$. Rearranging, we have $U_i(\phi_\sigma(x)) < x_i + \beta U_i(\phi_\sigma(x))$, which is equivalent to $U_i(\phi_\sigma(x)) < U_i(x)$. Thus, $\{i \in N : [\phi_\sigma^\infty(x)]_i < x_i\} \subset \{i \in N : U_i(\phi_\sigma(x)) < U_i(x)\}$, and hence $\{i \in N : [\phi_\sigma^\infty(x)]_i \geq x_i\} \supset \{i \in N : U_i(\phi_\sigma(x)) \geq U_i(x)\}$. By Part 3, the latter is winning, which, coupled with monotonicity, completes the proof.

Part 5. If $\phi_\sigma^\infty(x) \neq x$, then the set $J_x \neq \varnothing$. Assume, to obtain a contradiction, that $\phi_\sigma^\infty(x) \ntriangleright x$. Since $\{i \in N : [\phi_\sigma^\infty(x)]_i \geq x_i\} \in \mathcal{W}$ by Part 4, it must be that for all $i \in V$: $[\phi_\sigma^\infty(x)]_i = x_i$. Consider the agent $k = \pi_{\min J_x} \in V$ who makes the proposal at the stage $\min J_x$. Let us prove that $\left[\phi_\sigma^l(x)\right]_k = x_k$ for all $l \geq 1$. Indeed, suppose first that some l, $\left[\phi_\sigma^l(x)\right]_k > x_k$. But then Part 4 (given that $k \in V$) would imply that $[\phi_\sigma^\infty(x)]_k = \left[\phi_\sigma^\infty\left(\phi_\sigma^l(x)\right)\right]_k \geq \left[\phi_\sigma^l(x)\right]_k > x_k$, contradicting the earlier result. Now suppose that $\left[\phi_\sigma^l(x)\right]_k \leq x_k$ for all l, but $\left[\phi_\sigma^l(x)\right]_k < x_k$ for some l. But in that case, $U_k(\phi_\sigma(x)) < x_k/(1-\beta)$, implying $U_k(\phi_\sigma(x)) < U_k(x)$, which contradicts the result of Part 2. Hence, $\left[\phi_\sigma^l(x)\right]_k = x_k$ for all $l \geq 1$. This implies that $U_k(\phi_\sigma(x)) = U_k(x) = x_k/(1-\beta)$.

Consider the utility of agent k if he passes on his chance to propose. If $|J_x| = 1$, then in that stage x remains the status quo, and k gets continuation utility $U'_k = x_k + \beta U_k(x) = x_k + \beta x_k/(1-\beta) = x_k/(1-\beta) = U_k(x)$. If $|J_x| > 1$, then some proposal $y \neq x$ is accepted, and from Part 2 we know that in this case, $U'_k = U_k(y) \geq U_k(x)$. In either case, $U'_k \geq U_k(x)$, so his out-of-equilibrium continuation utility if he passes is at least as high as his utility from making the proposal. In other words, he does not strictly prefer to make the proposal, which is impossible in the set of equilibria that we consider. This contradiction completes the proof. ■

Lemma 3 *Any MPE σ is acyclic.*

Proof of Lemma 3. Part 1. Suppose that there is a cycle starting from x: $\phi_\sigma(x) \neq x$, but $\phi_\sigma^l(x) = x$ for some $l > 1$. Similarly to Part 2 of Lemma 2, we can prove that for any $i \in V$, $U_i\left(\phi_\sigma^{j+1}(x)\right) \geq U_i\left(\phi_\sigma^j(x)\right)$ for any j, which means that $U_i(x) = U_i(\phi_\sigma(x)) = \cdots = U_i\left(\phi_\sigma^{l-1}(x)\right)$, and therefore $x_i = [\phi_\sigma(x)]_i = \cdots = \left[\phi_\sigma^{l-1}(x)\right]_i$. Consider the agent $k = \pi_{\min J_x}$; as in the proof of Part 5 of Lemma 2, we can show that by passing on the opportunity to make his proposal, he can guarantee $U'_k \geq x_k/(1-\beta) = U_k(\phi_\sigma(x)) = U_k(x)$ which he gets if he follows the equilibrium play. But this means that he is not strictly better off by making a proposal, which is impossible. This contradiction completes the proof. ■

Proof of Proposition 2. Part 1, Part 2: Proven in the text.

Part 3. We let $\beta \in (\beta_0, 1)$, with β_0 defined above. Let $D = \{x \in \mathbf{A} : \phi_\sigma(x) = x\}$; we then must prove that $D = S$. To do so, we prove that D is vNM-stable with respect to the binary relation $\rhd$; given the results of Part 1 and Part 2, this will imply that $D = S$. We need to show that D satisfies internal stability and external stability. We also know, by Lemma 3, that ϕ_σ is acyclic, and thus mapping ϕ_σ^∞ is well-defined and D is nonempty.

External stability: Take any $x \notin D$. Let $y = \phi_\sigma^\infty(x) \neq x$ (because ϕ_σ is acyclic). By Part 5 of Lemma 2, $y \rhd x$, which proves external stability.

Internal stability: Suppose, to obtain a contradiction, that for some $x, y \in D$, $y \rhd x$. We know that if the current state is x, no alternative is accepted (because $x \in D$), and thus $|J_x| = \varnothing$. Let us show that this means that at no stage any agent makes a proposal. Indeed, suppose that at stage j, agent π_j makes a proposal that is rejected. Then equilibrium play gives him $U_{\pi_j}(x) = x_{\pi_j}/(1-\beta)$, whereas if he passes, then, by Part 2 of Lemma 2, he will get $U'_{\pi_j} \geq U_{\pi_j}(x)$, which means that proposing is not strictly better then passing. Given our equilibrium refinement, this means that no proposal is made at any stage when the state is x.

Now take $k \in V$ that satisfies $y_k > x_k$ and consider stage j with $\pi_j = k$. If, at this stage, agent k made proposal y, then it would be accepted, because $y \rhd x$ means, in particular, that $\{i \in N : y_i \geq x_i\} \in \mathcal{W}$, and thus $\{i \in N : U_i(y) \geq U_i(x)\} \in \mathcal{W}$ (as, of course, $U_i(y) = y_i/(1-\beta)$ and $U_i(x) = x_i/(1-\beta)$). We showed that in equilibrium, k does not make any proposal in this subgame, and gets $U_k(x)$. But then he would be able to get more by proposing y, as $U_k(y) = y_k/(1-\beta) > x_k/(1-\beta) = U_k(x)$. Hence, he has a profitable deviation in this subgame, which is impossible. This contradiction proves that D is also internally stable. Thus, it is vNM-stable, and given uniqueness, coincides with S. This completes the proof. ■

Proof of Proposition 1. Part 1. Let us prove a stronger result: for any mapping ϕ such that $\phi(x) = x$ for any $x \in \mathbf{S}$ and for any $x \notin \mathbf{S}$, $\phi(x) \in \mathbf{S}$ and $\phi(x) \rhd x$. (Existence of such a mapping follows from external stability of mapping $\mathbf{S}$ implying that for any $\mathbf{S}$ we can pick such $\phi(x) \in \mathbf{S}$; clearly, equilibrium σ with $\phi_\sigma = \phi$ will be simple because $\phi^2 = \phi$.)

To construct σ, take the following strategy profile. First, define continuation utilities of each agent i if the current period ends with state x, $U_i(x) = x_i + \sum_{t=1}^{\infty} \left[\phi^t(x)\right]_i$. If the current state is $x \in \mathbf{A} \setminus \mathbf{S}$, the last agenda-setter i with $[\phi(x)]_i > x_i$ proposes a transition to $\phi(x)$, and all agents $[\phi(x)]_j \geq x_j$ support this proposal while the rest vote against it, so it is accepted in equilibrium. Other agenda-setters — earlier and later — do not propose. (We do not need to define voting strategies for these proposals if they are made, or for any other proposal made by agent i, explicitly: the continuation equilibrium play is defined for any outcome of any given voting, either the proposal y or the status quo x, and thus the rule that agents support the proposal y whenever $U_i(y) \geq U_i(x)$ gives a unique set of strategies.) If the current state is $x \in \mathbf{S}$, then no agenda-setter makes a proposal; again, this defines voting strategies if some proposal is made. This set of strategies (call it σ) is clearly Markovian. We need to check that this is indeed an equilibrium, i.e., that there is no one-step deviation, and that it satisfies the requirements we imposed on agents' actions when they are indifferent.

If strategies in σ are played, mapping $\phi_\sigma = \phi$ is implemented, and therefore continuation utilities are indeed given by $U_i(x)$. This automatically implies that all voting strategies that we specified are best responses; moreover, we required agents to vote Yes whenever they are indifferent. We only need to consider agenda-setting strategies. First, suppose $x \in \mathbf{A} \setminus \mathbf{S}$, and agent j gets a chance to propose after i. By the choice of agent i, it must be that $[\phi(x)]_j \leq x_j$ (in fact, equality must hold), so $U_j(\phi(x)) \leq U_j(x_j)$, and the agent i is not better off making the proposal $\phi(x)$. Suppose he makes some other proposal y; this is a profitable deviation only if y is to be accepted (otherwise, passing yields the same utility). If y is accepted, Part 1 of Lemma 2 also implies that $\{k \in N : [\phi(y)]_k \geq [\phi(x)]_k\} \in \mathcal{W}$ (because mapping ϕ is simple). Since j

is strictly better off from this deviation, it must be that $U_j(y) > U_j(x) = x_j/(1-\beta)$, which means that either $y_j > x_j$ or $[\phi(y)]_j > x_j$; in either case, $[\phi(y)]_j > x_j = [\phi(x)]_j$. Now, we have shown that $\phi(y) \rhd \phi(x)$, but this is impossible for $\phi(x), \phi(y) \in \mathbf{S}$.

Let us now suppose that agent j gets a chance to propose before agent i. Then he is not strictly better off proposing $\phi(x)$, because i would do so along the equilibrium path if j passes. Suppose he proposes some $y \neq \phi(x)$; then again it is only profitable if y passes. For this to be true, it must be that $\{k \in N : [\phi(y)]_k \geq [\phi(x)]_k\} \in \mathcal{W}$. But if j is strictly better off, it must be that $[\phi(y)]_j > [\phi(x)]_j$, again implying $\phi(y) \rhd \phi(x)$, which cannot be true.

Consider the possibility that agent i deviates. He does not want to deviate by passing, since $[\phi(x)]_i > x_i$ and thus $U_i(\phi(x)) > U_i(x)$. Suppose that he deviates to proposing some $y \neq \phi(x)$; again, $\phi(x)$ must be accepted. We analogously get that $\{k \in N : [\phi(y)]_k \geq [\phi(x)]_k\} \in \mathcal{W}$ and $[\phi(y)]_j > [\phi(x)]_j$, leading to the same contradiction. Thus, there is no profitable deviation if $x \in \mathbf{A} \setminus \mathbf{S}$.

Finally, suppose that for some $x \in \mathbf{S}$, some agent $j \in V$ deviates and proposes some $y \neq x$. For this to be profitable, y must be accepted, which would imply, again by Part 1 of Lemma 2, that $\{k \in N : [\phi(y)]_k \geq x_k\} \in \mathcal{W}$. Again, similarly to earlier cases, we must have $[\phi(y)]_j \geq x_j$ if the deviation is to be profitable. But then $\phi(y) \rhd x$ for $x, \phi(y) \in \mathbf{S}$, which is impossible. This contradiction completes the proof that σ is a MPE and that it satisfies the refinements.

Part 2. Acyclicity of any MPE was proved in Lemma 3. Now take mapping $\chi = \phi_\sigma^\infty$ for some σ. From Part 3 of Proposition 2 it follows that $\phi_\sigma(x) = x \Leftrightarrow x \in \mathbf{S}$; since σ is acyclic, $\chi(x) = x \Leftrightarrow x \in \mathbf{S}$. By Part 5 of Lemma 2, $\phi_\sigma^\infty(x) = x$ or $\phi_\sigma^\infty(x) \rhd x$, which implies that for any $x \notin \mathbf{S}$, $\chi(x) \rhd x$. Consequently, mapping χ satisfies the conditions specified in the proof of Part 1, and there is a simple equilibrium σ' with $\phi_{\sigma'} = \chi = \phi_\sigma^\infty$. This proves Part 2. ■

Proof of Proposition 3. Part 1. Lemma 2 implies that $\phi(y) \rhd y$; in particular, for each $j \in V$, $[\phi(y)]_j \geq y_j$ and for at least one of them the inequality is strict. Suppose, to obtain a contradiction, that $\left|\left\{j \in M \setminus \{i\} : [\phi(y)]_j < y_j\right\}\right| < d-1$; then $\left|\left\{j \in M : [\phi(y)]_j < x_j\right\}\right| < d$. But we also have that for each $j \in V$, $[\phi(y)]_j \geq x_j$, with at least inequality strict. This means $\phi(y) \rhd x$, which is impossible, given that $x, \phi(y) \in \mathbf{S}$. Now suppose, to obtain a contradiction, that $\left|\left\{j \in M \setminus \{i\} : [\phi(y)]_j < y_j\right\}\right| > d-1$. But then for at least d agents $[\phi(y)]_j < y_j$, which contradicts $\phi(y) \rhd y$. This contradiction proves that $\left|\left\{j \in M \setminus \{i\} : [\phi(y)]_j < y_j\right\}\right| = d-1$. It remains to prove that $y_i \leq [\phi(y)]_i < x_i$. Suppose not, i.e., either $[\phi(y)]_i < y_i$ or $[\phi(y)]_i \geq x_i$. In the first case, we would have that at least d agents have $[\phi(y)]_j < y_j$, contradicting $\phi(y) \rhd y$. In the second case, $[\phi(y)]_i \geq x_i$, coupled with the already established $\left|\left\{j \in M \setminus \{i\} : [\phi(y)]_j < y_j\right\}\right| = d-1$, would mean $\left|\left\{j \in M : [\phi(y)]_j < x_j\right\}\right| = d-1$, and therefore $\phi(y) \rhd x$. This is impossible, and this contradiction completes the proof.

Part 2. This proof is similar to the proof of internal stability in the proof of Proposition 2. Denote $\phi(y) = z$; then $z \rhd y$ and $x, z \in \mathbf{S}$. We know that x and z have the group structure by Part 2 of Proposition 2; then let the r groups be $G_1, \ldots, G_r$ for x and $H_1, \ldots, H_r$ for z, respectively. Without loss of generality, we can assume that each set of groups are ordered so that x_{G_j} and z_{H_j} are nonincreasing in j for $1 \le j \le r$. Suppose, to obtain a contradiction, that for some agent $i' \in M$ with $x_{i'} \le y_i < x_i$, $z_{i'} < y_{i'}$. In that case, among the set $\{j \in M : x_j \ge x_i\}$ there are at most $d-1$ agents with $z_j < y_j$; similarly, among the set $\{j \in M : x_j < x_i\}$ there are at most $d-1$ agents with $z_j < y_j$.

We can now proceed by induction, similarly to the proof of Proposition 2, and show that $x_{G_j} \le z_{H_j}$ for all j. Base: suppose not, then $x_{G_1} > z_{H_1}$; then $x_{G_1} > z_s$ for all $s \in M$. But this mean that for all agents $l \in G_1$ have $x_l > z_l$; since their total number is d, we get a contradiction. Step: suppose $x_{G_l} \le z_{H_l}$ for $1 \le l < j$, and suppose, to obtain a contradiction, that $x_{G_j} > z_{H_j}$. Given the ordering of groups, this means that for any l, s such that $1 \le l \le j$ and $j \le s \le r$, $x_{G_l} > z_{H_s}$. Consequently, for a agent $i'' \in \bigcup_{l=1}^{j} G_l$ to have $z_{i''} \ge x_{i''}$, he must belong to $\bigcup_{s=1}^{j-1} H_s$. This implies that for at least $jd - (j-1)d = d$ agents in $\bigcup_{l=1}^{j} G_l \subset M$, $z_{i''} \ge x_{i''}$ does not hold (denote this set by D. If that is true, it must be that $\bigcup_{l=1}^{j} G_l$ includes all the agents in D, including agents i and i' found earlier, and in particular, $x_{G_j} \le y_i < x_i$. But on the other hand, these d agents are not in $\bigcup_{s=1}^{j-1} H_s$. In particular, this implies that for any $i'' \in D$, $z_{i''} < x_{G_j}$, but $x_{i'} \ge x_{G_j}$, which means $z_i < x_{i'}$. But $z_i \ge y_i$ by Part 1 of this Proposition, so $y_i < x_{i'}$. But this contradicts with the way we chose i' to satisfy $x_{i'} \le y_i < x_i$. This proves that such i' cannot exist, and thus the $d-1$ agents other than i who are made worse off satisfy $x_j \ge x_i$. ■

Proof of Proposition 4. This result immediately follows from the formulas $m = n - v$, $d = n - k + 1$, $r = \lfloor m/d \rfloor$ and from Proposition 2. ■

Proof of Proposition 5. Part 1. If $k < n$, then $d > 1$. An allocation x is stable only if $|\{j \in M : x_j > 0\}|$ is divisible by d. If x is stable and some agent i with $x_i > 0$ is made a veto agent, then the set $|\{j \in M' : x_j > 0\}| = |\{j \in M : x_j > 0\}| - 1$ and is not divisible by d, thus x becomes unstable. At the same time, if $x_i = 0$, then the group structure for all he groups with a positive amount is preserved, thus x remains a stable allocation.

Part 2. In this case, the size of each group in x is $d > 2$, and every positive amount is possessed by either none or d non-veto players. If k increases by 1, d decreases by 2. Then allocation x becomes unstable, except for the case $x|_M = 0$. ■

Proof of Proposition 6. To be completed. ■

Proof of Proposition 7. From Proposition 6, it follows that

$$g = p + dp^d = p + (n - k + 1)\, p^{n-k+1}.$$

Therefore, g is increasing in p. Consider

$$\frac{(d+1)\, p^{d+1}}{dp^d} = \frac{d+1}{d} p \geq \frac{3}{2} p,$$

because $d \geq 2$. Since we are considering small p, this ratio is less than 1. Consequently, g is decreasing in d, and thus is increasing in k. This completes the proof. ■