The Cost of Human Capital Depreciation during Unemployment

Lien Laureys[1]

Job Market Paper

November 2012

Abstract. There is evidence that workers' skills erode during periods of unemployment. This issue seems of particular concern now that unemployment duration has increased in the Great Recession. I argue that loss of skill during unemployment generates an externality in job creation: firms ignore how their hiring decisions affect the skill composition of the unemployment pool, and hence the output produced by new hires. Overall, job creation is too low, but contrary to conventional wisdom less so in recessions than in booms. The larger share of job-seekers with eroded skills in the unemployment pool in recessions lowers the social cost of having a worker losing her skills because it decreases the expected difference in productivity between a new hire and a job-seeker with eroded skills. As a consequence, everything else equal, loss of skill during unemployment may warrant procyclical employment subsidies.

Keywords: long-term unemployment, skill erosion, inefficiency, optimal labor market policy

JEL Classification: E24, J24, J41, J64.

---

[1] Universitat Pompeu Fabra.

E-mail address: lien.laureys@upf.edu; Contact address: Ramon Trias Fargas 25-27; Telephone: (+34) 93 542 2672.

I am especially grateful to my advisor Jordi Galí for his many helpful comments and guidance throughout. I would also like to thank Regis Barnichon,Vasco Carvalho, Jan Eeckhout, Jonathan Heathcote, Timothy Kehoe, Edouard Schaal, Robert Shimer, Kjetil Storesletten, Thijs van Rens, participants of the CREI-macrobreakfast seminar, the 2011 Workshop on Dynamic Macroeconomics, the EEA Meeting Oslo 2011 and the Annual Search and Matching meeting 2012 in Cyprus for valuable comments and suggestions at various stages of this project. I am also grateful to the University of Minnesota for their hospitality. I acknowledge financial support from the Spanish Ministry of Education.

1 Introduction

"*...One concern we do have, of course, is the fact that more than 40 percent of the unemployed have been unemployed for six months or more. Those folks are either leaving the labor force or having their skills eroded. Although we haven't seen much sign of it yet, if that situation persists for much longer then that will reduce the human capital that is part of our growth process going forward.*" (Ben Bernanke, 2012)[2]

The Great Recession has brought back the specter of long-term unemployment. In the U.S. the average unemployment duration has increased from an average of around 15 weeks in the period 1960-2008 to an average of close to 40 weeks in 2012.[3] This increase has been of great concern to policy makers. One reason is the widely held belief that long unemployment spells lead to the depreciation of a worker's human capital. [4]

But does the presence of skill loss during unemployment call for policy intervention? This paper looks at this issue through the lens of an otherwise standard random search model with aggregate uncertainty in which loss of skill is introduced. Workers who had their skills eroded while being unemployed are less productive upon re-employment than workers whose skills were not affected. At the same time, I allow for learning-by-doing such that workers with eroded skills can regain their initial skill level while being employed.

In the presence of skill loss during unemployment, firms' hiring decisions do not only affect the unemployment rate but also the share of workers with eroded skills in the unemployment pool. Hiring influences workers' chance of finding a job, average unemployment duration, and thus the extent of skill erosion. For example, when firms hire less, unemployed workers have a smaller chance of finding a job, which increases their unemployment duration. Longer unemployment spells in turn raise the probability that their skills erode. As a result, a drop in hiring increases the relative share of job-seekers with eroded skills in the unemployment pool.

The skill composition of the unemployment pool determines how likely it is that job-seekers with or without eroded skills show up for job interviews. Thus, the pool's composition determines the average productivity of job candidates. Consequently, firms' hiring decisions, through their effect on job-seekers' skills, affect the output that can be generated by new matches.

Comparing hiring decisions in the laissez-faire economy and the constrained-efficient economy reveals that skill erosion is a source of inefficiency. This is the consequence of a composition externality related to job creation which arises in addition to the familiar congestion externality following from the search

[2]Question and answer session of the Senate Banking Committee hearing on the 1st of March 2012 with Federal Reserve chairman Ben Bernanke testifying on monetary policy and the U.S. economy: http://www.reuters.com/article/2012/03/01/usa-fed-bernanke-idUSL2E8E13KI20120301.

[3]Source: Bureau of Labor Statistics series LNS13008275.

[4]Empirical evidence for human capital depreciation is provided by e.g. Addison and Portugal (1989), Keane and Wolpin (1997), Gregory and Jukes (2001), and Neal (1995).

frictions. The composition externality arises because firms ignore how their hiring decisions affect the average skills of next period's job-seekers, and hence the output that can be generated by new matches. Firms neglect that through hiring they prevent workers from being unemployed and exposed to skill erosion. At the same time, firms do not take into account that by employing a worker, this worker keeps her skills or regains her skills, and hence that there is an additional job-seeker without eroded skills when the match separates. In other words, when skills erode during unemployment, there are gains from job creation which are not fully internalized.

I analyze if constrained-efficiency can be attained in the absence of policy intervention. Given that I assume that wages are set every period through Nash bargaining, I examine, in the spirit of Hosios (1990), if there exists a parameter condition for workers' bargaining power which restores efficiency. I find this is not the case when aggregate uncertainty is present and workers' bargaining power is constant across states. Thus, policy intervention is required to restore constrained-efficiency.

The optimal labor market policy which offsets the composition externality takes the form of a time-varying employment subsidy. This reflects that job creation in the laissez-faire economy is on average too low. Surprisingly, I find that the optimal subsidy is procyclical. Put differently, when skill erosion during unemployment is the only source of inefficiency employment should be subsidized less in recessions. This finding indicates that the composition externality matters more in booms than in recessions. The intuition behind that finding is the following. The magnitude of the composition externality hinges on the extent to which job creation affects the average skills of the unemployment pool, and hence the expected productivity of new hires. The impact of hiring on this expected productivity depends on the pool's composition. The larger the share of unemployed workers with eroded skills, the smaller the impact of having an additional unemployed worker with eroded skills in the pool. Thus, the larger share of unemployed workers with eroded skills in recessions explains why the composition externality matters less. Through the same mechanism, one can explain why the composition externality matters more during good times. In booms, the fraction of workers in the unemployment pool with eroded skills decreases. Therefore, the impact on the expected productivity of new hires of having an additional unemployed worker with eroded skills is larger. As a result, the social cost of letting a worker be unemployed and lose skills increases, which makes the composition externality more important.

Next, I look at how the presence of skill loss during unemployment changes job creation relative to an economy without skill loss. Job creation and hence labor market outcomes are expected to change relative to an economy where the unemployed are not exposed to skill erosion because its presence affects the workers' and the firms' problem. However, whether on average more or less jobs are created is not clear because of two opposing effects. On the one hand, the expected gain from job creation drops because the expected productivity of a new hire decreases relative to a world without skill loss. On the other hand, workers' outside option becomes worse when they face the possibility of losing some of their skills when being unemployed. This deterioration of their outside option leads to lower wages, which stimulates job creation. I find that the presence of skill loss can lower the average unemployment rate in the decentralized allocation. However, the unemployment rate is still too high from a social point of view because of the composition externality.

Finally, I seek to quantify the efficiency cost of human capital depreciation during unemployment. In particular, I look at the extent to which labor market outcomes should change to reach constrained-efficiency. To calibrate the model to the U.S. economy I make use of its prediction that workers' wages are on average negatively affected by the length of their unemployment spells. Empirical evidence on the effect of unemployment duration on workers' wages is taken from the displacement literature. This literature investigates the effect of displacement on workers' earnings and wages.[5] The cost in terms of efficiency is considerable. When skill loss is the only source of inefficiency, restoring constrained-efficiency entails a drop in the average unemployment rate of up to 1 percentage point.

The remainder of the paper is organized as follows. Section 2 discusses the related literature. Section 3 describes the model. Section 4 compares the job creation decision in the decentralized and the constrained-efficient allocation. Next, section 5 explores whether constrained-efficiency can be attained through the wage setting mechanism, and what the implications are for optimal labor market policy. Section 6 contains two exercises. The first exercise shows how labor market outcomes in the presence of skill loss change relative to an economy without skill loss. In the second exercise, the model is calibrated to the U.S. economy and it is analyzed what the cost of human capital depreciation is in terms of efficiency by looking at the extent to which labor market outcomes should change to attain constrained-efficiency. Section 7 discusses an extension. Finally, section 8 concludes.

2 Related Literature

This paper relates to three main strands of the literature. First of all, it relates to the literature investigating how the presence of human capital depreciation during unemployment should affect the design of policy. The literature has focused in the first place on how policies providing insurance for risk-averse workers in an economy with incomplete markets are affected by the presence of skill loss. For example, Pavoni (2009) and Shimer and Werning (2006) have computed the optimal scheme for unemployment benefits in the presence of skill loss. Pavoni and Violante (2007) have derived an optimal welfare-to-work program in the presence of skill loss. Such a program consits of a mix of policy instruments targeted at the unemployed, including among others unemployment insurance and job search monitoring. Spinnewijn (2010) has analyzed how optimal training schemes for unemployed workers should be designed when workers' are exposed to both skill loss at the moment of displacement and during their unemployment spell.

My contribution to this strand of the literature lies primarily in its different focus. I focus on whether loss of skill affects the efficiency of aggregate labor market outcomes. I show that those outcomes are not constrained-efficient in the presence of skill loss, and hence policy intervention is required. Moreover, I find that loss of skill during unemployment is an argument for procyclical employment subsidies.

[5] See e.g. Addison and Portugal (1989), Couch and Placzek (2010), Jacobson et al. (1993), and Neal (1995).

Second, this paper relates to the literature looking at how skill erosion during unemployment affects labor market outcomes. The main findings in this literature are the following. Pissarides (1992) has shown that skill loss can be a potential explanation for the observed persistence of unemployment fluctuations. This effect has also been explored by Esteban-Pretel and Faraglia (2010) in a model with both labor market and nominal frictions. Furthermore, Ljungqvist and Sargent (1998) have found that the presence of skill loss, together with differences between welfare systems, is important to understand labor market outcomes in the U.S. versus Europe. Related work has been done by den Haan et al. (2005) and Ljungqvist and Sargent (2004, 2007, 2008). Moreover, Pissarides (1992) and Coles and Masters (2000) have shown in a framework without aggregate uncertainty that multiple equilibria can arise when unemployed workers are exposed to skill loss. Additionally, Coles and Masters (2000) provide the composition externality as an explanation for the existence of multiplicity, and have argued that those multiple equilibria are Pareto rankable.

My main focus differs from this strand of the literature, but I provide some additional insights into how loss of skills affects labor market outcomes. My paper's findings indicate that attention should be paid to the wage setting mechanism if one wants to understand how human capital depreciation during unemployment affects labor market outcomes. In particular, the influence of a worker's outside option on the wage is crucial. This is because a worker's outside option deteriorates when she faces the possibility of losing some of her human capital when being unemployed. This in turn leads to lower wages, which has a positive effect on job creation. When this positive effect is sufficiently strong, it outweighs the negative effect on job creation induced by part of the job candidates having eroded skills.[6] However, from a social point of view job creation is always too low in the laissez-faire economy because of the composition externality. Moreover, I show that taking into account that the unemployed are exposed to skill erosion does not only matter for understanding unemployment outcomes but also for understanding the welfare costs related to unemployment. Loss of skill alters those welfare costs because it gives rise to a composition externality, whose magnitude varies over the cycle.

Finally, this paper relates to other work showing that a composition externality can arise when the pool of searchers is heterogeneous. Burdett and Coles (1997, 1999) have shown that a sorting externality arises in an environment characterized by random search and two-sided heterogeneity. This externality arises because agents do not take into account how their decision to match with an agent of a particular type affects the matching possibilities of the remaining agents. Moreover, Shimer and Smith (2001) have shown that the decentralized-allocation is inefficient in an environment characterized by random search where heterogeneous agents have to decide about their search intensity. Their finding follows from workers who choose their search intensity ignoring how their search intensity affects the probability that matches of a certain type will be formed. Furthermore, Albrecht, Navarro and Vroman (2010) have shown that in a framework with random search and workers who

[6] The importance of workers' outside option in the presence of skill loss during unemployment has also been pointed out in a recent paper by Ortego-Marti (2012). He shows that the presence of skill loss during unemployment generates more wage dispersion among otherwise identical workers in a random search model because the possibility of losing skills while being unemployed makes workers willing to accept lower wages to avoid long unemployment spells.

are heterogeneous with respect to their market productivity, the decentralized allocation is no longer constrained-efficient under the standard Hosios condition when workers' participation decision is endogeneous. Their result follows from workers not internalizing how their participation decision affects the average productivity of newly formed matches. Finally, Fernandez-Blanco and Preugschat (2011) have shown in a framework with directed search, worker heterogeneity, and imperfect information about the worker's type, that labor market outcomes are no longer constrained-efficient because firms do not internalize how their hiring decisions affect the unemployment's pool composition in terms of worker types.

My paper contributes to this strand of the literature in the following aspects. First, I show that in a market characterized by random search the decentralized allocation is not constrained-efficient in the presence of human capital depreciation during unemployment because the latter gives rise to a composition externality. This composition externality is driven by firms ignoring how their hiring decisions affect job-seekers' skills and hence the output that can be produced by other firms' newly formed matches. Moreover, I show that this finding hinges on both workers with and without eroded skills searching for jobs in the same market. When workers with and without eroded skills search for jobs in separate markets, with each of those markets characterized by random search, and firms choose in which market to post vacancies, the decentralized allocation is constrained-efficient if the standard Hosios condition holds in each market. Finally, and in contrast to previous work, I explore the composition externality in an environment subject to aggregate shocks. Those shocks make the composition of the pool of searchers time-varying. This in turn allows me to analyze whether and how the externality's magnitude depends on the composition of the pool of searchers.

3 The Economy

This section outlines the model. It is an extension of a discrete-time search and matching model à la Diamond-Mortensen-Pissarides with aggregate uncertainty. In this framework unemployed workers face the risk of losing some of their human capital, making them less productive upon re-employment.[7] The longer the unemployment spell, the more likely a worker has eroded skills. At the same time the model allows for learning-by-doing such that workers can regain their human capital while being employed.[8]

3.1 Population and Technology

There is continuum of infinitely-lived, risk-neutral workers on the unit interval. These workers maximize their expected discounted utility, which is defined over consumption and home production. Employed workers earn a wage w^i depending on their skills, whereas unemployed workers engage in

[7] This framework abstracts from firm specific human capital. Therefore, workers only face the risk of losing some of their human capital while being unemployed and not when the match separates.

[8] In what follows I use the term "skill" and "human capital" interchangeably.

home production which generates a value b. The latter can be thought of as the opportunity cost of working and is assumed to be the same for all workers.

Workers are heterogeneous in their skills because skill erosion during unemployment and learning-by-doing on the job makes workers' human capital depend on their employment history. To keep the analysis simple, workers' human capital can only take two values, and is either high (H) or low (L). A worker's skills determine her productivity: high-skilled workers have high productivity, whereas low-skilled workers have low productivity. The transition between skill types occurs as follows. In each period, an unemployed high-skilled worker becomes low-skilled with probability $l \in (0, 1]$. Thus the longer a worker's unemployment duration, the larger the chance that her human capital has depreciated. At the same time, when being low-skilled, she can regain her productivity while being employed through learning-by-doing. In each period, an employed low-skilled worker becomes high-skilled with probability $g \in (0, 1]$.

A large measure of risk-neutral, profit-maximizing firms employs workers. As is standard in the literature, each firm consists of a single-worker production unit. Its output depends on the worker's human capital and aggregate productivity A. The latter follows the process

$$log\left(A\right) = \rho_a log\left(A_{-1}\right) + \varepsilon \tag{1}$$

where ε is an *iid* shock. Given the production technology, output produced by matches with a high and low-skilled worker is determined by equations (2) and (3) respectively

$$y^H\left(A\right) = A \tag{2}$$

$$y^L\left(A\right) = \left(1 - \delta\right) A \tag{3}$$

where the skill level of a high-skilled worker is normalized to one, and that of a low-skilled worker is defined by $1-\delta$. The parameter $\delta \in [0, 1)$ can be interpreted as the rate of human capital depreciation. When $\delta \in (0, 1)$ skill erosion is present, making some workers less productive upon re-employment.[9] When $\delta = 0$, the model boils down to the standard model.

3.2 Labor Market

The labor market is characterized by random search à la Diamond-Mortensen-Pissarides. I assume that all workers search in the same market. Thus when a firm opens a vacancy at cost $\kappa > 0$, both workers with and without eroded skills can apply for the job opening. Since a firm meets at most

[9] The interpretation that workers who have suffered from human capital depreciation during unemployment are less productive upon re-employment has also been used by Pissarides (1992). Alternatively, Coles and Masters (2000) and Esteban-Pretel and Faraglia (2010) assume that these workers are equally productive as workers without skill loss once a fixed training cost has been paid.

one worker at each round of interviews, which is a standard assumption in this class of models, all interviews lead to successful hiring as long as the match surplus is non-negative. In every period, the total number of interviews in the economy is determined by a matching function. This function is strictly increasing and concave in both arguments and displays constant returns to scale. It is given by

$$m(v, u) = Bv^{1-\xi}u^{\xi}$$

where B represents the efficiency of the matching process, $1-\xi$ is the elasticity of vacancies where $\xi \in (0, 1)$, v is the total number of vacancies posted by firms, and u is the total number of job-seekers weighted by their search effectiveness. Because I assume that unemployment duration does not affect workers' search effectiveness, and normalizing search effectiveness to one, the relevant measure of job-seekers in the matching function is given by the total number of unemployed u. The latter is defined as the sum of high-skilled $\left(u^H\right)$ and low-skilled $\left(u^L\right)$ unemployed

$$u \equiv u^H + u^L \tag{4}$$

Labor market tightness is defined as $\theta(x) \equiv \frac{v(x)}{u(x)}$, where x denotes the state of the economy and is defined below. The probability for a firm posting a vacancy to meet a job-seeker is defined as

$$q(\theta(x)) \equiv \frac{m(v(x), u(x))}{v(x)} = B\theta(x)^{-\xi} \tag{5}$$

where $q(\theta(x))$ is decreasing in labor market tightness. The probability that a job-seeker gets a job interview is given by

$$p(\theta(x)) \equiv \frac{m(v(x), u(x))}{u(x)} = B\theta(x)^{1-\xi} \tag{6}$$

where $p(\theta(x))$ is increasing in labor market tightness. The job finding probability is the same for both worker types because unemployment duration has no effect on search effectiveness. When the match surplus is non-negative for both skill types, workers also have the same hiring probability.

Timing. At the beginning of the period, a shock to aggregate productivity A is realized. After observing the economy's state, firms post vacancies and hire workers. Next, production takes place using both the existing and newly hired workers. After production some workers' type changes: unemployed high-skilled workers become low-skilled with probability l, and employed low-skilled workers become high-skilled with probability g. Next exogenous separation takes place, and a fraction γ of the matches breaks up.[10]

Labor market flows and the economy's state. The unemployment pool's heterogeneity affects the economy's state x compared to the standard model. In addition to aggregate productivity A, the

[10] The timing assumption in this model is standard in the business cycle literature, see e.g. Blanchard and Galí (2010). The difference compared to the standard DMP setting is that newly hired workers become productive immediately upon hiring.

number of vacancies posted by firms also depends on the fraction of low-skilled job-seekers in the unemployment pool. The latter is given by

$$s(x) \equiv \frac{u^L(x)}{u(x)} \tag{7}$$

The number of high and low-skilled job-seekers evolve according to

$$u^H(x) = (1-l)\,\tilde{u}^H_{-1} + \gamma\left(n^H(x_{-1}) + gn^L_{-1}\right) \tag{8}$$

$$u^L(x) = \tilde{u}^L_{-1} + l\tilde{u}^H_{-1} + \gamma(1-g)\,n^L_{-1} \tag{9}$$

where $\tilde{u}^i$ are the job-seekers of type $i = \{H, L\}$ who remain unemployed after hiring takes place $\tilde{u}^i \equiv (1-p(\theta(x)))\,u^i(x)$, and n^i is the number of workers of type i employed in a given period. Equation (8) shows that the high-skilled job-seekers are all previous period's unemployed high-skilled workers who have not lost their skills, and all the high-skilled workers who just got fired. The latter consists on the one hand of those who were operating in the previous period as high-skilled workers, and on the other hand of those who were low-skilled but regained their skills because of learning-by-doing. Similarly, equation (9) shows that the low-skilled job-seekers are last period's unemployed low-skilled workers and high-skilled workers who have lost some of their skills, and all the low-skilled workers who were employed last period but did not regain skills and just lost their job.

High-skilled and low-skilled employment are given by

$$n^H(x) = (1-\gamma)\left[n^H(x_{-1}) + gn^L_{-1}\right] + p(\theta(x))\,u^H(x)$$

$$n^L = (1-\gamma)(1-g)\,n^L_{-1} + p(\theta(x))\,u^L(x)$$

So high-skilled employment is given by the high-skilled and low-skilled employees with regained skills who kept their job, and the high-skilled new hires. Similarly, the low-skilled employed are on the one hand those who did neither regain skills nor got fired, and on the other hand the newly hired low-skilled unemployed.

Given the labor market flows, keeping track of the fraction of low-skilled workers in the unemployment pool (equation (7)), implies keeping track of the distribution of worker types across employment states. However, when taking into account the definition of the total labor force, it can be seen that either the number of high or low-skilled unemployed after hiring takes place ($\tilde{u}^i$) or the number of high or low-skilled employed (n^i) can be expressed as a function of the other three. Normalizing the total size of the labor force to one and abstracting from labor force participation decisions gives

$$1 = \tilde{u}^H + \tilde{u}^L + n^L + n^H(x)$$

As a result, workers and firms can keep track of the composition of the pool of job-seekers, by for example keeping track of $\tilde{u}^H, \tilde{u}^L$ and n^L. Therefore, the economy's state is given by $x = \left\{A, \tilde{u}^H_{-1}, \tilde{u}^L_{-1}, n^L_{-1}\right\}$.

In the following sections, I will focus on the special case where $g = l = 1$.[11] Thus a low-skilled worker's productivity is restored with probability 1 after having worked for one period, and a high-skilled worker's productivity deteriorates with probability 1 after having been out of work for one period. The latter implies that human capital depreciation during unemployment can only be avoided when a worker who loses her job in a given period finds a new one during that same period. The reason for focusing on this specific case is because it allows me to derive analytical expressions which provide insights into this economy. Numerical analysis shows that the same insights hold in the general case.

Imposing the parameter restriction $g = l = 1$, the number of high and low-skilled job-seekers respectively is defined as

$$u^H(n_{-1}) = \gamma n_{-1} \tag{10}$$

$$u^L(n_{-1}) = 1 - n_{-1} \tag{11}$$

where n represents total employment, which is defined as $n \equiv n^H + n^L$ and evolves according to:

$$n = (1-\gamma)\, n_{-1} + p(\theta(x))\,(1 - (1-\gamma)\, n_{-1}) \tag{12}$$

As can bee seen from equations (10) and (11), both the number of high and low-skilled job-seekers can be written as a function of previous period's employment, and hence so can be the fraction of low-skilled job-seekers in the unemployment pool. Consequently, the economy's state is now given by $x = \{A, n_{-1}\}$.

3.3 Firm's Problem

The firm's value of employing a worker of type $i = \{H, L\}$ is given by

$$J^i(x) \quad = \quad y^i(A) - w^i(x) + (1-\gamma)\,\beta E_x\left\{J^H(x')\right\} \tag{13}$$

where $\beta \in (0,1)$ represents the discount factor. The firm's value of employing a worker depends on the generated output, the wage cost $w^i(x)$, where wages are set through Nash bargaining as discussed in section 3.5, and the continuation value of the match. The worker-firm pair keeps on producing with probability $1-\gamma$. Because $g = 1$ all low-skilled workers regain their productivity after having been employed for one period. As a result, the continuation value of the match is the value of employing a high-skilled worker.

The firm's value of posting a vacancy is given by

$$V(x) \quad = \quad -\kappa + q(\theta(x))\left[(1 - s(n_{-1}))\, J^H(x) + s(n_{-1})\, J^L(x)\right] + (1 - q(\theta(x)))\,\beta E_x\{V(x')\} \tag{14}$$

[11] Derivations for the generalized version of the model with $g \in (0,1]$ and $l \in (0,1]$ can be found in Appendix A.6.

When imposing the free-entry condition $V(x) = 0$, equation (14) becomes

$$\frac{\kappa}{q(\theta(x))} = (1 - s(n_{-1})) J^H(x) + s(n_{-1}) J^L(x) \tag{15}$$

This reflects that firms create jobs such that the expected hiring cost (LHS) equals the expected gain of hiring (RHS). The latter is a function of the unemployment pool's composition because the composition determines the probability that a job-seeker of a particular type shows up for the job interview.

Given free-entry, the firm's value of employing a worker can also be expressed as

$$J^i(x) = \frac{\kappa}{q(\theta(x))} + G^i(x) \tag{16}$$

where

$$G^i(x) \equiv \left(y^i(A) - w^i(x)\right) - (\bar{y}(x) - \bar{w}(x)) \tag{17}$$

and where $\bar{y}(x) \equiv (1 - s(n_{-1})) y^H(A) + s(n_{-1}) y^L(A)$ represents the expected output of a new hire. Note that because of random matching it equals the weighted average of the output produced by each worker type. Each type's share in the unemployment pool is sufficient to determine this type's weight because all job-seekers have the same hiring probability; and $\bar{w}(x) \equiv (1 - s(n_{-1})) w^H(x) + s(n_{-1}) w^L(x)$ is the expected wage cost of a new hire.

Relative to the standard model without skill loss an additional term (G^i) shows up in the firm's value of employing a worker (equation (16)). The intuition is straightforward. In the standard model, the value from employing a worker equals the expected hiring cost $\left(\frac{\kappa}{q(\theta(x))}\right)$ because an employee can be replaced at this cost. However, in the presence of worker heterogeneity this is no longer the case because a new hire is not necessarily of the same type. As a result, the value of employment contains the additional term G^i, capturing the gain from employing a worker of type i. This gain compares the output and wage of a given worker type to the output and wage of an average worker from the unemployment pool.

To provide some insights into this gain, expression (17), can be written as

$$G^H(x) = s(n_{-1}) \left[J^H(x) - J^L(x)\right]$$

$$G^L(x) = -(1 - s(n_{-1})) \left[J^H(x) - J^L(x)\right]$$

First, whether the gain from employing a high-skilled worker is positive depends on the wage setting mechanism. This can be seen as follows: $J^H(x) - J^L(x) = \left(y^H(A) - y^L(A)\right) - \left(w^H(x) - w^L(x)\right)$.[12] A firm is only better off employing a high-skilled worker, i.e. $J^H(x) > J^L(x)$, when the higher productivity of a high-skilled worker is not fully offset by a wage increase, i.e. $\delta A > w^H(x) - w^L(x)$.

[12] The difference between the firm's value of employing a high and a low-skilled worker only depends on the value generated during the first period of production because low-skilled workers regain their skills after one period, i.e. $g = 1$.

Note that even though this implies that $G^L(x) < 0$, as long as $J^L \geqslant 0$ firms are willing to hire the low-skilled. Second, the unemployment pool's composition also affects this benefit. This is because it is precisely a given worker type's fraction in the unemployment pool which determines how likely it is that you can replace an employee by someone of the same type. For example, when $G^H(x) > 0$ and $G^L(x) < 0$, an increase in the fraction of low-skilled searchers increases the benefits of employing a high-skilled worker because the probability that a new hire would also be high-skilled decreases. At the same time, the loss of being matched with a low-skilled worker decreases because the larger the share of low-skilled workers in the unemployment pool the more likely it would have been to be matched with a low-skilled worker.

3.4 Worker's Problem

A worker of type i's value of being employed W^i is given by

$$W^i(x) = w^i(x) + \beta E_x \left\{ (1 - \gamma + \gamma p(\theta(x'))) W^H(x') + \gamma (1 - p(\theta(x'))) U(x') \right\} \tag{18}$$

This value depends on the wage and the continuation value. The latter is made up of three parts. With probability $1-\gamma$ the match survives separation and the worker continues working at the same firm as a high-skilled worker. Also the low-skilled workers will now be high-skilled because of learning-by-doing and the probability of regaining skills being equal to one. With probability γ the match breaks up, and the worker finds a new job with probability $p(\theta)$. Given that the worker gets hired in the same period her skills are not eroded yet, and hence she continues producing as a high-skilled worker. If she does not get hired, she gets the value of being unemployed U. This value is given by

$$U(x) = b + \beta E_x \left\{ p(\theta(x')) W^L(x') + (1 - p(\theta(x'))) U(x') \right\} \tag{19}$$

It depends on the value of home production b, and the probability of finding a job next period now as a low-skilled worker because of skill erosion during unemployment. Note that the value of being unemployed is the same for both worker types because by the time the high-skilled unemployed can start searching again for jobs they all have eroded skills.

Both value functions show that workers take into account that in the presence of skill erosion during unemployment and learning-by-doing their employment status affects their productivity. But whether workers without eroded skills are better off than workers with eroded skills depends on the wage setting mechanism. This is because the difference between the surplus from being employed as a high and low-skilled worker only depends on the wage difference. Thus workers with eroded skills are worse off when being matched only if they receive a lower wage, which can be seen as follows. Defining the surplus from being in a match as $X^i \equiv W^i(x) - U(x)$, worker type i' s surplus becomes

$$X^i(x) \quad = \quad w^i(x) - o(x) + (1-\gamma) \beta E_x \left\{ X^H(x') \right\} \tag{20}$$

where $o(x)$ represents a worker's outside option

$$o(x) \equiv b + \beta E_x \left\{ p(\theta(x')) \left[X^L(x') - \gamma X^H(x') \right] \right\} \tag{21}$$

The outside option reflects that a worker cannot engage in home production b and cannot search for another job. However, if she had not been employed, her skills would have deteriorated, and hence she would have been a low-skilled job-seeker. At the same time, the worker takes into account that being in a job guarantees that she keeps her productivity or regains her productivity. Thus, she keeps in mind that when the match separates she will be able to search as a high-skilled worker. Note that the outside option is the same for both worker types because $l = g = 1$.

3.5 Wages

Wages are renegotiated in every period. Following the literature, I assume generalized Nash bargaining between a worker and firm. Consequently, total match surplus is split between the worker and the firm so that each of them gets a fraction of the total match surplus given by their bargaining power parameter. Defining total match surplus as $M^i(x) \equiv X^i(x) + J^i(x)$, the surplus for a worker and a firm from being in a match is given by[13]

$$X^i = \eta M^i(x) \tag{22}$$

$$J^i(x) = (1-\eta) M^i(x) \tag{23}$$

where η and $1-\eta$ measure respectively the worker's and firm's bargaining power.

The solution to the Nash bargaining problem leads to the following expression for the wage of a worker of type i

$$w^i(x) = \eta y^i(A) + (1-\eta) o(x) \tag{24}$$

Note that the wage difference is given by $w^H(x) - w^L(x) = \eta\delta A > 0$. So the wage difference is a function of the difference in output generated by each match.

As discussed in section 3.3 and 3.4, the wage setting mechanism plays a crucial role in determining both how employing a low-skilled worker affects firms, and how losing skills during unemployment affects workers. When both workers and firms have some bargaining power $\eta \in (0,1)$, the following implications can be derived:

1. *Firms are strictly better off employing a high-skilled worker than employing a low-skilled worker.*

[13] Note that the surplus for a firm from being in a match equals the value of having a worker employed because free-entry drives the value of having a vacancy to zero.

This follows directly from combining equation (13) and (24)

$$J^H(x) - J^L(x) = \left(y^H(A) - y^L(A)\right) - \left(w^H(x) - w^L(x)\right) = (1-\eta)\,\delta A > 0$$

Firms are better off employing a high-skilled worker because the output gains from employing a high-skilled instead of a low-skilled worker are not fully offset by a higher wage. This also implies that the benefits of employing a specific worker type (equation (17)) are given by $G^H = s(n_{-1})(1-\eta)\,\delta A > 0$ and $G^L = -(1 - s(n_{-1}))(1-\eta)\,\delta A < 0$.

2. *High-skilled workers are better off than low-skilled workers.*

   From equation (20) it results that the difference in terms of surplus from being employed as a high and a low-skilled worker only depends on the wage difference. Given that the wage of a high-skilled worker is strictly higher than that of a low-skilled worker when $\delta > 0$, it follows that the surplus from being employed as a high-skilled worker is strictly larger than the surplus from being employed as a low-skilled worker.

$$X^H(x) - X^L(x) = w^H(x) - w^L(x) = \eta\delta A > 0 \tag{25}$$

3. *The presence of skill loss during unemployment affects the wage of workers without eroded skills through its effect on their outside option.*

   In the presence of skill loss during unemployment a high-skilled worker takes into account that part of her outside option is searching for a job as a low-skilled worker (see equation (21)). Given that the surplus of being in a match as a high-skilled worker is higher than the surplus of being in a match as a low-skilled worker (see equation 25), the high-skilled worker's outside option is affected by the presence of skill loss. Given that the wage depends on the outside option (see equation(24)), it follows that the high-skilled worker's wage will also be affected by the presence of skill loss during unemployment.

3.6 Equilibrium

In equilibrium, the surplus from a match with a worker of type i is given by

$$M^i(x) = y^i(A) - b + \beta E_x\left\{\left(1-\gamma+\eta\gamma p\left(\theta\left(x^{'}\right)\right)\right)M^H\left(x^{'}\right) - \eta p\left(\theta\left(x^{'}\right)\right)M^L\left(x^{'}\right)\right\} \tag{26}$$

The above expression is obtained from combining firms' and workers' value functions (equations (13), (18) and (19) respectively) with the wage setting rule (equations (22) and (23)).

From equation (26) it follows that the surplus from a match with a low-skilled worker can be expressed as

$$M^L(x) = M^H(x) - \delta A \tag{27}$$

Consequently, by combining equation (26) for $i = H$ with equation (27), the surplus from a match with a high-skilled worker can be written as

$$M^H(x) = A - b + \beta E_x \left\{ (1-\gamma)\left(1 - \eta p\left(\theta\left(x'\right)\right)\right) M^H(x') + \eta p\left(\theta\left(x'\right)\right) \delta A' \right\} \tag{28}$$

The equilibrium vacancy creation condition follows from combining equations (15), (23) and (27), and is given by

$$\frac{\kappa}{q(\theta(x))} = (1-\eta)\left[M^H(x) - s(n_{-1})\,\delta A\right] \tag{29}$$

where $s(n_{-1}) = \frac{\gamma n_{-1}}{1-(1-\gamma)n_{-1}}$.

Definition 1. *For a given state* $x = \{A, n_{-1}\}$, *this economy's equilibrium consists of a value for labor market tightness* $\theta(x)$ *which satisfies the vacancy creation condition* (*equation* (28)), *given the surplus from a match with a high-skilled worker* (*equation* (29)), *and taking into account the expressions for the job filling and finding probability* (*equations* (5) *and* (6)).

Given this period's state and the equilibrium value of labor market tightness, next period's state is determined by the law of motion for employment (equation (12)), and the law of motion for aggregate technology (equation (1)).

4 Skill Erosion as a Source of Inefficiency

In the previous section I have outlined the model and discussed how the presence of skill loss during unemployment affects the firms' and the workers' problem. I now turn to analyzing whether there is a cost in terms of efficiency.

When the unemployed are exposed to skill loss, and both workers with and without eroded skills search for jobs in the same labor market, a *composition effect* arises in addition to the standard congestion effect. The congestion effect follows from the presence of search frictions, and is well-understood in the literature. It refers to vacancy posting decisions affecting labor market tightness, and hence job filling and finding probabilities. The composition effect refers to today's hiring decisions affecting the output that can be generated by new matches through their influence on the skill composition of the unemployment pool. The pool's composition affects output because it determines the probability that a new match will be of a particular type.

Hiring affects the unemployment pool's composition through two channels. First, hiring affects the job finding probability, the average unemployment duration, and hence the extent to which job-seekers' skills erode. In particular, for the case where $g = l = 1$, when workers become unemployed and do not get rehired in the same period, their human capital will have depreciated by the time they face a new opportunity to find a job. Thus, hiring more workers today lowers the fraction of job-seekers with eroded skills in next period's unemployment pool. Second, hiring today implies that when this match separates, another worker of a given type will enter the unemployment pool. In particular, for the case where $g = l = 1$, all low-skilled workers will have regained their skills by the time they can become unemployed again because of learning-by-doing. As a result, hiring today implies that another high-skilled worker will be searching for a job when the match separates.

In what follows, I first solve the social planner's problem to understand how the presence of skill erosion influences job creation. Next, I compare job creation in the constrained-efficient and the decentralized allocation to analyze if skill erosion during unemployment is a source of inefficiency. Finally, I discuss the role of having both workers with and without eroded skills searching for jobs in the same labor market.

4.1 Constrained-Efficient Allocation

The social planner is subject to the same technological constraints, the same pattern of losing and regaining skills, and the same labor market frictions as in the decentralized economy. The social planner's problem consists of choosing the optimal amount of jobs to create such that the utility of the representative worker is maximized. Given workers' risk neutrality, this coincides with maximizing total output net of vacancy posting costs.

The planner's problem is given by

$$V^P(x) = \max_{\theta} \left[An^H(x) + (1-\delta) An^L(x) - \kappa\theta(x)(1-(1-\gamma)n_{-1}) + b(1-n) + \beta E_x \left\{ V^P(x') \right\} \right]$$

subject to the process for aggregate technology (equation (1)), and the the law of motion for the endogenous state variable employment (equation (12)), where $n^H(x) = (1-\gamma)\, n_{-1} + B\theta(x)^{1-\xi}\gamma n_{-1}$ and $n^L(x) = B\theta(x)^{1-\xi}(1-n_{-1})$. Note that the relevant state for the social planner (x) is the same as in the decentralized allocation $x = \{A, n_{-1}\}$.[14]

The first order condition is

$$\frac{\kappa}{q(\theta(x))} = (1-\xi)\left[\bar{y}(x) - b + \beta E_x \left\{ \frac{\partial V^P(x')}{\partial n} \right\} \right] \tag{30}$$

The envelope condition for employment is

$$\frac{\partial V^P(x)}{\partial n_{-1}} = \left[\begin{array}{c} (1-\gamma+\gamma p(\theta(x)))\, A - p(\theta(x))(1-\delta) A + (1-\gamma)\kappa\theta(x) \\ -(1-\gamma)(1-p(\theta(x)))\left[b - \beta E_x \left\{ \frac{\partial V^P(x')}{\partial n} \right\} \right] \end{array} \right] \tag{31}$$

[14] This follows from the assumption that new hires become productive upon hiring in combination with the parameter restriction $l = g = 1$. As discussed in section 3.2, those conditions imply that the fraction of low-skilled searchers (s) can be expressed as a function of last period's employment. Moreover, those conditions imply that all employed workers will be high-skilled by the time the planner has to decide again about optimal job creation. As a result, keeping track of how many workers are employed is also sufficient to know the quality of the employed workers. If production in a given period were to take place with both last period's employed and last period's new hires, the social planner would also have to keep track of the quality of last period's new hires, since the low-skilled wouldn't have been able yet to regain their productivity. Therefore, even though the social planner needs to keep track of the total number of high-skilled and low-skilled employed workers in addition to the share of low-skilled job-seekers in the unemployment pool, this can still be done just be keeping track of last period's total employment.

Combining the first order condition (equation (30)) and the envelope condition for employment (equation (31)) gives the following expression for job creation in the constrained-efficient allocation

$$\frac{\kappa}{q(\theta(x))} = (1-\xi)\left[\bar{y}(x) - b + \beta E_x\left\{\Lambda^P(x')\right\}\right] \tag{32}$$

where Λ^P represents the continuation value of the match and is given by

$$\Lambda^P(x) \equiv \Lambda_1^P(x) + \Lambda_2^P(x) + \Lambda_3^P(x) \tag{33}$$

where

$$\Lambda_1^P(x) \equiv (1-\gamma)\left(\frac{\kappa}{q(\theta(x))(1-\xi)} + y^H(A) - \bar{y}(x)\right)$$

$$\Lambda_2^P(x) \equiv p(\theta(x))\left[\gamma\left(\frac{\kappa}{q(\theta(x))(1-\xi)} + y^H(A) - \bar{y}(x)\right) - \left(\frac{\kappa}{q(\theta(x))(1-\xi)} + y^L(A) - \bar{y}(x)\right)\right]$$

$$\Lambda_3^P(x) \equiv (1-\gamma)(1-\xi)\,p(\theta(x))\left(\frac{\kappa}{q(\theta(x))(1-\xi)}\right)$$

Job creation (equation(32)) is such that the expected hiring cost (LHS) equals the expected gain from job creation (RHS). As in the standard model, the expected hiring cost depends on the vacancy posting cost κ and the vacancy filling probability $q(\theta)$. The expected gain from job creation depends on the expected output produced by the new hire $\bar{y}$, the loss in home production b, and the continuation value of the match (equation (33)). Note that the expected gain from job creation is weighted by $1-\xi$ because the planner takes into account how posting an additional vacancy affects the vacancy filling probability.

The continuation value consists of three parts: the value when the match continues producing $\left(\Lambda_1^P\right)$, the worker's outside option $\left(\Lambda_2^P\right)$, and the congestion effect $\left(\Lambda_3^P\right)$. Even though the overall structure is the same as in the standard model, the presence of the skill loss alters the continuation value. I now discuss each part of the continuation value in detail to gain insight into how skill loss precisely influences the job creation decision.

The continuation value's first term $\left(\Lambda_1^P\right)$ captures that creating a job today guarantees that next period a match will be operating with a high-skilled worker if this match does not separate. The value of a match with a high-skilled worker is given by the savings in expected hiring costs[15], and a term representing the ouput benefit of employing a high-skilled worker. There is an output benefit related to employing a specific worker type because a new hire would not necessarily be of the same type.[16]The new hire would be an average job-seeker from the unemployment pool. Therefore, the expected output gain realized next period from hiring a worker today is defined as the output generated by this worker

[15]The savings in expected hiring costs depend on the vacancy posting cost κ and the vacancy filling probability $q(\theta)$ weighted by $(1-\xi)$. The term $(1-\xi)$ shows up because the planner takes into account the effect on the vacancy filling probability that would be caused by posting an additional vacancy: $\frac{\partial q(\theta)}{\partial v} = -\xi\frac{q(\theta)}{v}$

[16]In the standard model with homogeneous workers the value of a match equals the savings in expected hiring costs because an employee can be replaced by an identical worker when paying this cost.

$\left(y^H\right)$ relative to the expected output of a new hire $(\bar{y})$.

The second term $\left(\Lambda_2^P\right)$ represents the worker's outside option and reflects that today's job creation affects next period's output through its influence on the composition of next period's unemployment pool, i.e. the composition effect. The planner takes into account that upon separation there will be a high-skilled worker searching for a job, following from employment allowing a worker to regain or keep her productivity. When this worker finds a new job in the same period, which happens with probability $p(\theta)$, a high-skilled match starts operating leading to an output gain $\left(y^H - \bar{y} > 0\right)$. At the same time, the planner also considers that if the worker had been unemployed, she would have lost skills or remained low-skilled. If this now low-skilled worker had found a job, which would have happened with probability $p(\theta)$, this worker would have produced y^L whereas if another worker had been hired, the expected output would have been $\bar{y}$, creating an output loss $\left(y^L - \bar{y} < 0\right)$. The prospect of skill loss when not being employed lowers the worker's outside option, and hence increases the continuation value of a match.

The third term $\left(\Lambda_3^P\right)$ reflects the positive congestion effect induced by having a job-seeker less when the match survives separation, making it easier for the other job seekers to get hired.[17] The value generated by this positive effect is expressed as the value of a match with an average worker, which is given by the expected hiring cost. This follows from a change in labor market tightness affecting each worker's hiring probability in exactly the same way because it is independent of her type. [18]

Thus, the continuation value shows that the presence of skill erosion during unemployment influences the job creation decision. The planner takes into account that today's job creation generates output gains in the next period through two channels: the effect on workers' skills in existing matches, reflected in Λ_1^P, and on job-seekers' skills, reflected in Λ_2^P.

The *magnitude of the composition effect* ultimately hinges on the extent to which today's job creation affects the expected productivity of new hires. Importantly, the impact of hiring on this expected productivity depends on the pool's composition. In particular, the larger the share of unemployed workers with eroded skills, the smaller the impact of having an additional unemployed worker with eroded skills in the pool. As can be seen from the continuation value, having another worker with eroded skills in the unemployment pool because of not hiring her generates an output effect given by the difference between the output that would have been produced by this worker and the expected output produced if another, random worker from the pool were to be hired. This output effect depends on the unemployment pool's composition, and hence so does the magnitude of the composition effect.

For example, an increase in the share of low-skilled workers in the unemployment pool affects the magnitude of the composition effect in two opposite ways. On the one hand, it lowers the expected productivity difference between a new hire and a low-skilled worker. Therefore, it decreases the extent to which hiring affects the output of new matches by preventing workers from being unemployed and

[17]Note that $-\frac{\partial p(\theta)}{\delta u} = (1-\xi)\,\frac{p(\theta)}{u}$

[18]Note that the congestion effect following from today's job creation affecting today's labor market tightness is reflected by the expected surplus of a match with a new hire being weighted by the elasticity of vacancies in the matching function $(1-\xi)$.

having their skills eroded. On the other hand, it increases the expected productivity difference between a new hire and a high-skilled worker. Therefore, it increases the extent to which hiring affects the output of new matches by having another high-skilled workers searching for a job when a match separates. The opposite holds for a decrease in the share of low-skilled workers in the unemployment pool.[19]

In conclusion, the social planner's problem shows that the presence of the composition effect arising in this economy influences the job creation decision because there are additional output gains related to job creation. However, the magnitude of this composition effect depends on the unemployment pool's composition. Given that this composition is state-dependent, so is the extent to which the composition effect influences job creation.

4.2 Constrained-Efficient versus Decentralized Allocation

To detect whether skill erosion during unemployment is a source of inefficiency I compare the job creation decision in the decentralized and the constrained-efficient allocation under the standard Hosios condition. This condition sets workers' bargaining power (η) equal to the elasticity of unemployment in the matching function (ξ). The reason for doing so is because in the absence of skill erosion the decentralized allocation is constrained-efficient when the standard Hosios condition holds. This follows from the congestion externality being internalized when workers' bargaining power satisfies this parameter condition.

Proposition 1. *In the presence of skill erosion during unemployment, the decentralized allocation is no longer constrained-efficient under the standard Hosios condition for workers' bargaining power* $\eta = \xi$.

Proof. Combining equations (28) and (29) gives the job creation equation in the decentralized allocation

$$\frac{\kappa}{q\left(\theta\left(x\right)\right)} = \left(1-\eta\right)\left[\bar{y}\left(x\right) - b + \beta E_x\left\{\Lambda^D\left(x'\right)\right\}\right] \tag{34}$$

where $\Lambda^D\left(x\right)$ represents the continuation value of the match and is given by

$$\Lambda^D\left(x\right) \equiv \Lambda_1^D\left(x\right) + \Lambda_2^D\left(x\right) + \Lambda_3^D\left(x\right) \tag{35}$$

where

$$\Lambda_1^D\left(x\right) \equiv \left(1-\gamma\right)\left(\frac{\kappa}{q\left(\theta\left(x\right)\right)\left(1-\eta\right)} + y^H\left(A\right) - \bar{y}\left(x\right)\right)$$

[19] The output difference between an average and a low-skilled unemployed worker is decreasing in the share of low-skilled job-seekers: $\frac{\partial\left[\bar{y}(x)-y^L(A)\right]}{\partial s(n_{-1})} = \frac{\partial\left[1-s(n_{-1})\right]\delta A}{\partial s(n_{-1})} < 0$. The output difference between a high-skilled and an average unemployed worker is increasing in the share of low-skilled job-seekers: $\frac{\partial\left[y^H(A)-\bar{y}(x)\right]}{\partial s(n_{-1})} = \frac{\partial s(n_{-1})\delta A}{\partial s(n_{-1})} > 0$

$$\Lambda_2^D(x) \equiv -\eta(1-\gamma)\,p(\theta(x))\left(\frac{\kappa}{q(\theta(x))(1-\eta)}\right)$$

$$\Lambda_3^D(x) \equiv \eta p(\theta(x))\left[\gamma\left(y^H(A)-\bar{y}(x')\right)+\left(\bar{y}(x)-y^L(A)\right)\right]$$

The job creation equation in the constrained-efficient allocation is represented below in a slightly different way relative to the expression in section 4.1 to facilitate comparison with the job creation equation in the decentralized allocation.[20]

$$\frac{\kappa}{q(\theta(x))} = (1-\xi)\left[\bar{y}(x)-b+\beta E_x\left\{\Lambda^P(x')\right\}\right] \tag{36}$$

where the continuation value of the match is given by

$$\Lambda^P(x) \equiv \Lambda_1^P(x)+\tilde{\Lambda}_2^P(x)+\tilde{\Lambda}_3^P(x) \tag{37}$$

where

$$\Lambda_1^P(x) \equiv (1-\gamma)\left(\frac{\kappa}{q(\theta(x))(1-\xi)}+y^H(A)-\bar{y}(x)\right)$$

$$\tilde{\Lambda}_2^P(x) \equiv -\xi(1-\gamma)\,p(\theta(x))\left(\frac{\kappa}{q(\theta(x))(1-\xi)}\right)$$

$$\tilde{\Lambda}_3^P(x) \equiv p(\theta(x))\left[\gamma\left(y^H(A)-\bar{y}(x')\right)+\left(\bar{y}(x)-y^L(A)\right)\right]$$

By comparing equation (34) evaluated at $\eta=\xi$ and equation (36), it follows that the decentralized allocation does not replicate the constrained-efficient allocation under the standard Hosios condition $\eta=\xi$. □

In the presence of skill erosion during unemployment the decentralized allocation is no longer constrained-efficient under the standard Hosios condition because a *composition externality* arises which is not internalized by this condition. The composition externality refers to firms ignoring how their hiring decisions affect the skill composition of the unemployment pool, and hence the output that can be generated by other firms' newly formed matches. Comparing the term Λ_3^D and $\tilde{\Lambda}_3^P$ of the continuation values (equations (35) and (37) respectively) shows that the output gain from preventing a worker to

[20] The term reflecting the worker's outside option Λ_2^P and the term reflecting the congestion effect Λ_3^P of the continuation value of the match have been rearranged in the terms $\tilde{\Lambda}_2^P$ and $\tilde{\Lambda}_3^P$. This separates those terms which are in terms of hiring costs $\left(\tilde{\Lambda}_2^P\right)$ and those which are in terms of next period's output gains following from today's job creation affecting the skills of next period's job-seekers $\left(\tilde{\Lambda}_3^P\right)$

lose skills $\left(\bar{y}-y^L\right)$, and in case of separation, the output gain from employment guaranteeing that a worker keeps her productivity or regains her productivity when she had eroded skills upon hiring $\left(y^H-\bar{y}\right)$, show up only to a fraction of the worker's bargaining power η in the decentralized allocation, whereas those output gains show up fully in the planner's allocation. This reflects that the social benefits of today's job creation are larger than the private ones.

The *magnitude of the composition externality* depends on the extent to which firms' hiring decisions affect the average skills of the unemployment pool, and hence the expected productivity of new hires. As discussed in the previous section, the impact of hiring on this expected productivity depends on the pool's composition. The larger the share of unemployed workers with eroded skills, the smaller the impact of having an additional unemployed worker with eroded skills in the pool. Given that the share of unemployed workers with eroded skills is state-dependent, so is the magnitude of the composition externality.

How the composition externality arises can be seen from the firms' job creation decision. The latter is obtained by combining equation (13) and (15)

$$\frac{\kappa}{q\left(\theta\left(x\right)\right)} = \bar{y}\left(x\right)-\bar{w}\left(x\right)+\left(1-\gamma\right)\beta E_x\left\{\frac{\kappa}{q\left(\theta\left(x'\right)\right)}+G^H\left(x'\right)\right\} \tag{38}$$

The above expression shows that vacancy posting is such that the expected hiring cost (LHS) equals the expected output produced by a new hire, taking into account the expected wage cost and the continuation value (RHS). The latter shows that firms internalize how today's hiring decisions affect workers' productivity when the worker remains employed. However, it also shows that firms ignore their effect on workers' skills if the match with a worker separates or if a worker had never been hired by the firm in the first place. Those output effects are still partially accounted for in equilibrium (equation (34)) through the wage setting mechanism. Workers recognize that being in a job affects their skills, and hence their outside option. Therefore, as being discussed in section 3.5, it is reflected in the wage. Note, however, that even though those output effects are only partially internalized, the value of a match with a worker of a specific type in equilibrium is evaluated in the same way as in the planner's allocation for $\eta=\xi$. [21]

The reason why the standard Hosios condition does not internalize both the congestion and the composition externality is the following. The social planner takes into account both the worker's outside option and the congestion effect following from having a job-seeker less if the match does not separate. The former lowers the continuation value because a matched worker cannot be employed elsewhere. The latter increases the continuation value because when a worker stays in the match, it is easier for the other searchers to find a job. What matters is the net effect. In the standard model

[21] This is because of the wage setting mechansim: the value of employment for the firm is given by $J^i\left(x\right)=\frac{\kappa}{q(\theta(x))}+G^i\left(x\right)$ (see section (3.3)), and the wage setting mechanism implies that $G^H=\left(1-\eta\right)s\left(n_{-1}\right)\delta A=\left(1-\eta\right)\left(y^H\left(A\right)-\bar{y}\left(x\right)\right)$ and $G^L=-\left(1-\eta\right)\left(1-s\left(n_{-1}\right)\right)\delta A=\left(1-\eta\right)\left(y^L\left(A\right)-\bar{y}\left(x\right)\right)$ (see section (3.5)). Given that $M^i\left(x\right)=\frac{J^i(x)}{(1-\eta)}$ (see expression (23)), the value of a match with a worker of type i is just as in the social planner's allocation given by the sum of the expected hiring cost and the difference in output generated by this worker $\left(y^i\right)$ and an average worker from the unemployment pool $\left(\bar{y}\right)$.

without skill erosion during unemployment, where $\bar{y} = y^L = y^H$, the net effect can be expressed as a fraction ξ of the worker's outside option (net of the value of home production b). Therefore, the decentralized allocation is efficient when $\eta = \xi$, even though only the worker's outside option is considered up to a fraction η instead of both the outside option and the congestion effect. However, in the presence of skill erosion the net effect can no longer be expressed as a fraction ξ of the worker's outside option (net of the value of home production b). Consequently, for $\eta = \xi$ the net effect is not fully taken into account, and hence the decentralized allocation is not constrained-efficient under the standard Hosios condition.[22]

4.3 The Role of a Heterogeneous Unemployment Pool

The heterogeneity of the unemployment pool plays a crucial role for the previous section's result. If workers with and without eroded skills were searching for jobs in different labor markets, the decentralized allocation would be constrained-efficient as long as the standard Hosios condition would hold in each market.

Proposition 2. *When unemployed workers with and without eroded skills search for jobs in separate markets, with each of those markets characterized by random search, and firms can choose in which market to post vacancies, the decentralized allocation is constrained-efficient as long as the standard Hosios condition holds in each market.*

Proof. See Appendix A.2 □

Proposition 2 follows from the composition externality no longer being present when firms can decide which type of workers show up for job interviews. In this environment job creation still affects unemployed workers' skills, but it no longer affects how likely it is that job-seekers with or without eroded skills show up for job interviews. Thus, job creation no longer determines the average productivity of job candidates, and hence the output that can be generated by a new match. As a result, with segmented labor markets for workers with and without eroded skills the decentralized allocation is efficient when the standard Hosios condition holds in each market, i.e. the workers' bargaining power equals the elasticity of job-seekers in the respective labor market. However, given the absence of empirical evidence showing that workers with different unemployment durations search for jobs in separate labor markets, I consider the framework where the unemployment pool contains both workers with and without eroded skills the relevant one to focus on.

---

[22] The net effect in the presence of skill erosion is given by the second and third term of expression (37)

$$\Upsilon(x) \equiv E_x\left\{-\xi(1-\gamma)p(x')\left(\frac{\kappa}{q(x')(1-\xi)}\right) + \gamma p(x')\left(y^H(A') - \bar{y}(x')\right) + p(x')\left(\bar{y}(x') - y^L(A')\right)\right\}$$

and the outside option net of the value of home production b is given by the second term in expression (33)

$$\tilde{o}(x) \equiv E_x\left\{-(1-\gamma)p(x')\left(\frac{\kappa}{q(x')(1-\xi)}\right) + \gamma p(x')\left(y^H(A') - \bar{y}(x')\right) + p(x')\left(\bar{y}(x') - y^L(A')\right)\right\}$$

Comparing both expressions shows that $\Upsilon(x) = \xi\tilde{o}(x)$ iff $\delta = 0$.

5 Attaining Constrained-Efficiency

5.1 Wage Setting Mechanism

The decentralized allocation is no longer constrained-efficient under the standard Hosios condition as shown in Proposition 1. However, just as in the absence of skill erosion, this condition internalizes the congestion externality.

Proposition 3. *In the presence of skill erosion during unemployment the standard Hosios condition $\eta = \xi$ internalizes the congestion externality when workers have the same hiring probability.*

Proof. See Appendix A.3 □

The intuition behind Proposition 3 is the following. The same parameter condition internalizes the congestion externality because there is no interaction between the congestion effect and the unemployment pool's composition. When labor market tightness changes, the hiring probability for all workers is affected in exactly the same way because they all have the same hiring probability. Therefore, the effect of job creation on labor market tightness, and hence on the probability that a match starts producing, is independent of the unemployment pool's composition.

Next, I analyze if the standard Hosios condition can be modified such that both the congestion and the composition externality are internalized. Put differently, I examine whether there exists a parameter condition for the workers' bargaining power for which the decentralized allocation is constrained-efficient.

Proposition 4. *In the presence of skill erosion during unemployment and aggregate shocks, there is no constant bargaining power η such that constrained-efficiency is attained in all states of the world.*

Proof. By comparing equation (34) and (36), it follows that internalizing the composition externality requires a value of $\eta = 1$. At the same time, Proposition 3 showed that internalizing the congestion externality requires $\eta = \xi$. Because both conditions are mutually exclusive, there exists no longer a condition for the parameter η, for which the decentralized alloction replicates the constrained-efficient allocation in all states of the world. □

Proposition 4 implies that some form of policy intervention is required to restore constrained-efficiency. However, constrained-efficiency can still be attained without policy intervention when workers' bargaining power is allowed to be state-dependent, i.e. $\eta(x)$. This can be explained as follows. The only decision which has to be made is how many vacancies to post. Therefore, the decentralized allocation is constrained-efficient when the expected gain from job creation is the same as in the planner's allocation, i.e. the RHS of equation (34) and (36) are equalized. The latter depends on both the expected surplus of a new match and the share of this surplus that goes to the firm $(1 - \eta(x))$. From this it follows immediately that when the share is allowed to be state-dependent the constrained-efficient allocation can be attained without policy intervention. Because even if the expected surplus of a new match differs across allocations the share that the firm receives can be adjusted such that the expected gain from job creation is the same in both allocations.[23] Given that in the literature the workers' bargaining power is considered being a constant parameter I do not explore this further, and turn to the policy implications.

5.2 Optimal Labor Market Policy

As discussed in the previous section, some form of policy intervention is required to restore constrained-efficiency. In this section I analyze the implications for optimal labor market policy.[24] The policy maker is subject to the same technological constraints and labor market frictions as the planner. I assume throughout that the policy maker can implement a labor market policy through non-distorting revenue sources.

The optimal labor market policy takes the form of an employment subsidy. The derivation is described in Appendix A.4. This subsidy is received in every period by firms when they employ a worker independently of the worker's type. Although this subsidy might be difficult to implement in practice, it sheds light on the nature of the composition externality. Note also that in the presence of an employment subsidy, workers' skills are still only affected by their employment history, leaving the trade-offs related to job creation unaffected.

Since the congestion externality is already well-understood in the literature, I focus on the case where that externality is fully internalized ($\eta = \xi$). When only the composition externality remains, the optimal employment subsidy $\Phi(x)$ is given by

$$\Phi(x) = (1 - \xi)\,\beta E_x \left\{p\left(\theta\left(x'\right)\right)\Omega\left(x'\right)\right\} > 0 \tag{39}$$

where

$$\Omega(x) \equiv \left(\bar{y}(x) - y^L(A)\right) + \gamma\left(y^H(A) - \bar{y}(x)\right) \tag{40}$$

First, expression (39) shows that employment should be subsidized to attain constrained-efficiency. This reflects that not enough jobs are created in the laissez-faire economy, following from the expected social benefits of today's job creation being larger than the private ones.

Second, expression (39) shows that the optimal employment subsidy is state-dependent. This follows immediately from the magnitude of the composition externality varying with the economy's state, as discussed in section 4.2. In particular, the extent to which employment should be subsidized in a given period is a function of next period's expected economic situation. The reason is that the output gains caused by the impact of today's hiring decisions on the skill composition of the unemployment pool are only realized next period, and those gains depend on next period's expected economic situation. In particular, they are a function of both the expected fraction of low-skilled job-seekers and the

[23] Note that for the same reason in the steady state there also exists a parameter condition for workers' bargaining power such that constrained-efficiency is attained.

[24] See Laureys (2012) for the implications for optimal monetary policy.

expected level of technology.[25]

Third, expression (39) shows that the optimal subsidy is a function of the output gains caused by the impact of today's hiring decisions on the skill composition of the unemployment pool (Ω). The first term in expression (40) captures that hiring generates an output gain because it prevents a worker from being unemployed, and hence having an additional low-skilled job-seeker in the unemployment pool. The second term in expression (40) captures the output gain related to employment guaranteeing that there will be another high-skilled job-seeker when the match separates.

Finally, I expect the optimal subsidy to be decreasing in the share of workers with eroded skills in the unemployment pool because the output gains following from today's job creation affecting the skill composition of the unemployment pool (Ω) are decreasing in it:

$$\frac{\partial \Omega(x)}{\partial s(n_{-1})} = \frac{\partial A\left(1-(1-\gamma)\, s(n_{-1})\right)}{\partial s(n_{-1})} < 0$$

Those gains are decreasing in the share of low-skilled job-seekers because the output gain from hiring and preventing workers from being unemployed and having their skills eroded has a weight of one, whereas the weight on the output gain from retraining equals the separation rate $\gamma \in (0,1)$. As discussed in section 4.1, a larger share of job-seekers with eroded skills lowers the impact on output of hiring and preventing skill deterioration because it decreases the expected productivity difference between a new hire and a low-skilled worker.

6 Labor Market Outcomes

This section contains two different exercises. Section 6.2 sheds light on how the presence of skill loss changes job creation relative to an economy without skill loss, whereas in section 6.3 I seek to quantify the efficiency cost of skill loss during unemployment.

As discussed in section 3, the presence of skill loss affects the workers' and the firms' problem. Therefore, labor market outcomes are expected to change relative to an economy where the unemployed are not exposed to skill loss. However, whether on average more or less jobs will be created in the presence of skill loss is not clear because the equilibrium job creation equation (equation (34)) shows that there are two opposing effects at work. On the one hand, the expected gain from job creation drops because the expected output produced by a new hire decreases relative to a world without skill loss. On the other hand, there are additional expected output gains following from today's job creation affecting workers' skills. By looking at how job creation changes relative to an economy without skill loss, it can be understood which of the opposing effects dominates. This is the focus of section 6.2.

I perform this exercise not only for the decentralized but also for the constrained-efficient allocation. Even though it is clear that job creation in the decentralized allocation is lower than in the

[25] The optimal subsidy also depends on the expected job finding probability. This is because ultimately today's job creation influences next period's output only to the extent that the unemployed workers get hired.

constrained-efficient allocation when the adequate policy is absent (see section 5), comparing labor market outcomes in both allocations provides an insight into whether the change in job creation in the decentralized allocation induced by the presence of skill loss goes in the same direction as the change in job creation in the constrained-efficient allocation.

As discussed in section 4 and 5, loss of skill during unemployment generates an efficiency cost. This cost ultimately depends on the extent to which labor market outcomes should change to attain constrained-efficiency, which in turn depends on the magnitude of the composition externality. To provide an insight into this cost, I perform a quantitative analysis in section 6.3. I calibrate the model to the U.S. economy, and look at the extent to which key labor market outcomes, such as the average unemployment rate and the job finding probability should change to attain constrained-efficiency.

For both exercises I use the model's generalized version where workers lose and regain skills with some probability in every period, as outlined in section 3.1 and 3.2. For a description of the model, see Appendix A.6. The model is solved numerically by taking a first-order approximation.[26]

6.1 Calibration

The parameterization strategy is the following. The length of a period is one month. I use parameter values standard in the literature for the U.S. economy. I set the discount factor β to 0.996 implying an annual interest rate of 4%; I set the elasticity of unemployment in the matching function ξ to 0.5 following the evidence in Petrongolo and Pissarides (2001). I set $\eta = \xi = 0.5$ such that the congestion externality is internalized. The value of home production b is set to 0.71 following Hall and Milgrom (2008) and Pissarides (2009). I target the following long-run values, which correspond to the model's steady state values: a job finding probability of 35% following Fujita and Ramey (2012) and an unemployment rate of 5% following Blanchard and Gali (2010) where total unemployment is given by all those searchers who did not find a job $\tilde{u} \equiv (1 - p(\theta))\, u$. This implies an exogenous separation rate of $\gamma = \tilde{u} p(\theta) / ((1 - \tilde{u})(1 - p(\theta))) = 0.0283$, which lies in the range of values used in the literature.[27] I follow Pissarides (2009) and set $\theta = 0.72$, implying a value for matching efficiency $B = p(\theta)\, \theta^{\xi - 1} = 0.412$. Steady state aggregate productivity A is normalized to 1. Once the values of the parameters governing the skill loss process are determined, the value of the vacancy posting κ is obtained from the equilibrium conditions.

[26] Alternatively, the model could be solved through value function iteration. The drawback of this method is the computional time following from the economy's state space being 4-dimensional. One of the advantages of solving this model with a non-linear solution method, however, is the possibility of exploring the importance of non-linearities. The latter are potentially important in an environment with skill loss as the shape of the economy's response to a given shock might depend on its size. This can be explained as follows. The initial response of the job finding probability depends on the size of the shock hitting the economy. As a result, the initial response of the average unemployment duration, and hence the change of the unemployment pool's composition also depends on it. The latter in turn affects the expected gains from job creation in future periods. Consequently, the shape of the economy's response to a shock might depend on the extent to which the shock hitting the economy affects the unemployment pool's composition. I leave the exploration of this channel for future research.

[27] For example, Fujita and Ramey (2012) target an average monthly separation rate of 0.02 , Pissarides (2009) one of 0.036, and Shimer (2005) one of 0.033

The parameters governing the skill loss process are δ, l, and g. The parameters δ and l determine the degree to which an unemployment spell erodes workers' skills: the human capital depreciation rate δ determines how many skills a high-skilled worker loses conditional upon losing, whereas the probability that a high-skilled worker will lose some of its skills in each period that she spends in unemployment depends on l. The parameter g determines how long it takes on average for a worker with eroded skills to regain those skills. Importantly, the difference in nature between the exercises performed in section 6.2 and 6.3 requires a different strategy for pinning down δ, l, and g.

Section 6.2 explores how the presence of skill erosion changes job creation relative to an economy without skill erosion. This exercise needs a baseline economy. I choose the baseline economy to be an economy without skill loss, and proceed as follows. I use the parameterization strategy described above for the baseline economy. Thus, I set $\delta = 0$. Therefore, the value of the vacancy posting cost κ is obtained from the vacancy creation condition (equation (34)) when setting $\delta = 0$. This gives $\kappa = \frac{(1-\eta)p(\theta)(y-b)}{\theta(1-\beta(1-\gamma)(1-\eta p(\theta)))} = 0.3497$.

Next, to analyze how the presence of skill erosion during unemployment affects labor market outcomes relative to an economy without skill loss, I keep the parameter values β, ξ, η, b, γ, B, and κ fixed at their value for the baseline economy and vary only those related to the human capital process: δ, l, and g. I also keep the steady state value for aggregate technology A equal to one. However, the steady state values of unemployment ($\tilde{u}$), the job finding probability ($p(\theta)$) and labor market tightness (θ) will vary. Note that it is precisely this variation that will shed light on how job creation changes when skill loss is present. Finally, given that there is no quantitative aspect related to this exercise and direct empirical evidence on those three values is absent, I explore the effect of skill loss on labor market outcomes for an entire range of parameter values for δ, l and g. In particular, when I vary one of those parameters over a certain range, I keep the other parameters fixed at an arguably reasonable baseline value, namely $\delta = 0.3$, and $l = g = 0.167$, which implies that workers both lose and regain skills after six months on average.[28]

In section 6.3, in contrast, I parameterize the economy taking into account skill loss during unemployment. Therefore, the main difference relative to the parameterization strategy for section 6.2, is that now the steady state values of the unemployment rate ($\tilde{u} = 5\%$), the job finding rate ($p(\theta) = 0.35$), and the aggregate labor market tightness ($\theta = 0.72$) are reached when skill loss is present. Thus, the value of the vacancy posting cost κ is now computed from the equilibirum conditions of an economy with skill loss. Given that direct empirical evidence on δ, l, and g is absent, I propose a strategy to obtain values for these parameters. This strategy is discussed in section 6.3.1.

[28]For those parameter values the Blanchard-Kahn (1980) conditions are satisfied which implies that the solution is stable and unique. Pissarides (1992) and Coles and Masters (2000) have shown, in a model without aggregate uncertainty, that multiple equilibria can arise in the presence of skill loss during unemployment. For a discussion about the possibility of multiple equilibria in this model see Appendix A.5.

6.2 Comparative Statics

Figure 1 shows that in the presence of skill loss the social planner lowers the average unemployment duration relative to a world without skill loss, i.e. for $\delta = 0$. The increase in job creation needed to attain this drop in the average unemployment duration also translates into a lower average unemployment rate and a smaller fraction of both low-skilled job-seekers and employed workers. This indicates that even though the expected gain from job creation drops because the expected output produced by a new hire decreases relative to a world without skill loss, this drop is more than offset by the expected output gains following from today's job creation affecting workers' skills. Put differently, the increase in the expected gains from job creation induced by the presence of skill loss outweigh the cost. Moreover, the higher δ, the lower the optimal average unemployment duration. The intuition behind this pattern is that an unemployment spell becomes more costly when the skill loss that can occur during that spell becomes more severe. As a result, it is optimal to reduce the average unemployment duration, and hence lower the chance that workers' skills erode.

The same pattern is present for all values of δ in the decentralized allocation although less pronounced. Thus, the presence of low-skilled job-seekers leads to more job creation, even though a firm can now be matched with a low-skilled worker which generates a lower value than being matched with a high-skilled worker (see section 3.5). This implies that the value of being matched with a high-skilled worker has to increase to the extent that it more than offsets the negative effect induced by having workers with eroded skills as potential new hires. It turns out that the value of being in a match with a high-skilled worker increases because high-skilled workers' wages decrease with δ. As discussed in section 3.5, and shown in the lower panel of figure 1, when workers face the risk of losing skills during unemployment, their outside option worsens. This lowers wages, making job creation more attractive for firms.[29] The composition externality, however, prevents job creation to reach the constrained-efficient level.

Figure 2 shows the effect of the probability of skill loss l on labor market outcomes in the decentralized and the constrained-efficient allocation relative to an economy without skill loss. Note that the latter would be attained in the limiting case where the time it takes on average for workers to lose skills goes to infinity. It can be seen that the social planner lowers average unemployment duration when skills erode faster on average. This follows directly from an unemployment spell of a given length being more costly when it is more likely that a high-skilled worker will lose her skills during that spell. The same pattern is also present in the laissez-faire economy, albeit to a lesser extent. As shown in the lower panel of figure 2, this can again be explained by high-skilled workers being willing to accept lower wages because of a deterioration of their outside option. Moreover, figure 2 shows that the share of low-skilled job-seekers increases when workers' skills erode faster on average. This implies that even

[29] For this mechanism to be at work it is of course crucial that workers' bargaining power is sufficiently high. The lower workers' bargaining power, the smaller the effect of workers' outside option on wages, and hence the less likely it becomes that the positive effect on job creation induced by lower wages more than offsets the negative effect induced by having low-skilled workers searching for jobs. However, this exercise shows that when the workers' bargaining power satisfies the standard Hosios condition the positive effect is sufficiently strong to dominate the negative one.

though a lower average unemployment duration is desirable for higher values of l, it should not drop to the extent that it prevents an increase in the share of low-skilled job-seekers.

Figure 3 shows how labor market outcomes depend on the time it takes on average for low-skilled workers to regain their skills. The longer it takes on average for workers to regain their productivity, the lower average unemployment duration is in the social planner's allocation. This reflects that the social cost of letting a worker lose her skills is larger when she regains her skills on average more slowly. More job creation also lowers the unemployment rate, and the fraction of low-skilled job-seekers. Despite the increase in job creation, low-skilled employment still increases as a fraction of total employment the longer it takes on average for workers to regain their skills. In the decentralized allocation labor market outcomes are similar because of the effect on a worker's outside option. As shown in the lower panel of figure 3, the faster a worker regains her skills on average, the smaller is the cost of skill loss in the first place, which leads to an improvement of the worker's outside option. Consequently, workers are less willing to accept a wage cut, making job creation less attractive for firms.

Next, I focus on how the presence of skill loss affects the shape of the economy's dynamic response to shocks relative to an economy without skill loss. Figures 4-6 show the response of the unemployment rate and the job finding probability to a persistent negative aggregate technology shock. The shock is such that aggregate technology decreases by 1% on impact relative to its steady state level and the autoregressive parameter ρ_a is set to 0.95. The job finding rate is represented in terms of relative deviation from its steady state, whereas the unemployment rate is represented in terms of absolute deviation from its steady state. It can be seen that in the presence of skill erosion the unemployment rate shows a humped shaped response to a negative technology shock similar to that of the economy without skill loss.[30] Moreover, the decentralized economy's response has a shape similar to that of the constrained-efficient allocation.

To sum up, relative to an economy without skill erosion, steady state labor market outcomes in the laissez-faire economy change in a way similar to those in the constrained-efficient allocation when skill loss is introduced: the more costly skill loss (i.e. the higher δ, the higher l, and the lower g) the shorter average unemployment duration and the lower the unemployment rate. Nevertheless, labor market outcomes differ across allocations because of the composition externality. Furthermore, both in the decentralized and the constrained-efficient allocation, the presence of skill erosion does not seem to alter the shape of the economy's dynamic response to an aggregate technology shock much relative to an economy without skill erosion.

[30] Pissarides (1992) has shown that skill loss can be a potential explanation for the observed persistence of unemployment fluctuations. The intuition behind this finding is that when the economy is hit by a negative shock, the increase in the average unemployment duration leads to a deterioration of the unemployment pool's quality. This in turn makes job creation even less attractive, leading to more persistent unemployment fluctuations. This effect has also been explored by Esteban-Pretel and Faraglia (2010) in a model with both labor market and nominal frictions. They find that even though the presence of skill loss improves the performance of the model in terms of the magnitude of the response of unemployment to a monetary shock, it does not improve the model in terms of explaining the observed persistence in unemployment.

6.3 Cost of Skill Loss during Unemployment

6.3.1 Parameters of the Skill Loss Process

To analyze the cost of skill loss in terms of efficiency, the values of the parameters governing the skill loss process δ, l, and g need to be pinned down. Recall that the parameters δ and l determine the degree to which an unemployment spell erodes workers' skills: the human capital depreciation rate δ determines how many skills a high-skilled worker loses conditional upon losing, whereas the probability that a high-skilled worker will lose some of its skills in each period that she spends in unemployment depends on l. The parameter g determines how long it takes on average for a worker with eroded skills to regain those skills. Empirical evidence on those three values is absent. Therefore, I propose a strategy to pin down these parameter values by making use of two model predictions.

First, the model predicts that a productivity gap arises between an average new hire and an incumbent worker, where the latter is interpreted as being high-skilled. There is a productivity gap between new hires and incumbent workers because a fraction of the new hires has eroded skills. Over time, however, this productivity gap closes because those workers with eroded skills regain them. The training literature provides empirical evidence on those issues. Barron et al. (1997) look at how long it takes on average to become fully trained and qualified. However, the evidence depends on the type of survey used. Using employer based training survey data (the Employment Opportunity Pilot Project survey (EOPP) and the Small Business Administration survey) they find that it takes on average 20 weeks to become fully trained. However, when making use of household survey data (the PSID) they find that workers report that it takes on average 19.9 months to become fully trained. Using the EOPP, and eliminating the outliers, Cairó and Cajner (2011) find that it takes on average 13.4 weeks to become fully trained. [31]

I use the empirical evidence on how long it takes on average for a worker to be fully trained and qualified to pin down g. This provides a first insight even though the evidence refers to both the time to acquire firm specific skills and to regain those skills lost because of being unemployed. Given that the estimates for the average time it takes to become fully trained range from 13.4 weeks to 19.9 months, I set $g = 0.083$ such that it takes on average 12 months to regain skills, i.e. $1/g = 12$.

The second prediction of the model is that the presence of skill loss during unemployment translates into wage differences across workers because workers with eroded skills earn less then workers without eroded skills. Whether a worker has eroded skills depends on the length of their unemployment spell because it is precisely the spell's length which determines the chance that workers lose skills. As a result, the model predicts that on average the length of a worker's past unemployment spell lowers a worker's wage relative to the situation where this worker had not been unemployed in the first place.[32]

[31] Cairó and Cajner (2011) also find that the productivity gap between an average new hire and an incumbent worker is 39.1%. However, the size of the productivity gap does not allow to pin down uniquely the values of δ and l. This is driven by the productivity of an average new hire depending both on the productivity gap between a worker with and without eroded skills and on the share of new hires with eroded skills.

[32] In this model the mere fact of displacement does not affect workers' skills. No skills are lost at the moment of

The presence of learning-by-doing in turn implies that the effect of an unemployment spell on workers' re-employment wages will faid away over time as those workers with eroded skills regain their skills.

The displacement literature has provided empirical evidence on the effect of job loss on both earnings and wages. It has been found that job loss significantly lowers both of them. In addition, those losses are documented to be long lasting. [33] The empirical evidence also shows that at least part of the wage loss is caused by the length of the unemployment spell, which is in line with the predictions of the model. For example, by using data from the Displaced Worker Survey, Addison and Portugal (1989) find that an increase in the unemployment duration by 10% reduces wages between 0.8% and 1.4%. Using the same data, Neal (1995) finds that an additional week of unemployment reduces the wages by 0.37%, implying a monthly rate of wage loss of 1.5%. By using data from the PSID, Ortego-Marti (2012) finds that an additional month of unemployment lowers wages by 1.2%.

The empirical evidence on wage losses due to displacement cannot be used to pin down uniquely the parameters δ and l. The degree to which the length of an unemployment spell lowers workers' wages is determined both by how long it takes on average before unemployment affects workers' skills (l) and if workers lose skills, how much they lose (δ). Hence, a given effect of unemployment duration on wages can be caused by a range of combinations of the parameters l and δ. Thus, computing a precise value for the extent to which labor market outcomes would change if the composition externality was eliminated is not possible. Nevertheless, it is possible to provide a range of parameter combinations for δ and l in line with the empirical evidence. This in turn provides a first insight into the cost of skill loss during unemployment in terms of efficiency.

To obtain a range of parameter combinations for δ and l in line with the empirical evidence I proceed as follows. First, I pin down the time it takes on average for workers' skills to erode when they are unemployed. Given the lack of empirical evidence, I look at a range of 3 to 6 months. An average pace of skill loss of 3 months can be thought of as a natural upperbound because given the high turnover in the U.S. labor market it implies that an unemployment spell of an average length already leads to skill erosion. At the same time, 6 months can be thought of as a natural lowerbound because it implies that on average the skills of a long-term unemployed worker are eroded.[34] In order to be in

becoming unemployed but only during the unemployment experience. Note that it is precisely this type of skill loss that gives rise to the composition externality. This follows from the externality's nature because it only arises when firms affect the unemployment pool's composition in terms of job-seekers' skills. If all the skill loss, and hence wage loss, was driven by skill loss at the moment of displacement then there would be no composition externality. This because job-seekers' skills would be unaffected by their unemployment duration, and hence by firms' hiring decisions.

[33]For example, by using administrative data from Pennsylvania Jacobson et al. (1993) find earnings losses for workers displaced through mass layoff up to 40% in the period immediately following displacement. Six years after displacement those workers' earnings remain 25% lower than the earnings of workers who have not been displaced. Couch and Placzek (2010) obtain similar results by using administrative data from Connecticut. They find initial earnings losses of around 32%, while six years later those losses are reduced to around 14%. By using data from the Panel Study of Income Dynamics (PSID), Stevens (1997) finds that in the initial year of job loss, wages decrease by around 12%. Six years after displacement wages are still around 9% below their expected level. Gregory and Jukes (2001) report similar evidence obtained from British data. They show that an unemployment spell lowers wages by around 20% relative to the wage that would have been obtained without the unemployment experience.

[34]In the U.S. a worker is typically referred to as being long-term unemployed if this worker has been unemployed for at least 26 weeks.

line with the empirical evidence I set the value for the human capital depreciation rate δ such that I match the estimated effect of the length of an unemployment spell on wages. In particular, for a given average pace of skill loss $1/l$ and an initial guess for δ, I generate artificial employment histories and the according wage paths from the model.[35] Next, I use those panel data obtained from the model to regress wages on the length of the unemployment spell

$$ln\left(Wage\right) = \alpha * Length\ Spell + \varepsilon \tag{41}$$

I update the value of δ and repeat the same procedure until the regression coefficient equals that of the empirical finding, i.e. $\hat{\alpha} = -0.013$. Note that in the empirical literature the regression (equation (41)) contains additional control variables. Given that in this model workers are homogenous aside from their skill level determined by their unemployment duration, no additional controls are added. Moreover, no time fixed effects are added because the model is time-stationary.

Finally, the process for aggregate technology is set such as to match the empirical evidence on the volatility and the autocorrelation of quarterly U.S. labor productivity.[36] This implies a value for the autoregressive parameter for technology $\rho_a = 0.984$ and a standard deviation of the innovations to technology of $\sigma_a = 0.0050$. Note, however, that another process for aggregate technology could be chosen since the goal of this exercise is not to see how well the model performs in replicating the patterns observed in the data.

6.3.2 Findings

Table 1 denotes the average labor market outcomes in the decentralized versus the constrained-efficient allocation for different values of the average pace of skill loss $(1/l)$ and the corresponding rate of human capital depreciation (δ). The fraction of skills that workers lose while being unemployed is relative small, ranging between 14 and 17%. Despite this relatively small loss, however, the cost in terms of efficiency generated by not fully taking into account how job creation can prevent this skill loss is considerable. In addition, the cost is higher the faster unemployed workers lose their skills on average. Looking at both the upper and lowerbound of the pace of skill loss, the unemployment rate should be between 0.92 and 0.60 percentage points lower to reach constrained-efficiency. This would require an increase in the average job finding probability in the range of 14.17 to 8.83%, implying a drop in the average unemployment duration of between 1.5 and 1.1 weeks. In the constrained-efficient allocation

[35] I generate an employment history and wage path for 10.000 worker by using the steady state values for the job finding probability and wages. For each worker I compute 400 periods and discard the first 150 periods to avoid an effect from the initial type and employment status of the worker. In the model, those workers who get fired and immediately rehired in the same period are not exposed to skill loss. Therefore, I compute the length of a given unemployment spell as the number of periods where the unemployed worker was exposed to skill loss.

[36] Quarterly data for seasonally-adjusted real output per person in the non-form business sector is obtained from the Bureau of Labor Statistics series PRS85006163. I restrict the sample to data in the period 1975-2005 to be in line with the period for the empirical evidence regarding the job finding probability provided by Fujita and Ramey (2012). After taking logs and detrending the data by using an HP-filter with smoothing parameter 10^5 as in Shimer (2005), the values for the standard deviation and the quarterly autocorrelation are 0.017 and 0.882 respectively.

Time for skills to erode on average in months ($1/l$)		3	4	5	6
Human capital depreciation rate (δ)		0.14	0.15	0.16	0.17
		Mean (%)			
Unemployment rate ($\tilde{u}$)	D.	5.00			
	C.E.	4.08	4.22	4.32	4.40
Job finding probability ($p(\theta)$)	D.	35.00			
	C.E.	39.96	39.15	38.54	38.09
Share low-skilled job-seekers (s)	D.	44.81	37.88	32.78	28.94
	C.E.	39.65	33.79	29.52	26.28
Share low-skilled employment $\left(n^L/n\right)$	D.	11.65	9.85	8.52	7.53
	C.E.	10.31	8.79	7.67	6.83

Table 1: The cost of skill loss during unemployment

the share of low-skilled job-seekers would be between 13.01 and 10.12% lower relative to the share in the laissez-faire economy. Similarly, the fraction of low-skilled employed workers would decrease by between 13.00 and 10.25%. [37]

As discussed in section 5.2, constrained-efficiency is attained when the adequate employment subsidy is implemented. Table 2 shows some of the properties of this subsidy. First of all, to get a better idea about the magnitude of the average optimal employment subsidy, the subsidy is expressed as a fraction of the average output per worker, which is denoted by $\tilde{\Phi}$. This shows that the optimal subsidy ranges on average between 8.19 and 6.00% of average ouput per worker. Second, as discussed in section 5.2, the optimal subsidy is time-varying. Looking at the standard deviation of the optimal subsidy relative to unemployment shows that the optimal subsidy becomes relatively less volatile the longer it takes for workers' human capital to erode. Longer periods for workers' skills to erode also imply lower volatility in the share of low-skilled job-seekers in the unemployment pool.

In section 5.2 I have argued that the optimal subsidy is expected to be decreasing in the share of workers with eroded skills in the unemployment pool. Simulating the model, shows that this leads the optimal employment subsidy to have a strong positive correlation with aggregate productivity, a strong negative correlation with the unemployment rate, and a strong negative correlation with the fraction of low-skilled searchers in the unemployment pool. This shows that the optimal employment subsidy is *procyclical*. The procyclicality of the subsidy reflects that the composition externality matters less in recessions than in booms. This can be explained as follows. The larger share of job-seekers with eroded skills in recessions lowers the social cost of letting a worker lose her skills because it decreases the expected productivity difference between a new hire and a job-seeker with eroded skills.[38] Put

[37]The vacancy posting cost κ varies with the values for the parameters governing the skill loss process because, as explained in section 6.3.1, the value for κ is obtained from the equilibrium conditions of the model. On average the resources spent on vacancy posting $\left(\frac{\kappa v}{A(n^H+(1-\delta)n^L)}\right)$ lies in the range of $2.67-2.81\%$ of total output.

[38]Note that this finding does not hinge on the source of economic fluctuations. It is important to point out, however, that when aggregate technology shocks are the driving force behind economic fluctuations, the argument for procyclical employment subsidies is reinforced. This is because, for a given skill composition of the unemployment pool, the expected productivity difference between a new hire and a worker with eroded skills is increasing in aggregate technology.

Time for skills to erode on average in months ($1/l$)		3	4	5	6
Human capital depreciation rate (δ)		0.14	0.15	0.16	0.17
Unemployment rate ($\tilde{u}$)	mean (%)	4.08	4.22	4.32	4.40
Optimal employment subsidy (Φ)	mean	0.0808	0.0703	0.0619	0.0553
	$\tilde{\Phi}$ (%)	8.19	7.12	6.27	6.00
	vol. rel. to $\tilde{u}$	1.8480	1.3252	0.9917	0.7715
	corr. with A	0.9946	0.9969	0.9984	0.9992
	corr. with $\tilde{u}$	−0.9919	−0.9891	−0.9857	−0.9819
	corr. with s	−0.9394	−0.9245	−0.9101	−0.8958
Share of low-skilled job-seekers (s)	vol. rel to $\tilde{u}$	5.7378	5.1596	4.6600	4.2448

Note: $\tilde{\Phi}$ denotes the subsidy as a fraction of the average output per worker. The first-order approximation of the model has been simulated for 301.000 periods and the first 1.000 observations have been eliminated. The volatilities are defined as the standard deviation of the quarterly average of the monthly data, which has been detrended by using an HP-filter with smoothing parameter 10^5.

Table 2: The optimal employment subsidy

differently, the impact of letting a worker lose her skills on the average skills of the unemployment pool is smaller in recessions. As a result, in recessions firms' hiring decisions have a smaller impact on the average skills of the unemployment pool, and hence on the expected productivity of new hires. This in turn drives the procyclicality of the employment subsidy.

The labor market dynamics in the decentralized versus the constrained-efficient allocation are shown in figure 7. The simulated series of the labor market outcomes reveal that the difference between labor market outcomes in both allocations is in the first place driven by differences in the mean.

In what has been discussed so far, the congestion externality has been fully internalized. This naturally raises the question how big the cost of skill loss during unemployment would be if in addition to the composition externality the congestion externality was present. Therefore, I analyze how the previous findings depend on the relation between the workers' bargaining power (η) and the elasticity of unemployment in the matching function (ξ).[39] Results are reported in table 3. First of all, I look at the importance of the value of ξ while the Hosios condition still holds. Petrolongo and Pissarides (2001) have estimated a range of values for ξ. When ξ is set to a higher value, namely 0.72, the drop in the average unemployment rate needed to reach constrained-efficiency is only between 0.17 and 0.12 percentage points.[40] This is not surprising since, as discussed in section 4.2, the expected gains from output following from job-seekers' skills being affected by their employment status is

Therefore, if economic fluctuations are driven by shocks in aggregate technology, there will be another channel, in addition to the unemployment pool's composition, through which the expected productivity difference between a new hire and a job-seeker with eroded skills decreases during recessions.

[39]Changes in the parameters ξ and η also imply a variation in the vacancy posting costs κ in order to attain the same targets for the decentralized allocation. For the first and second column of ξ and η, the total resources spent on recruitment vary between 0.7 and 1.12% of total output. In the last column however, total resources spent on recruitment are as much as 5.81%. This is driven by a sharp increase in κ which can be explained by vacancy posting having to be more costly to support an average unemployment rate of 5% when vacancy posting becomes more attractive because of the low bargaining power for workers.

[40]This value has been used by Shimer (2005).

		$\eta = \xi$		$\eta > \xi$			$\eta < \xi$		
Elasticity of unemployment in m.f. (ξ)		0.72			0.5			0.5	
Bargaining power workers (η)		0.72			0.72			0.3	
Time for skills to erode on av. in months $(1/l)$		3	6	/	3	6	/	3	6
Human capital depreciation rate (δ)		0.11	0.14	0	0.11	0.14	0	0.21	0.23
Congestion externality		no	no	yes	yes	yes	yes	yes	yes
Composition externality		yes	yes	no	yes	yes	no	yes	yes
Unemployment rate $(\tilde{u})$ %	D.	5.00			5.00			5.00	
	C.E.	4.83	4.88	2.13	1.14	1.60	8.59	7.75	7.90
Job finding probability $(p(\theta))$ %	D.	35.00			35.00			35.00	
	C.E.	35.85	35.58	56.48	71.06	63.61	23.18	25.24	24.84
Share low-skilled job-seekers (s) %	D.	44.81	28.94	/	50.26	28.94	/	44.81	28.94
	C.E.	43.90	28.42	/	15.11	11.15	/	56.43	39.88
Share low-skilled employment (n^L/n) %	D.	11.65	7.53	/	13.07	7.53	/	11.65	7.53
	C.E.	11.41	7.39	/	3.93	28.98	/	14.67	10.37

Table 3: The role of ξ and η

taken into account up to a fraction of the workers' bargaining power η. Therefore, the higher the workers' bargaining power, the closer average labor market outcomes are to what they would be if the composition externality was fully offset.

Second, to understand how the presence of the congestion externality affects the results two cases need to be considered. In the first case workers' bargaining power is higher than its optimal value implied by the Hosios condition, i.e. $\eta > \xi$. As a result, average job creation is too low even in the absence of skill loss during unemployment. To reach constrained-efficiency the average unemployment rate should be 2.87 percentage points lower. The presence of skill loss, and hence the presence of the composition externality reinforce this pattern. Optimal unemployment is an additional 0.99 to 0.53 percentage points lower relative to the case without skill loss. In the second case workers' bargaining power is lower than its optimal value implied by the Hosios condition, i.e. $\eta < \xi$. This implies that in the absence of skill loss, too many jobs would be created, implying that the average unemployment rate should increase by 3.59 percentage points to reach constrained-efficiency. The presence of skill loss only partially offsets this increases needed for the unemployment rate to be at its constrained-efficient level. Optimal unemployment is between 0.84 and 0.69 percentage points lower relative to the case where only the congestion externality is present.

These results suggest that the efficiency cost generated by the congestion externality outweighs the cost generated by the composition externality because the former requires a larger change in the average unemployment rate to attain constrained-efficiency than the latter. Moreover, whether unemployment should increase or decrease when both the congestion and the composition externality are present, depends in the first place on whether the congestion externality leads to too much or not enough job creation. Nevertheless, the presence of skill loss calls for a drop in the optimal unemployment rate relative to the case without skill loss.

7 Extension: Effect of Match Specific Productivity

In the economy analyzed so far match output is only a function of aggregate productivity and the worker's skills. When match output also depends on match specific productivity, in the spirit of Mortensen and Pissarides (1994), some matches will separate for endogenous reasons. In particular, only those matches whose productivity is above an endogenously determined threshold value will start producing. As a result, some job interviews will not result in hiring and some of the existing matches will separate.[41]

Given that some matches separate for endogenous reasons, the skill composition of the unemployment pool is affected through two channels instead of one: the vacancy posting decision and the decision on whether a match will start operating. Vacancy creation determines how many job-seekers will get a job interview. But it is the threshold value for match specific productivity which ultimately determines whether workers will be unemployed and exposed to skill erosion.[42]

Just as in the absence of match specific productivity, loss of skill generates a composition externality: firms do not take into account how both their vacancy posting decisions and their decisions concerning separation affect the skill composition of the unemployment pool, and hence the expected productivity of new hires. Thus, in the laissez-faire economy both vacancy posting and the threshold values for match specific productivity are not constrained-efficient. Note that the composition externality still matters less in recessions than in booms because the larger share of job-seekers with eroded skills in recessions decreases the difference between the expected productivity of a new hire and a job-seeker with eroded skills.

In contrast to the finding in section 5, I find that the congestion externality is no longer internalized by the standard Hosios condition. Intuitively, this can be explained as follows. Even though all job-seekers still have the same probability of getting a job interview, they no longer have the same hiring probability. This because the threshold value for match specific productivity depends on the worker's human capital. As a result, a change in labor market tightness no longer affects the hiring probability of all workers in the same way. Consequently, there is an interaction between the congestion effect and the unemployment pool's composition.

Finally, in this framework workers with a higher unemployment duration have on average a lower hiring probability. Given that low-skilled job-seekers are less productive, match specific productivity has to be higher for those workers than for high-skilled workers for matches to have a non-negative value. Therefore, the threshold value for match specific productivity of low-skilled job-seekers is higher than that of high-skilled job-seekers. This in turn lowers the hiring probability of the low-skilled job-seekers. This prediction is in line with the empirical evidence on negative duration dependence.[43]

---

[41]The model with match specific productivity for $g = l = 1$ is described in the online Appendix.

[42]This threshold value determines both whether a worker gets hired and whether an existing match continues operating. This follows from the worker's and the firm's outside option being the same in the case of "not hiring" and in the case of "firing": a worker's outside option is being unemployed, while a firm's outside option is having an unfilled vacancy.

[43]However, there is a debate in the literature about the extent to which the observed decrease in the job finding

8 Conclusion

This paper shows that the depreciation of human capital during unemployment calls for policy intervention when both workers with and without eroded skills search for jobs in the same labor market. This is the consequence of a composition externality related to job creation: when making their hiring decisions, firms do not take into account how their decisions affect the skill composition of the unemployment pool, and hence the expected productivity of new hires. Hiring affects this composition because it prevents having additional workers with eroded skills in the unemployment pool.

Constrained-efficiency can be attained by implementing a procyclical employment subsidy. Employment should be subsidized because some gains from job creation are not fully taken into account, making job creation in the laissez-faire economy too low. The optimal subsidy is time-varying because the magnitude of the composition externality depends on the unemployment pool's composition. This magnitude is determined by the extent to which job creation affects the average skill composition of the unemployment pool. The impact on the latter of having an additional unemployed worker with eroded skills in the pool is smaller, the larger the fraction of workers with eroded skills in the unemployment pool. Consequently, the composition externality matters less in recessions than in booms. This in turn drives the procyclicality of the optimal employment subsidy.

Calibrating the model to the U.S. economy by making use of the empirical evidence regarding the effect of unemployment duration on workers' wages leads to the finding that the efficiency cost of human capital depreciation is considerable. If human capital depreciation was the only source of inefficiency, the average unemployment rate would be up to 1 percentage point lower in the constrained-efficient allocation.

This paper has focused on an environment where search is random. I have shown that loss of skill during unemployment only gives rise to a composition externality when unemployed workers with and without eroded skills search for jobs in the same labor market. When workers with and without eroded skills search for jobs in separate markets, and firms can direct their vacancy posting to a particular market, loss of skill does not generate an externality. Therefore, I expect that loss of skill will give rise to a composition externality in other search environments when the environment is such that the expected productivity of a new hire depends on the unemployment pool's composition. I also expect this paper's finding that the composition externality matters less in recessions than in booms to carry over to other search environments. In particular, I expect this to happen when the environment is such that a larger share of workers with eroded skills in the unemployment pool increases the probability of hiring a worker with eroded skills, and hence decreases the expected productivity difference between a new hire and a job-seeker with eroded skills.

probability for higher unemployment durations reflects "true" negative duration dependence. The lower job finding probability for workers with higher average unemployment duration could also result from unobserved heterogeneity. Those workers with intrinsically lower job finding probabilities make up a larger share of the long-term unemployed causing the observed negative duration dependence. See e.g. Barnichon and Figura (2011), Layard et al. (2005), and Machin and Manning (1999).

Finally, this paper has focused on the case where the depreciation of human capital during unemployment is the only source of inefficiency besides the search frictions characterizing the labor market. I believe that this environment is appropriate for understanding the nature of the efficiency loss generated by skill erosion. However, I also believe that introducing skill erosion into a richer environment is important because its presence might alter the efficiency loss generated by other frictions in the economy. One example is wage rigidity. This paper has emphasized that firms are still willing to hire the unemployed with eroded skills because those workers' wages are adjusted to bring them in line with their productivity. Therefore, I expect the efficiency loss generated by rigid wages to be larger once it is taken into account that the unemployed are exposed to skill erosion because firms might no longer be willing to hire unemployed workers with eroded skills. I leave the exploration of this avenue for future research.

References

Addison, John, and Pedro Portugal. 1989. "Job Displacement, Relative Wage Changes, and Duration of Unemployment." Journal of Labor Economics, 7(3): 281-302.

Albrecht, James, Lucas Navarro, and Susan Vroman. 2010. "Efficiency in a search and matching model with endogenous participation." Economics Letters, 106(1): 48-50.

Barnichon, Régis, and Andrew Figura. 2011. "What drives matching efficiency? a tale of composition and dispersion." Finance and Economics Discussion Series 2011-10, Board of Governors of the Federal Reserve System (U.S.).

Barron, John M., Mark C. Berger, and Dan A. Black. 1997. "How Well Do We Measure Training?" Journal of Labor Economics, 15(3): 507-528.

Blanchard, Olivier, and Jordi Galí. 2010. "Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment." American Economic Journal: Macroeconomics, 2(2): 1-30.

Blanchard, Olivier, and Charles M. Kahn. 1980. "The Solution of Linear Difference Models under Rational Expectations." Econometrica, 48(5):1305-1312.

Burdett, Kenneth, and Melvyn G. Coles. 1997. "Marriage and Class." Quarterly Journal of Economics, 112(1): 141-168.

Burdett, Kenneth, and Melvyn G. Coles. 1999. "Long-term partnership formation: marriage and employment." Economic Journal, 109(456): 307-334.

Cairó, Isabel and Tomaz Cajner. 2011. "Human capital and unemployment dynamics: why more educated workers enjoy greater employment stability." Working Paper.

Coles, Melvyn G., and Adrian Masters. 2000. "Retraining and long-term unemployment in a model of unlearning by not doing." European Economic Review, 44(9): 1801-1822.

Couch, Kenneth A. and Dana W. Placzek. 2010. "Earnings Losses of Displaced Workers Revisited." American Economic Review, 100(1): 572-589.

den Haan, Wouter J., Christian Haefke, and Garey Ramey. 2005."Turbulence and Unemployment in a Job Matching Model." Journal of the European Economic Association, 3(6):1360–85.

Esteban-Pretel, Julen, and Elisa Faraglia. 2010. "Monetary Shocks in a Model with Skill Loss." Journal of Money, Credit and Banking, 42(7):1235-1265.

Fernández-Blanco, Javier and Edgar Preugschat. 2011. "On the Effects of Ranking by Unemployment Duration." Working Paper.

Fujita, Shigeru and Garey Ramey. 2012. "Exogenous versus Endogenous Separation." American Economic Journal: Macroeconomics, 4(4): 68-93.

Gregory, Mary, and Robert Jukes. 2001. "Unemployment and Subsequent Earnings: Estimating Scarring Among British Men 1984-94." Economic Journal, 111(474): 607-625.

Hall, Robert E., and Paul R. Milgrom. 2008. "The Limited Influence of Unemployment on the Wage Bargain." American Economic Review, 98(4): 1653-1674.

Hosios, Arthur J. 1990. "On the Efficiency of Matching and Related Models of Search and Unemployment." Review of Economic Studies, 57(2): 279–298.

Jacobson, Louis S., Robert J. Lalonde, and Daniel G. Sullivan.1993."Earning Losses of Displaced Worksers," American Economic Review, 83(4): 695–709.

Keane, Michael P., and Kenneth I. Wolpin. 1997. "The Career Decisions of Young Men," Journal of Political Economy 105(3): 473–522.

Laureys, Lien. 2012. "Optimal Monetary Policy in the Presence of Human Capital Depreciation during Unemployment". Working Paper.

Layard, P. Richard G., Nickell, Stephen and Richard Jackman. 2005. "Unemployment: macroeconomic performance and the labour market." Oxford University Press Inc., New York.

Ljungqvist, Lars, and Thomas J. Sargent. 1998. "The European Unemployment Dilemma." Journal of Political Economy, 106(3): 514-550.

Ljungqvist, Lars, and Thomas J. Sargent. 2004. "European Unemployment and Turbulence Revisited in a Matching Model." Journal of the European Economic Association, 2(2-3) :456–68.

Ljungqvist, Lars, and Thomas J. Sargent. 2007. "Understanding European unemployment with matching and search-island models." Journal of Monetary Economics, 54(8): 2139 – 2179.

Ljungqvist, Lars, and Thomas J. Sargent. 2008. "Two Questions about European Unemployment." Econometrica, 76(1):1-29.

Mortensen, Dale. T., and Christopher A. Pissarides. 1994. "Job Creation and Job Destruction in the Theory of Unemployment." Review of Economic Studies, 61(3): 397-415.

Mortensen, Dale. T., and Christopher A. Pissarides. 2002. "Taxes, Subsidies and Equilibrium Labor Market Outcomes," CEP Discussion Papers 0519, Centre for Economic Performance, LSE.

Machin, Stephen, and Alan Manning. 1999. "The Causes and Consequences of Long-Term Unemployment in Europe." In Handbook of Labor Economics, Vol. 3, Part 3, edited by Orley C. Ashenfelter and David Card, 3085–3139. Amsterdam, Holland: Elsevier Science.

Neal, Derek. 1995. "Industry-Specific Human Capital: Evidence from Displaced Workers." Journal of Labor Economics, 13(4): 653-677.

Ortego-Marti, Victor. 2012. "Unemployment History and Frictional Wage Dispersion." Working Paper.

Pavoni, Nicola, and Gianluca Violante. 2007. "Optimal Welfare-to-Work Programs." Review of Economic Studies, 74(1): 283-318.

Pavoni, Nicola. 2009. "Optimal Unemployment Insurance, with Human Capital Depreciation, and Duration Dependence." International Economic Review, 50 (2): 323-362.

Petrongolo, Barbara, and Christopher A. Pissarides. 2001. “Looking into the Black Box: A Survey of the Matching Function.” Journal of Economic Literature, 39(2): 390-431.

Pissarides, Christopher A. 1992. “Loss of Skill During Unemployment and the Persistence of Employment Shocks.” Quarterly Journal of Economics, 107(4):1371-1391.

Pissarides, Christopher A. 2009. “The Unemployment Volatility Puzzle: is Wage Stickiness the Answer?” Econometrica, 77(5): 1339-1369.

Shimer, Robert, and Lones Smith. 2001. “Matching, Search and Heterogeneity.” Advances in Macroeconomics, 1 (1): 1-16.

Shimer, Robert. 2005. “The Cyclical Behavior of Equilibrium Unemployment and Vacancies.” American Economic Review, 95(1): 25-49.

Shimer, Robert and Ivan Werning. 2006. “On the Optimal Timing of Benefits with Heterogeneous Workers and Human Capital Depreciation.” NBER working paper no. 12230, May.

Spinnewijn, Johannes. 2010. “Training and Search During Unemployment.” Working Paper.

Stevens, Ann Huff. 1997. “Persistent Effects of Job Displacement: The importance of Multiple Job Losses.” Journal of Labor Economics, 15(1): 165-188.

A Appendix

A.1 The Economy with Separate Labor Markets

A.1.1 Labor Market

The labor market is characterized by two separate markets: a market for workers with eroded skills and one for workers without. So workers' skill type determines the labor market in which they are searching. Firms can decide in which market they post vacancies. Each submarket is characterized by a specific matching technology. I define market 1 as the market for the high-skilled and market 2 as the market for the low-skilled. The matching function is assumed to be the same in both markets. The number of matches in each submarket $k = \{1, 2\}$ is given by

$$m_k = Bv_k^{1-\xi} u_k^{\xi}$$

Defining labor market tightness $\theta_k(x) \equiv \frac{v_k(x)}{u_k(x)}$ the job finding probability, and the job filling probability are given by

$$q(\theta_k(x)) \equiv \frac{m(v_k(x), u_k(x))}{v_k(x)} = B\theta_k(x)^{-\xi}$$

$$p(\theta_k(x)) \equiv \frac{m(v_k(x), u_k(x))}{u_k(x)} = B\theta_k(x)^{1-\xi}$$

where $x = \{A\}$ is the economy's state. The latter is only a function of aggregate technology just as in the standard model because each submarket i contains only one type of unemployed.

Timing. The timing is the same as outlined in section 3.2.

Labor market flows.

Job-seekers in submarket 1

$$u_1 = \gamma n(x_{-1})$$

Job-seekers in submarket 2

$$u_2 = 1 - n(x_{-1})$$

High-skilled employment

$$n^H(x) = (1-\gamma) n(x_{-1}) + p(\theta_1(x)) u_1$$

Low-skilled employment

$$n^L(x) = p(\theta_2(x)) u_2$$

Total employment

$$n(x) = (1-\gamma) n(x_{-1}) + p(\theta_1(x)) u_1 + p(\theta_2(x)) u_2$$

The labor market flows show that in the presence of skill erosion during unemployment and learning-by-doing there is an interaction between both submarkets. The number of job-seekers in submarket 2 will not only depend on the amount of jobs created in the previous period in this submarket but also on the amount of jobs that was created in submarket 1. The reason is that workers who did not get hired in submarket 1, lose a fraction of their skills and become job-seekers in submarket 2.

A.1.2 Decentralized Alloction

Firm's problem

The value from having a worker of type $i = \{H, L\}$ producing is given by

$$J^i(x) = y^i(x) - w^i(x) + (1-\gamma)\beta E_x\left\{J^H(x')\right\}$$

The value form having a vacancy in submarket 1 is given by

$$V_1(x) = -\kappa + q(\theta_1(x))\, J^H(x) + (1 - q(\theta_1(x)))\,\beta E_x\{V_1(x')\}$$

The value form having a vacancy in submarket 2 is given by

$$V_2(x) = -\kappa + q(\theta_2(x))\, J^H(x) + (1 - q(\theta_2(x)))\,\beta E_x\{V_2(x')\}$$

Free-entry in both submarkets implies

$$V_1(x) = V_2(x) = 0$$

Note that the value from being in a match only depends on the expected hiring cost, and no longer on the unemployment pool's composition: $J^H(x) = \frac{\kappa}{q(\theta_1(x))}$ and $J^L(x) = \frac{\kappa}{q(\theta_2(x))}$. The reason is that by posting a vacancy firms will be able to hire a worker of the same type, because the unemployment pool in each submarket is homogeneous.

Worker's problem

The value from being employed as type $i = \{H, L\}$ is given by

$$W^i(x) = w^i(x) + \beta E_x\left\{(1-\gamma+\gamma p(\theta_1(x')))\, W^H(x') + \gamma(1 - p(\theta_1(x')))\, U(x')\right\}$$

The value from being unemployed is given by

$$U(x) = b + \beta E_x\left\{p(\theta_2(x'))\, W^L(x') + (1 - p(\theta_2(x')))\, U(x')\right\}$$

The worker's surplus from being in a match becomes

$$X^i(x) = w^i(x) - o(x) + (1-\gamma)\beta E_x\left\{X^H(x')\right\}$$

where $o(x)$ represents the worker's outside option

$$o(x) \equiv b + \beta E_x \left\{ p(\theta_2(x')) X^L(x') - \gamma p(\theta_1(x')) X^H(x') \right\}$$

Wages

Wages are determined through Nash bargaining between workers and firms in every period. When workers' bargaining power in submarket k is given by η_k, Nash bargaining implies

$$X^H(x) = \eta_1 M^H(x)$$

$$J^H(x) = (1-\eta_1) M^H(x)$$

$$X^L(x) = \eta_2 M^L(x)$$

$$J^L(x) = (1-\eta_2) M^L(x)$$

where $M^i(x) \equiv X^i(x) + J^i(x)$ represents the surplus of being in a match with a worker of type i. The solution to the Nash bargaining problem results in the following expressions for the wage of a high-skilled and low-skilled worker respectively

$$w^H(x) = \eta_1 y^H(x) + (1-\eta_1) o(x)$$

$$w^L(x) = \eta_2 y^L(x) + (1-\eta_2) o(x) + (1-\gamma) \beta E_x \left\{ \eta_2 J^H(x') - (1-\eta_2) X^H(x') \right\}$$

Equilibrium

Submarket 1

$$\frac{\kappa}{q(\theta_1(x))} = (1-\eta_1) M^H(x)$$

$$M^H(x) = A - b + \beta E_x \left\{ \left[1 - \gamma + \eta_1 \gamma p\left(\theta_1\left(x^{'}\right)\right)\right] M^H(x') - \eta_2 p\left(\theta_2\left(x^{'}\right)\right) M^L(x') \right\}$$

Combining both equations gives

$$\frac{\kappa}{q(\theta_1(x))} = (1-\eta_1) \left[A - b + \beta E_x \left\{ \begin{array}{c} \left[1-\gamma+\eta_1\gamma p\left(\theta_1\left(x^{'}\right)\right)\right] \frac{\kappa}{q(\theta_1(x'))(1-\eta_1)} \\ -\eta_2 p\left(\theta_2\left(x^{'}\right)\right) \frac{\kappa}{q(\theta_2(x'))(1-\eta_2)} \end{array} \right\} \right] \tag{42}$$

Submarket 2

$$\frac{\kappa}{q(\theta_2(x))} = (1-\eta_2) M^L(x)$$

$$M^L(x) = (1-\delta) A - b + \beta E_x \left\{ \left[1 - \gamma + \eta_1 \gamma p\left(\theta_1\left(x^{'}\right)\right)\right] M^H(x') - \eta_2 p\left(\theta_2\left(x^{'}\right)\right) M^L(x') \right\}$$

Combining both equations gives

$$\frac{\kappa}{q(\theta_2(x))} = (1-\eta_2) \left[(1-\delta) A - b + \beta E_x \left\{ \begin{array}{c} \left[1-\gamma+\eta_1\gamma p\left(\theta_1\left(x^{'}\right)\right)\right] \frac{\kappa}{q(\theta_1(x'))(1-\eta_1)} \\ -\eta_2 p\left(\theta_2\left(x^{'}\right)\right) \frac{\kappa}{q(\theta_2(x'))(1-\eta_2)} \end{array} \right\} \right] \tag{43}$$

A.1.3 Constrained-Efficient Allocation

The social planner's problem consists of choosing labor market tightness in both submarkets such that total output in the economy, net of vacancy posting costs, is maximized. The planner's problem is given by

$$V^P\left(x^P\right) = \max_{\theta_1,\theta_2}\left[\begin{array}{c} An^H\left(x^P\right) + (1-\delta)\,An^L\left(x^P\right) - \kappa\theta_1\left(x^P\right)\gamma n_{-1} \\ -\kappa\theta_1\left(x^P\right)(1-n_{-1}) + b\,(1-n) + \beta E_{x^P}\left\{V^P\left(x^{P\prime}\right)\right\} \end{array}\right]$$

with $n^H\left(x^P\right) = (1-\gamma)\,n_{-1} + B\theta_1\left(x^P\right)^{1-\xi}\gamma n_{-1}$ and $n^L\left(x^P\right) = B\theta_2\left(x^P\right)^{1-\xi}(1-n_{-1})$

and subject to the law of motion for employment

$$n = (1-\gamma)\,n_{-1} + B\theta_1\left(x^P\right)^{1-\xi}\gamma n_{-1} + B\theta_2\left(x^P\right)^{1-\xi}(1-n_{-1})$$

The relevant state for the social planner is given by $x^P = \{A, n_{-1}\}$.

The first order condition for labor market tightness in submarket 1 and 2 are respectively given by

$$\frac{\kappa}{q\left(\theta_1\left(x^P\right)\right)} = (1-\xi)\left[A - b + \beta E_{x^P}\left\{\frac{\partial V^P\left(x^{P\prime}\right)}{\partial n}\right\}\right]$$

$$\frac{\kappa}{q\left(\theta_2\left(x^P\right)\right)} = (1-\xi)\left[(1-\delta)\,A - b + \beta E_{x^P}\left\{\frac{\partial V^P\left(x^{P\prime}\right)}{\partial n}\right\}\right]$$

The envelope condition for employment is given by

$$\frac{\partial V^P\left(x^P\right)}{\partial n_{-1}} = \left[\begin{array}{c} \left[1-\gamma+\gamma p\left(\theta_1\left(x^P\right)\right)\right]A - p\left(\theta_2\left(x^P\right)\right)(1-\delta)\,A + \kappa\left(\theta_2\left(x^P\right) - \gamma\theta_1\left(x^P\right)\right) \\ -\left[1-\gamma+\gamma p\left(\theta_1\left(x^P\right)\right) - p\left(\theta_2\left(x^P\right)\right)\right]\left[b - \beta E_{x^P}\left\{\frac{\partial V^P\left(x^{P\prime}\right)}{\partial n}\right\}\right] \end{array}\right]$$

Combining the first order condition for submarket i with the envelope condition, gives the following expressions for job creation in each submarket

Submarket 1

$$\frac{\kappa}{q\left(\theta_1\left(x^P\right)\right)} = (1-\xi)\left[A - b + \beta E_{x^P}\left\{\begin{array}{c} \left[1-\gamma+\xi\gamma p\left(\theta_1\left(x^{P\prime}\right)\right)\right]\frac{\kappa}{q\left(\theta_1\left(x^{P\prime}\right)\right)(1-\xi)} \\ -\xi p\left(\theta_2\left(x^{P\prime}\right)\right)\frac{\kappa}{q\left(\theta_2\left(x^{P\prime}\right)\right)(1-\xi)} \end{array}\right\}\right] \quad (44)$$

Submarket 2

$$\frac{\kappa}{q\left(\theta_2\left(x^P\right)\right)} = (1-\xi)\left[(1-\delta)\,A - b + \beta E_{x^P}\left\{\begin{array}{c} \left[1-\gamma+\xi\gamma p\left(\theta_1\left(x^{P\prime}\right)\right)\right]\frac{\kappa}{q\left(\theta_1\left(x^{P\prime}\right)\right)(1-\xi)} \\ -\xi p\left(\theta_2\left(x^{P\prime}\right)\right)\frac{\kappa}{q\left(\theta_2\left(x^{P\prime}\right)\right)(1-\xi)} \end{array}\right\}\right] \quad (45)$$

Note that labor market tightness in each submarket is such that

$$\frac{\kappa}{q\left(\theta_2\left(x^P\right)\right)(1-\xi)}=\frac{\kappa}{q\left(\theta_1\left(x^P\right)\right)(1-\xi)}-\delta A$$

This implies that even though it is equally costly to post vacancies in submarket 1, in equilibrium labor market tightness is higher in the first submarket because there is an output gain δA related to posting in submarket 1 relative to submarket 2.

A.2 Proof of Proposition 2

Proof. Comparing equations (42) and (43) with equations (44) and (45), shows that the decentralized allocation is constrained-efficient if the standard Hosios condition is satisfied in each submarket. □

A.3 Proof of Proposition 3

Proof. By comparing equation (34) and equation (36), it follows that the total effect of this period's vacancy posting on the number of matches that can be formed this period is taken into account when $\eta=\xi$. When the latter is satisfied, the expected surplus of a new match in the decentralized allocation is weighted by the elasticity of vacancies in the matching function $1-\xi$, just as in the social planner's allocation. The effect on next period's labor market tightness following from having a job-seeker less when this period's match survives separation is also internalized for $\eta=\xi$. This can be shown as follows. For $\delta=0$, this part of the congestion effect is internalized in the decentralized allocation because the net effect, generated by the congestion effect and a worker's outside option, can be expressed as a fraction of the worker's outside option. From the constrained-efficient allocation it follows that the net effect is given by (see the second term of expression (37))

$$\Upsilon(x)\equiv\tilde{o}(x)+\Omega(x)=-\frac{\xi}{1-\xi}(1-\gamma)\kappa E_x\left\{\theta\left(x'\right)\right\} \tag{46}$$

where the worker's outside option net of home production is given by (see the second term of expression (33))

$$\tilde{o}(x)\equiv-\left(\frac{1-\gamma}{1-\xi}\right)\kappa E_x\left\{\theta\left(x'\right)\right\} \tag{47}$$

and where the congestion effect is given by (see the third term of expression (33))

$$\Omega\equiv(1-\gamma)\kappa E_x\left\{\theta\left(x'\right)\right\} \tag{48}$$

Comparing equation (46) and the decentralized allocation (equation (34)) shows immediately that for $\eta=\xi$ the congestion effect is internalized, even though only the worker's outside option is taken into account (see equation (**??**)). This follows from the net effect being a fraction ξ of the worker's outside option (net of the value of home production) (equation (47)), and the latter being taken into account up to a fraction η in the decentralized allocation.

For $\delta > 0$, the net effect is now given by (see the second and third term of expression (37))

$$\Upsilon(x) = -\frac{\xi}{1-\xi}(1-\gamma)\kappa E_x\{\theta(x')\} + E_x\{A'p(\theta(x'))[1-(1-\gamma)s(n)]\} \tag{49}$$

where the outside option net of home production is now given by (see the second term of expression (33))

$$\tilde{o}(x) = -\left(\frac{1-\gamma}{1-\xi}\right)\kappa E_x\{\theta(x')\} + E_x\{A'p(\theta(x'))[1-(1-\gamma)s(n)]\} \tag{50}$$

and where the congestion effect is still given by equation (48). The congestion effect is still expressed as a function of the match with an average job-seeker because they all have the same hiring probability, and hence a change in labor market tightness affects all job-seekers' hiring probability in the same way.

Comparing equation (49) and the decentralized allocation (equation (34)) shows that the standard Hosios condition $\eta = \xi$ still internalizes the congestion effect even though only the worker's outside option is taken into account. This follows from the fact that this part of the net effect containing the congestion effect (given by the first term in equation (49)) can still be expressed as a fraction ξ of part of the worker's outside option (given by the first term in equation (50)). This in turn follows from both part of the net effect and part of the worker's outside option being able to be expressed as a function of the hiring probability and match value of an average job-seeker.

□

A.4 Optimal labor market policy

A.4.1 The Economy

Changes relative to the setup without an employment subsidy/tax

In the presence of an employment subsidy/tax only the firm's problem is affected. When employment is subsidized/taxed $\Phi(x)$, a firm's value of employing a worker of type i is given by

$$J_s^i(x) = y^i(A) - w^i(x) + \Phi(x) + (1-\gamma)\beta E_x\left\{J_s^H(x')\right\} \tag{51}$$

Equation (51) shows that the firm will receive the employment subsidy/tax $\Phi(x)$ independently of which worker type the firm employs.

Note that wages are also affected by the introduction of an employment subsidy/tax through its effect on the surplus of a match. The solution to the Nash bargaining problem is now given by

$$w^i(x) = \eta\left(y^i(A) + \Phi(x)\right) + (1-\eta)o(x)$$

where $o(x)$ is the worker's outside option defined by equation (21). It can be seen from the wage expression that in the presence of an employment subsidy $\Phi(x) > 0$, wages are higher than in the absence of it. The positive effect of employment subsidies on wages has also been emphasized by Mortensen and Pissarides (2001).

Equilibrium

The surplus of a match with a high-skilled worker (equation (52)) and the vacancy creation equation (equation (53)) can be obtained in the same way as described in section 3.6. However, the firm's value function is now given by equation (51) instead of by equation (13).

$$M_s^H(x) = A - b + \Phi(x) + \beta E_x \left\{ (1-\gamma)\left(1 - \eta p\left(\theta\left(x'\right)\right)\right) M_s^H(x') + \eta p\left(\theta\left(x'\right)\right)\delta A' \right\} \tag{52}$$

$$\frac{\kappa}{q(\theta(x))} = (1-\eta)\left[M_s^H(x) - s(x)\delta A\right] \tag{53}$$

Combining equation (52) and (53), job creation in the decentralized allocation in the presence of the employment subsidy is given by

$$\frac{\kappa}{q(x)(1-\eta)} = \bar{y}(x) + \Phi(x) - b + \beta E_x\left\{\Lambda^D(x')\right\} \tag{54}$$

where $\Lambda^D(x)$ is defined by equation (35) in section 4.2.

A.4.2 Optimal Employment Subsidy/Tax

The optimal employment subsidy/tax $\Phi(x)$ internalizes both the congestion and the composition externality if job creation in the decentralized allocation (equation (54)) replicates job creation in the constrained efficient allocation (equation (32)). It follows that the optimal employment subsidy/tax is given by

$$\Phi(x) = \begin{bmatrix} \frac{(\eta-\xi)}{(1-\xi)(1-\eta)q(\theta(x))}\kappa \\ +\beta E_x\left\{\frac{(\eta-\xi)(1-\gamma)(\eta p(\theta(x'))-1)}{(1-\xi)(1-\eta)q(\theta(x'))}\kappa + \delta(1-\eta)p(\theta(x'))A'(1-(1-\gamma)s(n))\right\} \end{bmatrix} \tag{55}$$

A.5 Multiple Equilibria

Pissarides (1992) has shown that in an overlapping generations model where the unemployed are exposed to skill loss the steady state is not unique for certain parameter conditions. This finding has been confirmed by Coles and Masters (2000) in a different setup. The intuition behind this finding is that the expected gain from vacancy posting is no longer strictly decreasing in labor market tightness

θ. In an economy without skill erosion during unemployent, this gain is strictly decreasing in θ because the job filling probability is a decreasing function of θ. In a model where skills erode, an increase in θ still implies a decrease in the job filling probability but there is an additional effect. Now an increase in θ also leads to an improvement in the skill distribution of the unemployment pool. The latter follows from a higher θ implying a lower average unemployment duration, and hence a smaller fraction of job-seekers with eroded skills. As pointed out by Pissarides (1992), when this effect is strong enough, an increase in θ can lead to an increase in the expected gain from vacancy posting, and hence multiple equilibria can arise.

Also in this model multiple steady state potentially arise. Evaluating the vacancy creation condition (29) in steady state shows that the expected gains are not necessarily strictly decreasing in θ. However, for a reasonable parameterization, as described in section 6.1 and 6.3.1, the Blanchard-Kahn (1980) conditions are satisfied which implies that the model has a stable and unique solution.

A.6 Generalized Version of the Model

This section describes the decentralized allocation and the constrained-efficient allocation when workers lose their skills with probability $0 < l \leq 1$, and regain their skills upon reemployment with probability $0 < g \leq 1$.

A.6.1 Decentralized Allocation

Firm's problem

Value of having a high-skilled worker employed

$$J^H(x) = y^H(A) - w^H(x) + (1-\gamma)\beta E_x\left\{J^H(x')\right\}$$

Value of having a low-skilled worker employed

$$J^L(x) = y^L(A) - w^L(x) + (1-\gamma)\beta E_x\left\{gJ^H(x') + (1-g)J^L(x')\right\}$$

Value of having a vacancy

$$V(x) = -\kappa + q(\theta(x))\left[(1-s(x))J^H(x) + s(x)J^L(x)\right] + (1-q(\theta(x)))\beta E_x\{V(x')\}$$

Imposing free entry $V(x) = 0$ leads to the following expression for vacancy creation:

$$\frac{\kappa}{q(\theta(x))} = (1-s(x))J^H(x) + s(x)J^L(x)$$

Worker's problem

Value of being employed as a high-skilled worker

$$W^{H}(x)=w^{H}(x)+\beta E_{x}\left\{(1-\gamma+\gamma p(x'))\,W^{H}(x')+\gamma(1-p(x'))\,U^{H}(x')\right\}$$

Value of being employed as a low-skilled worker

$$W^{L}(x)=w^{L}(x)+\beta E_{x}\left\{\begin{array}{c}(1-\gamma+\gamma p(x'))\left[gW^{H}(x')+(1-g)\,W^{L}(x')\right]\\ \gamma(1-p(x'))\left[gU^{H}(x')+(1-g)\,U^{L}(x')\right]\end{array}\right\}$$

Value of being unemployed as a high-skilled worker

$$U^{H}(x)=b+\beta E_{x}\left\{\begin{array}{c}l\left[p(x')\,W^{L}(x')+(1-p(x'))\,U^{L}(x')\right]\\ (1-l)\left[p(x')\,W^{H}(x')+(1-p(x'))\,U^{H}(x')\right]\end{array}\right\}$$

Value of being unemployed as a low-skilled worker

$$U^{L}(x)=b+\beta E_{x}\left\{p(x')\,W^{L}(x')+(1-p(x'))\,U^{L}(x')\right\}$$

Wages

Wages are set through Nash bargaining between worker and firm. Denoting the bargaining power of the workers by η, the value from being in match of a specific type for workers and firms respectively is given by

$$W^{i}(x)-U^{i}(x)=\eta M^{i}(x)$$

$$J^{i}(x)=(1-\eta)\,M^{i}(x)$$

The solution to the Nash bargaining problem leads to the following expression for wages

$$w^{i}(x)=\eta y^{i}(x)+(1-\eta)\,o^{i}(x)$$

where the outside option for each worker type are given by

$$o^{H}(x)\equiv b+\beta E_{x}\left\{\begin{array}{c}\eta p(x')\left[(1-l)\,M^{H}(x')+lM^{L}(x')\right]\\ -\eta\gamma p(x')\,M^{H}(x')\\ -l\left(U^{H}(x')-U^{L}(x')\right)\end{array}\right\}$$

$$o^{L}(x)\equiv b+\beta E_{x}\left\{\begin{array}{c}\eta p(x')\,M^{L}(x')\\ -\eta\gamma p(x')\left[gM^{H}(x')+(1-g)\,M^{L}(x')\right]\\ -g\left(U^{H}(x')-U^{L}(x')\right)\end{array}\right\}$$

Equilibrium

Given the match surplus from a high-skilled worker (56), the match surplus from a low-skilled worker (57), and the unemployment values (58) and(59), for a given state of the economy x, labor market tightness θ is defined by the vacancy creation condition (60)

$$M^{H}(x) = A - b + \beta E_x \left\{ \begin{array}{c} \left(1-\gamma+\eta\gamma p\left(\theta\left(x'\right)\right)\right) M^{H}\left(x'\right) \\ -\eta p\left(\theta\left(x'\right)\right)\left[l M^{L}\left(x'\right)+(1-l) M^{H}\left(x'\right)\right] \\ +l\left(U^{H}\left(x'\right)-U^{L}\left(x'\right)\right) \end{array} \right\} \tag{56}$$

$$M^{L}(x) = A(1-\delta) - b + \beta E_x \left\{ \begin{array}{c} \left(1-\gamma+\eta\gamma p\left(\theta\left(x'\right)\right)\right)\left(g M^{H}\left(x'\right)+(1-g) M^{L}\left(x'\right)\right) \\ -\eta p\left(\theta\left(x'\right)\right) M^{L}\left(x'\right) \\ +g\left(U^{H}\left(x'\right)-U^{L}\left(x'\right)\right) \end{array} \right\} \tag{57}$$

$$U^{H}(x) = b + \beta E_x \left\{\eta p\left(\theta\left(x'\right)\right)\left[(1-l) M^{H}\left(x'\right)+l M^{L}\left(x'\right)\right]+(1-l) U^{H}\left(x'\right)+l U^{L}\left(x'\right)\right\} \tag{58}$$

$$U^{L}(x) = b + \beta E_x \left\{\eta p\left(\theta\left(x'\right)\right) M^{L}\left(x'\right)+U^{L}\left(x'\right)\right\} \tag{59}$$

$$\frac{\kappa}{q(\theta(x))} = (1-\eta)\left[(1-s(x)) M^{H}(x)+s(x) M^{L}(x)\right] \tag{60}$$

where the law of motions of the endogenous state variables $\tilde{u}^{H}_{-1}, \tilde{u}^{L}_{-1}, n^{L}_{-1}$ are given by

$$\tilde{u}^{H} = (1-p(\theta(x)))\left[(1-l) \tilde{u}^{H}_{-1}+\gamma\left(1-(1-g) n^{L}_{-1}-\tilde{u}^{H}_{-1}-\tilde{u}^{L}_{-1}\right)\right] \tag{61}$$

$$\tilde{u}^{L} = (1-p(\theta(x)))\left[\tilde{u}^{L}_{-1}+l \tilde{u}^{H}_{-1}+\gamma(1-g) n^{L}_{-1}\right] \tag{62}$$

$$n^{L} = (1-\gamma)(1-g) n^{L}_{-1}+p(\theta(x))\left[\tilde{u}^{L}_{-1}+l \tilde{u}^{H}_{-1}+\gamma(1-g) n^{L}_{-1}\right] \tag{63}$$

A.6.2 Constrained-Efficient Allocation

The constrained-efficient allocation is obtained by solving the social planner's problem. The latter is defined as

$$V^{P}(x) = \max_{\theta}\left[A n^{H}(x)+(1-\delta) A n^{L}(x)-\kappa \theta(x)\left(1-(1-\gamma) n\left(x_{-1}\right)\right)+b(1-n(x))+\beta E_x\left\{V^{P}\left(x'\right)\right\}\right]$$

subject to the law of motions of the endogenous state variables $\tilde{u}^{H}, \tilde{u}^{L}$ and n^{L} (see equations(61),(62), and (63) respectively), and the process for aggregate technology (equation (1)), and where $n^{H}(x) = 1 - n^{L}(x) - \tilde{u}^{L}(x) - \tilde{u}^{H}(x)$; $n = n^{H}(x) + n^{L}(x)$; and the economy's state is given by $x = \left\{A, \tilde{u}^{H}_{-1}, \tilde{u}^{L}_{-1}, n^{L}_{-1}\right\}$.

The first order condition for vacancy creation is

$$\frac{\kappa}{q(\theta(x))} = (1-\xi)\left[\bar{y}(x) - b + \beta E_x\left\{-(1-s(x))\frac{\partial V^P(x')}{\partial u^H} - s(x)\frac{\partial V^P(x')}{\partial u^L} + s(x)\frac{\partial V^P(x')}{\partial n^L}\right\}\right] \tag{64}$$

The envelope condition for high-skilled unemployment is

$$\frac{\partial V^P(x)}{\partial \tilde{u}^H_{-1}} = \left\{\begin{array}{c} -\underbrace{\left[(1-\gamma+\gamma p(\theta(x)))\, y^H(A) + \gamma(1-p(\theta(x)))\,\beta E_x\left\{\frac{\partial V^P(x')}{\partial \tilde{u}^H}\right\}\right]}_{no\ HS\ worker} \\ +\underbrace{p(\theta(x))}_{hiring}\left[(1-l)\, y^H(A) + l\left(y^L(A) + \beta E_x\left\{\frac{\partial V^P(x')}{\partial n^L}\right\}\right)\right] \\ +\underbrace{(1-p(\theta(x)))}_{no\ hiring}\beta E_x\left\{(1-l)\frac{\partial V^P(x')}{\partial \tilde{u}^H} + l\frac{\partial V^P(x')}{\partial \tilde{u}^L}\right\} \\ +(1-\gamma)(1-p(\theta(x)))\, b - (1-\gamma)\,\kappa\theta(x) \end{array}\right\} \tag{65}$$

The envelope condition for low-skilled unemployment is

$$\frac{\partial V^P(x)}{\partial \tilde{u}^L_{-1}} = \left\{\begin{array}{c} -\underbrace{\left[(1-\gamma+\gamma p(\theta(x)))\, y^H(A) + \gamma(1-p(\theta(x)))\,\beta E_x\left\{\frac{\partial V^P(x')}{\partial \tilde{u}^H}\right\}\right]}_{no\ HS\ worker} \\ +\underbrace{p(\theta(x))}_{hiring}\left[y^L(A) + \beta E_x\left\{\frac{\partial V^P(x')}{\partial n^L}\right\}\right] + \underbrace{(1-p(\theta(x)))}_{no\ hiring}\beta E_x\left\{\frac{\partial V^P(x')}{\partial \tilde{u}^L}\right\} \\ +(1-\gamma)(1-p(\theta(x)))\, b - (1-\gamma)\,\kappa\theta(x) \end{array}\right\} \tag{66}$$

The envelope condition for low-skilled employment is

$$\frac{\partial V^P(x)}{\partial n^L_{-1}} = \underbrace{(1-g)}_{no\ regaining}\left\{\begin{array}{c} -\underbrace{\left[(1-\gamma+\gamma p(\theta(x)))\, y^H(A) + \gamma(1-p(\theta(x)))\,\beta E_x\left\{\frac{\partial V^P(x')}{\partial \tilde{u}^H}\right\}\right]}_{no\ HS\ worker} \\ +(1-\gamma+\gamma p(\theta(x)))\left[y^L(A) + \beta E_x\left\{\frac{\partial V^P(x')}{\partial n^L}\right\}\right] \\ +\gamma(1-p(\theta(x)))\,\beta E_x\left\{\frac{\partial V^P(x')}{\partial \tilde{u}^L}\right\} \end{array}\right\} \tag{67}$$

The constrained-efficient allocation is defined by equations (64)-(67) together with the law of motions of the endogenous state variables (equations (61),(62), and (63)), and the process for aggregate technology (equation (1)).

B Figures

Figure 1: Role of the human capital depreciation rate in shaping labor market outcomes

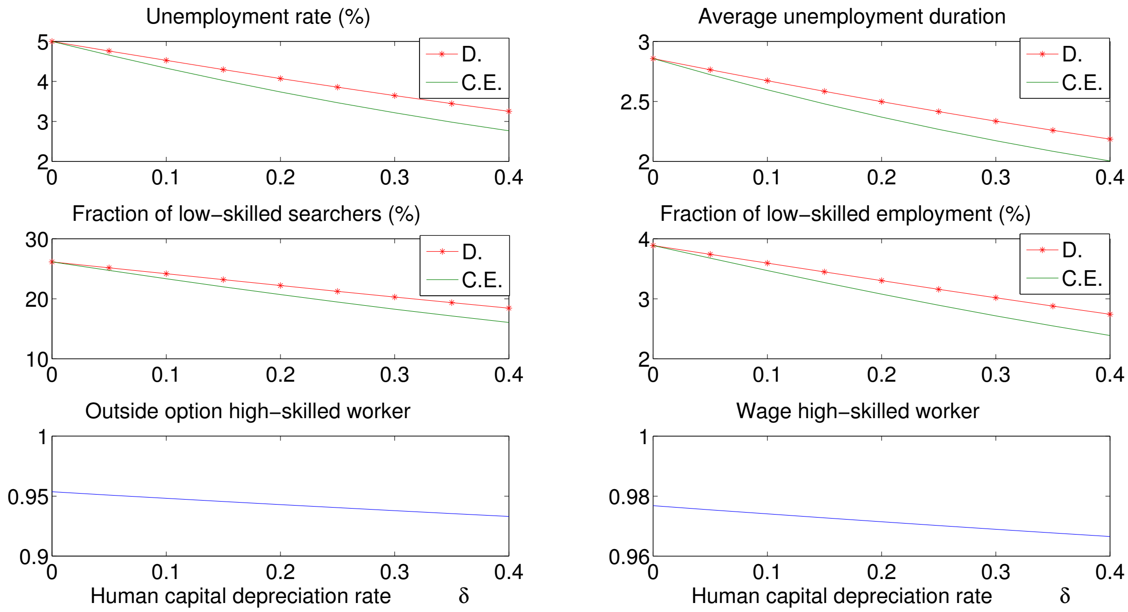

Figure 2: Role of the probability of skill loss in shaping labor market outcomes

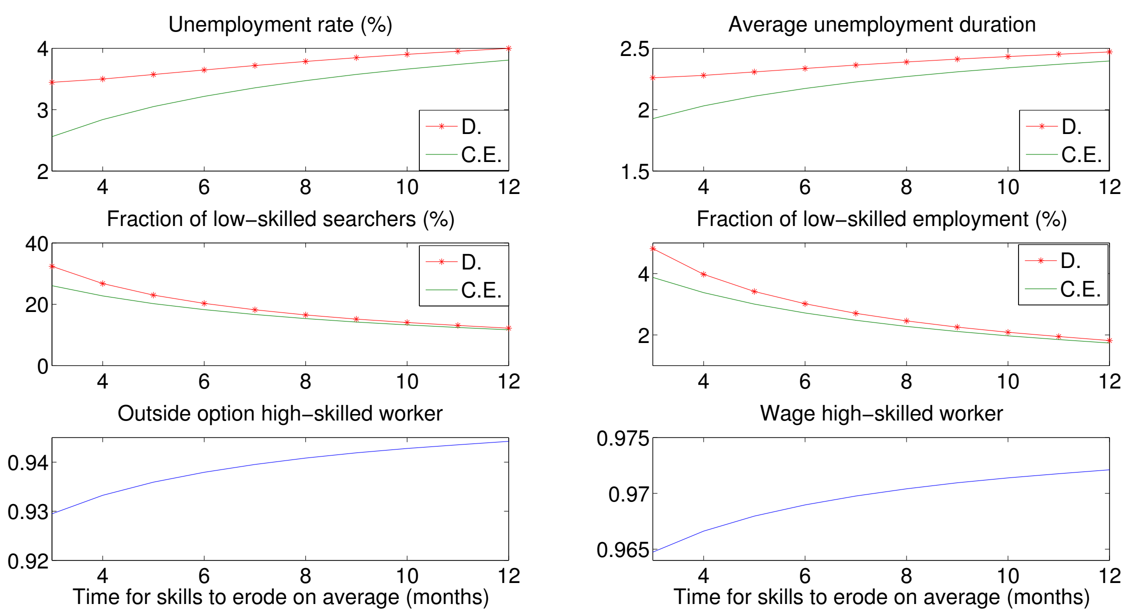

Figure 3: Role of the probability of regaining skills in shaping labor market outcomes

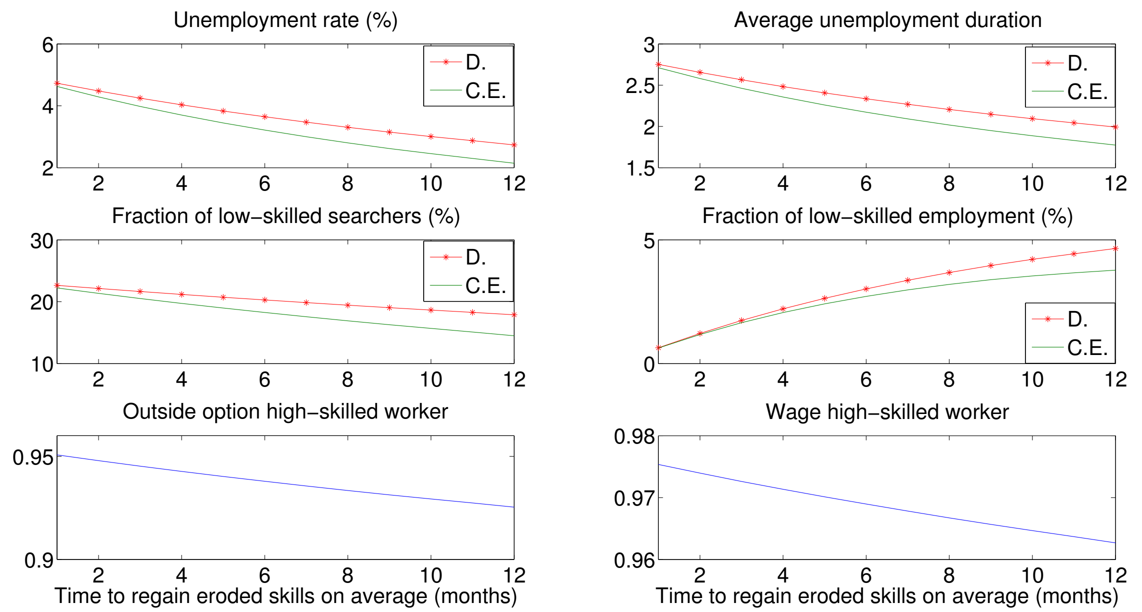

Figure 4: Role of human capital depreciation rate in shaping labor market dynamics

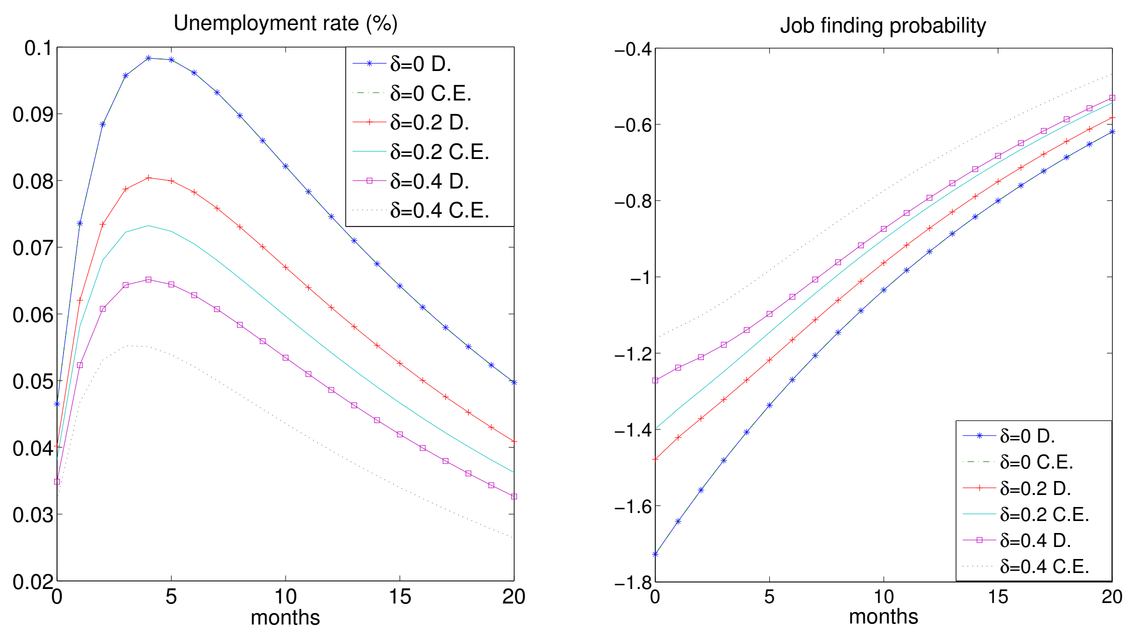

Figure 5: Role of probability of skill loss in shaping labor market dynamics

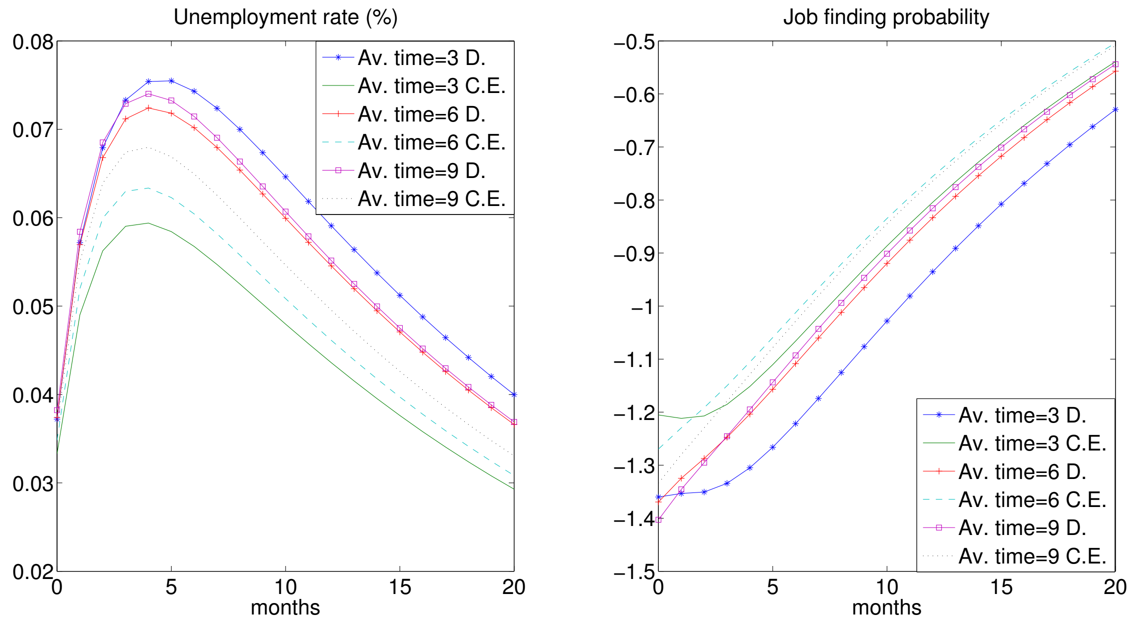

Figure 6: Role of probability of regaining skills in shaping labor market dynamics

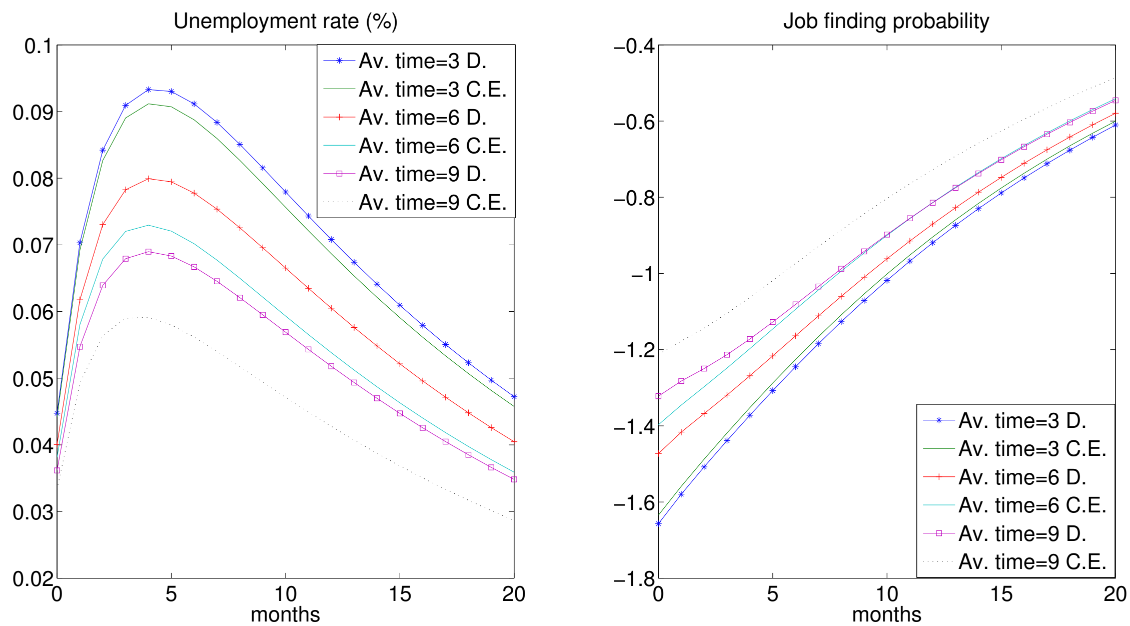

Figure 7: Labor market dynamics in the decentralized versus the constrained-efficient allocation for $1/l = 4$