Multiproduct Retailing

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November 7th 2012

We study the pricing behavior of a multiproduct retailer, when consumers must pay a search cost to learn its prices. Equilibrium prices are high because rational consumers understand that visiting the store exposes them to a hold-up problem. However a firm with more products charges lower prices, because it attracts consumers who are more price-sensitive. We also show that when the retailer advertises the price of one product, it provides consumers with some indirect information about all of its other prices. The firm therefore exploits this and sets a relatively low advertised price, in order to build a store-wide 'low-price image'.

Keywords: retailing, multiproduct search, advertising **JEL:** D83, L0, M37

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1 Introduction

Retailing is inherently a multiproduct phenomenon. Most firms sell a wide range of products, and consumers frequently travel to a store with the intention of purchasing several items. Whilst consumers often know a lot about a retailer's products, they are poorly-informed about the prices of individual items unless they buy them on a regular basis. Therefore a consumer must spend time visiting a retailer in order to learn its prices. The theoretical literature has mainly focused on single-product retailing. However once we take into account the multiproduct nature of retailing, a whole new set of important questions arises. For example, what are the advantages to a retailer from stocking a wider range of products? How do pricing incentives change when a firm sells more products? Some retailers eschew advertising and charge 'everyday low prices' across the whole store. Others use a so-called HiLo strategy, charging high prices on many products but advertising steep discounts on others. When is one strategy better than the other? In addition advertisements usually can only contain information about the prices of a small proportion of a retailer's overall product range. This naturally raises the question of how much information consumers can learn from an advert, about the firm's overall pricing strategy. Related, when an advertisement contains a good deal on one product, should consumers expect the firm to compensate by raising the prices of other goods?

In order to answer these and other related questions, we focus on the pricing behavior of a monopolist who sells a number of independent products. Consumers have different valuations for different products, and would like to buy one unit of each. Every consumer is privately informed about how much she values the products, but does not know their prices. The retailer can inform consumers about one of its prices by paying a cost and sending out adverts. Consumers observe whether or not an advert was sent, and form (rational) expectations about the price of every product within the store. Each consumer then decides whether or not to pay a search cost and visit the retailer. After searching, a consumer learns actual prices and makes purchases. We think this set-up closely approximates several product markets. For instance local convenience shops and drugstores sell mainly standardized products which do not change much over time. A consumer's main reason for visiting them is not to learn whether their products are a good 'match', but instead to buy products that she already knows are suitable. On the other hand studies have repeatedly found that consumers have only limited recall of the prices of products which they recently bought (see for example Monroe and Lee (1999) pp. 212-3 for a survey of several such studies). Therefore in many cases consumers are not well-informed about prices before they go shopping.

In order to characterize the retailer's optimal prices, we first provide a new and elegant solution to the following hold-up problem, which was first elucidated by Diamond (1971) and then by Stiglitz (1979). Suppose that consumers have unit demands, that firms sell only one product and do not advertise its price, and that travelling to the store costs $s > 0$. Under these assumptions no consumer will search, and firms gets zero profit. The reason is that if consumers expect retailers to charge a price p^e, only people with a valuation above $p^e + s$ will search. After paying the search cost, all these consumers will buy the product provided its actual price is below $p^e + s$. Hence there cannot be an equilibrium in which some consumers search, because retailers would always charge more than was expected.

Many papers have suggested possible ways to overcome this surprising result. For example in Burdett and Judd (1983) consumers sometimes learn several prices during a single search. Alternatively some consumers may enjoy shopping, or know all retailers' prices before searching (Varian 1980, Stahl 1989). In both cases competition for the better-informed consumers leads to somewhat lower prices. Secondly firms might send out adverts which commit them to charging a particular price, and thereby guarantee consumers some surplus (Wernerfelt 1994, Anderson and Renault 2006). Thirdly consumers might only learn their match for a product after searching for it. Anderson and Renault (1999) show that the market does not collapse provided there is enough preference for variety.

In this paper we show that when a retailer sells enough products, this hold-up problem disappears. This is true even when all prices are unadvertised, and every consumer has a strictly positive search cost. Intuitively in the single-product case, only consumers with a high valuation decide to search, so retailers exploit this and charge a high price. However in the multiproduct case, somebody with a low valuation on one product may search because she has a high valuation on another. When the firm increases one of its prices, some consumers with a low valuation for that product stop buying it. This reduces the retailer's incentive to surprise consumers by charging more than they expected. Consequently there can exist equilibria in which consumers search.

Building on this observation, we then use the model to characterize a retailer's optimal pricing strategy. We begin by examining the relationship between the size of a firm's product range, and the prices it charges on individual items. There is lots of evidence, both empirical and anecdotal, that larger stores generally charge lower prices. For example in the United Kingdom, the two major supermarkets Tesco and Sainsbury's, charge higher prices in their small stores (Competition Commission 2008). Also after reviewing 14 prior studies, Kaufman et al (1997) concluded that prices were almost always highest at smaller outlets. Our model provides a novel explanation for this phenomenon. In particular we show that larger stores attract a broader mix of consumers, who on average have more low product-valuations. As a result starting with the same pool of *potential* customers, when a retailer is larger its *actual* customers (who turn up at its store) are more price-elastic. This naturally gives a larger retailer an incentive to charge lower prices. Later on we discuss some empirical evidence which is consistent with this explanation.[1]

We then seek to characterize a multiproduct retailer's optimal advertising strategy. Typically adverts can only contain information on a very small number of prices. Although there is a large literature on advertising (see Bagwell 2007), an important but much-neglected question, is how a retailer's few (but typically very

[1]Villas-Boas (2009) and Zhou (2012) also investigate how a multiproduct firm's pricing strategy depends upon its product range. One important difference is that in these papers, consumers search for both price and product match information. In Villas-Boas's model a monopolist sells many substitute products. It charges higher prices when it sells more products, because it provides consumers with a better product match. In Zhou's model firms sell two independent products, and a consumer's match realizations are independent across retailers. Prices are typically lower than in the single-product version of the model, because a price reduction on one product causes more consumers to stop searching and buy both products immediately.

low) advertised prices are related to the prices that it charges on its other products. In order to answer this question, we allow the retailer to pay a cost and send out an advert which directly informs consumers about one of its prices. We show that when the firm cuts its advertised price, some new consumers with relatively low valuations decide to search. The firm then finds it optimal to also reduce its unadvertised prices, in order to sell more products to these new searchers. Therefore consumers (rationally) expect a positive relationship between the firm's advertised and unadvertised prices. Whilst the firm cannot commit in advance to its unadvertised prices, it can indirectly convey information about them via its advertised price. This implies that a low advertised price on one product can build a store-wide 'low price-image', even on completely unrelated products.

This result is related to but different from Lal and Matutes (1994) and Ellison (2005), in which two firms are located on a Hotelling line and sell two products but advertise only one of them. In Lal and Matutes all consumers have an identical willingness to pay H for one unit of each good. In Ellison all consumers value the advertised (base) product the same, but have either a high or a low valuation for the unadvertised (add-on) product.[2] As in Diamond's model, the unadvertised price is driven up to H in Lal and Matutes, and (typically) up to the high-types' willingness to pay in Ellison's model. Firms use their advertised price to compete for store traffic, but unlike in our model, cannot use it to credibly convey information about their unadvertised price. The difference arises because in our paper valuations are heterogeneous and continuously distributed. Therefore when a firm changes its advertised price, the pool of searchers also changes and this alters the firm's pricing incentives on its other products. Simester (1995) also finds that prices are positively correlated, but due to cost rather than preference heterogeneity. In his model a low-cost firm charges a lower unadvertised price, and may signal its cost advantage to consumers by advertising a lower price on another good as well.

In our model the retailer advertises a low price on one product to attract con-

[2]More precisely consumers in Ellison's model have either a high or a low marginal utility of income (and therefore either a low or a high valuation for the add-on). This induces correlation in horizontal and vertical attributes, and raises industry profits when add-on prices are not advertised.

sumers into its store. It then charges consumers higher prices on its remaining (unadvertised) products. This pattern of pricing is also found in other papers such as Hess and Gerstner (1987) and Lal and Matutes (1994). They show that retailers compete for store traffic through low advertised prices, because once consumers arrive at a store they end up paying high prices on other products.[3] Our model suggests a different explanation for low advertised prices - namely a retailer's desire to build a low-price image. This low-price image persuades consumers that they will not be overcharged too much on the retailer's other products, and ultimately leads to higher store-wide profits. Also related to our paper are Bliss (1988) and Ambrus and Weinstein (2008), who study models in which consumers are fully-informed about all prices. Bliss shows that firms cover their overheads by using Ramsey pricing, and he argues that some mark-ups could be small or even negative. However Ambrus and Weinstein prove that loss-leading is only possible when there are very delicate demand complementarities between products. Our model on the other hand can generate low and even below-cost advertised prices, even when all products are symmetric and independent.[4]

Finally we extend the model in several directions. Most importantly, we show that the main results generalize to the competitive case where two retailers sell the same products. Most of the time a retailer does not advertise, and charges a high regular product across its store. Occasionally it holds a sale, advertising a low price on one product and also offering a somewhat lower price on another product. In order to prevent its rival from undercutting it, a retailer randomizes over both which product it advertises and also over the price it chooses for that product. Each firm's advertised and unadvertised prices are therefore random but positively correlated.

[3]Konishi and Sandfort (2002) also show that a monopolist selling substitute products, often advertises a low price on one of them to attract consumers to its store. The firm hopes that once inside the store, some of these consumers will switch to a more expensive substitute.

[4]Low advertised prices can arise for other reasons. In DeGraba (2006) some consumers are more profitable than others, and firms target the more profitable consumers by offering loss-leaders on products that are (primarily) bought by them. In Chen and Rey (2010) consumers differ in terms of their shopping cost, such that a retailer may use a loss-leader on one product in order to better discriminate between multi-stop and one-stop shoppers.

This contrasts with McAfee (1995) and Hosken and Reiffen (2007) who find that multiproduct firms' prices are negatively correlated, and with Shelegia (2012) who finds that prices are uncorrelated. However these papers are quite different from ours because all prices are unadvertised, and instead competition is driven by the assumption that some consumers can search costlessly.

The rest of the paper proceeds as follows. Section 2 outlines the model, whilst Section 3 characterizes prices when the firm decides not to advertise. Section 4 then characterize's the retailer's optimal advertising strategy. Finally Section 5 extends the model in different directions, whilst Section 6 concludes.

2 Model

There is a unit mass of consumers interested in buying N goods. These N products are neither substitutes nor complements, and consumers demand at most one unit of each. We let v_j denote a typical consumer's valuation for product j. Each v_j is drawn independently across both products and consumers, using a distribution function $F(v_j)$ whose support is $[a, b] \subset \mathbb{R}^+$. The corresponding density $f(v_j)$ is strictly positive, continuously differentiable, and logconcave.[5] There is a single retailer who produces $n \leq N$ goods, denoted by $j = 1, \dots, n$, at constant marginal cost c where $0 \leq c < b$. We have in mind a situation where $N - n$ is relatively small, such that the retailer is able to satisfy most of its customers' needs. Note that in the textbook model without search frictions, each good's profit function is strictly quasiconcave and has a unique maximizer $p^m = \arg\max (p - c)[1 - F(p)]$. To simplify matters we focus on the case $p^m > a$, although our results also hold when $p^m = a$.

As discussed in the introduction, consumers often have a good idea about which products a firms sells. However they only learn the prices of most of these products after they have travelled to the retailer's store. We therefore assume that consumers have full information about which n products the retailer stocks. In addition each

[5]Bagnoli and Bergstrom (2005) show that many common densities (and their truncations) are logconcave. Logconcavity ensures that the hazard rate $f(v_j)/(1 - F(v_j))$ is increasing.

consumer knows her individual valuations $(v_1, v_2, ..., v_n)$, but initially does not know any of the firm's prices. In order to buy anything consumers must travel to the store at cost $s > 0$; once inside the store they learn all the retailer's prices. We can therefore interpret s as both a search and a shopping cost, and we use these terms interchangeably throughout the paper. The model has three stages. In the first stage the retailer privately chooses the prices of every product, which we denote by $(p_1, p_2, ...p_n)$. It also has the opportunity to inform consumers about the price of one its products, by buying an advertisement at cost $\kappa > 0$. The advert must be truthful, and is received by all consumers. In the second stage consumers observe whether or not an advert was sent (and if so, learn the advertised price) and then form expectations $\boldsymbol{p^e} = (p_1^e, p_2^e, ...p_n^e)$ about the prices of each product. Consumers then visit the retailer if and only if their expected surplus $\sum_{j=1}^{n} \max\left(v_j - p_j^e, 0\right)$ exceeds the shopping cost s. Finally in the third stage, consumers who have travelled to the store learn actual prices and make their purchases.

3 No advertising

This section analyzes the benchmark case in which the firm has decided not to advertise any of its prices.

3.1 Solving for equilibrium prices

Consumers learn that the firm has not advertised, and then form expectations $\boldsymbol{p^e} = (p_1^e, p_2^e, \ldots, p_n^e)$ about the price of each product. Clearly if these expected prices satisfy $\sum_{j=1}^{n} \max\left(b - p_j^e, 0\right) \leq s$ no consumer searches and the retailer earns zero profit. However we are interested in finding equilibria where $\sum_{j=1}^{n} \max\left(b - p_j^e, 0\right) > s$ such that some consumers do search. Note that once a consumer is in the store, she buys product i provided that her valuation v_i exceeds the actual price p_i. Therefore demand for unadvertised product i is

$$D_i\left(p_i; \boldsymbol{p^e}\right) = \int_{p_i}^{b} f\left(v_i\right) \Pr\left(\sum_{j=1}^{n} \max\left(v_j - p_j^e, 0\right) \geq s \,\middle|\, v_i\right) dv_i \tag{1}$$

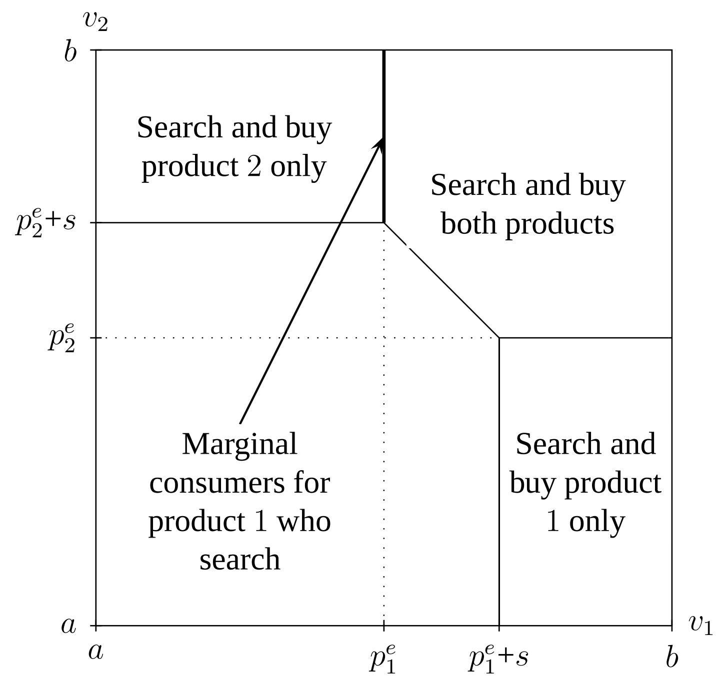

Figure 1: Search behavior when $n = 2$.

The firm chooses the actual price p_i to maximize its profit $(p_i - c)\, D_i\,(p_i; \boldsymbol{p^e})$ on good i. In equilibrium consumer price expectations must be correct, so we require that $p_i^e = \underset{p_i}{\text{argmax}}\ (p_i - c)\, D_i\,(p_i; \boldsymbol{p^e})$. Imposing this condition, we have the following lemma (Note that proofs generally appear in the appendices.)

Lemma 1 *The equilibrium price of unadvertised good i satisfies*

$$D_i\,(p_i = p_i^e; \boldsymbol{p^e}) - (p_i^e - c)\, f\,(p_i^e) \Pr\left(\sum_{j \neq i} \max\left(v_j - p_j^e, 0\right) \geq s\right) = 0 \tag{2}$$

To understand (2), consider a small increase in p_i above the expected level p_i^e. The firm gains revenue on those who continue to buy, and they have mass equal to demand. It loses $p_i^e - c$ on those who stop buying good i, and they have mass $f\,(p_i^e) \Pr\left(\sum_{j \neq i} \max\left(v_j - p_j^e, 0\right) \geq s\right)$. Intuitively, each consumer who stops buying the good a). has a marginal valuation $v_i = p_i^e$ for it, and b). has searched. Any consumer who is marginal for good i only searches if her expected surplus on the remaining $n - 1$ goods, $\sum_{j \neq i} \max\left(v_j - p_j^e, 0\right)$, exceeds the search cost s.

Figure 1 illustrates search and purchase behavior when $n = 2$ and consumers rationally anticipate prices p_1^e and p_2^e. Consumers in the top-right corner have both a high v_1 and a high v_2, and therefore definitely search and buy both products. Consumers in the bottom-right corner have $v_2 \leq p_2^e$ and therefore do not expect to buy product 2; however they still search provided that $v_1 \geq p_1^e + s$ because they expect product 1 alone to give enough surplus to cover the search cost. If the firm considered increasing the actual price p_1 slightly above the expected level p_1^e, only those consumers on the thick line would stop buying product 1. All other marginal consumers for product 1 have $v_2 < p_2^e + s$, so they don't respond to changes in p_1.

An equilibrium with consumer search does not always exist. When $n = 1$ equation (2) simplifies to $D_1\left(p_1 = p_1^e; p_1^e\right) = 0$ and it is immediate that:

Remark 2 (Diamond hold-up problem) *If $n = 1$ any equilibrium has $p_1^e \in [b - s, b]$, and no consumer visits the retailer.*

This is a famous result which can be traced back to Diamond (1971). Intuitively suppose the expected price were in fact below $b - s$; consumers with $v_1 \geq p_1^e + s$ would then find it worthwhile to visit the retailer. However since all the consumers in the store would be willing to pay at least $p_1^e + s$ for the product, the retailer would hold them up by charging $p_1 > p_1^e$. This hold-up problem can only be avoided when $p_1^e \in [b - s, b]$, because then the expected price is so high that no consumer finds it worthwhile to search. However once we take seriously the multiproduct nature of retailing, we find that:

Proposition 3 *If n is sufficiently large, there exists at least one ('non-Diamond') equilibrium in which consumers search and the retailer makes positive profit.*

Our paper therefore provides a simple yet elegant solution to the above hold-up problem. Building on this result, we then investigate how the multiproduct nature of a retailer's offering affects the prices it chooses for individual products. The intuition behind the proposition itself is straightforward. A multiproduct firm attracts a broad mix of consumers, not all of whom have high valuations for every product in the store. In particular many consumers who are marginal for product 1 say, will search when n is large because there are many other products in the store which collectively offer them a surplus exceeding s. If the retailer tried to 'hold up' consumers and charge more than was expected for product 1, it would then lose a lot of demand from these marginal consumers. This deters the firm from holding people up, making it possible to have an equilibrium where consumers have rational expectations and search. Note that consumer valuations must be heterogeneous for this argument to work. If instead all consumers attached the same valuation $\bar{v}$ to each of the n products, the retailer would certainly hold them up by charging $\bar{v}$ on every product if they ever travelled to its store. Therefore to have an equilibrium where consumers search (and are not held-up), *both* consumer heterogeneity and multiple products are required.

According to Proposition 3 only retailers with sufficiently many products, are able to attract consumers and make a positive profit. In general the critical number of products needed for this to happen, varies with underlying parameters of the model in a natural way.[6] Firstly more products are required when either the search or production cost is higher. Intuitively fewer marginal consumers search when s is higher, so the firm's incentive to hold-up consumers is greater. Similarly marginal consumers are less valuable to the firm when c is higher, so raising prices is again more attractive. To offset this increased desire to hold-up consumers, larger n is required – because this draws more marginal consumers to the store, and makes demand more elastic. Secondly suppose valuations are drawn from $[a+q, b+q]$ using distribution function $F(v-q)$. The parameter q can be interpreted as a measure of the quality of a firm's products. When quality is higher, fewer products are required to get a non-Diamond equilibrium. Intuitively a consumer who is marginal for one product, is (ceteris paribus) more likely to search when other products in the store have higher valuations. Consequently a firm with higher-quality products has less incentive to surprise consumers by raising its prices.

Table 1 provides numerical examples of the number of products required for the retailer to be searched and have positive profits, when valuations are uniformly distributed. For instance when $a = 1$, $b = 2$, $c = 1/2$ and $s = 10^{-6}$, a non-

[6] These results require that $(p-c)[1-F(p)]$ is concave, or equivalently that $f'(v)$ is not too negative. This ensures that any solutions to the first order conditions are global maximizers. See Lemma 16 in the appendix.

Table 1: Number of products required for an equilibrium with search.

		Support of (uniformly distributed) valuations					
		$[0,1]$	$[1/2,3/2]$	$[1,2]$	$[0,2]$	$[1,3]$	$[0,3]$
$c=\frac{1}{4}$	$s=10^{-6}$	3	2	2	3	2	3
	$s=1/2$	12	6	4	6	3	4
	$s=1$	20	9	6	9	5	7
	$s=2$	36	15	9	15	8	11
$c=\frac{1}{2}$	$s=10^{-6}$	4	3	2	3	2	3
	$s=1/2$	22	8	5	7	4	4
	$s=1$	39	13	7	12	6	7
	$s=2$	73	22	11	20	9	12

Diamond equilibrium exists if and only if $n \geq 2$. More generally the number of products required exceeds two, and as explained above increases in both the search and production costs. In addition an equilibrium is more likely to exist when the distribution is shifted towards higher valuations.

To gain further understanding of how the retailer chooses its prices, it is convenient to introduce the notation $t_j \equiv \max\left(v_j - p_j^e, 0\right)$ for the expected surplus on good j. Only consumers with $\sum_{j=1}^n t_j \geq s$ actually visit the store, and can therefore respond to changes in the actual price of product i. These consumers naturally split into two groups. We use the term *'shoppers for product i'* to denote those consumers with $\sum_{j\neq i} t_j \geq s$. They search irrespective of how much they value product i.[7] We use the term *'Diamond consumers for product i'* to denote those people with $\sum_{j=1}^n t_j \geq s > \sum_{j\neq i} t_j$. They only search because they anticipate a strictly positive surplus on product i. Equation (2), which is the first order condition for product i,

[7]We use the term 'shoppers for product i' because these consumers act *as if* they have no search cost when it comes to buying good i. This terminology mimics the existing literature, in which somebody is a 'shopper' if they have no search cost. See for example Stahl (1989).

can be rewritten as:

$$\underbrace{\Pr\left(\sum_{j\neq i} t_j \geq s\right)}_{\text{Probability of a shopper}} \left[1-F\left(p_i^e\right)-\left(p_i^e-c\right)f\left(p_i^e\right)\right]+\underbrace{\Pr\left(\sum_{j=1}^{n} t_j \geq s\right)-\Pr\left(\sum_{j\neq i} t_j \geq s\right)}_{\text{Probability of a Diamond consumer}}=0 \tag{3}$$

The lefthand side of this equation decomposes into two parts, the change in profits caused by a small change in p_i around p_i^e. Firstly shoppers for product i search irrespective of their v_i, which therefore continues to be distributed on $[a, b]$ with the usual density $f(v_i)$. Consequently profits on shoppers are simply $(p_i - c)[1 - F(p_i)]$ (the same as in a standard zero-search-cost monopoly problem), and so small changes in p_i around p_i^e affect profits by $1 - F(p_i^e) - (p_i^e - c) f(p_i^e)$. Secondly Diamond consumers for product i have $v_i - p_i^e > 0$, so they would all buy product i even if the price were slightly higher than expected. Consequently a small increase in p_i above p_i^e, causes profits on Diamond consumers to increase by 1. Just like consumers at a single-product retailer, their demand is locally perfectly inelastic.

Recall from earlier that $p^m = \arg\max (p - c)[1 - F(p)]$ is the standard monopoly price which arises when there is no search cost. The following result is immediate:

Remark 4 *The equilibrium price of any unadvertised good i strictly exceeds p^m.*

This follows because equation (3) can only be satisfied when the gains on Diamond consumers (from increasing p_i above p_i^e) are offset by losses on shoppers. A small increase in p_i above p_i^e only leads to losses on shoppers if $1 - F_i(p_i^e) - (p_i^e - c) f(p_i^e) < 0$, which is equivalent to $p_i^e > p^m$ because $(p_i - c)[1 - F(p_i)]$ is strictly quasiconcave. Intuitively the firm faces the following sample selection problem: only consumers with several high valuations search, and the firm will naturally try to somewhat exploit this ex post. This drives equilibrium prices above p^m.

We have been assuming that the search cost is constant, though in fact we might expect search to be more time-consuming in a larger store. This can be incorporated into the model without changing the results. To do this, let $s(n)$ denote the search cost and $\mu_p = \int_p^b (x - p) f(x)\, dx$ be the expected surplus on a good whose price is p. Conditional on an equilibrium existing, prices still exceed p^m. The statement in Proposition 3 concerning existence, also remains valid provided that there exists $\delta > 0$ and an n', such that $s(n)/n \leq \mu_{p^m} - \delta$ for all $n \geq n'$. This allows the average search cost $s(n)/n$ to be both increasing and decreasing in store size. As an illustration of the latter, suppose that consumers incur a fixed cost ω_0 when travelling to the store, and then a variable cost $\omega_1 \xi(n)$ to navigate it, where $\xi(n)$ is increasing in n but sublinear. In this case Proposition 3 holds so long as $\omega_1 < \mu_{p^m}$.

A common alternative assumption within the literature is that the firm sells only one product, but that each consumer wishes to buy a continuous amount of it. Each consumer is then assumed to have an identical downward-sloping demand for the product. Using our earlier notation, we could represent this by saying that faced with a price p, each consumer wishes to buy $1 - F(p)$ units of the product. How does this compare with our (arguably more realistic) assumptions that the firm sells multiple products, and that consumers can only buy a discrete amount of each one? It turns out that when consumers are fully-informed about prices and can costlessly visit the firm, the two approaches are equivalent. This is because in both cases, the firm earns profit $(p - c)[1 - F(p)]$ on any product and therefore optimally charges p^m.[8] However when consumers must pay $s > 0$ to learn about prices, the two approaches differ in several respects. Firstly they make very different predictions when $n = 1$. We showed earlier in Remark 2 that with unit demand, the market completely unravels. In contrast provided s is not too large, Stiglitz (1979) shows that with continuous demand, there exists an equilibrium in which consumers all search and the firm charges p^m.[9] Secondly although both approaches give rise to an equilibrium with trade, they achieve this in somewhat different ways. Suppose consumers have continuous demand and the firm sells one product. Consumers expect to pay p^m on each unit, and to earn positive surplus on inframarginal units. Therefore provided s is not too large, everybody searches. The firm then faces the same demand as it would when $s = 0$, and so charges p^m as expected. Our multiproduct model is different because (for any finite n) there are always some

[8]With continuous demand the firm sells $1 - F(p)$ units, whereas with unit demand the firm sells one unit with probability $1 - F(p)$.

[9]As usual there are also equilibria in which consumers expect a very high price and therefore do not search. There is no other possible equilibrium in this model.

consumers who don't search. Instead multiple products 'work' by inducing some *marginal* consumers to search. For example even a single-product firm would be searched by consumers with $v_1 \geq p_1^e + s$, but the problem is that their demand is completely inelastic. A multiproduct retailer can avoid this problem because it is also searched by some consumers who are marginal for product 1, which then makes demand sufficiently elastic to support an equilibrium. Third and finally, equilibrium prices differ across the two approaches. When each consumer has a continuous demand curve, equilibrium price is p^m and is not affected by small changes in s. Our model differs because i). prices always strictly exceed p^m, and ii). a change in s (or n) modifies the pool of consumers who search, which causes equilibrium prices to change. Sections 3.2 and 3.3 now show in more detail that in our model, equilibrium prices respond to changes in s and n in a way that is both intuitive and economically interesting.

3.2 Equilibrium multiplicity and selection

There are two distinct types of equilibrium. Firstly as discussed earlier, the Diamond outcome remains an equilibrium for all values of n. This is because if consumers expect sufficiently high prices, they rationally decide not to search; the firm is then unable to do better than simply charge the prices that consumers expect. Secondly Proposition 3 showed that when n is large enough, at least one non-Diamond equilibrium also exists. There can be several of these equilibria as well. Intuitively this is because a change in consumers' price expectations from say $\boldsymbol{p'}$ to $\boldsymbol{p''}$, leads to a change in the composition of the pool of consumers who search. In principle the firm's optimal choice of prices may also change from $\boldsymbol{p'}$ to $\boldsymbol{p''}$, in which case $\boldsymbol{p'}$ and $\boldsymbol{p''}$ are both equilibrium price vectors. We now discuss how to select amongst these various Diamond and non-Diamond equilibria.

As a first step note that the Diamond outcome is worst for everybody, because the firm makes no profit and consumers get zero surplus. Next suppose that there are two non-Diamond equilibria $\boldsymbol{p'}$ and $\boldsymbol{p''}$ which satisfy $p''_j < p'_j$ for each $j = 1, \ldots, n$ (the relevance of this supposition will become clear shortly). Consumers are clearly better off if the 'lower' equilibrium $\boldsymbol{p''}$ is played, and surprisingly so too is the firm.

Intuitively this is because prices are already too high from the firm's perspective. More formally if the $\boldsymbol{p'}$ equilibrium is played, the firm's total profit is

$$\sum_{i=1}^{n}\left[(p_i'-c)\times\int_{p_i'}^{b} f(v_i)\Pr\left(\sum_{j=1}^{n}\max\left(v_j-p_j',0\right)\geq s\right)dv_i\right] \tag{4}$$

When consumers expect the $\boldsymbol{p''}$ equilibrium they search if $\sum_{j=1}^{n}\max\left(v_j-p_j'',0\right)\geq s$, and then buy product i provided that it gives positive surplus. Therefore if the firm 'deviates' and sets its prices equal to $\boldsymbol{p'}$ rather than to $\boldsymbol{p''}$, its total profit equals

$$\sum_{i=1}^{n}\left[(p_i'-c)\times\int_{p_i'}^{b} f(v_i)\Pr\left(\sum_{j=1}^{n}\max\left(v_j-p_j'',0\right)\geq s\right)dv_i\right] \tag{5}$$

which strictly exceeds (4). However when consumers expect the $\boldsymbol{p''}$ equilibrium, by definition the firm prefers to set prices equal to $\boldsymbol{p''}$ rather than to $\boldsymbol{p'}$. So by revealed preference profits in the lower $\boldsymbol{p''}$ equilibrium must exceed (5), and therefore also exceed profits in the $\boldsymbol{p'}$ equilibrium.

In fact if our model does have multiple non-Diamond equilibria, one of them is lower than the others. It is simple to prove that, since valuations are drawn from the same distribution, in any equilibrium all goods must have the same price. Existence of a lowest non-Diamond equilibrium then immediately follows. However it is possible to relax the assumption of identical distributions, and still have a lowest equilibrium. Suppose for a moment that each v_i is drawn independently according to its own (strictly positive, continuous and logconcave) density $f_i(v_i)$. At least one non-Diamond equilibrium exists whenever n is sufficiently large (c.f. Proposition 3), although of course different products will generally have different prices. Nevertheless provided the search cost s is sufficiently small, we can also prove that there will be a lowest equilibrium.

We will select the lowest non-Diamond equilibrium when performing comparative statics in the next section. One justification is that since this equilibrium is best for both the firm and its customers, it may be a natural focal point on which to coordinate. A second justification is that the firm may be able to actively signal its intention to play this equilibrium. For example suppose that the firm must pay a cost F to enter the market (or otherwise stay out, and earn zero). Suppose further that $F \in (\Pi', \Pi'')$ where Π'' is profit in the lowest equilibrium, and Π' is the highest profit achievable in any other equilibrium. There is one subgame perfect equilibrium in which the firm enters and gets $\Pi'' - F$; in all other subgame perfect equilibria the firm stays out of the market and gets zero. However in the spirit of forward induction, if the firm did enter the market, consumers should rationalize this as a signal that the firm expects to play the lowest-price equilibrium. Since $\Pi'' - F > 0$ it is in the firm's interest to enter and send this signal. Numerical simulations also suggest that $\Pi'' - \Pi'$ can be relatively large, so this argument may apply widely.

3.3 Comparative statics

Throughout this section we assume that n is sufficiently large to guarantee existence of an equilibrium with trade (c.f. Proposition 3). We study how the equilibrium price p^* is affected by changes in the search cost and product range. To do this recall that $t_j \equiv \max(v_j - p^*, 0)$ is the (equilibrium) expected surplus on good j. Substituting $p_j^e = p^*$ for $j = 1, 2, \ldots, n$ into equation (3) and then rearranging, p^* satisfies

$$1 - F(p^*) - (p^* - c) f(p^*) + \frac{\Pr\left(\sum_{j=1}^{n} t_j \geq s\right) - \Pr\left(\sum_{j=2}^{n} t_j \geq s\right)}{\Pr\left(\sum_{j=2}^{n} t_j \geq s\right)} = 0 \qquad (6)$$

As stated in the previous section we focus on the lowest p^* which solves equation (6). The final term in equation (6) is the ratio of Diamond consumers to shoppers for a single product. We use Karlin's (1968) Variation Diminishing Property to prove in the appendix that $\Pr\left(\sum_{j=1}^{n} t_j \geq s\right) \Big/ \Pr\left(\sum_{j=2}^{n} t_j \geq s\right)$ is increasing in s, or equivalently that $\Pr\left(\sum_{j=1}^{n} t_j \geq s\right)$ is log-supermodular in s and n. This means that as search becomes costlier, the ratio of Diamond consumers to shoppers increases, and demand for each product becomes less elastic. In light of this, it is intuitive that:

Proposition 5 *The equilibrium price increases in the search cost.*

After controlling for product range and production costs, a multiproduct monopolist charges higher prices when search is more costly. This is a very natural result, and single-product search models such as Anderson and Renault (1999) also find a positive relationship between price and search cost. The usual intuition is that higher search costs deter consumers from looking around for a better deal, which gives firms more market power. However the mechanism which drives Proposition 5 is different. In our model a multiproduct firm faces a sample selection problem, which becomes worse when the search cost is larger. When s is small, the surplus from any single product is less pivotal in determining a consumer's search decision, so the ratio of shoppers to Diamond consumers is relatively large. This means that each product's demand is relatively elastic, and consequently equilibrium prices are relatively low. However as the search cost becomes larger, the retailer is increasingly searched only by consumers who have many high valuations. Since its potential customers now have on average a higher willingness-to-pay, the retailer responds by increasing its prices.

The firm's product range also has an important impact on the prices that it charges for individual items. In particular suppose we fix the number of products N that consumers care about, but increase the number of products n that the retailer stocks. Provided $n < N$ we have the following result:

Proposition 6 *The equilibrium price decreases in the number of products stocked.*

When the retailer stocks a wider range of products, its customers are on average more price-sensitive. To illustrate why, compare what happens when the retailer stocks n' and $n'' \supset n'$ products respectively. Consider a thought experiment in which consumers expect the retailer to charge the same price irrespective of how large is its product range. Clearly any consumer who would search when $n = n'$, will also search when $n = n''$. In addition there are some consumers who would not search when $n = n'$, but who will search when $n = n''$ because they like some of the extra $n'' - n'$ products. Note that these additional searchers must have relatively low valuations for the first n' products. Therefore in general a larger retailer attracts more low-valuation consumers. This implies that a small reduction in actual prices will lead to a (proportionately) larger increase in demand, for a retailer whose product range is broader. Equivalently, larger retailers face a more elastic demand curve, and so have a natural incentive to charge lower prices. Consumers rationally anticipate this and incorporate it into their shopping decision. Indeed in the extreme case where $n, N \to \infty$ demand is essentially as elastic as that faced by a standard monopolist, and therefore prices are arbitrarily close to p^m.[10]

As discussed in the introduction, there is lots of empirical and anecdotal evidence that larger stores generally have lower prices. One possible explanation is that larger stores have lower costs - either because they have buyer power, or because they can spread fixed costs more widely and therefore enjoy economies of scale. However according to our model, even after controlling for differences in costs, larger stores should still have lower prices because they face more elastic demand. Hoch *et al* (1995) found significant variation in store-level price elasticities, much of which they attributed to differences in local demographics. However after controlling for demographic factors, they also found evidence that larger stores had more elastic demand. In a similar vein Shankar and Krishnamurthi (1996) find that stores with an everyday low-price (EDLP) strategy, attract consumers who are more responsive to regular (i.e. non-promotional) price changes. Since the EDLP format is generally favored by larger stores (Ellickson and Misra 2008), this is again consistent with the model's prediction that larger stores should face a more elastic demand.[11]

The model also predicts that the profit earned on any single product, is increasing in the size of the retailer's product range. Although a larger retailer charges lower prices, it is also searched by more consumers and therefore makes higher sales. Since equilibrium prices exceed p^m, selling more units at a lower price is good for profits. On the other hand smaller retailers are 'trapped' into charging high prices and earning relatively low profits. More formally if the firm sells n' products and the

[10] As discussed earlier we are assuming that the shopping cost is independent of the number of products. Even if the search cost increases in the size of the firm's product range, the result in Proposition 6 will not change provided that $s(n)$ does not increase too rapidly.

[11] In addition Asplund and Friberg (2002) report evidence that after controlling for costs, larger stores have lower prices. However the authors do not observe firms' actual production costs, and therefore have to proxy them.

equilibrium price is p', the profit from any single product is

$$(p'-c)\times\int_{p'}^{b} f(v_i)\Pr\left(\sum_{j=1}^{n'}\max(v_j-p',0)\geq s\right)dv_i \tag{7}$$

Now suppose instead that the firm sells n'' products and the equilibrium price is p'', where $n''>n'$ and $p''<p'$. Consumers expect prices to be p'' and therefore search on that basis. However once inside the store, they will buy any product which they value more than its price. Therefore if the firm 'deviates' and charges p', it earns

$$(p'-c)\times\int_{p'}^{b} f(v_i)\Pr\left(\sum_{j=1}^{n''}\max(v_j-p'',0)\geq s\right)dv_i \tag{8}$$

on each product. This strictly exceeds (7) because $n''>n'$ and $p''<p'$. However by the definition of equilibrium, when consumers expect p'' the retailer's optimal response is to charge p'' on each product. Therefore by revealed preference, when $n=n''$ the profit earned on each product must exceed (8) and therefore also exceed (7).

Proposition 6 could also help explain why firms co-locate near to each other in shopping malls and highstreets. In particular we could interpret the multiproduct monopolist as a cluster of n single-product firms, each of which sells a different and unrelated product. Dudey (1990) and others have already shown that sellers of similar or identical products may cluster together. By doing this rival firms commit to fiercer competition and therefore lower prices. This makes consumers more likely to search, which increases the demand of every firm in the cluster. However Proposition 6 suggests a stronger result - even single-product firms that sell completely unrelated goods, can also commit to charging lower prices (and earn higher profits) by clustering together. This is despite the absence of any competition between firms in the cluster.

4 Advertising

Now suppose the retailer sends out an advert to consumers informing them about the price of one of its products. This advert is received by all consumers[12] and must be truthful. Therefore if the advert states that the price of good n is p_n^a, consumers use this information to i). conclude that $p_n^e = p_n^a$ and ii). form (rational) expectations about the prices of all other remaining unadvertised products. Given these price expectations $\boldsymbol{p^e} = (p_1^e, p_2^e, ...p_n^e)$ all the analysis from Section 3.1 can be straightforwardly applied. In particular demand for unadvertised product i can still be written as equation (1), and the equilibrium first order condition for unadvertised good i remains

$$D_i\left(p_i = p_i^e; \boldsymbol{p^e}\right) - \left(p_i^e - c\right) f\left(p_i^e\right) \Pr\left(\sum_{j \neq i} \max\left(v_j - p_j^e, 0\right) \geq s\right) = 0 \qquad (2)$$

It is simple to show that provided n is sufficiently large, there exists an equilibrium in which consumers search, unadvertised prices satisfy equation (2), and the profit function of each unadvertised good is quasiconcave (c.f. Proposition 3). Equilibrium unadvertised prices always (weakly) exceed p^m, because the firm again attracts a select sample of high-valuation consumers. As usual, for any particular p_n^a there is no guarantee that the equilibrium price vector is unique. However when there are multiple equilibria, one always Pareto dominates the others and is therefore selected for comparative statics.

When $p_n^a \leq a - s$ all consumers search so there is no sample selection problem. Equation (2) simplifies to $1 - F\left(p_i^e\right) - \left(p_i^e - c\right) f\left(p_i^e\right) = 0$ and each unadvertised price is p^m. However when $p_n^a > a - s$ not every consumer searches, and we have the following result:

Proposition 7 *If p_n^a decreases, so does the equilibrium price of all other products.*

Intuitively a reduction in p_n^a induces some new consumers to visit the retailer. These new consumers must have relatively low valuations, otherwise they would

[12]For tractability we assume that the advert is received by every consumer. Bester (1994) presents a single-product search model in which a retailer can pay to increase the reach of its advert. He shows that the firm randomizes over both its price and advertising reach.

have already been searching. Consequently it becomes profitable for the retailer to reduce its unadvertised prices a little, in order to sell more of these products to its new (low-valuation) customers. Whilst consumers do not naively expect every price to be p_n^a, they do anticipate that a fall in p_n^a is accompanied by a decrease in unadvertised prices. Therefore although the advert only contains direct information about one price, it also indirectly conveys some information about the retailer's other prices. This is despite the fact that products are completely unrelated in terms of use or valuation, and (unlike in Simester 1995) there is no private information about production costs. Consequently rational consumers who have no interest in buying the advertised product, should nevertheless account for its price when deciding whether or not they should search.

The result in Proposition 7 differs from both Lal and Matutes (1994) and Ellison (2005). As discussed earlier, they find that when a firm cuts its advertised price, it cannot credibly convince rational consumers that its unadvertised price is any lower. The key difference is that in our model consumers have heterogeneous valuations, drawn from a continuous distribution. This means that when the retailer's advertised price changes, so does the pool of consumers who search. Changes in the composition of this pool, drive the result in Proposition 7. We now discuss how an optimal advertising strategy can manipulate this positive relationship between advertised and unadvertised prices.

4.1 Optimal advertising behavior

The demand for good n when its price is advertised, is:

$$D_n\left(p_n^a;\boldsymbol{p^e}\right)=\int\limits_{p_n^a}^{b} f\left(v_n\right)\Pr\left(\sum_{j=1}^{n-1}\max\left(v_j-p_j^e,0\right)+v_n-p_n^a\geq s\right)dv_n \qquad (9)$$

A small reduction in p_n^a therefore affects demand for the product in two ways. Firstly some marginal consumers who would have searched anyway, now buy the product. Secondly some above-marginal consumers who would not have searched, now find it worthwhile to visit the retailer and buy this product precisely because they know its price is lower. Note that only the first of these effects is present when the price is not advertised. Therefore demand for a product should be more elastic, when its price is advertised. Kaul and Wittink (1995) argue on the basis of previous empirical studies that this is true, and call it an 'empirical generalization'.

If $\Pi_j\left(p_n^a;\boldsymbol{p^e}\right)$ denotes the equilibrium profit earned on good j, the retailer will choose p_n^a to maximize its store-wide profits $\sum_{j=1}^n \Pi_j\left(p_n^a;\boldsymbol{p^e}\right)$. Using the envelope theorem, this gives the following first order condition:

$$\frac{d\Pi_n\left(p_n^a;\boldsymbol{p^e}\right)}{dp_n^a}+\sum_{i=1}^{n-1}\left(p_i^e-c\right)\int\limits_{p_i^e}^{b} f\left(v_i\right)\left(\sum_{k=1}^{n-1}\frac{\partial\Pr\left(\sum_{j=1}^n t_j\geq s\right)}{\partial p_k^e}\frac{\partial p_k^e}{\partial p_n^a}\right)dv_i$$

$$+\sum_{i=1}^{n-1}\left(p_i^e-c\right)\left[-f\left(p_i^e+s\right)\Pr\left(\sum_{j\neq i}t_j=0\right)\frac{\partial p_i^e}{\partial p_n^a}+\int\limits_{p_i^e}^{b} f\left(v_i\right)\frac{\partial\Pr\left(\sum_{j=1}^n t_j\geq s\right)}{\partial p_n^a}dv_i\right]=0 \tag{10}$$

where as defined earlier $t_j\equiv\max\left(v_j-p_j^e,0\right)$.

Proposition 8 *The optimal advertised price is strictly below p^m.*

Proof. Suppose the statement is actually false, and start with the first term in (10). We know that for any $i=1,\ldots n-1$, $\partial\Pi_n/\partial p_i^e<0$ and $\partial p_i^e/\partial p_n^{ad}>0$. In order to sign $\partial\Pi_n/\partial p_n^a$ define $T=\sum_{j=1}^{n-1}\max\left(v_j-p_j^e,0\right)$ and rewrite Π_n as

$$\left(p_n^a-c\right)\left(\int_0^s\left[1-F\left(p_n^a+s-z\right)\right]d\mu\left(z\right)+\int_s^\infty\left[1-F\left(p_n^a\right)\right]d\mu\left(y\right)\right) \tag{11}$$

where μ is a measure over T with mass at zero. Since $f\left(p\right)$ is logconcave, it is straightforward to prove that $\left(p-c\right)\left[1-F\left(p+s-z\right)\right]$ is (weakly) decreasing in p whenever $p\geq p^m$. Therefore $d\Pi_n/dp_n^a<0$. The remaining terms in (10) are also negative, therefore (10) cannot hold for any p_n^a weakly above p^m. ■

Proposition 8 can be explained as follows. Suppose hypothetically that the retailer chooses p_n^a to maximize *only* its profits from the advertised good. The advertised price then solves $d\Pi_n/dp_n^a=0$ (c.f. equation 10), and we can show that it lies below p^m. Intuitively the shopping cost depresses demand for the advertised product, so the firm partly offsets this by reducing p_n^a below p^m (which is the price it would charge absent any search cost). Of course the retailer actually chooses p_n^a to maximize profits across the entire store, and this makes it optimal to reduce p_n^a

even further below p^m. By doing this more consumers are encouraged to search, either because they like the advertised product directly, or because they like other products which they now believe to be cheaper. Once inside the store most of these additional consumers buy some unadvertised products, and the subsequent boost to profits is captured by the second and third sets of terms in equation (10).

According to the model the retailer therefore uses a low advertised price (below p^m) to attract consumers into the store, and then charges a high price (above p^m) on its remaining unadvertised products. A similar pattern also arises in the papers by Lal and Matutes and Ellison. As discussed in the introduction, there is also much empirical evidence that advertised products have lower prices than products which are not advertised. In Lal and Matutes' model a low advertised price entices consumers into the store, whereupon they are charged very high unadvertised prices which leave them with no additional surplus. The latter is possible because consumers all have the same willingness-to-pay. Our model is different because consumers are heterogeneous, and consequently the retailer would actually earn higher profits on its unadvertised products, if it could convince consumers that their prices were lower. In order to build this 'low-price image', the retailer takes advantage of the mechanism identified in Proposition 7, and advertises a relatively low price. Our model therefore gives an additional explanation for why a retailer would use low-price advertising. Note there is no reason why the advertised price should exceed marginal cost - although not common, it is possible to construct examples where loss-leading is optimal.[13]

At the beginning of the game the retailer can choose whether or not to advertise. Without advertising, we showed earlier that a non-Diamond equilibrium exists provided n is large enough, and that each good costs p^u (say) where $p^u > p^m$. Now suppose the retailer advertises. If it chooses $p_n^a = p^u$, by inspection of equation (2) equilibrium prices satisfy $p_1^e = ... = p_{n-1}^e = p^u$, and therefore profits are the same as when all prices are unadvertised. However in fact the retailer chooses $p_n^a < p^m < p^u$ and so by revealed preference advertising must strictly increase its store-wide profits.

[13]One such example is the following: $a = 0$, $b = 1$, $f(v) = 4e^{-4v+2} / \left(e^2 - e^{-2}\right)$, $c = 1/2$ and $s \to 0$. The optimal advertised price is approximately $0.426 < c$.

Clearly then the retailer will choose to advertise provided that the cost κ of doing so is small enough.

It is natural to ask how a retailer's product range should influence its advertising strategy. For example, is the benefit from advertising larger when a firm stocks more products? How does the optimal advertised price vary with the number of products stocked? Unfortunately equation (10) is not sufficiently tractable to answer these questions in the general case. However Figures 2 and 3 show by example that both the optimal advertised price and the gains from advertising, can be non-monotonic in the size of the firm's product range. Consider first the optimal advertised price. We know that when p_n^a is lower, the firm charges less on its unadvertised products and earns more profit from them. On the one hand for a given p_n^a, a larger retailer's prices are already closer to p^m and therefore cannot fall very quickly if p_n^a is further reduced. Therefore a larger retailer probably gains less additional profit on any individual product, if it reduces p_n^a. However on the other hand a larger retailer sells more products on which these smaller gains can accrue. Figure 3 suggests that the latter effect dominates when n is small whilst the former dominates when n is larger. Now consider the benefits from advertising. When n is small the retailer must advertise in order to make positive profit, and conditional on advertising, its profits naturally increase in the size of its product range. In this example once n is large enough to support a non-Diamond equilibrium, even without advertising prices fall quickly towards p^m as n increases, and hence the added benefits from advertising decrease rapidly.

Analytic results on optimal advertising strategy can be derived when the number of products stocked is large. In particular for any small $\epsilon_1, \epsilon_2, \epsilon_3 > 0$, there exists an $\bar{n}$ such that whenever $n \geq \bar{n}$, i). the gains from advertising are less than ϵ_1, ii). conditional on advertising, the optimal advertised price is above $p^m - \epsilon_2$ and iii). conditional on not advertising, unadvertised prices are below $p^m + \epsilon_3$. The model therefore suggests that retailers with a large product range are unlikely to advertise, but will offer consistently low prices across the store. Smaller retailers on the other hand charge higher prices on most products, but advertise a very low price on one product (or more generally on a few products - see below). Equivalently using

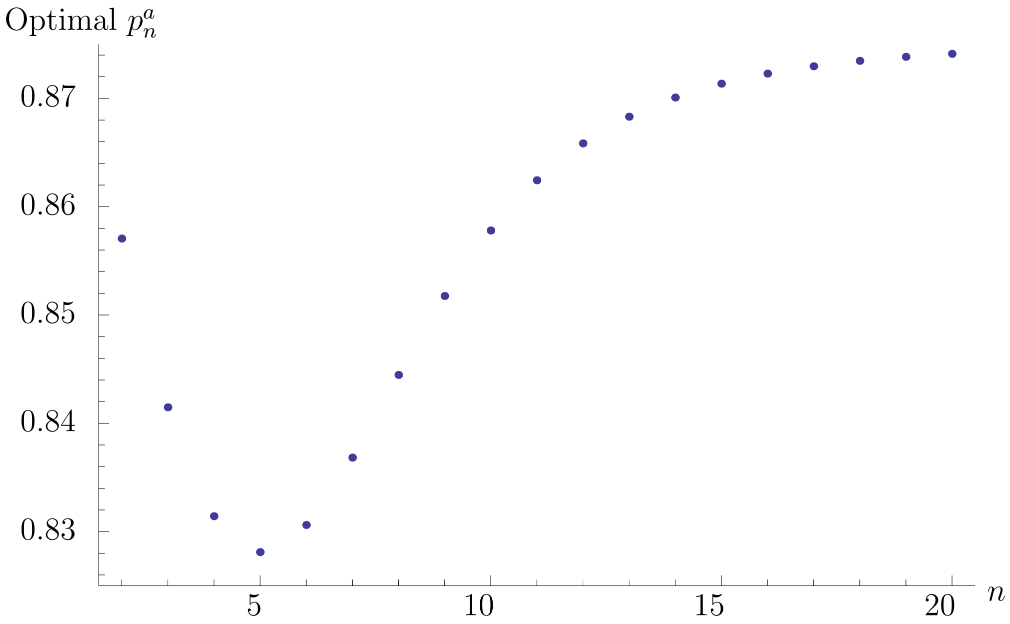

Figure 2: Non-monotonicity of the optimal advertised price.

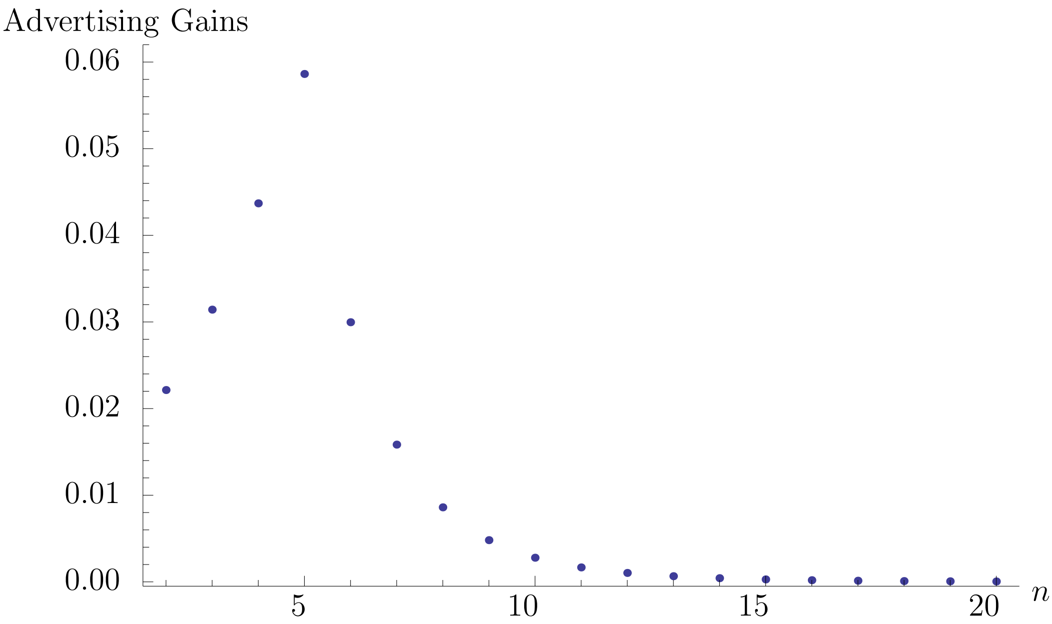

Figure 3: Non-monotonicity of the gains from advertising

terminology from the marketing literature, larger stores should follow an 'everyday low-price' (EDLP) strategy whilst smaller stores are more likely to employ 'Hi-Lo' pricing. This is consistent with Ellickson and Misra's (2008) investigation of US supermarkets. In their dataset firms are classified as either EDLP or Hi-Lo, or a mixture of the two which they call 'hybrid'. They find that larger supermarkets are more likely to have an everyday low-price focus, and are less likely to use a Hi-Lo strategy.[14]

Finally collecting some results from earlier in this section, we have the following observation:

Corollary 9 *When a retailer chooses to advertise, it charges strictly lower prices on every product.*

Using some earlier notation, without any advertising the retailer charges a price $p^u > p^m$ on each product. When the retailer does advertise, it chooses $p_n^a < p^m$ and this draws lower-valuation consumers into the store, which makes it optimal to then charge less than p^u on the remaining unadvertised products as well. Advertising therefore leads to store-wide lower prices. To our knowledge there is very little empirical evidence on the store-wide effects of price advertising. Cox and Cox (1990) created a supermarket flyer which listed discounts on a few products. They found that consumers also believed other products in the store would be somewhat cheaper. However Milyo and Waldfogel (1999) studied firm prices before and after the Supreme Court overturned a ban on alcohol advertising in Rhode Island. They found that some stores began advertising, but that they did not reduce their other unadvertised prices by a statistically significant amount (Table 5, page 1091). However one explanation might be that prior to the Court ruling some retailers were advertising prices of non-alcoholic items; after the ruling they may have substituted

[14]Shankar and Bolton (2004) find that price-promotions are less likely at smaller stores and at retailers with more (regular) price-elastic consumers. However our model implies that price elasticity is endogenously determined by store size. In particular larger stores are less likely to use price promotions precisely because they attract more price-sensitive consumers. This may explain why Shankar and Bolton's finding is different from Ellickson and Misra's.

to advertising liquor products instead, in which case the net effect on store-wide prices would be ambiguous.

A comment on multiple advertised prices Suppose the retailer is able to advertise the prices of products $n-k,\ldots,n$, and that advertising an additional product costs $\kappa > 0$. In order to make the model tractable, we focus only the case of a very small search cost. Rewriting equation (3) from earlier (the first order condition for unadvertised product i), we find that when $s \to 0$ but remains positive, $p_i^e \to \bar{p}_i^e$ where

$$\frac{\left(\bar{p}_i^e - c\right) f\left(\bar{p}_i^e\right)}{1 - F\left(\bar{p}_i^e\right)} = \frac{1}{1 - \prod_{j \neq i} F\left(p_j^e\right)} \tag{12}$$

Fixing $p_{n-k}^a,\ldots,p_{n-1}^a$ a simple fixed point argument can be used to show that all equilibrium unadvertised prices lie above p^m and are increasing in p_n^a. We can also prove that the retailer optimally sets $p_j^a < p^m$ for $j = n-k,...,n$, and the intuition is the same as described earlier. If κ were arbitrarily small the retailer would choose to advertise as many prices as it could, because it would then have more control over them. However fixing κ, we can show that a very large firm would not find it worthwhile to advertise even one price. Therefore again firms with very large product selections find an EDLP strategy to be optimal; small retailers are more likely to follow a HiLo strategy and advertise several low prices, but also have higher prices on their remaining unadvertised goods.

5 Extensions

5.1 Competition

Suppose there are two firms A and B each selling the same n products at zero marginal cost. p_{ij} and p_{ij}^e denote respectively the actual and expected prices charged by firm i on product j. There is a unit mass of consumers. Each firm has a fraction $\lambda \in (0, 1/2)$ of the consumers who are loyal to it: this means they choose between not searching, or paying s and visiting that particular firm. The remaining $1-2\lambda$ 'non-loyal' consumers are willing to buy from either firm. They can search sequentially with costless recall, paying s each time they visit a new firm. However non-loyals are one-stop shoppers, meaning that ultimately they make all their purchases in the same store. The timing of the game is as follows. In the first stage each firm exogenously decides to advertise with some probability α, choosing both which good to advertise and at what price.[15] In the second stage both retailers observe their rival's advertising behavior, and then simultaneously choose prices on their unadvertised products. Consumers also observe advertised prices, and decide whether or not to search. The interpretation behind this two-stage set-up is that taking out an advert requires planning, whereas changing instore prices is easier and quicker to do. It is convenient to introduce the following notation. Let $p^u_{(n)}$ be the price that a non-advertising monopolist would charge on each product, and let $\pi^u_{(n)} > 0$ be the corresponding profit. In addition if a monopolist advertises one product at price p^a, let $\phi_{(n)}(p^a)$ be the price it charges on the remaining $n-1$ products. Recall from earlier that $\phi_{(n)}(\cdot)$ is increasing and that by definition $p^u_{(n)} = \phi_{(n)}\left(p^u_{(n)}\right)$.

Before presenting our main result (Proposition 10, below) we describe equilibrium play in each of the second-stage subgames. Firstly, suppose that neither firm is advertising. It is straightforward to show that there is an equilibrium where each firm charges the monopoly price $p^u_{(n)}$ on all products. Consumers search if and only if $\sum_{j=1}^n \max\left(v_j - p^u_{(n)}, 0\right) \geq s$, with non-loyals randomly choosing which retailer they should visit. Charging $p^u_{(n)}$ is an equilibrium because, given consumers' beliefs, each firm attracts the same mix of consumers as a monopolist. Consequently each retailer maximizes its profits by charging $p^u_{(n)}$ as expected. This has the same flavor as Diamond's original result, except that here the market does not unravel even though consumers have unit-demand.[16]

Now suppose instead that both retailers have advertised the price of good k, offering it at p^a_{Ak} and p^a_{Bk} respectively. It is simple to show that in equilibrium, firm

[15] One way to endogenize this probability is to assume that when a retailer sends out an advert it incurs a cost κ_a. In the spirit of Baye and Morgan's (2001) paper on internet advertising, for each α there will exist some κ_a such that each retailer is indifferent over whether or not to advertise. Consequently each is happy to randomly advertise with probability α.

[16] As usual there are also equilibria where consumers expect prices to be so high that they don't search. In addition there are also equilibria where firm k is expected to charge very high prices (and is therefore not searched), but firm $l \neq k$ is expected to charge $p^u_{(n)}$ and is therefore searched.

A charges $\phi_{(n)}\left(p^a_{Ak}\right)$ on its remaining unadvertised products, whilst firm B charges $\phi_{(n)}\left(p^a_{Bk}\right)$ on them. Recall from earlier that $\phi_{(n)}(\cdot)$ is an increasing function. This implies that the firm with the lowest advertised price, also has the lowest unadvertised prices. Consequently *all* non-loyal consumers who search, go to the firm with the lowest advertised price. This means that conditional upon being searched, each retailer attracts the same mix of consumers as it would were it a monopolist. As a result the retailers find it optimal to charge $\phi_{(n)}\left(p^a_{Ak}\right)$ and $\phi_{(n)}\left(p^a_{Bk}\right)$ as expected. This can be easily extended to the case where only one retailer advertises. For example suppose firm A advertises a price p^a_{Ak} but firm B does no advertising. It follows that A charges $\phi_{(n)}\left(p^a_{Ak}\right)$ on all its unadvertised products, whilst firm B charges $p^u_{(n)}$ across the whole of its store. Therefore non-loyal consumers who search, patronize firm A if $p^a_{Ak} < p^u_{(n)}$ and patronize firm B if $p^a_{Ak} > p^u_{(n)}$.

Finally and most interestingly, suppose that the retailers are advertising the prices of different products. In order to simplify the analysis we restrict attention to $n = 2$. Suppose that firm A advertises good 1 at price p^a_{A1}, whilst firm B advertises good 2 at price p^a_{B2}. Also, assume without loss of generality that $p^a_{A1} < p^a_{B2}\,(\leq p^m)$, and look for an equilibrium in which $(p^m) \leq p^e_{A2} < p^e_{B1}$. We now argue that in such an equilibrium, $p^e_{A2} < \phi_{(2)}\left(p^a_{A1}\right)$ whilst $p^e_{B1} = \phi_{(2)}\left(p^a_{B2}\right)$. Start with firm B's pricing problem. Note that all consumers with $v_1 \geq p^e_{B1}$ can get $p^e_{B1} - p^a_{A1}$ additional expected surplus by searching firm A rather than firm B. Since $p^e_{A2} - p^a_{B2} < p^e_{B1} - p^a_{A1}$, there is no value of v_2 that persuades non-loyal customers with $v_1 \geq p^e_{B1}$ to search firm B. Consequently this retailer's demand for product 1 consists (locally) only of its loyal consumers. This then explains why firm B optimally charges the monopoly price $\phi_{(2)}\left(p^a_{B2}\right)$ on its unadvertised product. Retailer A's pricing problem is more complicated. However what is crucial is that the presence of non-loyal consumers makes A's demand more price-elastic. Note that non-loyal consumers with $v_2 \geq p^e_{A2}$ can gain $p^e_{A2} - p^a_{B2}$ additional expected surplus by searching retailer B rather than retailer A. Consequently these consumers only search firm A if i). $v_1 - p^a_{A1} \geq p^e_{A2} - p^a_{B2}$ and ii). $v_1 - p^a_{A1} \geq s - (v_2 - p^e_{A2})$. Therefore for small changes in p_{A2}

around p^e_{A2}, A's demand for product 2 from non-loyal consumers is:

$$(1-2\lambda)\int_{p_{A2}}^{b} f(v_2)\Pr\left(v_1 - p^a_{A1} \geq \max\left\{p^e_{A2}+s-v_2, p^e_{A2}-p^a_{B2}\right\}\right) dv_2 \tag{13}$$

To interpret this, when $s < p^e_{A2} - p^a_{B2}$ and $p_{A2} \approx p^e_{A2}$ equation (13) is proportional to $1-F(p_{A2})$ i.e. the same demand curve that a standard monopolist faces. When instead $s > p^e_{A2} - p^a_{B2}$, i). consumers with $v_2 \in [p^e_{A2}, p^a_{B2}+s]$ are equally likely to search firm A irrespective of whether they are loyal or non-loyal, but ii). consumers with $v_2 > p^a_{B2}+s$ are strictly less likely to search firm A if they are non-loyal. As a result A's non-loyal consumers are more price-elastic than its loyal consumers, and this gives it an incentive to charge below the monopoly price $\phi_{(2)}(p^a_{A1})$. Intuitively this is because some non-loyal consumers with a high v_2 but low v_1 will buy from firm B instead, which causes the upper tail of firm A's demand to shift inwards.

Putting all these steps together, and letting $p^u \equiv p^u_{(2)}$ and $\phi \equiv \phi_{(2)}$, we find:

Proposition 10 *Suppose $n = 2$. There is a mixed strategy equilibrium in which (conditional on advertising) each retailer randomizes both over which product to advertise, and over the price of that product. In addition:*
(a). A non-advertising firm charges p^u on both products.
(b). If the firms advertise the same good, a firm with advertised price q charges $\phi(q)$ on the other product.
(c). If the firms advertise different goods, the firm with the higher advertised price q_h charges $\phi(q_h)$ on the other product, and the firm with the lowest advertised price q_l charges less than $\phi(q_l) < \phi(q_h)$ on the other product.

The retailers randomly decide both which product to advertise and what price to charge for it, in order to prevent their rival from undercutting them and stealing business from the non-loyal customers. This intuition is familiar from single-product models such as Varian (1980). However one difference in our model is that since the retailers sell more than one product, mixing over their advertised price automatically induces a mixed distribution over their unadvertised price as well. Another important difference is that single-product models such as Varian (1980) and Baye and Morgan (2001), typically find that each firm draws its price from a continuous distribution. However empirical evidence suggests that most products stay at a 'regular price' for long periods of time, and that any deviation from that price tends to be downward (see for example Pesendorfer 2002 and Hosken and Reiffen 2004). A natural way to interpret Proposition 10 is to think about p^u as the regular price; occasionally a firm holds a sale, during which time the prices of both products are marked down. As in the model presented in Section 2, consumers then use this advertisement to form a rational expectation about other prices in the marketplace.

5.2 Heterogeneous downward-sloping demands

In our model consumers have unit-demand. A common alternative approach within the search literature is to assume that each consumer has an identical downward-sloping demand curve. We believe that our modelling choice is more appropriate in many retail contexts. However to show how our results fit into the wider literature, we now briefly demonstrate that they can be generalized when consumers have *heterogeneous* downward-sloping demands.

To illustrate this as simply as possible, we model consumer heterogeneity using a single parameter θ which is drawn from $\left[\underline{\theta}, \overline{\theta}\right]$ using a strictly positive density function $g(\theta)$. A consumer's demand for good j is $p_j = \theta + P(q_j)$ where q_j is the quantity consumed and $P(q_j)$ is continuously differentiable, strictly decreasing and concave. Consequently consumers with higher θ both earn higher surplus and have less elastic demands. When there is no search cost and types are not too different, the firm sells to everybody and charges the standard monopoly price p^m which is defined as

$$p^m = \underset{x}{\operatorname{argmax}} \int_{\underline{\theta}}^{\overline{\theta}} g(\theta) \left\{ (x - c) P^{-1} (x - \theta) \right\} d\theta \tag{14}$$

Now suppose the search cost is positive, and assume for simplicity that all n prices are unadvertised. Equilibrium varies according to two thresholds $\underline{s}_n$ and $\overline{s}_n$ which satisfy $0 < \underline{s}_n < \overline{s}_n$. Firstly when $s \leq \underline{s}_n$ there is an equilibrium where each good is priced at p^m. Intuitively because the search cost is relatively small every consumer finds it worthwhile to search. The firm then faces the same problem as it does when $s = 0$, and consequently charges p^m. Note that unlike in the earlier unit-demand model, $\underline{s}_1 > 0$ and therefore even a single-product retailer may be able to generate trade. Secondly however when $s > \underline{s}_n$ such an equilibrium is no longer possible. This is because even if consumers expect to pay p^m on each product, those with low θ cannot earn enough surplus to make search worthwhile. Therefore the firm is only searched by consumers with relatively high θ, and since their demands are less elastic than in the population as a whole, the firm no longer finds it optimal to charge p^m. Nevertheless

Proposition 11 *When $s \in (\underline{s}_n, \overline{s}_n)$ there exists an equilibrium in which some consumers search and the price strictly exceeds p^m. The (Pareto dominant) equilibrium price increases in the search cost and decreases in the number of products stocked.*

As in the earlier unit-demand model, the firm is only searched by a select sample of high-type consumers, and this pushes the equilibrium price above the frictionless benchmark p^m. An increase in the retailer's product range causes some new low-type consumers to search. Since these new consumers have relatively elastic demands, the firm optimally reduces its unadvertised prices. An increase in the search cost (and more generally an increase in any advertised price) have the opposite effect. Third and finally when $s \geq \overline{s}_n$ the search cost is too high to support any equilibrium except the Diamond outcome. Since $\overline{s}_n$ increases in n the hold-up problem is again easier to overcome when s is small or n is large.

5.3 Relaxing independence

Up until now we have assumed that products are neither substitutes nor complements, and that valuations are drawn independently. We now discuss how relaxing these assumptions might change our results.

5.3.1 Substitutes and complements

To begin with suppose that all n products are perfect substitutes. Each consumer is willing to pay at most v for one unit of whichever product has the lowest price. Consumers search if and only if $v \geq \min p_i^e + s$, and once inside the store make a purchase provided that $v \geq \min p_i$. It follows that when the firm does not advertise, it can set $\min p_i = \min p_i^e + s$ and still sell to everybody who searches. Consumers anticipate this hold-up problem and the market breaks down. Since consumers only wish to buy one of the products, it is very natural that the equilibrium outcome should be the same as when $n = 1$.

Now consider the opposite extreme in which all n products are perfect complements. If a consumer buys only some of the products, she gets zero utility. If she buys all n products her utility is $\sum_{i=1}^{n} v_i$, where each v_i is again drawn independently from $[a, b] \subset \mathbb{R}^+$. Consumers search if and only if $\sum_{i=1}^{n} v_i \geq \sum_{i=1}^{n} p_i^e + s$, and once inside the store buy all the products provided that $\sum_{i=1}^{n} v_i \geq \sum_{i=1}^{n} p_i$. It follows that if one or more prices are unadvertised, the firm can set $\sum_{i=1}^{n} p_i = \sum_{i=1}^{n} p_i^e + s$ and still sell to everybody who searches. Consequently the market breaks down unless *every* price is advertised. Again the problem is that since consumers always buy everything or nothing, the firm effectively sells only one product (whose price is $\sum_{i=1}^{n} p_i$).

Nevertheless the degree of substitutability or complementarity may be much weaker. For example the products may be substitutes and yet some consumers wish to buy all of them. To explore the consequences of this we make the following assumptions. The firm sells two products. If a consumer buys only product i her utility is v_i, whereas if she buys both products her utility is $v_{12} = \max(v_1, v_2, v_1 + v_2 - \gamma)$. We assume that v_1 and v_2 are drawn independently from $[a, b] \subset \mathbb{R}^+$ using a density function $f(v)$ which satisfies all the assumptions from Section 2 and is also weakly increasing (this can be relaxed, provided $f(v)$ does not decrease too rapidly). The parameter γ is identical across consumers: when $\gamma > 0$ the goods are substitutes, and when $\gamma < 0$ they are complements. We impose $\gamma < b - c$ because otherwise no consumer would pay more than marginal cost for a second product. We also assume $c \geq a$, and that a non-Diamond equilibrium exists when products are neither substitutes nor complements i.e. when $\gamma = 0$.

Consumers search provided that $\max(v_1 - p_1^e, v_2 - p_2^e, v_{12} - p_1^e - p_2^e) \geq s$, as illustrated by Figures 4 and 5. Compared to when products are independent (recall Figure 1), search behavior is similar when $\gamma < s$ (complements or 'mild' substitutes) but different when $\gamma > s$ ('strong' substitutes). We can again derive an equilibrium pricing condition by supposing that the firm 'deviates', and increases the price for

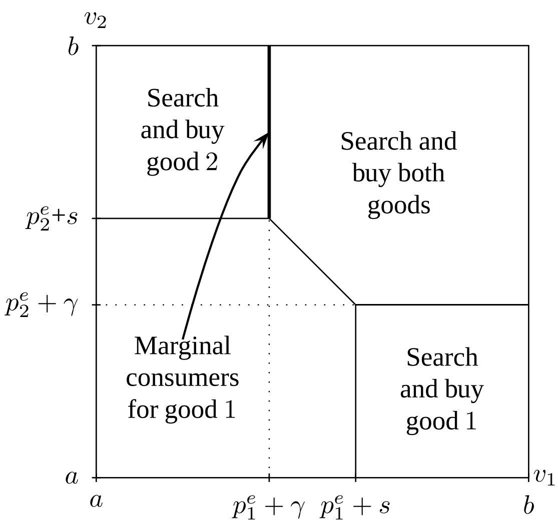

Figure 4: Search behavior when $\gamma < s$

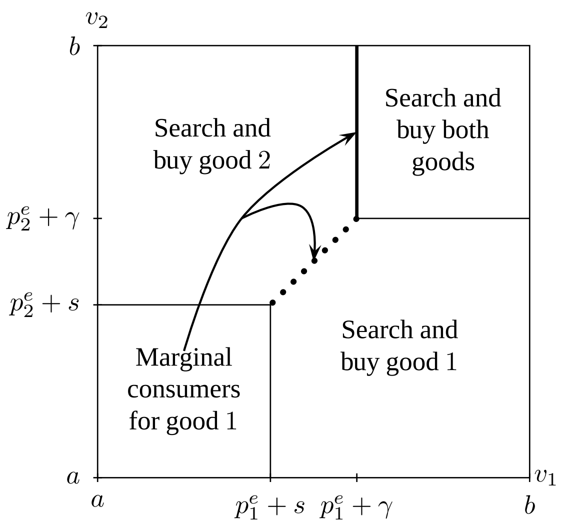

Figure 5: Search behavior when $\gamma > s$

(unadvertised) good 1 slightly above p_1^e. Consumers who are indifferent between buying both products or just buying product 2, have $v_1 = p_1^e + \gamma$. When $\gamma < s$ these marginal consumers only search if $v_2 \geq p_2^e + s$. When $\gamma > s$ everybody with $v_1 = p_1^e + \gamma$ searches, but only those with $v_2 \geq p_2^e + \gamma$ are actually indifferent between buying both products or buying only product 2. Therefore a small increase in p_1 causes consumers on the thick lines in Figures 4 and 5 to stop buying product 1. Interestingly when $\gamma < s$ a small increase in p_1 has no other effect - in particular there is no impact on the demand for product 2, even though it is a substitute or complement for product 1. However when $\gamma > s$ everybody on the diagonal line in Figure 5 is indifferent between buying 1 or 2; they all substitute towards product 2 following a small increase in p_1. Therefore compared to the earlier model, pricing incentives are similar when $\gamma < s$ but more different when $\gamma > s$.

Claim 12 *Without advertising, a non-Diamond equilibrium exists provided $|\gamma|$ is not too large. If the firm then advertises one of its products at a lower price, it also optimally reduces the price of its other (unadvertised) product.*

As usual if the firm does not advertise, there is an equilibrium in which consumers expect very high prices and do not search. However if the products are not too substitutable or complementary, there also exists a non-Diamond equilibrium. As in the earlier model, the firm attracts a relatively large number of consumers who are indifferent between buying both products or just one. Consequently each product's demand is elastic enough to support an equilibrium. We find that when products become more complementary, the equilibrium price increases. The combination of high prices and high complementarity means that few consumers would contemplate buying only one of the two products. Therefore eventually a non-Diamond equilibrium fails to exist, just as when the products are perfect complements. Similarly when the products become too substitutable, most consumers who search want to buy only one of the two products, and so just like with perfect substitutes the market collapses.

Suppose $|\gamma|$ is not too large and select the lowest non-Diamond equilibrium (adapting the argument on page 16, this equilibrium is shown to be Pareto dominant). If the firm then advertises one of its products at a lower price, it attracts some new consumers into the store. After accounting for the substitutability or complementarity between products, these consumers tend to have a relatively low willingness-to-pay for the unadvertised product. Consequently the firm has an incentive to somewhat reduce its unadvertised price. When $\gamma > s$ the firm has an additional incentive to reduce its unadvertised price, namely to prevent too many consumers from substituting towards its (cheaper) advertised product.

5.3.2 Correlated valuations

We assume throughout that products are neither substitutes nor complements.

Start with perfect correlation i.e. a consumer attaches the same valuation v to every good. v is distributed on $[a, b] \subset \mathbb{R}^+$ according to a logconcave density $f(v)$. Recall that $p^m = \arg\max (p - c) [1 - F(p)]$ and that $(p - c) [1 - F(p)]$ is strictly quasiconcave. Since the expected benefits from search are $\sum_{i=1}^{n} \max (v - p_i^e, 0)$, there is a threshold $\tilde{v}$ such that consumers search if and only if $v \geq \tilde{v}$. Therefore the firm optimally charges $\max (p^m, \tilde{v})$ on each unadvertised product. However this means that if all prices are unadvertised, consumers with $v = \tilde{v}$ should not search because they will get zero surplus. Thus an equilibrium only exists if $\tilde{v} = b$ i.e. if (expected) prices are so high that nobody searches. The sample selection problem is particularly bad because everybody who searches necessarily has a high valuation on every product. Now suppose the firm advertises good n at price p_n^a:

Remark 13 *When $p_n^a \leq b - s$ there is a unique equilibrium with $p_1^e = \ldots = p_{n-1}^e = \max(p^m, p_n^a + s)$. When $p_n^a > b - s$ a vector of prices $\left(p_1^e, \ldots, p_{n-1}^e\right)$ is an equilibrium if and only if no consumer searches i.e. $\sum_{i=1}^{n-1} \max(b - p_i^e, 0) + b - p_n^a < s$.*

A small reduction in p_n^a attracts new consumers into the store. Since valuations are perfectly correlated, all these new consumers have a lower willingness-to-pay than those already searching. Therefore whenever the firm's unadvertised prices are relatively high (in particular above p^m) the firm has a strong incentive to reduce them. This makes advertising a very effective commitment device. Indeed when $s \to 0$ advertising allows the firm to commit to charge (very close to) p^m on every product, something which would not generally be possible if valuations were independent.

Now suppose valuations are less-than-perfectly correlated. In particular they are drawn from $[a, b]^n$ using a strictly positive and continuous joint density function $f(v_1, \ldots, v_n)$ whose marginals $f_j(v_j)$ are logconcave.

Proposition 14 *Even when valuations are not independent, a non-Diamond equilibrium exists provided that n is sufficiently large.*

Indeed introducing a small amount of positive correlation may even make it easier to overcome the hold-up problem. This is because consumers who are marginal for one product, are more likely to have high valuations on other products and therefore find it worthwhile to search. The firm's demands may therefore be more elastic compared to when valuations are independent. To illustrate this fix $c = 0$ and $n = 2$. Suppose v_1 and v_2 are both uniformly distributed on $[0, 1]$, and jointly distributed according to the FGM copula $F(v_1, v_2) = v_1 v_2 + \theta v_1 v_2 (1 - v_1)(1 - v_2)$. When $\theta = 0$ valuations are independent and a non-Diamond equilibrium does not exist for any $s > 0$. However for each $\theta \in (0, 1)$ valuations are positively correlated, and a non-Diamond does exist for some strictly positive search costs. Continuing with this example, suppose that the firm then decides to advertise one of the products. When $\theta \in (0, 1)$ a unique equilibrium exists, and as usual there is a positive relationship between the firm's advertised and unadvertised prices.

6 Conclusion

We have studied the pricing and advertising behavior of a multiproduct retailer, when consumers must pay a cost in order to visit its store and learn its prices. Only when the retailer stocks sufficiently many products, does there exist an equilibrium in which consumers search. The critical number of products required to get such an equilibrium, varies with parameters of the model in a natural way. For example we showed that retailers with lower-quality products, generally need to have a larger product range in order to convince consumers to visit their store. We also demonstrated that all retailers suffer from a sample selection problem. In particular, a firm sets its unadvertised prices to maximize the profits it earns on the consumers who show up at its store. However those consumers generally have above-average valuations, and this feeds into high equilibrium unadvertised prices. Indeed these prices are so high that, if a retailer could commit to charge slightly less, it would certainly do so.

To credibly convince consumers that its unadvertised prices are low, a retailer can take one of two actions. Firstly it can stock a wider range of products, or secondly it can advertise low prices on a subset of its products. We showed that a firm with a larger product selection, attracts more consumers with low valuations. This endogenously causes it to face a more elastic demand curve on each individual product, meaning that it ends up charging lower prices across its entire store. Larger retailers therefore eschew price advertising, choosing instead to cultivate a store-wide low-price image. On the other hand smaller retailers actively use price advertising. We demonstrated that if a firm advertises a low price on one product, it endogenously changes the composition of the pool of consumers who search. In particular, the pool of searchers changes in such a way that the retailer ends up charging lower prices on its unadvertised products as well. Smaller retailers take advantage of this mechanism, and advertise very low prices on one or more products. Overall this implies that larger retailers have lower prices on most products, but that by virtue of advertising, smaller retailers can be cheaper on some items.

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A Main Proofs

A.1 Useful lemmas

Lemma 15 *If p_i^e is an equilibrium price, then $1 - F(p) - (p-c) f(p)$ is weakly decreasing in p for all $p \in [p^m, p_i^e)$.*

Proof. We prove the contrapositive of this statement. Suppose that $1 - F(p) - (p-c) f(p)$ is *not* weakly decreasing in p for all $p \in [p^m, p_i^e)$.
Step 1. Note that for all $p > c$, the derivative of $1-F(p)-(p-c) f(p)$ is proportional to $-2/(p-c) - f'(p)/f(p)$, which is strictly increasing in p for all $p > c$. This follows because $f(p)$ is logconcave and therefore $f'(p)/f(p)$ decreases in p.
Step 2. Since by assumption $1-F(p)-(p-c) f(p)$ is not weakly decreasing in p for every $p \in [p^m, p_i^e)$, there will exist a $\bar{p} \in [p^m, p_i^e)$ such that $-2/(\bar{p}-c) - f'(\bar{p})/f(\bar{p}) > 0$. Step 1 then implies that $-2/(p-c) - f'(p)/f(p) > 0$ for all $p \in [\bar{p}, p_i^e)$.
Step 3. When $p_i < p_i^e$ the derivative of (18) with respect to p_i is

$$[-2f(p_i) - (p_i - c) f'(p_i)] \Pr(T \geq s) \tag{15}$$

Step 2 implies that (15) is strictly positive for every $p_i \in [\bar{p}, p_i^e)$. However by definition (18) is zero when $p_i = p_i^e$, so (18) is strictly negative for every $p_i \in [\bar{p}, p_i^e)$. Given the definition of (18), this means that $(p_i - c) D_i(p_i; \boldsymbol{p^e})$ is strictly decreasing in p_i for every $p_i \in [\bar{p}, p_i^e)$ - so the firm would do better to charge some $p_i \in [\bar{p}, p_i^e)$ rather than charge p_i^e. This implies that p_i^e cannot be an equilibrium price. ■

Lemma 16 *$(p_i - c) D_i(p_i; \boldsymbol{p^e})$ is quasiconcave in p_i if either:*

1. *$1 - F(p) - (p-c) f(p)$ is strictly decreasing in p for all $p \in [a, b]$. Densities which do not decrease too rapidly - such as the uniform - satisfy this.*
2. *$\Pr(T \geq s) > 1/2$, where $T = \sum_{j \neq i} \max(v_j - p_j^e, 0)$ as defined earlier.*

Proof. We must show that $(p_i - c) D_i(p_i; \boldsymbol{p^e})$ strictly increases in p_i when $p_i \in [a, p_i^e)$, and strictly decreases in p_i when $p_i \in (p_i^e, b]$. Equivalently we must check that (18) is strictly positive when $p_i \in [a, p_i^e)$ and strictly negative when $p_i \in (p_i^e, b]$.

Condition 1. Rewrite (18) as

$$D_i(p_i;\boldsymbol{p^e}) - (p_i - c) f(p_i) \Pr(T \geq s) + (p_i - c) f(p_i) [\Pr(T \geq s) - \Pr(T + \max(p_i - p_i^e, 0) \geq s)] \quad (16)$$

The derivative of the first term with respect to p_i is

$$\begin{aligned} & -f(p_i) \Pr(T + \max(p_i - p_i^e, 0) \geq s) - [f(p_i) + (p_i - c) f'(p_i)] \Pr(T \geq s) \\ \leq \; & \Pr(T \geq s) [-2f(p_i) - (p_i - c) f'(p_i)] \end{aligned}$$

which is strictly negative because $1 - F(p) - (p - c) f(p)$ is strictly decreasing in p by assumption.[17] Since by definition the first term in (16) is zero when $p_i = p_i^e$, it must be strictly positive when $p_i \in [a, p_i^e)$ and strictly negative when $p_i \in (p_i^e, b]$. The second term of (16) is 0 when $p_i \leq p_i^e$, and negative otherwise. Therefore (18) is strictly positive when $p_i \in [a, p_i^e)$ and strictly negative when $p_i \in (p_i^e, b]$, as required.

Condition 2. Note that (16) is proportional to:

$$\left[\frac{D_i(p_i;\boldsymbol{p^e})}{f(p_i)} - (p_i - c) \Pr(T \geq s)\right] + (p_i - c) [\Pr(T \geq s) - \Pr(T + \max(p_i - p_i^e, 0) \geq s)] \quad (17)$$

The second term is again 0 when $p_i \leq p_i^e$, and negative otherwise. The derivative of the first term with respect to p_i is

$$-\Pr(T + \max(p_i - p_i^e, 0) \geq s) - \Pr(T \geq s) - \frac{D_i(p_i;\boldsymbol{p^e}) f'(p_i)}{f(p_i)^2} < -1 + \frac{D_i(p_i;\boldsymbol{p^e})}{1 - F(p_i)} \leq 0$$

where the first inequality follows because $\Pr(T \geq s) > 1/2$ (by assumption), and because v_i has an increasing hazard rate and this implies that $-f'(p_i)/f(p_i)^2 \leq 1/[1 - F(p_i)]$. The second inequality follows because $D_i(p_i;\boldsymbol{p^e}) \leq 1 - F(p_i)$. Using the same arguments as for Condition 1, the fact that the first term of (17) is strictly decreasing in p_i, means that (18) is strictly positive when $p_i \in [a, p_i^e)$ and strictly negative when $p_i \in [a, p_i^e)$. ■

Lemma 17 *In a non-Diamond equilibrium all unadvertised prices are the same.*

[17] Note that the first term is differentiable everywhere except at the point $p_i = p_i^e + s$ (assuming $p_i^e + s < b$), because $D_i(p_i;\boldsymbol{p^e})$ kinks at that point. However this does not affect the argument that the first term of (16) is strictly decreasing in p_i.

Proof. Suppose to the contrary that there exists an equilibrium where two different goods g and h have prices satisfying $p_g^e < p_h^e$. Applying Lemma 15 $1 - F(p_h^e) - p_h^e f(p_h^e)$ is (weakly) less than $1 - F(p_g^e) - p_g^e f(p_g^e)$, which is itself negative since (as proved in the text) $p_g^e > p_m$. Clearly also $\Pr\left(\sum_{j \neq h} t_j \geq s\right) > \Pr\left(\sum_{j \neq g} t_j \geq s\right)$. Therefore equation (3) cannot hold for both $i = g$ and $i = h$ - a contradiction. ■

A.2 Proofs for Section 3

Proof of Lemma 1. Differentiate $(p_i - c) D_i(p_i; \boldsymbol{p^e})$ with respect to p_i to get

$$D_i(p_i; \boldsymbol{p^e}) - (p_i - c) f(p_i) \Pr(T + \max(p_i - p_i^e, 0) \geq s) \tag{18}$$

where $T = \sum_{j \neq i} \max\left(v_j - p_j^e, 0\right)$. Substitute $p_i = p_i^e$ and set (18) to zero. Lemma 16 (above) provides sufficient conditions under which $(p_i - c) D_i(p_i; \boldsymbol{p^e})$ is quasiconcave in p_i. ■

Proof of Proposition 3. Let $\tilde{p}$ be the unique solution to $1 - F(p) - pf(p)/2 = 0$, and $\tilde{n}$ be the smallest n such that $\Pr\left(\sum_{j=2}^{n} \max(v_j - \tilde{p}, 0) \geq s\right) > 1/2$. According to Lemma 17 (above) we must look for a symmetric equilibrium in which $p_j^e = p^e \forall j$. Using equation (2) p^e satisfies

$$D_i(p_i = p^e; \boldsymbol{p^e}) - (p^e - c) f(p^e) \Pr\left(\sum_{j \neq i} \max(v_j - p^e, 0) \geq s\right) = 0 \tag{19}$$

The lefthand side of (19) is strictly positive when evaluated at $p^e = p^m$, because $D_i(p_i = p^e; \boldsymbol{p^e}) > [1 - F(p^e)] \Pr\left(\sum_{j \neq i} \max(v_j - p^e, 0) \geq s\right)$ and $1 - F(p^m) - (p^m - c) f(p^m) = 0$. Assuming $n \geq \tilde{n}$, the lefthand side of (19) is strictly negative when evaluated at $p^e = \tilde{p}$, because $D_i(p_i = p^e; \boldsymbol{p^e}) \leq 1 - F(p^e)$ and $1 - F(\tilde{p}) - \tilde{p} f(\tilde{p})/2 = 0$. Therefore since the lefthand side is continuous in p^e, there exists (at least one) $p^e \in (p^m, \tilde{p})$ which solves (19). According to Lemma 16 the profit on each good is quasiconcave because $\Pr\left(\sum_{j=2}^{n} \max(v_j - p^e, 0) \geq s\right) > 1/2$. ■

Proof of Propositions 5 and 6. Equation (6) is copied here for convenience

$$\phi(p^*; s, n) = 1 - F(p^*) - (p^* - c) f(p^*) + \frac{\Pr\left(\sum_{j=1}^{n} t_j \geq s\right)}{\Pr\left(\sum_{j=2}^{n} t_j \geq s\right)} - 1 = 0$$

where $t_j \equiv \max(v_j - p^*, 0)$. **Firstly** fix n and decrease s from s_0 to $s_1 < s_0$. Let $p_{s_0}^*$ denote the lowest solution to $\phi(p^*; s_0, n) = 0$. Lemma 18 (below) shows that

$\phi(p^*; s, n)$ increases in s, so we conclude that $\phi\left(p^*_{s_0}; s_1, n\right) \leq 0$. Since $\phi(p^m; s_1, n) > 0$ and $\phi(p^*; s_1, n)$ is continuous in p^*, this implies that when $s = s_1$ the lowest equilibrium price is (weakly) below $p^*_{s_0}$. So the lowest equilibrium price increases in s. **Secondly** fix s and increase n from n_0 to n_1. Let $p^*_{n_0}$ be the lowest solution to $\phi(p^*; s, n_0)$. Lemma 19 (below) shows that $\phi(p^*; s, n)$ decreases in n, so we conclude that $\phi\left(p^*_{n_0}; s, n_1\right) \leq 0$. This again implies that when $n = n_1$, the lowest equilibrium price is (weakly) below $p^*_{n_0}$, so the lowest equilibrium price falls in n. ■

Lemma 18 *$\phi(p^*; s, n)$ increases in s.*

Proof of Lemma 18. Define $\tilde{v}_j = v_j - p^*$, and note that $\tilde{v}_j$ has a logconcave density function $\tilde{f}(\tilde{v}_j)$ defined on the interval $\left[\tilde{a}, \tilde{b}\right]$ where $\tilde{a} = a - p^*$ and $\tilde{b} = b - p^*$. Recall that the $\tilde{v}_j$ are *iid* and that $t_j \equiv \max(\tilde{v}_j, 0)$.
Define $\Omega(s, n) = \Pr\left(\sum_{j=1}^n t_j \geq s\right) \Big/ \Pr\left(\sum_{j=2}^n t_j \geq s\right)$. $\phi(p^*; s, n)$ increases in s if and only if $\Omega(s, n)$ increases in s. We now prove that $\Omega(s, n)$ increases in s.
Begin with $n = 2$ Consider $\Pr(t_1 + t_2 \geq x)$ for some $x > 0$. If $\tilde{v}_1 \geq x$ then $t_1 + t_2 \geq x$ since $t_2 \geq 0$; if $\tilde{v}_1 \in (0, x)$ then $t_1 + t_2 \geq x$ if and only if $t_2 = \tilde{v}_2 \geq x - \tilde{v}_1$; if $\tilde{v}_1 \leq 0$ then $t_1 + t_2 \geq x$ if and only if $t_2 = \tilde{v}_2 \geq x$. Therefore

$$\begin{aligned}\frac{\Pr(t_1 + t_2 \geq x)}{\Pr(t_2 \geq x)} &= \frac{\Pr(\tilde{v}_1 \leq 0)\Pr(\tilde{v}_2 \geq x) + \int_0^x \tilde{f}(z)\Pr(\tilde{v}_2 \geq x - z)\,dz + \Pr(\tilde{v}_1 \geq x)}{\Pr(\tilde{v}_2 \geq x)} \qquad (20)\\ &= \Pr(\tilde{v}_1 \leq 0) + \int_0^x \tilde{f}(z)\frac{\Pr(\tilde{v}_2 \geq x - z)}{\Pr(\tilde{v}_2 \geq x)}dz + 1\end{aligned}$$

which increases in x. This is because $\tilde{v}_2$ is logconcave and so has an increasing hazard rate, which means that $\Pr(\tilde{v}_2 \geq x - z) / \Pr(\tilde{v}_2 \geq x)$ increases in x.
Now proceed by induction We show that if $\Omega(w, n-1)$ increases in w, then $\Omega(s, n)$ increases in s. To prove this, let $k > 1$ and write:

$$\Omega(s, n) - k = \frac{\Pr\left(\sum_{j=1}^n t_j \geq s\right)}{\Pr\left(\sum_{j=2}^n t_j \geq s\right)} - k = \frac{\Pr\left(\sum_{j=1}^n t_j \geq s\right) - k\Pr\left(\sum_{j=2}^n t_j \geq s\right)}{\Pr\left(\sum_{j=2}^n t_j \geq s\right)} \qquad (21)$$

Then using the same principles used to derive equation (20), expand *only the top* of equation (21) to get

$$\Omega(s,n)-k=\Pr(\tilde{v}_n\le 0)\left[\frac{\Pr\left(\sum_{j=1}^{n-1}t_j\ge s\right)-k\Pr\left(\sum_{j=2}^{n-1}t_j\ge s\right)}{\Pr\left(\sum_{j=2}^{n}t_j\ge s\right)}\right]$$
$$+\frac{\int_0^s\tilde{f}(z)\left[\Pr\left(\sum_{j=1}^{n-1}t_j\ge s-z\right)-k\Pr\left(\sum_{j=2}^{n-1}t_j\ge s-z\right)\right]dz+\int_s^{\tilde{b}}\tilde{f}(z)[1-k]\,dz}{\Pr\left(\sum_{j=2}^{n}t_j\ge s\right)} \tag{22}$$

Since t_1 and t_n are *iid*, the first term in (22) simplifies to $\Pr(\tilde{v}_n\le 0)[1-k/\Omega(s,n-1)]$, which is weakly increasing in s because of the inductive assumption that $\Omega(w,n-1)$ increases in w. The second term in (22) is proportional to

$$\int_0^s\tilde{f}(z)\left[\Pr\left(\sum_{j=1}^{n-1}t_j\ge s-z\right)-k\Pr\left(\sum_{j=2}^{n-1}t_j\ge s-z\right)\right]dz+\int_s^{\tilde{b}}\tilde{f}(z)[1-k]\,dz$$

Using the change of variables $y=s-z$, this can be rewritten as

$$\int_{s-\tilde{b}}^{0}\tilde{f}(s-y)[1-k]\,dy+\int_0^s\tilde{f}(s-y)\left[\Pr\left(\sum_{j=1}^{n-1}t_j\ge y\right)-k\Pr\left(\sum_{j=2}^{n-1}t_j\ge y\right)\right]dy$$

and then written more compactly as

$$\int_{-\infty}^{\infty}\mathbf{1}_{0\le s-y\le\tilde{b}}\tilde{f}(s-y)\,\Gamma(y)\,dy \tag{23}$$

where $\mathbf{1}_{0\le s-y\le\tilde{b}}$ is an indicator function taking value 1 when $0\le s-y\le\tilde{b}$, and 0 otherwise; and where

$$\Gamma(y)=\begin{cases}1-k & \text{if } y\le 0\\ \Pr\left(\sum_{j=1}^{n-1}t_j\ge y\right)-k\Pr\left(\sum_{j=2}^{n-1}t_j\ge y\right) & \text{if } y>0\end{cases}$$

Now let S be an interval in $\mathbb{R}_{++}$ such that $\Pr\left(\sum_{j=1}^{n-1}t_j\ge s\right)>0\forall s\in S$, and choose $s_0,s_1\in S$ where $s_1>s_0$. Also choose a constant k_0 such that (23) is zero when evaluated at $s=s_0$ and $k=k_0$. (Zero terms being discarded) $\Gamma(y)$ has one sign-change from negative to positive: this follows directly from the definition of $k_0\,(>1)$ and the inductive assumption that $\Omega(w,n-1)$ increases in w.

Using the definition in Karlin (1968) §1.1, $\mathbf{1}_{0\le s-y\le\tilde{b}}$ is totally positive of order 2 (TP_2) in (s,y). Since $\tilde{f}(z)$ is logconcave, $\tilde{f}(s-y)$ is also TP_2 in (s,y). Therefore $\mathbf{1}_{0\le s-y\le\tilde{b}}\tilde{f}(s-y)$ is also TP_2 in (s,y).

Applying Karlin's Variation Diminishing Property (Karlin §1.3, Theorem 3.1), (23) changes sign once in s (and from negative to positive) when it is evaluated at $k = k_0$. Since by assumption (23) is zero when evaluated at $s = s_0$ and $k = k_0$, it is positive when evaluated at $s = s_1$ and $k = k_0$. Therefore returning to equation (22) it follows that $\Omega(s_1, n) - k_0 \geq \Omega(s_0, n) - k_0$, or equivalently that $\Omega(s, n)$ increases in s. In summary we have shown that if $\Omega(w, n-1)$ increases in w, then $\Omega(s, n)$ increases in s. Since $\Omega(w, n)$ increases in w for $n = 2$, $\Omega(s, n)$ increases in s for any integer-valued n. ∎

Lemma 19 *$\phi(p^*; s, n)$ decreases in n.*

Proof of Lemma 19. $\phi(p^*; s, n)$ decreases in n if and only if $\Omega(s, n)$ decreases in n. To show that $\Omega(s, n)$ decreases in n, write out $\Omega(s, n)$ as

$$\frac{\Pr(\tilde{v}_n \geq s) + \int_0^s \tilde{f}(\tilde{v}_n) \Pr\left(\sum_{j=1}^{n-1} t_j \geq s - \tilde{v}_n\right) d\tilde{v}_n + \Pr(\tilde{v}_n \leq 0) \Pr\left(\sum_{j=1}^{n-1} t_j \geq s\right)}{\Pr(\tilde{v}_n \geq s) + \int_0^s \tilde{f}(\tilde{v}_n) \Pr\left(\sum_{j=2}^{n-1} t_j \geq s - \tilde{v}_n\right) d\tilde{v}_n + \Pr(\tilde{v}_n \leq 0) \Pr\left(\sum_{j=2}^{n-1} t_j \geq s\right)}$$

This is less than $\Omega(s, n-1)$ because $\Pr\left(\sum_{j=1}^{n-1} t_j \geq x\right) \Big/ \Pr\left(\sum_{j=2}^{n-1} t_j \geq x\right)$ exceeds 1 and (from Lemma 18) is increasing in x. Therefore we know that $\Omega(s, n) \leq \Omega(s, n-1)$ for any n, or alternatively $\Omega(s, n)$ decreases in n. ∎

A.3 Proof for Section 4

Proof of Proposition 7. Let p' be the equilibrium unadvertised price. Let $t_j = \max(v_j - p', 0)$ for $j < n$, and $t_n = \max(v_n - p_n^a, 0)$. Then using equation (6) again, p' satisfies

$$\Phi(p'; p_n^a) = 1 - F(p') - (p' - c) f(p') + \frac{\Pr\left(\sum_{j=1}^{n-1} t_j + t_n \geq s\right)}{\Pr\left(\sum_{j=2}^{n-1} t_j + t_n \geq s\right)} - 1 = 0$$

We focus on the smallest p' which solves $\Phi(p'; p_n^a) = 0$. Note that $\Phi(p'; p_n^a)$ is continuous in p', and that $\Phi(p^m; p_n^a) \geq 0$. Therefore as with comparative statics in s and n, *if* $\Pr\left(\sum_{j=1}^{n} t_j \geq s\right) \Big/ \Pr\left(\sum_{j=2}^{n} t_j \geq s\right)$ increases in p_n^a, the lowest p' (that solves $\Phi(p'; p_n^a) = 0$) also increases in p_n^a. To prove that $\omega(p_n^a) =$

$\Pr\left(\sum_{j=1}^{n} t_j \geq s\right) \big/ \Pr\left(\sum_{j=2}^{n} t_j \geq s\right)$ increases in p_n^a, write:

$$\omega\left(p_n^a\right) - k = \frac{\Pr\left(\sum_{j=1}^{n} t_j \geq s\right) - k \Pr\left(\sum_{j=2}^{n} t_j \geq s\right)}{\Pr\left(\sum_{j=2}^{n} t_j \geq s\right)} \tag{24}$$

Just as in the proof of Lemma 18, the numerator of (24) can be rewritten as

$$\begin{aligned}
&\int_a^{p_n^a} f\left(v_n\right)\left[\Pr\left(\sum_{j=1}^{n-1} t_j \geq s\right) - k \Pr\left(\sum_{j=2}^{n-1} t_j \geq s\right)\right] dv_n \\
&+ \int_{p_n^a}^{p_n^a+s} f\left(v_n\right)\left[\Pr\left(\sum_{j=1}^{n-1} t_j \geq s + p_n^a - v_n\right) - k \Pr\left(\sum_{j=2}^{n-1} t_j \geq s + p_n^a - v_n\right)\right] dv_n \\
&\qquad + \int_{p_n^a+s}^{b} f\left(v_n\right)\left[1-k\right] dv_n
\end{aligned} \tag{25}$$

Using the change of variables $y = p_n^a - z$ (25) can be written as

$$\int_{-\infty}^{\infty} \mathbf{1}_{a \leq p_n^a - y \leq b} f\left(p_n^a - y\right) \gamma\left(y\right) dy \tag{26}$$

where $\gamma\left(y\right) = \begin{cases} 1-k & \text{if } \ y \leq -s \\ \Pr\left(\sum_{j=1}^{n-1} t_j \geq s+y\right) - k \Pr\left(\sum_{j=2}^{n-1} t_j \geq s+y\right) & \text{if } \ y \in (-s, 0) \\ \Pr\left(\sum_{j=1}^{n-1} t_j \geq s\right) - k \Pr\left(\sum_{j=2}^{n-1} t_j \geq s\right) & \text{if } \ y \geq 0 \end{cases}$
and $\mathbf{1}_{a \leq p_n^a - y \leq b}$ is again an indicator function that is TP_2 in (p_n^a, y). Now choose $p_n^{a,0}, p_n^{a,1} \in [a-s, b]$ such that $p_n^{a,0} < p_n^{a,1}$. Also define k_0 such that (26) is zero when evaluated at $p_n^a = p_n^{a,0}$ and $k = k_0$. (Zero terms being discarded) $\gamma\left(y\right)$ is piecewise continuous and changes sign once from negative to positive: this follows from the definition of $k_0\,(>1)$, and because $\Pr\left(\sum_{j=1}^{n-1} t_j \geq x\right) \big/ \Pr\left(\sum_{j=2}^{n-1} t_j \geq x\right)$ increases in x (from Lemma 18). Karlin's variation diminishing property says that (26) is single-crossing from negative to positive in p_n^a. Therefore $\omega\left(p_n^{a,1}\right) - k_0 \geq \omega\left(p_n^{a,0}\right) - k_0$, or equivalently $\omega\left(p_n^a\right)$ increases in p_n^a as required. ■

B Selected additional proofs

Any omitted proofs are available on request.

Proof of Proposition 10. Most of the details were already explained in the text. Consider part (c) and firm A's pricing decision when $p_{A1}^a < p_{B2}^a$. We can write A's equilibrium first order condition for product 2 as

$$\lambda\times(2)|_{i=2}+\left[(13)|_{p_{A2}=p^e_{A2}}-(1-2\lambda)\left(p^e_{A2}-c\right)f\left(p^e_{A2}\right)\Pr\left(v_1-p^a_{A1}\geq\max\left\{s,p^e_{A2}-p^a_{B2}\right\}\right)\right] \tag{27}$$

where equation (2) is the firm's first order condition in the monopoly case, and equation (13) is the firm's demand from non-loyal consumers in the duopoly case. The lefthand side of equation (27) is i). continuous in p^e_{A2}, ii). strictly positive for all $p^e_{A2}\leq p^m$ and iii). negative at $p^e_{A2}=\phi\left(p^a_{A1}\right)$ [this follows because the first term is by definition zero at $p^e_{A2}=\phi\left(p^a_{A1}\right)$, whilst the second term is lower than the first]. Therefore we can conclude that at least one $p^e_{A2}\in\left(p^m,\phi\left(p^a_{A1}\right)\right)$ solves equation (27).

Now consider the first part of the proposition. If firm k randomizes over which product to advertise, it is also a best response for firm $l\neq k$ to randomize. Let p^a_A and p^a_B denote the retailers' advertised prices. It is simpler to prove the result if we assume that at the first stage both firms draw an advertised price from a distribution; then with probability $1-\alpha$ a retailer's chance to advertise vanishes, and its simply charges p^u on both products. Let $\tilde{\Pi}_i\left(p^a_A,p^a_B\right)$ be firm i's profit if the firms advertises prices p^a_A and p^a_B respectively. Using Theorem 5b from Dasgupta and Maskin (1986), a mixed equilibrium in advertised prices exists provided that for all $\{p^a_A,p^a_B\}=\{\bar{p},\bar{p}\}$ there exists an $i\in\{A,B\}$ such that

$$\lim_{p^a_A\xrightarrow{-}\bar{p},\,p^a_B\xrightarrow{+}\bar{p}}\tilde{\Pi}_i\left(p^a_A,p^a_B\right)\;\geq\;\tilde{\Pi}_i\left(\bar{p},\bar{p}\right)\geq\lim_{p^a_A\xrightarrow{+}\bar{p},\,p^a_B\xrightarrow{-}\bar{p}}\tilde{\Pi}_i\left(p^a_A,p^a_B\right) \tag{28}$$

$$\lim_{p^a_A\xrightarrow{-}\bar{p},\,p^a_B\xrightarrow{+}\bar{p}}\tilde{\Pi}_j\left(p^a_A,p^a_B\right)\;\leq\;\tilde{\Pi}_j\left(\bar{p},\bar{p}\right)\leq\lim_{p^a_A\xrightarrow{+}\bar{p},\,p^a_B\xrightarrow{-}\bar{p}}\tilde{\Pi}_j\left(p^a_A,p^a_B\right),\qquad j\neq i \tag{29}$$

where the left (right) inequality in (28) is strict if and only if the right (left) inequality in (29) is strict. Clearly a firm's profits on its own loyal consumers are continuous in its own advertised price, and independent of its rival's price. Therefore to check these inequalities, we need to ensure that profits on non-loyal consumers jump in the correct direction. To do this, we must further specify what happens when $p^a_A=p^a_B$. Since this is a zero-probability event, we make the technical assumption that with probability 1/2 all non-loyal consumers who search go to firm A, and with probability 1/2 all non-loyal consumers who search go to firm B. Note that profits on non-loyal consumers are continuous everywhere except at points where $p^a_A=p^a_B$.

When the firms advertise the same price, the discontinuities clearly satisfy (28) and (29) with $i = A$. When the firms advertise different prices, it is also clear from part (c) that when their prices cross, the previously cheaper firm: i). loses all sales on its unadvertised product and ii). suffers a discrete jump down in the demand for its advertised product. Consequently discontinuities in profit are consistent with (28) and (29) with $i = A$ and $j = B$. ■

B.1 Proofs for Section 5.2

Define $Q(z) \equiv P^{-1}(z)$ and note that $Q', Q'' < 0$. Also define $\Pi(p;\theta) = (p-c)\,Q(p-\theta)$, $p^*(\theta) = \arg\max_x (x-c)\,Q(x-\theta)$, and let $CS(p_j;\theta)$ be the consumer surplus of a θ-type on one product when its price is p_j.

Lemma 20 *$p^*(\theta)$ and $CS(p^*(\theta);\theta)$ are both strictly increasing in θ.*

Proof. Differentiate $(p-c)\,Q(p-\theta)$ with respect to p to get a first order condition $Q(p-\theta) + (p-c)\,Q'(p-\theta) = 0$. Then totally differentiate with respect to θ to get

$$\frac{\partial p^*(\theta)}{\partial \theta} = \frac{Q'(p^*(\theta)-\theta) + (p^*(\theta)-c)\,Q''(p^*(\theta)-\theta)}{2Q'(p^*(\theta)-\theta) + (p^*(\theta)-c)\,Q''(p^*(\theta)-\theta)} \in (0,1)$$

Since $\partial p^*(\theta)/\partial\theta \in (0,1)$ and $P' < 0$ and (by definition) $p^*(\theta) = \theta + P(q^*(\theta))$, it follows that $\partial q^*(\theta)/\partial\theta > 0$. We can write consumer surplus as

$$CS(p^*(\theta);\theta) = \int_0^{q^*(\theta)} [p_j - p^*(\theta)]\,dq_j = \int_0^{q^*(\theta)} [P(q_j) - P(q^*(\theta))]\,dq_j$$

which is strictly increasing in θ because $P' < 0$ and $\partial q^*(\theta)/\partial\theta > 0$. ■

In equilibrium each good will have the same price - let p be the actual price and p^e the expected price. Consumers search if and only if $\theta \geq \tilde{\theta}$ where $\tilde{\theta}$ as the minimum $\theta \in [\underline{\theta}, \overline{\theta}]$ such that $n \times CS(p^e;\theta) \geq s$. Profit on good j is therefore $\int_{\tilde{\theta}}^{\overline{\theta}} g(\theta)\{(p-c)\,Q(p-\theta)\}\,d\theta$. Differentiating with respect to p and imposing $p = p^e$, we find that p^e satisfies[18]

$$\int_{\tilde{\theta}}^{\overline{\theta}} g(\theta)\,\Pi'(p^e;\theta)\,d\theta = \int_{\tilde{\theta}}^{\overline{\theta}} g(\theta)\{Q(p^e-\theta) + (p^e-c)\,Q'(p^e-\theta)\}\,d\theta = 0 \qquad (30)$$

Since $\partial\Pi'(p^e;\theta)/\partial\theta > 0$ equation (30) implies $\Pi'\left(p^e;\tilde{\theta}\right) < 0$, which further implies that $\Pi'(p^e;\theta) < 0 \forall\theta \leq \tilde{\theta}$. Consequently $\int_{\tilde{\theta}}^{\overline{\theta}} g(\theta)\,\Pi'(p^e;\theta)\,d\theta \geq \int_{\underline{\theta}}^{\overline{\theta}} g(\theta)\,\Pi'(p^e;\theta)\,d\theta$.

[18]Since $Q', Q'' < 0$ profit is quasiconcave in p provided that types are not too different.

Lemma 21 *In any equilibrium with trade, $p^e \geq p^m$.*

Proof. Proceed by contradiction and suppose that $p^e < p^m$. $\int_{\underline{\theta}}^{\overline{\theta}} g(\theta)\,\Pi'(p^e;\theta)\,d\theta > 0$ because $\int_{\underline{\theta}}^{\overline{\theta}} g(\theta)\,\Pi(p;\theta)\,d\theta$ is concave and maximized at $p = p^m$. This implies that $\int_{\tilde{\theta}}^{\overline{\theta}} g(\theta)\,\Pi'(p^e;\theta)\,d\theta > 0$ because (from the previous paragraph) $\int_{\tilde{\theta}}^{\overline{\theta}} g(\theta)\,\Pi'(p^e;\theta)\,d\theta \geq \int_{\underline{\theta}}^{\overline{\theta}} g(\theta)\,\Pi'(p^e;\theta)\,d\theta$. However this contradicts (30) and thus the supposition that $p^e < p^m$. ■

Define $\underline{s}_n = n \times CS(p^m;\underline{\theta})$ and $\overline{s}_n = n \times CS\left(p^*\left(\overline{\theta}\right);\overline{\theta}\right)$ and note that $\underline{s}_n < \overline{s}_n$.[19] It follows immediately that when $s \leq \underline{s}_n$ there is an equilibrium where the retailer charges p^m on every product.

Proof of Proposition 11. Existence is proved as follows. Firstly since $s > \underline{s}'$ then $\tilde{\theta} > \underline{\theta}$. Adapting the proof of Lemma 21 the lefthand side of equation (30) is strictly positive when evaluated at $p^e = p^m$. Secondly since $s < \overline{s}'$ consumers with $\theta = \overline{\theta}$ search, and by continuity so do nearby types. Adapting the proof of Lemma 21 we can also conclude that $\Pi'\left(p^*\left(\overline{\theta}\right);\theta\right) < 0$ for all $\theta < \overline{\theta}$. Therefore the lefthand side of equation (30) is strictly negative when evaluated at $p^e = p^*\left(\overline{\theta}\right)$. Thirdly putting these two facts together, and using the fact that the lefthand side of (30) is continuous in p^e, there exists at least one $p^e \in \left(p^m, p^*\left(\overline{\theta}\right)\right)$ such that (30) holds. Lemma 21 already shows that $p^e \geq p^m$.

Now suppose that the number of products stocked increases from n_0 to n_1 (the proof for an increase in s is virtually identical and is omitted). Let p_0^e be the lowest equilibrium price when $n = n_0$, and $\tilde{\theta}_{0,0}$ be the lowest type that searches when $n = n_0$ and price p_0^e is expected. Let $\tilde{\theta}_{0,1} < \tilde{\theta}_{0,0}$ be the lowest type who searches when $n = n_1$ but price p_0^e is *still* expected. Adapting earlier arguments we know that a). $\tilde{\theta}_{0,0} > \underline{\theta}$ and b). $\Pi'(p_0^e;\theta) < 0$ for all $\theta < \tilde{\theta}_{0,0}$. Therefore the lefthand side of (30) when evaluated at $p^e = p_0^e$ *but* $n = n_1$ is

$$\int_{\tilde{\theta}_{0,1}}^{\overline{\theta}} g(\theta)\,\Pi'(p_0^e;\theta)\,d\theta < \int_{\tilde{\theta}_{0,0}}^{\overline{\theta}} g(\theta)\,\Pi'(p_0^e;\theta)\,d\theta = 0$$

i.e. strictly negative. At the same time (30) is still weakly positive when evaluated at $p^e = p^m$ and $n = n_1$. Therefore (by continuity) the lowest solution to (30) when $n = n_1$, must be strictly lower than the lowest solution when $n = n_0$. ■

[19] It is straightforward to show that $p^m > p^*(\underline{\theta})$ which then implies $CS(p^*(\underline{\theta});\underline{\theta}) > CS(p^m;\underline{\theta})$. Since $CS\left(p^*\left(\overline{\theta}\right);\overline{\theta}\right)$ exceeds $CS(p^*(\underline{\theta});\underline{\theta})$ by Lemma 20, $\overline{s}_n$ exceeds $\underline{s}_n$.