Dynamic Decisions to Enter a Toll Lane on the Road

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Abstract

Technology advancements have made it possible to allow motorists to use under-utilized carpool lanes by paying a toll. Tolls generally follow a real-time pricing rule that fluctuates with daily traffic shocks. A typical toll lane has multiple entrances, which gives rise to a dynamic consumer-entry decision on the road. Entry decisions depend on not only current congestion and prices but also the expectation of congestion and prices further down the road. Real-time pricing informs consumers of the daily traffic shocks and allows them to update their expectations on downstream congestion and prices. Using a minute-level data set of the MnPASS program in Minneapolis, Minnesota, this paper estimates a model of consumer dynamic entry decisions and traffic flows. The estimation results provide the basis for evaluating the efficiency gain from toll lanes and comparing the effects of various pricing functions on consumer welfare. Before the implementation of real-time pricing, peak load pricing is common among toll lanes and is still used by some cities. Results suggest that allowing solo drivers to use the carpool lane significantly increases motorist welfare. The efficiency gain is considerably higher under the current MnPASS real-time pricing than under the optimal peak load pricing.

Keywords: dynamic demand, real-time pricing, peak load pricing

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1 Introduction

Traffic congestion is a prevalent problem among metropolitan areas. Technology advancements have made it possible to reallocate traffic on a highway from regular lanes to underutilized carpool lanes without slowing down traffic. Solo drivers can use the express lanes (the former carpool lanes) by paying a toll that follows a real-time pricing rule.\textsuperscript{1} Real-time pricing frequently varies with the traffic volume to ensure free flow traffic in the express lanes. Express lanes open policy discussions such as: What is the efficiency gain from converting carpool lanes to express lanes? How do various pricing functions change the welfare outcome?\textsuperscript{2} To evaluate these questions, it is necessary to estimate the demand for express lanes on the road.

Consumers make dynamic entry decisions on the road because a typical express lane has multiple entrances. Consumers can enter right away and pay the current price, which allows them to stay in for the rest of their trips. Alternatively, they can stay in the regular lanes and decide later as congestion and real-time prices often vary across entrances. Therefore, entry decisions depend on not only the price and congestion in the immediate vicinity, but also the expectations of prices and congestion further down the road. Real-time pricing affects demand in two ways. First, demand decreases with price because of the substitution between current time savings and money (the standard substitution effect). Second, consumers’ expectations of future prices and congestion vary with current price because real-time pricing reflects daily traffic shocks. In particular, demand may increase with price if expected price

\textsuperscript{1}Officially known as HOT (High-Occupancy Vehicle/Toll) lanes.

\textsuperscript{2}See Small, Winston, and Yan (2005), Yan, Small, and Winston (2006), and Levinson and Carrión-Madera (2011). Express lanes are costly to build and operate. They are either converted from existing carpool lanes or built as an addition to the highway. The construction of express lanes usually requires road reconfiguration and can be rather expensive. For example, the construction cost of the Capital Belt Highway in Virginia (40 miles) is 1.9 billion dollars. Once in operation, the express lanes barely make up enough revenue to cover up the marginal operation cost. Express lanes in the United States are divided between two types of pricing functions: real-time pricing and peak load pricing. The I-394 MnPASS Program in Minneapolis, Minnesota studied in this paper is an early adopter of real-time pricing. Other places that use real-time pricing include San Diego, Salt Lake City, Seattle, Atlanta, Los Angeles, San Jose, Miami and Fairfax County, Virginia. Orange County, California, Houston, and Denver still use peak load pricing today. Unlike real-time pricing that varies with the resolution of demand shocks, peak load pricing is set in advance and cannot respond to traffic shocks. Weather conditions and a surge in trips before a holiday are examples of traffic shocks that lead to variations in daily congestion. From a consumer’s perspective, real-time prices reveal the resolved traffic shock and signal downstream congestion and prices.
or congestion increases with current price (the forecasting effect). Variations in price and demand include the standard substitution effect and the forecasting effect. We need to separately identify the two effects to estimate consumers' willingness to pay for time.

This paper structurally estimates a model of consumer dynamic entry decisions on the road using a high-frequency data set on the I-394 MnPASS program in Minneapolis, Minnesota. The model incorporates expectations on downstream congestion and prices conditional on current price, which allows me to obtain unbiased estimates of consumers' willingness to pay. I find that without the structural model, consumers' willingness to pay is overestimated, which leads to an overestimation of the efficiency gain from the MnPASS program.

To study consumers' entry decisions on the road, a data set that includes what consumers observed at each entrance and their decisions is necessary. The MnPASS data set provides speed and prices at each minute and entrance, as well as the number of vehicles entering the express lane. The data set also includes the traffic flows into (out of) the highway on each entry (exit) ramp and the traffic flows at various locations along the highway. MnPASS uses a real-time pricing function that updates based on the speed and volume of the express lane. Preliminary evidence suggests that without modeling consumer expectations, the price coefficient is biased. Current price is positively correlated with downstream congestion and prices. Consumers are more likely to enter if they expect downstream congestion or price to rise. Therefore, demand may be upward-sloping if the forecasting effect is greater than the standard substitution effect.\footnote{Bias from unobserved variables is widely documented in the demand literature, including Trajtenberg (1989), Petrin (1998), Berry, Levinsohn, and Pakes (1995), Nevo (2001), Chintagunta, Dube, and Goh (2005), Villas-Boas and Winer (1999), Goolsbee and Petrin (2004), Crawford (2000) and Dhar, Chavas, and Gould (2003). Common approaches to control for the correlation between price and the unobserved attributes include instrumental variables (Berry (1994)) and control functions (Petrin and Train (2010) and Kim and Petrin (2010)).}

To obtain the unbiased demand estimates and to conduct welfare analysis, this paper models consumer entry decisions in a discrete choice framework and takes into account consumer expectations of downstream traffic and prices conditional on current price. Consumers solve an optimal stopping problem in which they decide which entrance to enter the express lane taking current congestion and prices as given. Once they enter, consumers pay a fixed
cost and stay in the express lane for the rest of their trips. At a given entrance, a consumer’s entry decision depends on her final utility flow from the trip. A consumer receives disutility from the total travel time and the tolls paid for the trip. She will enter the express lane at an entrance if the expected total utility of entering now is no less than the expected total utility of waiting. Consumers have rational expectations on the evolution of downstream traffic and prices, and take the transition probabilities as given.

To conduct welfare analysis, an equilibrium that incorporates the traffic inflows and how speed is determined (the speed functions) is necessary. Speed depends on subscribers’ endogenous entry decisions and the inflows of carpooler and nonsubscribers which are exogenous in the model. Given a pricing function and the inflows of motorists, in equilibrium, consumers (subscribers) take speed and prices as given and solve the optimal stopping problem described above. Equilibrium speed and prices are determined by the traffic volume, and the transition probabilities of congestion and prices are consistent with the policy functions, speed functions and the pricing functions.

Consumer preferences are estimated in two steps.\(^4\) First, I nonparametrically estimate the conditional transition probabilities of prices and speeds. Second, I estimate the structural parameters by using the method of simulated likelihood. Results suggest that consumers are heterogeneous in their willingness to pay (or value of time). The average value of time is $62 per hour with a standard deviation of $23 per hour.

To evaluate the degree to which consumer’s willingness to pay would be biased without the structural model, I estimate an alternative specification that does not take consumer expectations into account. MnPASS prices are updated every three minutes, and this structure of the pricing function provides a natural experiment for studying whether consumers are responsive on the fly. I obtain price elasticities by comparing the percentages of paying entrants immediately before and after a price change. The identification assumption is that conditional on price and congestion, the demand for express lanes is continuous within

a short time frame. This specification does not separately identify the substitution effect (negative on demand) from the forecasting effect (positive on demand), and it overestimates consumer’s willingness to pay by 21%.

With the model estimates, I investigate two policy questions. First, what is the efficiency gain from having an express lane? I find that motorists benefit significantly from the MnPASS program. Results show that closing down the express lane to subscribers for one day decreases consumer welfare by 8%. Subscribers benefit the most from the express lanes, and their welfare increases by 10%. Regular lane users also benefit due to the drop in regular lane congestion. Carpoolers are slightly worse off because the increase in traffic flow from subscribers.

Second, I compare the effects of real-time pricing and peak load pricing on consumer welfare. I first solve for the optimal peak load pricing for MnPASS. The optimal peak load pricing maximizes the expected consumer welfare with the constraint that the government makes no less than what it makes under the existing real-time pricing. The results show that the current real-time pricing function is more efficient than the optimal peak load pricing. Consumer welfare decreases by 3% when the current real-time pricing is switched to the optimal peak load pricing. The regular lane users lose most under the optimal peak load pricing. The welfare of regular lane users decreases by 5% when the current real-time pricing is switched to the optimal peak load pricing.

In addition, I explore alternatives to the current MnPASS real-time pricing function. The current pricing function solely depends on the traffic in the express lane. However, consumers’ entry decisions are based on the relative traffic conditions in the regular and the express lanes. Consumers enter the express lane when they can save travel time. Motivated by consumer behaviors, the alternative pricing function depends on the relative speed between the regular and the express lanes rather than the speed in the express lane alone. I find that the alternative pricing function improves consumer welfare by 5%.

This paper adds to the literature on estimating demand for toll lanes. Previous studies focus on the decisions made prior to a trip. These pretrip decisions include when to leave

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5GAO (2012) suggests that the current revenue from MnPASS just covers its operational costs. The constraint can be interpreted as the government is not making a profit from the express lanes.
for work, whether to purchase a transponder and whether to carpool.\textsuperscript{6} In particular, Small, Winston, and Yan (2005), Yan, Small, and Winston (2006), Small and Verhoef (2007) and Levinson and Carrion-Madera (2011) study the pre-trip decisions mentioned above. They also study consumer decisions to enter a toll lane in a static framework. This paper takes these decisions as given and focuses on the decisions on the road. It complements the previous studies by modeling the effect of different pricing functions on dynamic entry decisions. The dynamic framework captures the effect of real-time pricing to peak load pricing on the entry pattern, which attributes to the efficiency gain from real-time pricing to peak load pricing.

This paper is also complementary to the literature on congestion pricing, particularly road pricing with an untolled alternative\textsuperscript{7}. Previous studies find that welfare gain from real-time pricing is closely related to the distribution of demand elasticities\textsuperscript{8}. The estimation results of consumer elasticities on the road may be used in the discussion of the efficiency gain of various congestion pricing regimes.

The rest of the paper is organized as follows: section 2 describes the I-394 MnPASS program and its pricing schedule. Section 2 also summarizes the data analyzed in this paper, and the preliminary evidence that consumers respond to price changes on the fly. Section 3 presents a model of consumers’ dynamic entry decisions and determinants of speed. Section 4 discusses the estimation procedures. Section 5 presents the estimation results of consumer references and speed functions. Section 6 presents the policy experiments, and section 7 concludes.


\textsuperscript{7}Examples include Marchand (1998), Arnott, de Palma, and Lindsey (1992), Arnott and Kraus (1998) Verhoef, Nijkamp, Rietveld, et al. (1996) and Small and Yan (2001). Lindsey and Verhoef (2000) provides detailed literature review on congestion pricing. Small and Yan (2001) finds that the optimal toll rate with one route constrained to be free is far less efficient than the optimal toll rate with no constraints. The optimal toll rate with another constraint on the level of service is less efficient than the optimal toll rate with just the constraint that one route remains free.

\textsuperscript{8}Liu and McDonald (1998) uses a model with homogeneous agents and finds that tolling the express lane reduces consumer welfare. In a similar setup with heterogeneous agents, Verhoef and Small (2004) finds an efficiency gain from having an express lane.
2 Data

This paper uses a data set on the I-394 MnPASS program in Minneapolis, Minnesota. The program is among the first express lanes with real-time pricing in the United States.\textsuperscript{9} I-394 is the primary east/west bound highway between downtown Minneapolis and its western suburbs, and it also intersects with major highways that connect several business districts, the University of Minnesota, and Saint Paul.\textsuperscript{10} The original carpool lane on I-394 was underutilized before it was converted to the express lane in 2005.\textsuperscript{11}

The data set includes four major components: 1) the real-time pricing function used by MnPASS, 2) the tolls displayed before each entrance to the express lane, 3) the speeds of the express lane and the regular lanes measured at various locations of the highway, and 4) traffic flows measured at various locations of the highway, the number of vehicles entering (exiting) the highway at every entry (exit) ramp, and the number of subscribers entering the express lane at each entrance. All observations are on the minute level. The high-frequency sample reflects the price and traffic conditions observed by consumers at each entrance and their corresponding entry decisions. This paper focuses on eastbound traffic between 6:00 a.m. and 10:00 a.m. during weekdays.\textsuperscript{12}

2.1 MnPASS pricing function and the summary statistics of prices

Figure 1 illustrates the structure of the I-394 MnPASS and how consumers are tolled. The express lane is divided into two segments, each with its own price. A segment may have multiple entrances. On eastbound MnPASS, entrances 1 through 4 lead to the first segment and entrance 5 leads to the second segment.\textsuperscript{13} Tolls are displayed before each entrance (on the toll signs in figure 1). When a MnPASS subscriber\textsuperscript{14} enter the express lane, she electronically pays for the corresponding segment through the transponder in her vehicle.

\textsuperscript{9}Opened on May 16, 2005.

\textsuperscript{10}They are I-494, TH-100, Mn-169 and I-94.

\textsuperscript{11}See Zmud and Simek (2006) for more details.

\textsuperscript{12}This is when eastbound MnPASS is in operation.

\textsuperscript{13}Appendix A discusses the structure and operation hours in more detail.

\textsuperscript{14}A solo driver must purchase a transponder in advance if she intends to use the express lane.
(the tolling antenna in figure 1). She also locks down the price for the upcoming segment. If the subscriber remains in the HOT lane till the next segment, she pays the rate displayed when and where she initially entered the HOT lane.

MnPASS uses a real-time pricing function that depends on the speed of the express lane (see Table 2). The goal is to keep the express lane above 55 mph. In particular, the price of a segment displayed at an entrance decreases with the minimum speed of that segment. If the minimum speed is above 55 mph, which indicates that no congestion occurs in that segment of the express lane, then the toll equals to $0.25 (the minimum rate when the express lane is in operation). If the express lane speed falls below 55 mph, which indicates that congestion occurs at least one location of the express lane, then the price will increase to deter subscribers from entering. Prices increase in steps of $0.25 proportional to the change in speed. The maximum toll of a segment is $8.

Four features of this real-time pricing function are crucial in the modeling choice and estimation strategy of this paper. First, prices are exogenous to consumers at any given entrance. This is because the price for a segment at time $t$ is updated based on the previous price (at time $t - 3$), and the traffic conditions in that segment in the last three minutes (from $t - 3$ to $t - 1$). Consumer-entry decisions do not affect their own prices but aggregately affect the prices for the consumers behind. Second, individual entry decisions do not affect equilibrium prices. This is because the price only updates when there is a significant change in traffic volume in the express lane.

Third, expected downstream prices are correlated the current price. If the current price is high, then consumers are more likely to observe a higher price downstream. For example, let $p_{it}$ be the price at entrance 1 at time $t$. Let $\tau$ be the travel time from entrance 1 to entrance 2 at time $t$. When the consumer arrives at entrance 2, she observes $p_{2t+\tau}$ the price at entrance 2 at time $t + \tau$. If $p_{it}$ is high, and no price updates occurs between time $t$ and

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\[^{15}\] The exact amount paid by individuals may not equal to the rate displayed. The exact toll varies very half a minute while the displayed tolls vary every three minutes. Consumers do not know the exact toll but know that it is no more than the rate displayed.

\[^{16}\] 55mph is the speed limit of I-394.

\[^{17}\] Density is negative correlated with price and positively correlated with volume. For every unit of density increase over 1 vehicle per hour per mile, the toll rate increases by $0.25 (table 2). If the absolute value of the density change equals to 1, then the toll rate remains the same.
At time $t+\tau$, then $p_{2t+\tau} = p_{1t}$ will be high as well. If a price update occurs, then $p_{2t+\tau}$ is positively correlated with $p_{1t}$. This is because the new price depends on the previous price at entrance 2, which is positively correlated with the price observed at entrance 1.\textsuperscript{18}

Lastly, the price also reflects downstream congestion in the regular lanes. A high price at an entrance indicates a high demand for the express lane. Demand for the express lane increases with regular lane congestion. Therefore the price indirectly reflects the daily shock on regular lane traffic, and a consumer can infer the degree of congestion in the regular lane downstream from current price.

Tables 3 show the summary statistics of tolls displayed at an entrance. Entrance 1 through 4 (segment 1) is more expensive than entrance 5 (segment 2). The average toll is $1.24 for segment 1 and $0.85 for segment 2 at entrances 1 through 4. The maximum price for segment 1 is $8, and the maximum toll observed for segment 2 is $3.25. Price variations arise with fluctuations in the express-lane demand and carpool volumes over time. Figure 5 shows a typical example of the within-day price fluctuations. Price changes occur most of the time when the price is due for an update.

2.2 Speed

Speeds come from the detectors placed along the highway. 20 detector stations are placed on the 11-mile highway. Each station includes a set of detectors that measure the speed for each lane. Each observation is the speed for a lane at a location at a given minute. Figure 2 shows an example of the detectors. S271 is a detector station between entrances 2 and 3. L1 1661 and L2 1662 are the detectors for the regular lane at that location. L3 1663 is the detector for the express lane.

Consumers care about how quickly they can get to their destinations. To obtain the travel time from one entrance to the next at minute $t$, I calculate the average speed reported by the detectors between the two entrances. Travel time equals to the distance between two entrances divided by the average speed. For tractability, I assume that the speed observed at an entrance is the speed that a consumer travels at from that entrance to the next.

\textsuperscript{18}The express lane is mostly likely congested at entrances 4 and 5.
Table 3 shows the summary statistics of travel time at each entrance. The MnPASS pricing function generally keeps the express lane uncongested. The minimum travel time from entrance 1 to the end of the highway is 9 minutes. No significant difference exists between the average express lane travel time and the minimum travel time at most entrances. The average express lane travel time is 1 minute at entrances 1 and 4, 2 minutes at entrances 2 and 3, and 3.4 minutes at entrance 5. Time savings from entering the express lane is greatest at entrance 5. Regular lane congestion is most severe at entrance 5 (figure 4). The average travel time at entrance 5 is 4.7 minutes. Savings in travel time by entering the express increases from 7:00 a.m. to 9:00 a.m. (figure 3).

2.3 Volume and MnPASS demand

Traffic flows come from the detectors as well. Detectors are placed in various locations along the highway, at the entry and exit ramps to the highway, and at the express lane entrances. The detectors placed along the highway, such as S270, S271 and S272 in figure 2, provide the number of vehicles in each lane at an entrance and time \( t \) (the \( N^e \)'s and \( N^c \)'s in figure 5). Detectors on the entry and exit ramps to the highway, such as L1675merge and L1668exit in figure 2 provide traffic inflow and outflow to the highway (the \( IF \)'s and \( OF \)'s in figure 5). Detectors at each entrance of the express lane provide the number of subscribers who enter the express lane at an entrance at minute \( t \).

One challenge is that I do not observe the number of subscribers that remain in the regular lane at an entrance. The traffic volume reported by the detectors at an entrance (e.g. \( N^e \)) includes nonsubscribers that cannot enter the express and subscribers who can choose to enter the express lane. To identify the latter from the former, I assume the percentage of transponder holders at an entrance is 38% according to a survey by Zmud and Simck (2006). For tractability, I also assume that subscribers destination is the end of the highway. This is reasonable because the time saving is small at the first three entrances and is the greatest at the last entrance. Therefore, I also subtract the vehicles that will not travel to the end of the highway from the traffic volume in the regular lane to obtain the total number of subscribers.
To calculate the probability of a motorist at entrance $n$ exiting at entrance $n+1$, I assume the exit process follows a Poisson distribution. It is estimated using highway exit ramp data. On average, 25% of the motorists present at entrance 1 exit at entrance 2. 49% of the motorists present at entrance 2 exit at entrance 3. 11% of the motorists present at entrance 3 exit at entrance 4, and 14% of the motorists present at entrance 4 exit at entrance 5. I assume that the probability of exiting is independent over entrances to obtain the probability of a motorist at entrance $n$ traveling to the end of the highway. Therefore, the total number of subscribers at an entrance equals to 38% times the probability of going to the end of the highway times the total number of vehicles in the regular lane ($N^r$ in figure 5).

Tables 3 report the percentage of subscribers entering at an entrance. The percentage of subscribers entering at an entrance equals to the number of subscribers who entered the express lane (reported in the data) divided by the total number of subscribers at that entrance. Subscribers are more likely to enter at entrances 1 and 5. On average, 38% of subscribers at entrance 1 switches into the express lane. It decreases to 16% at entrance 2, 5% at entrance 3, 3% at entrance 4, and increases to 33% at entrance 5. The higher percentage of entry at entrance 5 is consistent with its high time savings and low cost.\textsuperscript{19}

2.4 Preliminary analysis

This section discusses the preliminary evidence that shows: 1) consumers are responsive to price on the fly; and 2) omitted variable bias exists without modeling consumer expectations. Table 4 presents the average demand by price at entrance 1. I report the variations for a specific time because a time-of-the-day effect may exist. For example, we might expect more consumers to use the express lane between 7:30 a.m. and 8:00 a.m. as motorists are trying to get to work on time. Demand may also vary with current congestion levels. To control for congestion, table 4 is constructed using a subsample such that no congestion occurs at the current entrance.

Table 4 shows that conditional on time-of-the-day and congestion, demand varies with

\textsuperscript{19} Despite high demand, the toll at entrance 5 is lower because two express lanes exist at this entrance, and one express lane exits from entrances 1 through 4. Traffic density decreases with capacity. The lower traffic density at entrance 5 explains the lower price
price. This implies that consumers are price elastic on the fly. For example, at 7:00 a.m., the average percentage of paying entrants is 70% when price is $.75, 60% when price increases to $1.00, 59% when price is $1.25, and 31% when price is $1.50. However, demand increases to 67% when price increases to $1.75. This provides preliminary evidence that price not only affects demand through the standard substitution effect (a negative effect) but also through the forecasting effect (a positive effect). In particular, a high price is associated severe congestion or a high price downstream. As consumers are more likely to enter the express lane if they expect worsened congestion or a higher price, the forecasting effect suggests that demand increases with price. The increase in demand at $1.75 in column 1 suggests that the forecasting effect is greater than the substitution effect. A similar pattern can be observed for 7:30 a.m., 8:00 a.m., 8:30 a.m., and 9:00 a.m. Therefore, evidence suggests that consumers make entry decisions based on expectations of downstream congestion and prices, which they infer from the current price. To separately identify the forecasting effect from the substitution effect and obtain an unbiased estimate for consumers' willingness to pay, I estimate a discrete choice model of entry decisions that incorporates the effect of current price on consumers' expectations.

3 Model

This section describes a model in which given a pricing function and the inflow of motorists, there is an equilibrium such that consumers solve a dynamic entry problem. Consumers have rational expectations on the evolution of downstream congestion and prices conditional on the current states. The equilibrium conditional transition probabilities of congestion and prices are consistent with consumer-entry decisions, speed functions and the pricing function. This framework allows me to compare the effect of different pricing schedules on the equilibrium outcome of traffic conditions and consumer welfare.

Consider a one-way $M$-mile highway with an express and a regular lane. Three types of motorists travel on this highway: MnPASS subscribers (subscribers or consumers), non-subscribers and carpoolers. Subscribers can pay to use the express lane. They are the
only decision makers in this model as they choose between the express and regular lanes. Carpoolers can use the express lane for free. They always travel in the express lane as the express lane is generally faster than the regular lane in the data. Nonsubscribers cannot use the express lane so they only travel in the regular lane.

The model focuses on consumer-entry decisions on the road and take exogenously the pre-trip decisions, such as when to leave for work, whether to carpool, and whether to purchase a transponder. Pretrip decisions determine the distributions of motorist types and the distributions of traffic inflows to the highway, which are exogenous to the model. In the long run, pricing schedules affect the number of subscribers, the number of carpoolers and when motorists leave for work. This model can be extended to incorporate the effect of pricing schedules on these pre-trip decisions.

3.1 Dynamic discrete choice model of consumer entry decisions

Consider a highway with \( N \) entrances to the express lane. The express lane speed is constant and equals to the free-flow speed at each entrance. The regular lane may be congested, and its speed fluctuates. Let \( x_{\ell n} \) be the travel time in lane \( \ell \) at entrance \( n \). The travel time in the express lane at entrance \( n \) is constant and equals to the free flow travel time, denoted by \( x_n^f \). The travel time in the regular lane at entrance \( n \), \( x_{0n} \), is bounded below by the free-flow travel time.

At each entrance, consumer \( i \) solves an optimal stopping problem of when and where to enter the express lane. If the consumer enters the express lane at entrance \( n \), she pays a toll and stays in the express lane to the end of the highway. If the consumer remains in the regular lane at entrance \( n \), she chooses whether to enter the express lane at the next entrance. Let \( p_n \) denote the toll for entering the express lane at entrance \( n \). Let \( d_{in} \) be consumer \( i \)'s entry decision at entrance \( n \). \( d_{in} \) equals to 0 if consumer \( i \) chooses the regular lane and 1 if she chooses the express lane.

The consumer problem at entrance \( n \) can be written recursively as

\[
V_{in} (x_{0n}, p_n, X_{in-1}, \epsilon_{in}) = \max_{d_{in} \in \{0,1\}} \left\{ V_{in}^1 (p_n, X_{in-1}) + \epsilon_{in} \right\} \tag{1}
\]
where $X_{in-1}$ is consumer $i$'s cumulative travel time up to entrance $n$ and $\epsilon_{in} = (\epsilon_{i1n}, \epsilon_{i0n})$ are unobserved (to econometricians) lane-specific individual shocks at entrance $n$. $V_{in}^1 (p_n; X_{in-1})$ is the choice-specific value function for choosing the express lane. It is given by

$$V_{in}^1 (p_n; X_{in-1}) = u \left( X_{in-1} + \sum_{n \geq n} x_{in}^f; p_n; \theta_i \right)$$  

where $u(x, p, \theta_i)$ is consumer $i$'s utility function. The first argument of the utility function is the total time the consumer has spent on the road. The second argument is the amount of tolls paid for that trip. When consumer $i$ enters the express, she remains in the express for the rest of her trip, and the express lane travel time is constant. The total travel time is the sum of the cumulated travel time to entrance $n$, $X_{in-1}$, and the remaining travel time to the end, $\sum_{n \geq n} x_{in}^f$. The consumer pays a toll of $p_n$ for entering the express at entrance $n$.

The choice-specific value function for choosing the regular lane is given by

$$V_{in}^0 (x_{0n}, p_n; X_{in-1}) = E [V_{in+1} (x_{0n+1}; p_{n+1}, X_{in-1} + x_{0n}, \epsilon_{in}) \mid x_{0n}, p_n]$$

$$= \int \int V_{in+1} (x_{0n+1}; p_{n+1}, X_{in}, \epsilon_{in+1}) \ dH_n (x_{0n+1}; p_{n+1}, x_{0n}, p_n) \ dF (\epsilon_{in+1})$$  

where $V_{in+1} (x_{0n+1}; p_{n+1}, X_{in}, \epsilon_{in+1})$ is the value function at entrance $n+1$. The value function for staying in the regular lane at entrance $n$ is the expected value function at the next entrance. The expectation is taken over next entrance's speed and prices conditional on the current aggregate states and the i.i.d. individual lane-specific shocks. This specification reflects that a consumer chooses whether to enter at the next entrance if she stays in the regular lane at entrance $n$.

Consumers have rational expectations on the evolution of downstream traffic and prices, and $dH_n (x_{0n+1}; p_{n+1}, x_{0n}, p_n)$ is the equilibrium conditional transition probability of entrance $n+1$'s speed and price on the aggregate states at entrance $n$. The conditional transition probabilities depend on individual policy functions, speed functions, the pricing function and the distribution of inbound traffic to the highway.
The choice specific value functions at the last entrance are given by

\[
V_{in}^1(p_N; X_{in-1}) = u_i(X_{in-1} + x^f_N, p_N; \theta_i) + \epsilon_{in},
\]

\[
V_{in}^0(x_{0n}, p_N; X_{in-1}) = u_i(X_{in-1} + x_{0n}, 0; \theta_i) + \epsilon_{0n}.
\]

(4)

If the consumer enters the express lane at the last entrance, she receives a disutility from travel time and toll, given by \(u_i(X_{in-1} + x^f_N, p_N; \theta_i)\). If consumer \(i\) stays in the regular lane at the last entrance, the total travel time for the trip is the sum of the cumulative travel time and the regular lane travel time at the last entrance. The consumer has not entered the express lane during the trip, so the toll for the trip is zero. Consumer \(i\) receives a disutility from travel time and toll, given by \(u_i(X_{in-1} + x_{0n}, 0; \theta_i)\).

Consumer \(i\) enters the express lane at entrance \(n\) if the value of entering exceeds the value of waiting. The policy function is that given the aggregate states \((x_{0n}, p_n)\) and the individual states \((X_{m-1}, \epsilon_{in})\),

\[
d_{in}(x_{0n}, p_n; X_{in-1}, \epsilon_{in}) = \begin{cases} 
1 & \text{if } V_{in}^1(p_n; X_{in-1}) + \epsilon_{in} \geq V_{in}^0(x_{0n}, p_n; X_{in-1}) + \epsilon_{0n} \\
0 & \text{otherwise}
\end{cases}
\]

(5)

The initial condition is that all consumers enter the highway with zero cumulative travel time.

Note that a consumer only receives utility flows at the end of a trip in this specification. An alternative specification is per-segment utility flows at each entrance. The former allows entry decisions to depend on how long the consumer has been on the road. The latter assumes that the current decision is independent on the previous states and decisions. As consumers are more likely to be concerned with the arrival time, which depends on the total travel time for a trip, it is reasonable to expect a consumer who has been in traffic for two hours to behave differently from someone who just entered the highway. The model incorporates this mechanism by using one utility flow for the entire trip instead of per-segment utility specifications.\(^{20}\)

\(^{20}\)For example, if the utility function is concave, then cumulative travel time will affect entry decisions.
Linear utility is a special case in which the two specifications are equivalent. Suppose the utility function is given by

\[ u(x, p; \alpha_i) = -x - \alpha_i p. \]

where \( \alpha_i > 0 \) is the marginal rate of substitution of time for price. \( 1/\alpha_i \) measures consumer \( i \)'s value of time—how much consumer \( i \) is willing to pay for one unit of gain in time. The function is negative to reflect the disutility from travel time and tolls. With this utility function, consumer-entry decisions do not depend on the cumulative travel time.$^{21}$

3.2 Determinants of speed

The travel time in lane \( \ell \) at entrance \( n \) and minute \( t \) depends on the number of vehicles that passed entrance \( n \) in lane \( \ell \) at the previous minute. Speed functions are lane- and entrance-specific to account for the differences in capacity at various sections of the highway. Let \( f^m_x : \mathbb{Z}^+ \rightarrow \{x_n, \ldots, x_m\} \) be the speed function for lane \( \ell \) at entrance \( n \). The travel time at time \( t \) and entrance \( n \) is given by \( x_{\ell nt} = f^m_x(v_{\ell nt-1}) \). \( v_{\ell nt-1} \) is the traffic volume in lane \( \ell \) at entrance \( n \) and time \( t - 1 \). Note that subscribers' entry decisions affect the speed for the following motorists and do not affect their own speed.

Traffic volume aggregated from individual decisions

Traffic volume in the express lane at entrance \( n \) and minute \( t \) includes three categories of motorists: carpoolers who arrived at that entrance at minute \( t \), subscribers who entered the express before entrance \( n \)

Consider the following preference

\[ u(x, p; \alpha, r) = - \frac{x^{1+r}}{1+r} - \alpha p \]

where consumer \( i \)'s value of time is still \( 1/\alpha_i \), and \( r > 0 \) is the relative risk aversive coefficient. Using backward induction to solve for the value functions, we get \( \partial (V_i^1 - V_i^0)/\partial X_{\ell nt} < 0 \) for any entrance \( n \). The probability of consumer \( i \) entering at entrance \( n \) is \( 1 - F(V_i^1 - V_i^0) \) where \( F(.) \) is the c.d.f of \( \epsilon_{0n} = \epsilon_{1n} \). Note that \( \partial (V_i^1 - V_i^0)/\partial X_{\ell nt} < 0 \). An increase in cumulative travel time increases the probability of entry.

$^{21}$Given the cumulative travel time \( X_{\ell Nt-1} \), the regular lane travel time \( x_{0N} \) and the toll \( p_N \) at entrance \( N \), consumer \( i \) enters the express lane if \( -X_{\ell Nt-1} - x_N - \alpha_i p_N + \epsilon_{1N} \geq -X_{\ell Nt-1} - x_{0N} + \epsilon_{0N} \). The decision at entrance \( N \) is independent of the cumulative travel time. Using backward induction, we can show that a consumer's decision at any entrance \( n \) is independent of the cumulative travel time.
and arrived at that entrance, and subscribers who switched into the express lane at entrance $n$ at minute $t$. Let $I_{nt}$ be the set of subscribers at entrance $n$ and time $t$, and let $v_{nt}^{HOV}$ be the corresponding number of carpoolers. Volume in the express lane at entrance $n$ and time $t$ is given by

$$v_{nt} = \sum_{i \in I_{nt}} 1 \{d_{int} = 1\} + v_{nt}^{HOV};$$  \hspace{1cm} (6)

where $d_{int-1}$ is consumer $i$’s entry decision at entrance $n$ and time $t$. Subscribers who entered the express lane before entrance $n$ always stay in the express lane, so $d_{int-1} = 1$ if $d_{in-1} = 1$.

The volume in the regular lane is the sum of non-subscribers and subscribers who stay in the regular lane. It is given by

$$v_{nt} = \sum_{i \in I_{nt}} 1 \{d_{int} = 0\} + v_{nt}^{NON};$$  \hspace{1cm} (7)

where $v_{nt}^{NON}$ is the number of non-subscribers holders at entrance $n$ and time $t$.

To calculate the traffic volumes, it is necessary to know where a motorist is at a given time. Let $g \in \{SUB, NON, HOV\}$ denote motorist types. Let $\lambda_{it}^g = (\mu_{it}^g, d_{it}^g) \in [0, M] \times \{0, 1\}$ be the location of motorist $i$ of type $g$ at time $t$. $\mu_{it}^g \in [0, M]$ denotes mileage mark of the location of the motorist. $\mu_{it} = 0$ implies that the motorist is at the beginning of the highway. $d_{it}^g$ denotes the lane which motorist $i$ travels in. Non-subscribers always travel in the regular lane, so $d_{it}^{NON} = 0, \forall i, t$. Carpoolers always travel in the express lane, so $d_{it}^{HOV} = 1, \forall i, t$. $d_{it}^{SUB}$ is subscriber $i$’s entry decision at the last entrance before her current location.

The volumes of carpoolers and non-subscribers in equations (6) and (7) are given by

$$v_{nt}^g = \sum_{i} 1 \{\mu_{it}^g = m_n\}, \text{ for } g \in \{HOV, NON\}$$  \hspace{1cm} (8)

where $m_n$ is the mileage mark of entrance $n$. Similarly, the set of subscribers at entrance $n$ and time $t$ is given by

$$I_{nt} = \{i : \mu_{it}^{TRANS} = m_n\}.$$  \hspace{1cm} (9)
New entrants to the highway and initial conditions  Motorists enter the highway at various times and locations. For simplicity, I assume that the entrances to the highway coincides with the entrances to the HOT lane, and subscribers can switch into the HOT lane as soon as they arrive on the highway. Non-subscribers and Subscribers arrive on the highway in the regular lanes. Carpoolers arrive in the express lanes. Therefore, the initial condition for motorist $i$ of type $g$ arriving on the highway at entrance $n$ and time $t$ is ($\mu_{it}^g = m_n$, $d_{it}^g = 0$) for $g \in \{\text{SUB}, \text{NON}\}$ and ($\mu_{it}^g = m_n$, $d_{it}^g = 1$) for $g = \text{HOV}$.

At each entrance, the inflow of non-subscribers is stochastic. The inflow of subscribers and carpoolers are deterministic at a given time. This assumption incorporates daily fluctuations in traffic conditions. At each entrance, the inflow of non-subscribers is stochastic. The inflow of subscribers and carpoolers are deterministic at a given time. This assumption incorporates daily fluctuations in traffic conditions with one aggregate shock\footnote{As subscribers and carpoolers are more likely to be commuters who follow a specific schedule, the arrival of subscribers and carpoolers are less likely to vary from one day to another. Non-subscribers may include the motorists who do not make regular trips at a certain time, and thus accounting for some of the variations in daily traffic conditions.}. Let $F_{nj}^g$ be the number of new motorists of type $g$ arriving at entrance $n$ and time $t$ on journey $j$. For entrance $n$ and time $t$, the inflow of each type is given by

\begin{align}
F_{nj}^g &= F_{nt}^g, \text{ for every date } j \text{ and motorist types } g \in \{\text{SUB}, \text{HOV}\} \tag{10} \\
F_{nj}^{\text{NON}} &\sim \Gamma_{nt}. \tag{11}
\end{align}

where $\Gamma_{nt}$ is the distribution of the inflow of non-subscribers at entrance $n$ and time $t$.

3.3 Equilibrium and model characteristics

Given a pricing function $f_p$\footnote{If $f_p$ is a constant function, then the pricing schedule is a flat rate. A peak load pricing implies that $f_p$ is a function of time. A real-time pricing implies that $f_p$ is a function of traffic conditions.}, together with the inflow of motorists given by equations (10-11), and the speed functions defined in section 3.2, an equilibrium includes value functions $V_{in}$ and policy functions $d_{in}$ for each transponder $i$ and the conditional transition probabilities $H_n(x_{0n}; p_n, x_{0n-1}; p_{n-1})$ for each entrance $n \in \{1, \ldots, N\}$ such that
1. (Consumers optimize) Value functions $V_{im}$ is given by equations (1-4), and the policy functions $d_{in}$ are given by equation (5).

2. (Consistency) Speeds and prices are determined by the speed functions and the pricing function. The transition probabilities $H_a(x', p' \ x, p)$ is consistent with the optimal decision rule, the pricing function, the speed functions and the distribution of non-subscriber inflows.

4 Estimation

To simulate the equilibrium defined in section 3.3, it is necessary to learn the parameters for consumer preferences, $\theta_i$, the speed functions, $f^n_i(.)$, the exogenous inflow of transponder holders and carpoolers, $F^{TRANS}_{nl}$, $F^{HOV}_{nl}$, and the distribution of the inbound flow of nonsubscribers $\Gamma_{FNON}$. Section 4.1 presents the estimation procedures for consumer preferences. Section 4.2 presents the estimation of the speed functions and the inflows of motorists.

4.1 Estimation of consumer preferences

**Econometric model** I consider linear utility functions for the baseline model. Linear utilities are a special case in which entry decisions are independent of consumers' accumulated travel time.\(^\text{24}\) Appendix B presents the econometric model and identification for a concave utility function.

The choice-specific value functions in equations (2-4) can be rewritten as: for lane $\ell \in \{0, 1\}$,

$$V_{in}^\ell (x_{0ntj}, p_{ntj}, X_{in-1tj}, \epsilon_{intj}; \theta_i) = V_{in}^\ell (x_{0ntj}, p_{ntj}, X_{in-1tj}; \theta_i) + \epsilon_{intj}, \quad (12)$$

where $\theta_i$ is consumer $i$'s utility parameters. In the linear specification, the parameter, $\alpha_i$, is consumer $i$'s marginal rate of substitution of price for time, and $1/\alpha_i$ is consumer's value

\(^{24}\)See 3.1. An individual's accumulated travel time depends on where the consumer initially entered the highway, the speed and prices observed at all the previous entrances, and her corresponding entry decisions/
of time. Since the entry decisions in the linear specification is independent of accumulated travel time, let \( V^n_\ell (x_{0ntj}^n, p_{ntj}; \alpha_i) \) be \( V^n_\ell (x_{0ntj}^n, p_{ntj}; X_{ntij} = 0; \alpha_i) \) for lane choice \( \ell \).

The unobserved shock \( \epsilon_{i\ell nij} \) for consumer \( i \) in lane \( \ell \) at entrance \( n \) and time \( t \) on date \( j \) can be written as the sum of an aggregate shock and its residual. Let \( \xi_{\ell nj} \) be an unobserved shock on lane \( \ell \) at entrance \( n \) and time \( t \) on date \( j \), and let \( \nu_{i\ell nij} \) be the residual. The error term in equation (12) can be written as

\[
\epsilon_{i\ell nij} = \xi_{\ell nj} + \nu_{i\ell nij}.
\]

The lane-entrance-date-specific shock \( \xi_{\ell nj} \) includes aggregate demand shocks such as weather conditions. On a rainy or snowy day, the road is likely to be slippery, thus increasing consumers' incentives to choose the congestion-free express lane over the congested regular lane. Another example is the structure of the highway, as we may expect that it is easier to access some entrances than others. In this case, the probability of entering the express lane may be consistently higher at that particular entrance.

Assume that the residual \( \nu_{i\ell nij} \) is i.i.d. following Type I extreme distribution. The market share simplifies to

\[
s^n_{\text{baseline}} (x_{0ntj}^n, p_{ntj}, \xi_{ntij}) = \int_{\alpha_i \in \Theta} \frac{\exp \left[ V^n_\ell (x_{0ntj}^n, p_{ntj}; \alpha_i) - V^n_\ell (x_{0ntj}^n, p_{ntj}; \alpha_i) + \xi_{ntij} \right]}{1 + \exp \left[ V^n_\ell (x_{0ntj}^n, p_{ntj}; \alpha_i) - V^n_\ell (x_{0ntj}^n, p_{ntj}; \alpha_i) + \xi_{ntij} \right]} d\Psi (\alpha_i)
\]

in which consumers vary in their value of time.

**Estimation strategy** I estimate consumer preferences in two stages. In the first stage, I estimate the conditional transition probabilities \( H^n_n \) for each entrance \( n \) nonparametrically using the bin estimator. In the second stage, I estimate the structural parameters of consumer preferences using method of simulated likelihood. The simulated log-likelihood function is
given by

\[
SLL = \sum_{j=1}^{J} \sum_{t=1}^{T} \sum_{\ell=1}^{\ell_{j}} N_{ntj}^c \log \left[ \frac{1}{S} \sum_{i=1}^{S} s_{\text{baseline}}^i (x_{0ntj}, p_{ntj}, X_{ntj}, \xi_{ntj}; \alpha_i) \right] + \frac{1}{S} \sum_{i=1}^{S} s_{\text{baseline}}^i (x_{0ntj}, p_{ntj}, X_{ntj}, \xi_{ntj}; \alpha_i) ;
\]

where \( N_{ntj}^c \) and \( N_{ntj}^r \) are the numbers of subscribers in the express and regular lanes respectively, at entrance \( n \) at time \( t \) on date \( j \). The price coefficient is drawn from a log-normal distribution: \( \alpha_i \sim \log N (\mu_\alpha, \sigma_\alpha) \).

**Identification** In the baseline model, the parameters are the distribution of price coefficients \( (\mu_\alpha, \sigma_\alpha) \). A consumer's value of time is \( 1/\alpha_i \). A high price coefficient indicates that the consumer has low value of time. Consumers with a low value of time are less likely to enter. The preliminary evidence in section ?? shows that consumers are responding to price changes, implying that \( \alpha_i > 0 \) for some consumers.

The distribution of the price coefficient is identified through the variations in price, travel time, and entry decisions within an entrance. The price elasticity of the homogeneous model is given by

\[
\eta_{ntj} = \frac{\partial s_{ntj}^\text{baseline}}{\partial p_{ntj}} \frac{p_{ntj}}{s_{ntj}^\text{baseline}} = -\alpha p_{ntj} \left( 1 - s_{ntj}^\text{baseline} \right).
\]

The function is convex and increasing in price. The price elasticity of the random coefficient model is given by

\[
\eta_{ntj} = \frac{\partial s_{ntj}^\text{baseline}}{\partial p_{ntj}} \frac{p_{ntj}}{s_{ntj}^\text{baseline}} = -\frac{p_{ntj}}{s_{ntj}^\text{baseline}} \int \alpha_i s_{ntj}^\text{baseline} \left( 1 - s_{ntj}^\text{baseline} \right) d\Psi (\alpha_i).
\]

Consumers who were indifferent between the express and regular lanes are more likely to change their entry decision with a change in price change than those with extremely high or low values of time. The price elasticity under the random coefficient model weighs sensitive consumers (who are most likely to change their entry behavior) more than those whose probability of entry is close to 0 or 1. For example, if most consumers have a high value of time, then price elasticity will increase at a high rate over low prices and level out over
high prices. Therefore, the distribution of the price coefficients is identified by the degree to which substitution patterns change over price.

The speed and price at an entrance at a given minute depend on the aggregate demand at the previous minute. Endogeneity of speed and prices may rise with serial correlation in any unobserved aggregate shocks on the demand for express lanes. Besides travel time and price, we might expect a few aggregate factors that affect the demand of express lanes. One example is weather. Consumers might be more inclined to enter the express lane on a rainy day due to slippery road conditions and low visibility. I control for weather conditions and other potential serial correlation aggregate shocks using date fixed effects. Since each date in the data is a four-hour period, it is reasonable to expect little variation in weather conditions during that period of time. In an alternative specification, I use visibility as one of the regressors to capture the effect of weather on the demand for express lanes.

4.2 Estimation of speed functions and traffic inflows

Speed functions relate individual entry decisions to travel time. The speed of a lane at an entrance depends on the capacity of the highway, the free-flow speed, and the traffic volume. To capture the capacity and structure variations that affect speeds, the speed functions are estimated separately for each entrance. I assume that speed follows a first-order ordinary differential equation such that

\[ \text{speed}_t = \text{speed}_{t-1} + G(\text{volume}_t - \text{volume}_{t-1}), \]

where \( G(0) = 0 \) and \( G' < 0 \). This specification implies that the change in speed is a function of the change in volume. If the traffic volume stays constant from one minute to the next, then the speed stays constant. If traffic volume increases, then speed will decrease. The speed at \( t = 1 \) is set to be the free flow speed.

It is also necessary to estimate the inflows of carpoolers and subscribers and the distribution of inbound nonsubscribers each entrance, \( \Gamma_{P_{NON}} \). I assume that the inflow of carpoolers and subscribers are 10% and 38% of the traffic inflows. I assume that the arrival of nonsub-
scribers holders follows a Poisson process. In particular, the inflow of nonsubscribers \( F_{ntj}^{NON} \) at entrance \( n \) and time \( t \) on date \( j \) equals to \( F_{ntj}^{NON} + k_j + k_{nj} \), where \( F_{t}^{NON} \) is the average flow of nonsubscribers at time \( t \) and entrance \( n \). \( k_j \) is a date-specific shock that applies to every entrance. \( k_{nj} \) is a date and entrance specific shock. The arrival process is estimated using highway entry ramp data for each entrance and every minute during morning rush hours.

5 Results

5.1 Model estimates

Figure 6 presents typical estimated conditional transition probabilities in the first stage. Figure 6a shows the distribution of the regular lane speed at entrance 5 conditional on the price at entrance 4, holding congestion at entrance 4 constant. The graph suggests that the probability of observing a higher travel time at entrance 5 increases with the price observed at entrance 4. For example, the probability that no congestion occurs at entrance 5 (travel time equals to 3 minutes) is 0.45 if the price at entrance 4 is $0.50, 0.18 if the price at entrance 4 is $0.75, and 0.8 if the price at entrance 4 is $1.00. Conversely, the probability that some congestion occurs at entrance 5 (travel time equals to 4 minutes) is 0.1 if the price at entrance 4 is $0.50, 0.3 if the price at entrance 4 is $0.75, and 0.4 if the price at entrance 4 is $1.00. Similarly, figure 6b show that the probability of observing a higher price at entrance 5 increases with the price at entrance 4. Therefore, consumers can infer downstream congestion and prices from the current price.

Table 5 presents the estimation results for consumer preferences in the second stage. Column 2 shows the estimates of a model in which consumers are homogenous in value of time, and column 4 shows the results of the random coefficient model. I use date fixed effects in both specifications to control for price and speed endogeneity. The likelihood ratio test shows the random coefficient model fits significantly better than the homogenous model. The mean price coefficient is -0.204 and the standard deviation is 0.056. The estimates of the random coefficient model imply that the mean price elasticity is 0.28.
To show that the dynamic model is necessary, I estimate a static model in which consumers can only enter the express lane when they first enter the highway (column 1). If consumers choose to stay in the regular lane at the initial entrance where they entered the highway, then they have to stay in the regular lane. Therefore, the choice specific value function of not entering in the static model is given by

\[ V_{in}^0(x_{0n}, p_n, X_{in-1}) = E \left[ u \left( X_{in-1} + \sum_{\hat{n} \geq n} x_{0\hat{n}}, 0; \theta_i, x_{0n}, p_n \right) \right]. \]

The expectation is taken over the regular lane travel time at all downstream entrances. Since consumers only make the entry decision once, I also refer to this model as the one-shot model.

The dynamic model fits significantly better than the one-shot model (comparing column 1 with columns 2 and 3 using likelihood-ratio test). The result suggests that some consumers wait to enter at later location. In particular, the static model overestimates the price coefficient (-0.367 vs. -0.204). This is because the opportunity cost of not entering the express lane is greater in the static model, conditional on the current price and travel time. Therefore, the static model will overestimate the price coefficient to explain the market share generated by a dynamic model.

To account for daily variations of aggregate demand shocks, I use date fixed effects in all specifications except for column 3. Column 3 uses visibility and weekday fixed effects. Visibility reflects how easy it is to drive on a particular day. It substitutes for weather conditions that may affect a consumer’s willingness to enter the express lane to account for unobserved shocks. The express lane is congestion-free, so it may be easier to drive in on a day with low visibility. I also control for day fixed effects in column 3. The estimate for visibility is negative. This is consistent with the fact that consumers are more likely to pay for the express lane when visibility is low. Compared with the specification in column 4, the likelihood and the implied annual income suggest that date fixed effects are a better control for unobserved aggregate demand shocks for the express lane.

Using the price coefficient estimates in table 5, we can calculate the value of time. The average value of time of the MnPASS subscribers is $62 per hour with a standard deviation
is $23 per hour (table 6). This is equivalent to an annual income of $120,624. According to the 2007-2011 American Community Survey, the mean household incomes are $116,668 and $131,035 of two neighborhoods at the first two entrances of MnPASS.

Table 7 shows the estimates for the speed functions, and figure 7 shows the mean inflow of nonsubscribers over time.

5.2 Model fit and robustness check

This section addresses two issues: 1) to evaluate the degree to which expectations affect demand estimates, I estimate an alternative demand specification in which the price coefficient incorporates the substitution effect and the forecasting effect. I compare the estimates from the alternative specification to the structural estimates presented in the previous section to how much demand estimates would be biased. 2) I simulate an equilibrium to show model fit.

As price updates are exogenous to consumers, this introduces a natural experiment to study the effect of price on demand. The identification assumption is the demand is continuous within five minutes, conditional on the location, date, and traffic conditions. Consider the following specification:

$$\log s_{ntj} = g(x_{0ntj}, p_{ntaj}) + \gamma 1 \{\Delta p \neq 0\} \ast \text{sign}(\Delta p) + \phi_n + \phi_j + \epsilon_{ntj},$$

where $s_{ntj}$ is the percentage of new paying entrants at entrance $n$ at time $t$ on date $j$. $x_{0ntj}$ is the regular lane speed at time $t$. $p_{ntaj}$ is the original price displayed (before the price change) at entrance $n$. $1 \{\Delta p \neq 0\} \ast \text{sign}(\Delta p)$ equals to 1 if there is an increase of $0.25$, 0 if there is no change and -1 if there is a decrease of $0.25$. $\gamma$ measures the treatment effect of the price change. $\phi_n$ and $\phi_j$ are entrance and date fixed effects. The date fixed effects control date-specific shocks such as weather that may lead to a surge in aggregate demand for the express lane. $\epsilon_{ntj}$ is the error term.

Table 8 shows that the treatment effect, $\gamma$, is statistically significant and negative. The

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25 The average number of working hours per year is 2,080.
mean elasticity is 0.14, which is significantly less than the mean elasticity from the structural estimates (0.28). The estimated value of time from the alternative specification is $75 compared to $62 from the structural model. The result is consistent with the fact that current price is positively correlated with expected downstream congestion and prices, which leads to a positive effect on demand. As the alternative demand specification cannot separately the positive forecasting effect from the negative substitution effect, it understates price elasticity and overstates consumer's willingness to pay.

A price change has two effects at the intermediate entrance. The first is the substitution effect, and demand decreases with price. Second, an increase in price affect the expectation of downstream traffic and prices. An increase in price indicates a higher probability of congestion and high tolls at later entrance, which may given consumers an incentive to enter right away. Thus, demand may increase with price. Therefore, price elasticities at the intermediate entrances depend on the transition of speed and prices as well as consumers' value of time.

It is possible to simulate the equilibrium traffic conditions and prices during morning rush hours given the estimates above. This section evaluates the goodness of fit of the baseline model by comparing the simulated equilibrium prices and speed with the observed prices and speed in the data. As a robustness check, the simulation in this section relaxes the assumption of a constant express lane speed.

Table 9 reports summary statistics of the simulated prices and speed. The table is constructed the same way as the summary statistics of the sample presented in table 3. The average regular lane travel time is similar to that observed in the data. The express lane is congestion-free the majority of the time. The simulated express lane travel time is constant from entrance 1 to entrance 4. Approximately 10% of the express travel time at entrance 5 deviates from the minimum travel time (three minutes).

The mean equilibrium price is $1.58 for the first segment and $1.21 for the second segment. The simulated prices are higher than the data ($1.21 for the first segment and $0.85 for the second segment). The probability of entering the express lane at entrance 1 through entrance 4 is higher than their data counterparts. However, the entry pattern across en-
trances persists. The probability of entering decreases from entrance 1 through entrance 4. The mean probability of entering the express lane at entrance 5 is 47%, which is close to its data counterpart at 49%. The higher probability of entering the express lane at the first four entrances is consistent with the increase in regular lane travel time at entrance 4 in the simulation.

Table 10 shows that some subscribers do not enter the express lane as soon as they enter the highway. Table 10 reports the probability of entering at each entrance where the subscriber initially entered the highway. Consumers prefer to enter early at entrance 1. Among the consumers who enter the highway at entrance 1, 49% enter immediately, 19% enter at entrance 2, 4% enter at entrance 3, 1% enter at entrance 4, and 13% at entrance 5. Consumers who initially enter the highway at entrance 2 prefer either entering right away (40%) or entering at entrance 5 (23%). Consumers who initially enter the highway at entrances 3 and 4 prefer to wait and enter at entrance 5. Approximately 39% of the consumers from entrance 3 wait to enter at entrance 5 (versus the 13% that enter at entrance 3). About 45% of the consumer from entrance 4 wait to enter at entrance 5 (versus the 4% that enter at entrance 4). This entry pattern suggests that consumers make dynamic entry decisions and sometimes wait and enter at later entrances and enter at later entrances.

6 Counterfactuals

The section uses the estimation results and attempts to answer the policy-related questions raised in the introduction. Section 6.1 evaluates the efficiency gain from the current express lane compared to a traditional carpool lane. Section 6.2 studies how different pricing functions change the welfare outcome.

6.1 The efficiency gain from MnPASS

To measure the efficiency gain from the current MnPASS program, I first simulate the model with the constraint that subscribers cannot access the carpool lane. The experiment is to measure the efficiency loss when the express lane unexpectedly shut down for one day. The
close-down is unanticipated, and consumers pretrip decisions (such as when to leave for work) likely remain the same.

The expected consumer welfare is composed of the subscriber surplus, carpooler surplus, and nonsubscriber surplus. Since the utility for subscribers is defined in travel time, the utilities of carpoolers and nonsubscribers are equal to their total travel time. The expectation of consumer welfare is taken over nonsubscribers inflows, consumers' value of time, and the i.i.d. shocks on subscribers at each entrance.

Column 1 in table 12 shows the expected total consumer welfare from only allowing carpoolers to access the express lane (HOV only). Consumer welfare is measured in hours per mile. The total consumer welfare is -574 hours per mile, which implies that on average, motorists spend a total of 574 hours per mile every day from 6:00 a.m. to 10 a.m. Subscribers spend a total of 242 hours per miles during morning rush hours. Carpoolers spend 48 hours. Subscriber surplus is greater than carpoolers because there are more subscribers than carpoolers on the highway, and each subscriber spends more time on the highway due to the congestion in the regular lane.

The express lane allows subscribers to save travel time by paying a toll. The first row in column 2 shows that giving subscribers access to the carpool lane improves their welfare. The total time saved is equivalent to $22,015 per day during morning rush hours (6:00 a.m. to 10 a.m.), for which they pay $14,470. This is consistent with subscriber surplus reported in hours per mile. The subscriber surplus increases by 10% under the current MnPASS program (217 hours per mile compared with 242 hours per mile). Note that 217 hours per mile reported for subscriber surplus include the disutility of paying a toll for using the express lane.

Nonsubscribers are also better off under the current MnPASS program. Regular lane users travel time decreases from 284 to 255 hours per mile, which accounts for 47% of the efficiency gain from the MnPASS program. Carpoolers are slightly worse off. Their travel time increases from 48 to 54 hours per mile. This is because occasionally, the current pricing fails to keep the express lane above 55mph. This phenomena is present in the data and the simulation. Overall, the current MnPASS program improves motorist welfare by 8%.
6.2 Alternative pricing functions: real-time pricing versus peak load pricing

The goal of this section is to compare the efficiency gain from using real-time pricing function to peak load pricing, which is also used by express lanes in the United States. I first calculate the optimal peak load pricing for the MnPASS program. The optimal peak load pricing maximizes the expected consumer welfare subject to the constraint that the government revenue does not decrease.\footnote{The Ramsey problem for the optimal peak load pricing is defined as follows. Given a pricing function, the deterministic inbound flows of carpoolers and subscribers, the distribution of the inbound flow of non-subscribers and the speed functions, there exists an equilibrium defined in section 3.3. Each equilibrium has a corresponding expected total consumer welfare. An optimal peak load pricing function maximizes the expected consumer welfare subject to the constraint that the government revenue does not decrease.} For tractability, I consider peak load prices that vary every hour.

The optimal peak load pricing is given as follows. From 6:00 a.m. to 7:00 a.m., the price for segment 1 is $0.25 and the price for segment 2 is $0.25. From 7:00 a.m. to 9 a.m., the price for segment 1 is $2.25 and the price for segment 2 is $1. From 9:00 a.m. to 10 a.m., the price for segment 1 is $2 and the price for segment 2 is $0.75. The optimal peak load prices are generally higher than real-time prices with a few exceptions. From 6:00 a.m. to 7:00 a.m., the average prices are $1.00 for segment 1 and $0.5 for segment 2 under the current real-time pricing. The average segment 2 prices are $1.9 from 8:00 a.m. to 9:00 a.m. and $1.5 from 9:00 a.m. to 10:00 a.m.

Table 13 suggests that the current real-time pricing is more efficient than the optimal peak load pricing (columns 1 and 2). The expected total consumer welfare decreases by 3\% when the current real-time pricing is switched to the optimal peak load pricing. Non-subscriber surplus decreases by 5\% from the current real-time pricing to the optimal peak load pricing. Carpooler surplus does not change, and subscriber surplus decreases by 2\%.

Though the current real-time pricing is more efficient than the optimal peak load pricing, it is not clear whether it is optimal. The current MnPASS pricing function solely depends on the express lane speed, which is motivated by the idea to keep the express lane flowing. The pricing function increases in the number of vehicles in the express lane to deter further entry. This paper proposes an alternative pricing function that depends on the time savings by
entering the express lane. The alternative pricing function directly corresponds to subscribers incentives to enter the express lane. For a fair comparison, this new function uses the same step as the current pricing function shown in table 2 except that it uses the relative traffic density between the express and regular lanes.

Column 3 of table 13 shows the result from the alternative real-time pricing function. Nonsubscribers benefit the most under the alternative pricing function (increases by 5%). Carpoolers and subscribers do not experience substantial efficiency gains from the alternative pricing function, and total motorist surplus increases.

7 Conclusion

This paper studies consumer dynamic decisions to enter express lanes on the road. A typical express lane has two features: multiple entrances and real-time pricing. Consumer-entry decisions at an intermediate entrance depend on the current congestion and price as well as the expectation on congestion and prices further down the road. Real-time pricing reflects the realized daily traffic shock, and consumer expectations of future congestion and prices depend on the current price. Variations in the demand for express lanes at an entrance and prices may be caused by the substitution between time and money or a change in the expected future price (the forecasting effect). The former is consumers’ willingness to pay that is crucial to evaluating the efficiency gain of the express lanes. To separately identify consumers’ willingness to pay from the forecasting effect, this paper structurally estimates a discrete choice model of consumer entry decisions on the road. Results suggest that a reduced form model overestimates consumer’s willingness to pay. Results also suggest that the dynamic model is necessary since not all consumers enter the express lane as soon as they arrive on the highway.

Using the estimation results, this paper evaluates the welfare gains from switching carpool lanes to express lanes. Results suggest that express lanes increase motorist welfare by 8%. This paper also compares the efficiency gain from real-time pricing and peak load pricing. The results of this paper suggest that even if the real-time pricing function is not optimal,
it may still be more efficient than the optimal peak load pricing. Highway authorities may consider to switch to real-time pricing if the operation cost of real-time pricing is similar to the cost of peak load pricing. This paper also proposes a more efficient real-time pricing function. The policy analysis in this paper takes consumer pretrip decisions as given. However, changes in the pricing rules may affect when consumers leave for work or the number of subscribers in the long run. The next step is to extend the framework to incorporate consumer pretrip decisions and study the welfare implications of various pricing rules. If the operation cost of real-time pricing is similar to the cost of peak load pricing. This paper also proposes a more efficient real-time pricing function. The policy analysis in this paper takes consumer pretrip decisions as given. However, changes in the pricing rules may affect when consumers leave for work or the number of subscribers in the long run. The next step is to extend the framework to incorporate consumer pretrip decisions and study the welfare implications of various pricing rules.
References


Table 1: List of HOT lanes in the United States in operation as of May 2013

<table>
<thead>
<tr>
<th>Highway</th>
<th>Location</th>
<th>Length (miles)</th>
<th>Pricing</th>
<th>Excess Capacity</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR 91</td>
<td>Orange country, CA</td>
<td>10</td>
<td>Peak-load</td>
<td></td>
<td>1995</td>
</tr>
<tr>
<td>I-15</td>
<td>San Diego, CA</td>
<td>16</td>
<td>Dynamic</td>
<td></td>
<td>1996</td>
</tr>
<tr>
<td>I-10</td>
<td>Houston, TX</td>
<td>12</td>
<td>Dynamic</td>
<td></td>
<td>1998</td>
</tr>
<tr>
<td>US 290</td>
<td>Houston, TX</td>
<td>15</td>
<td>Peak-load</td>
<td></td>
<td>2000</td>
</tr>
<tr>
<td>I-394</td>
<td>Minneapolis, MN</td>
<td>11</td>
<td>Dynamic</td>
<td></td>
<td>2005</td>
</tr>
<tr>
<td>I-25</td>
<td>Denver, CO</td>
<td>7</td>
<td>Peak-load</td>
<td></td>
<td>2006</td>
</tr>
<tr>
<td>I-15</td>
<td>Salt Lake City, UT</td>
<td>40</td>
<td>Dynamic</td>
<td></td>
<td>2006</td>
</tr>
<tr>
<td>SR 167</td>
<td>Seattle, WA</td>
<td>11</td>
<td>Dynamic</td>
<td></td>
<td>2008</td>
</tr>
<tr>
<td>I-95</td>
<td>Miami, FL</td>
<td>7</td>
<td>Dynamic</td>
<td></td>
<td>2008</td>
</tr>
<tr>
<td>I-35W</td>
<td>Minneapolis, MN</td>
<td>16</td>
<td>Dynamic</td>
<td></td>
<td>2009</td>
</tr>
<tr>
<td>I-680</td>
<td>San Jose, CA</td>
<td>14</td>
<td>Dynamic</td>
<td></td>
<td>2010</td>
</tr>
<tr>
<td>I-85</td>
<td>Atlanta, GA</td>
<td>15.5</td>
<td>Dynamic</td>
<td></td>
<td>2011</td>
</tr>
<tr>
<td>I-495</td>
<td>Fairfax county, VA</td>
<td>14</td>
<td>Dynamic</td>
<td></td>
<td>2012</td>
</tr>
<tr>
<td>IH 45</td>
<td>Houston, TX</td>
<td>15</td>
<td>Peak-load</td>
<td></td>
<td>2012</td>
</tr>
<tr>
<td>I-110</td>
<td>Los Angeles, CA</td>
<td>11</td>
<td>Dynamic</td>
<td>HOV only</td>
<td>2012</td>
</tr>
<tr>
<td>US 59</td>
<td>Houston, TX</td>
<td>15</td>
<td>Peak-load</td>
<td></td>
<td>2013</td>
</tr>
<tr>
<td>I-10</td>
<td>Los Angeles, CA</td>
<td>14</td>
<td>Dynamic</td>
<td>HOV only</td>
<td>2013</td>
</tr>
</tbody>
</table>

1 Additional policies in case the speed in the HOT lanes drop below the minimum requirement.
2 When I-15's first 8-mile stretch opened in 1996 in San Diego, it initially offered a monthly subscription for unlimited usage. In 1998, the pricing schedule was switched to real-time pricing (Supernak, Brownstone, Golob, Golob, Kaschade, Kazimi, Schreffer, and Steffey, 2001).
3 Houston used to have a flat-rate pricing schedule (Burris and Stockton, 2004).
4 Similar to the original pricing schedule used on I-15 in San Diego, CA, I-15 in Salt Lake City, UT used to have a monthly subscription with a flat rate.
5 HOV only implies that when the speed is below 45 mph, HOT lane is only open to carpoolers.

Table 2: The pricing function of I-394 MnPASS Minneapolis MN

<table>
<thead>
<tr>
<th>Density (TD)</th>
<th>[0, 12)</th>
<th>[12, 19]</th>
<th>[19, 30]</th>
<th>[30, 36]</th>
<th>[36, 46]</th>
<th>[46, 50]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>0.25</td>
<td>0.25</td>
<td>1.50</td>
<td>2.50</td>
<td>3.50</td>
<td>6.00</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.25</td>
<td>0.25</td>
<td>2.50</td>
<td>3.50</td>
<td>6.00</td>
<td>8.00</td>
</tr>
<tr>
<td>Default</td>
<td>0.25</td>
<td>0.25</td>
<td>2.50</td>
<td>3.50</td>
<td>5.00</td>
<td>8.00</td>
</tr>
</tbody>
</table>

| \Delta_{	ext{toll}}/(\Delta_{	ext{TD}} - 1) | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 |

2 Unit = Dollars
Table 3: Summary statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Regular lane travel time (min)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>entr 1</td>
<td>1.00</td>
<td>0.06</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>entr 2</td>
<td>2.01</td>
<td>0.10</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>entr 3</td>
<td>2.26</td>
<td>1.32</td>
<td>2</td>
<td>21</td>
</tr>
<tr>
<td>entr 4</td>
<td>1.08</td>
<td>0.31</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>entr 5</td>
<td>4.71</td>
<td>1.65</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td><strong>Express lane travel time (min)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>entr 1</td>
<td>1.00</td>
<td>0.13</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>entr 2</td>
<td>2.01</td>
<td>0.08</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>entr 3</td>
<td>2.02</td>
<td>0.22</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>entr 4</td>
<td>1.03</td>
<td>0.26</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>entr 5</td>
<td>3.53</td>
<td>0.51</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td><strong>Toll ($)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>segment 1</td>
<td>1.24</td>
<td>1.28</td>
<td>0.25</td>
<td>8.00</td>
</tr>
<tr>
<td>segment 2</td>
<td>0.85</td>
<td>0.62</td>
<td>0.25</td>
<td>3.50</td>
</tr>
<tr>
<td><strong>Percentage of entries (%)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>entr 1</td>
<td>.38</td>
<td>.34</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>entr 2</td>
<td>.16</td>
<td>.16</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>entr 3</td>
<td>.05</td>
<td>.09</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>entr 4</td>
<td>.03</td>
<td>.05</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>entr 5</td>
<td>.49</td>
<td>.24</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

The sample includes eastbound I-394 conditions from 6am to 10am in October 2009. The number of observations is 5,344 at the first entrance, 5,394 at the second entrance, 5,397 at the third entrance, 5,429 at the forth entrance and 5,434 at the fifth entrance.
Table 4: Average percentage of paying entrants at entrance 1

<table>
<thead>
<tr>
<th>price</th>
<th>7:00 a.m.</th>
<th>7:30 a.m.</th>
<th>8:00 a.m.</th>
<th>8:30 a.m.</th>
<th>9:00 a.m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>0.70</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.58</td>
</tr>
<tr>
<td>1.00</td>
<td>0.60</td>
<td>0.50</td>
<td>-</td>
<td>-</td>
<td>0.43</td>
</tr>
<tr>
<td>1.25</td>
<td>0.59</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.43</td>
</tr>
<tr>
<td>1.50</td>
<td>0.31</td>
<td>0.83</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1.75</td>
<td>0.67</td>
<td>0.38</td>
<td>-</td>
<td>0.50</td>
<td>0.33</td>
</tr>
<tr>
<td>2.00</td>
<td>0.63</td>
<td>0.45</td>
<td>-</td>
<td>0.40</td>
<td>-</td>
</tr>
<tr>
<td>2.25</td>
<td>0.55</td>
<td>0.75</td>
<td>0.50</td>
<td>0.57</td>
<td>0.50</td>
</tr>
<tr>
<td>2.50</td>
<td>-</td>
<td>0.60</td>
<td>0.67</td>
<td>0.35</td>
<td>-</td>
</tr>
<tr>
<td>2.75</td>
<td>-</td>
<td>0.58</td>
<td>0.80</td>
<td>0.54</td>
<td>-</td>
</tr>
<tr>
<td>3.00</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.88</td>
<td>-</td>
</tr>
<tr>
<td>3.25</td>
<td>-</td>
<td>-</td>
<td>0.71</td>
<td>0.50</td>
<td>-</td>
</tr>
<tr>
<td>3.50</td>
<td>-</td>
<td>0.86</td>
<td>0.60</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3.75</td>
<td>-</td>
<td>0.75</td>
<td>0.50</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4.00</td>
<td>-</td>
<td>0.58</td>
<td>0.62</td>
<td>0.67</td>
<td>-</td>
</tr>
<tr>
<td>4.25</td>
<td>-</td>
<td>-</td>
<td>0.60</td>
<td>0.63</td>
<td>-</td>
</tr>
<tr>
<td>4.50</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.50</td>
<td>-</td>
</tr>
<tr>
<td>4.75</td>
<td>-</td>
<td>-</td>
<td>0.65</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5.00</td>
<td>-</td>
<td>-</td>
<td>0.61</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5.25</td>
<td>-</td>
<td>-</td>
<td>0.60</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5.50</td>
<td>-</td>
<td>-</td>
<td>0.67</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

This table shows an example of demand and price variations at entrance 1 conditional on no current congestion.

Table 5: Second stage estimation results of consumer preferences

<table>
<thead>
<tr>
<th>variable</th>
<th>linear/static</th>
<th>linear/dynamic</th>
<th>linear/dynamic</th>
<th>linear/dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>price (mean)</td>
<td>0.367</td>
<td>0.108</td>
<td>0.075</td>
<td>0.204</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.050)</td>
<td>(0.023)</td>
<td>(0.091)</td>
</tr>
<tr>
<td>price (std dev.)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.058</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.001)</td>
</tr>
<tr>
<td>visibility</td>
<td>-</td>
<td>-</td>
<td>-1.028</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.000)</td>
<td>-</td>
</tr>
<tr>
<td>date fixed effect</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>day of the week</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>ln L</td>
<td>-159.206</td>
<td>-155.243</td>
<td>-158.956</td>
<td>-153.075</td>
</tr>
<tr>
<td>mean elasticities</td>
<td>.23</td>
<td>.09</td>
<td>.06</td>
<td>0.28</td>
</tr>
</tbody>
</table>

All standard errors are bootstrapped.
Table 6: Second stage estimation results of consumer preferences

<table>
<thead>
<tr>
<th>variable</th>
<th>linear/static</th>
<th>linear/dynamic</th>
<th>linear/dynamic</th>
<th>linear/dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>value of time (mean)</td>
<td>29</td>
<td>99</td>
<td>143</td>
<td>62</td>
</tr>
<tr>
<td>value of time (std. dev.)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>23</td>
</tr>
<tr>
<td>annual income (mean)</td>
<td>60,652</td>
<td>205,920</td>
<td>297,440</td>
<td>120,624</td>
</tr>
</tbody>
</table>

1 Value of time is measured in $ per hour.

2 Mean household income is 116,668 in Plymouth and 131,035 in Wayzata (2007-2011 American Community Survey)

Table 7: Speed function estimates

<table>
<thead>
<tr>
<th>Variable</th>
<th>Entrance 1</th>
<th>Entrance 2</th>
<th>Entrance 3</th>
<th>Entrance 4</th>
<th>Entrance 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>lagged speed</td>
<td>0.83***</td>
<td>0.74***</td>
<td>0.95***</td>
<td>0.86***</td>
<td>0.93***</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>change in volume</td>
<td>-.03***</td>
<td>-.03*</td>
<td>-.01***</td>
<td>-.05***</td>
<td>-.02***</td>
</tr>
<tr>
<td></td>
<td>(.00)</td>
<td>(.00)</td>
<td>(.00)</td>
<td>(.00)</td>
<td>(.00)</td>
</tr>
<tr>
<td>Intercept</td>
<td>9.57***</td>
<td>16.14***</td>
<td>2.84***</td>
<td>8.46***</td>
<td>4.24***</td>
</tr>
<tr>
<td></td>
<td>(.06)</td>
<td>(.07)</td>
<td>(.03)</td>
<td>(.05)</td>
<td>(.04)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.72</td>
<td>0.56</td>
<td>0.91</td>
<td>0.73</td>
<td>0.86</td>
</tr>
<tr>
<td>Obs.</td>
<td>370,636</td>
<td>370,449</td>
<td>371,134</td>
<td>370,851</td>
<td>371,161</td>
</tr>
</tbody>
</table>

* The sample period spans from 2008 to 2010.
Table 8: The effect of a price change on the percentage of paying trips starting at entrance

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>treatment effect</td>
<td>-0.217***</td>
<td>(0.066)</td>
</tr>
<tr>
<td>regular lane speed</td>
<td>-0.045***</td>
<td>(0.008)</td>
</tr>
<tr>
<td>initial price</td>
<td>1.257***</td>
<td>(0.046)</td>
</tr>
<tr>
<td>intercept</td>
<td>-5.326***</td>
<td>(0.549)</td>
</tr>
<tr>
<td>entrance fixed effects</td>
<td>y</td>
<td>y</td>
</tr>
<tr>
<td>date fixed effects</td>
<td>y</td>
<td>y</td>
</tr>
<tr>
<td>mean elasticity</td>
<td>-0.141***</td>
<td>(0.006)</td>
</tr>
<tr>
<td>obs.</td>
<td>11,787</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>.238</td>
<td></td>
</tr>
</tbody>
</table>

1 The regressions only consider subsamples with price changes of 25 cents.
2 ***, **, * indicate statistical significance of 1%, 5%, 10% respectively.
3 The initial prices are the price before the change. The result reports the marginal effects of initial prices and regular lane speed.
Table 9: Summary statistics of the simulated equilibrium

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Regular lane travel time (min)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>entr 1</td>
<td>1.00</td>
<td>0.00</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>entr 2</td>
<td>2.00</td>
<td>0.00</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>entr 3</td>
<td>2.40</td>
<td>0.49</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>entr 4</td>
<td>1.00</td>
<td>0.00</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>entr 5</td>
<td>4.07</td>
<td>0.61</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td><strong>Express lane travel time (min)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>entr 1</td>
<td>1.00</td>
<td>0.00</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>entr 2</td>
<td>2.00</td>
<td>0.00</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>entr 3</td>
<td>2.00</td>
<td>0.00</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>entr 4</td>
<td>1.00</td>
<td>0.00</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>entr 5</td>
<td>3.63</td>
<td>0.48</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td><strong>Toll ($)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>segment 1</td>
<td>1.58</td>
<td>0.97</td>
<td>0.25</td>
<td>5.75</td>
</tr>
<tr>
<td>segment 2</td>
<td>1.21</td>
<td>0.72</td>
<td>0.25</td>
<td>3.25</td>
</tr>
<tr>
<td><strong>Percentage of entries</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>entr 1</td>
<td>.49</td>
<td>.14</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>entr 2</td>
<td>.39</td>
<td>.13</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>entr 3</td>
<td>.14</td>
<td>.07</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>entr 4</td>
<td>.06</td>
<td>.03</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>entr 5</td>
<td>.47</td>
<td>.10</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

The table is constructed the same way as table 3 using equilibrium simulation outcomes.

Table 10: Probability of entering the HOT lane by the initial entrance to the highway (%)  

<table>
<thead>
<tr>
<th>Highway</th>
<th>express lane</th>
<th>Entrance 1</th>
<th>Entrance 2</th>
<th>Entrance 3</th>
<th>Entrance 4</th>
<th>Entrance 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entrance 1</td>
<td>.49</td>
<td>.19</td>
<td>.04</td>
<td>.01</td>
<td>.13</td>
<td></td>
</tr>
<tr>
<td>Entrance 2</td>
<td>-</td>
<td>.40</td>
<td>.08</td>
<td>.03</td>
<td>.23</td>
<td></td>
</tr>
<tr>
<td>Entrance 3</td>
<td>-</td>
<td>-</td>
<td>.13</td>
<td>.04</td>
<td>.39</td>
<td></td>
</tr>
<tr>
<td>Entrance 4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>.04</td>
<td>.45</td>
<td></td>
</tr>
<tr>
<td>Entrance 5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>.46</td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>.49</td>
<td>.39</td>
<td>.14</td>
<td>.06</td>
<td>.47</td>
<td></td>
</tr>
</tbody>
</table>

*This table reports the probability of entering at an entrance given the consumer’s initial entrance to the highway.*

40
Table 11: The effect of a price change on the percentage of paying trips starting at entrance

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Err.</th>
</tr>
</thead>
</table>
treatment effect  | -0.252***   | (0.047)  |
regular lane speed| -0.048***   | (0.004)  |
initial price     | 0.045***    | (0.018)  |
intercept         | 2.224***    | (0.251)  |
entrance fixed effects | y   | y        |
date fixed effects | y       | y        |
obs.              | 10,709      |          |
$R^2$             | .231        |          |

This table replicates the regression in Table 8.

Table 12: Comparing consumer welfare between HOV only and the current pricing schedule (6:00a.m. - 10:00a.m.)

<table>
<thead>
<tr>
<th></th>
<th>unit</th>
<th>carpooler only</th>
<th>current real-time</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>subscriber welfare</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
time savings         | $    | -               | 22,015            |
cost                 | $    | -               | 14,407            |
|**total travel time** |      |                 |                   |
total CS             | hours per mile | 574              | 526               |
subscriber           | hours per mile | 242              | 217               |
carpooler            | hours per mile | 48               | 54                |
non-subscriber        | hours per mile | 284              | 255               |
government revenue   | $    | -               | 14,407            |

1 The unit is person-hours per mile per morning.
2 HOV lanes only allow carpooler to access.
3 Government revenue is measured in dollars per morning rush hours (a four-hour period).
Table 13: Comparing consumer welfare from the current pricing function and optimal peak load pricing function

<table>
<thead>
<tr>
<th>unit</th>
<th>current real-time</th>
<th>optimal peak load</th>
<th>alternative</th>
</tr>
</thead>
<tbody>
<tr>
<td>total CS</td>
<td>hours per mile</td>
<td>-526</td>
<td>-544</td>
</tr>
<tr>
<td>subscriber</td>
<td>hours per mile</td>
<td>-217</td>
<td>-222</td>
</tr>
<tr>
<td>carpooler</td>
<td>hours per mile</td>
<td>-54</td>
<td>-54</td>
</tr>
<tr>
<td>non-subscriber</td>
<td>hours per mile</td>
<td>-255</td>
<td>-268</td>
</tr>
<tr>
<td>government revenue</td>
<td>$</td>
<td>14.407</td>
<td>14,410</td>
</tr>
</tbody>
</table>

1 The optimal peak load pricing varies by the hour. From 6:00a.m. to 7:00a.m., the price for segment 1 is $0.25 and the price for segment 2 is $0.25. From 7:00a.m. to 9a.m., the price for segment 1 is $2.25 and the price for segment 2 is $1. From 9:00a.m. to 10a.m., the price for segment 1 is $2 and the price for segment 2 is $0.75.

# Government revenue is measured in dollars per morning rush hours (a four-hour period).
Figure 1: Simple illustration of highway structure and traffic flows

Figure 2: An example of detectors
Figure 3: Average speed from 7am to 9am

![Graph showing average speed from 7am to 9am at different entrances.]

---

Figure 4: Distribution of speed by entrances

![Graphs showing distribution of speed for each entrance.]

---

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Figure 5: An example of price changes

Figure 6: Examples of the estimated transition probabilities of travel time (top) and prices (bottom) at entrance 5 conditional on the price at entrance 4, holding congestion at entrance 4 constant.
Figure 7: Estimates of the mean inflow of non-subscriber inflow

Figure 8: Average travel time at entrance 5 by pricing functions
A Background Appendix

A.1 Segments, entrances and hours of operation

There are two segments and five entrances on the MnPASS of I-394. The two segments are known as the “diamond” section and the “reversible” section. The segments are divided at Trunk Highway 100 (TH-100, see Figure 9). The segment to the east of TH-100 has two reversible lanes located in the median of eastbound and westbound regular lanes. This segment is open to eastbound traffic from 6a.m. to 1p.m. and westbound traffic from 2p.m. to 12a.m. A toll is imposed during the afore mentioned operation times for this segment, and the lanes are closed at the other times. Due to its reversed directions at different operation times, this segment is known as the “reversible section.

Figure 9: Map of I-394 MnPASS

The segment to the west of TH-100 has one-lane is each direction. On a weekday, a toll is imposed on the eastbound from 6a.m. to 10a.m. and on the westbound from 2p.m. to 7p.m. Other than these times, this segment is open to general traffic. Since its operation and structure resembles the original HOV lane, this segment is known as the “diamond section.

From east to west, the five entrances to the HOT lane are located at I-494, Ridgedale, Mn-169, Carlson, TH-100. The entrance at TH-100 is the only entrance to the reversible section on eastbound, and the other four entrances belong to the diamond section. Since the rest of this paper focuses on eastbound traffic, I rename the segments and entrances...
in the order of traffic direction. The diamond segment is denoted as the first segment and the reversible segment as the second segment. The entrance at I-494 is denoted as the first entrance and so forth.

A.2 When and how motorists pay for MnPASS

Before each entrance, motorists observe the toll rate of the segment that the entrance belongs to and that of the upcoming segment if applicable. For example, the first entrance (I-494) displays two prices: one for the first segment (diamond) and one for the second segment (reversible). The fifth entrance (TH-100) only displays the price for the second segment.

At a given time, the toll rate for the same segment may vary from entrance to entrance. This is because the pricing schedule only considers the most congested portion of the segment (maximum downstream density), which does not necessarily occur at the end of a segment. Another reason is that the initial updates are not necessarily synchronized. We might see the tolls at entrance 4 update at 6:02 and that at entrance 5 updates at 6:03.

When a MnPASS subscriber switches into the HOT lane where multiple prices are displayed, she locks down the prices for all segments but is only charged the toll of the segment the entrance belongs to. For example, if a motorist enters the HOT lane at the first entrance, then she is immediately charged for the first segment. If she remains in the HOT lane till the fifth entrance and chooses to stay in, then she will be charged for the second segment at the rate displayed at the first entrance when she initially entered the HOT lane. If she switches out of the HOT lane at an intermediate entrance, then the cost of using the second segment will be reset at the rate displayed at where and when she chooses enter again.

B Estimation Appendix

This section presents the econometric model of the concave utility function. The probability of consumer $i$ switching to the express lane at entrance $n$ and time $t$ on date $j$ is given by

$$s_{in}(x_{0nj}; p_{nj}; X_{nj}; \xi_{nj}; \theta_{i}) \left[ \frac{\exp \left[ V_{in}^{1} (x_{0nj}; p_{nj}; X_{nj}; \theta_{i}) - V_{in}^{0} (x_{0nj}; p_{nj}; X_{nj}; \theta_{i}) + \xi_{nj} \right]}{1 + \exp \left[ V_{in}^{1} (x_{0nj}; p_{nj}; X_{nj}; \theta_{i}) - V_{in}^{0} (x_{0nj}; p_{nj}; X_{nj}; \theta_{i}) + \xi_{nj} \right]} \right],$$

where the choice specific value function, $V_{in}^{t}$, is presented in equation 12.

Let $s_n(x_{0nj}; p_{nj}; \xi_{j}, \xi_{n})$ be the aggregate demand at entrance $n$ given the aggregate states $x_{0nj}$, $p_{nj}$, and $\xi_{nj}$. It is the share of consumers switching to the express lane from the regular lane at entrance $n$. To obtain aggregate demand from individual demand, it is necessary to
integrate over the heterogeneous taste parameters and the individual state-the accumulated travel time. The aggregate demand be written as

\[ s_n (x_{0ntj}, p_{ntj}, \xi_{ntj}) = \int \int s_{in} (x_{0ntj}, p_{ntj}, X_{intj}, \xi_{ntj}; \theta_i) dF_{in} (X_{in-1} - x_{0n}, p_n, d_{in-1} = 0; \theta_i) d\Psi (\theta_i). \tag{15} \]

where \( \Theta \) is the space of possible parameters. \( dF_{in} (X_{in-1} - x_{0n}, p_n, d_{in-1} = 0; \theta_i) \) is the conditional distribution of accumulated travel time on the current speed and price as well as the fact that consumer \( i \) arrived in the regular lane.

Given a structural parameter \( \theta_i \), the conditional distribution of accumulated travel time can be derived from the distribution of consumers’ initial entrances, the conditional transition probabilities of aggregate states, and the policy functions of consumers with parameter \( \theta_i \). Let \( \phi_n \) be the ratio of the inflow of subscribers at entrance \( n \) to that at entrance 1. Let \( dF_{in}^\hat{n} \) be the conditional distribution of the accumulated travel of consumer \( i \) who entered the highway at entrance \( \hat{n} \). The conditional distribution of accumulated travel time at entrance \( n \) can be written as

\[ dF_{in} (X_{in-1} - x_{0n}, p_n, d_{in-1} = 0; \theta_i) = \frac{\sum_{\hat{n}=1}^{n} dF_{in}^\hat{n} (X_{in-1} - x_{0n}, p_n, d_{in-1} = 0; \theta_i) \phi_{\hat{n}}}{\int_{X_{in-1}} \sum_{\hat{n}=1}^{n} dF_{in}^\hat{n} (X_{in-1} - x_{0n}, p_n, d_{in-1} = 0; \theta_i) \phi_{\hat{n}}}. \tag{16} \]

A special case is for consumers who just arrived on the highway at entrance \( n \). The initial condition defined in the model states that all consumers arrive on the highway in the regular lane with zero accumulated travel time. The conditional distribution of accumulated travel time for consumers who just arrived on the highway at entrance \( n \) can be written as

\[ dF_{in}^n (X_{in-1} - x_{0n}, p_n, d_{in-1} = 0; \theta_i) = \begin{cases} 1 & \text{if } X_{in-1} = 0 \\ 0 & \text{if otherwise} \end{cases}. \tag{17} \]

The function holds for all consumer types \( \theta_i \), all current current prices and speed \( x_{0n}, p_n \).

The conditional distribution of the accumulated travel for those who arrived the highway before entrance \( n \), \( dF_{in}^\hat{n} \), for every entrance \( 1 \leq \hat{n} < n \), can be written as

\[ dF_{in}^\hat{n} (X_{in-1} - x_{0n}, p_n, d_{in-1} = 0; \theta_i) = \int_{x_{0n}, \ldots, x_{0\hat{n}}, p_{n-1}, \ldots, p_n} 1 \left\{ \sum_{\hat{n}=\hat{n}}^{n-1} x_{0\hat{n}} = X_{in-1} \right\} d\hat{G}_n (x_{0n-1}, \ldots, x_{0\hat{n}}, p_{n-1}, \ldots, p_n \times x_{0n}, p_n, d_{in-1} = 0), \tag{18} \]
where the conditional distribution of previous speed and prices \( d\hat{G}_n(x_{0n-1}, \ldots, x_{0n}, p_{n-1}, \ldots, p_{n}, x_{0\hat{n}}, p_{\hat{n}}, d_{in-1} = 0) \) can be obtained using Bayes Theorem and the first-order Markov transition probabilities:

\[
\frac{d\hat{G}_n(x_{0n-1}, \ldots, x_{0\hat{n}}, p_{n-1}, \ldots, p_{\hat{n}}, x_{0n}, p_{n}, d_{in-1} = 0; \theta_i)}{d\hat{H}_n(x_{0n}, p_{0n}, x_{0n-1}, p_{n-1})} = \frac{\left[1 - s_{in-1}(x_{0n-1}, p_{n-1}; \sum_{0i=1}^{n-2} x_{0i}; \theta_i)\right] \hat{G}_{n-1}(x_{0n-1}, p_{n-1}, \ldots, x_{0\hat{n}}, p_{\hat{n}})}{\int x_{0n-1}, p_{n-1}, \ldots, x_{0\hat{n}}, p_{\hat{n}} 1 - s_{in-1}(x_{0n-1}, p_{n-1}; \sum_{0i=1}^{n-2} x_{0i}; \theta_i) \hat{G}_{n-1}(x_{0n-1}, p_{n-1}, \ldots, x_{0\hat{n}}, p_{\hat{n}})
\frac{d\hat{H}_n(x_{0n}, p_{0n}, x_{0n-1}, p_{n-1})}{d\hat{H}_n(x_{0n}, p_{0n})}.
\]

(19)

Recall \( d\hat{H}_n(x_{0n}, p_{0n}, x_{0n-1}, p_{n-1}) \) is the conditional transition probabilities of speed and prices at entrance \( n-1 \). \( d\hat{H}_n(x_{0n}, p_{0n}) \) is the unconditional distribution of speed and prices at entrance \( n \). \( d\hat{G}_{n-1}(x_{0n-1}, p_{n-1}, \ldots, x_{0\hat{n}}, p_{\hat{n}}) \) is the unconditional joint distribution of aggregate states from entrance \( \hat{n} \) to entrance \( n-1 \). Both unconditional distributions can be calculated from the conditional transition probabilities, and the unconditional transition probability of speed and prices at entrance \( 1 \).

Note that \( \left[1 - s_{in-1}(x_{0n-1}, p_{n-1}; \sum_{0i=1}^{n-2} x_{0i}; \theta_i)\right] \hat{G}_{n-1}(x_{0n-1}, p_{n-1}, \ldots, x_{0\hat{n}}, p_{\hat{n}}) \) is the joint probability of consumer \( i \) observing speed and prices \( (x_{0n-1}, p_{n-1}, \ldots, x_{0\hat{n}}, p_{\hat{n}}) \) and staying in the regular lane till entrance \( n \). Consumer \( i \)'s previous decisions affect the distribution of accumulated travel time at entrance \( n \) because the potential entrants are those who still remain in the regular lane. The probability of a high accumulated travel time might increase due to more consumers entering at early entrances. It might also increase due to congestion upstream. However, if congestion upstream is severe, then more consumers will enter the express lane early on. This will reduce the total number of consumers in the regular lanes from upstream entrances, and thus lowering the probability of a high accumulated travel time present in the regular lane at entrance \( n \).

B.1 Identification

The curvature of the utility from travel time is identified if the probability of entry depends on consumer's previous states and decisions. If \( r \) is equal to 0, then the consumers' decisions are independent of accumulated travel time. If \( r \) is greater than 0, then a consumer with a high accumulated travel time is more likely to enter the express lane. The degree to which a consumer's entry decision depends on the accumulated travel time is positively related to \( r \). Conditional on current travel time and prices, \( r \) is identified if the probability of entry depends on the traffic conditions at the previous entrance and at the previous time. Table 14
shows a regression of the probability of entry at entrance 5 on the average regular lane speed at previous entrances in the last 10 minutes. The coefficient of previous traffic conditions is statistically significant. An increase in the average regular lane speed at entrances 3 and 4 in the previous 10 minutes leads to a 0.002% decrease of market share. This is consistent with the model implication, in which the probability of entry increases with the accumulated travel time.

Table 14: Identification of the curvature of the utility function

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>(Std. Err.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>previous regular lane speed</td>
<td>-0.002</td>
<td>(0.000)</td>
</tr>
<tr>
<td>current price</td>
<td>0.195</td>
<td>(0.008)</td>
</tr>
<tr>
<td>current regular lane speed</td>
<td>-0.003</td>
<td>(0.000)</td>
</tr>
<tr>
<td>intercept</td>
<td>0.250</td>
<td>(0.016)</td>
</tr>
</tbody>
</table>

  date fixed effects  yes  
  \[ R^2 \] 0.341  
  observations 5,434