

PARENTAL TIME INVESTMENT AND HUMAN
CAPITAL FORMATION: A QUANTITATIVE
ANALYSIS OF INTERGENERATIONAL MOBILITY

Minchul Yum¹

The Ohio State University

Job Market Paper

January 2015

¹e-mail: yum.7@osu.edu; I am indebted to Julia Thomas and Aubhik Khan for invaluable advice. I would also like to thank David Blau for helpful discussions and suggestions. I am grateful for comments from Laurence Ales, Nick Bloom, Benjamin Bridgman, Andrew Chen, Mariacristina De Nardi, Paul Evans, George-Levi Gayle, Seonghoon Kim, Nobuhiro Kiyotaki, Pok-sang Lam, John Leahy, Tim Lee, Yongseok Shin, Stephen Terry, Joseph Vavra, Venky Venkateswaran, Bruce Weinberg and seminar participants at OSU macro lunch seminars, 9th Economics Graduate Student Conference at Washington University St. Louis, and the 2014 European Winter Meeting of Econometric Society in Madrid.

Abstract

A large literature has documented low intergenerational mobility in the U.S. over the last few decades, prompting a growing interest in understanding mechanisms underlying intergenerational mobility. In this paper, I construct a quantitative general equilibrium model that explores parental time investment in preschool-aged and younger children as a channel through which economic status can be transmitted intergenerationally. Altruistic parents differ in their own human capital and assets, and in the human capital of their children. They each decide how to split their time across investment in their child's human capital, market work, and leisure. My calibrated model reproduces several measures of intergenerational income mobility, as well as the lifecycle inequality seen in U.S. data. Decomposing its results, I find that the parental time investment channel accounts for nearly 50 percent of the observed persistence in intergenerational income. Despite their higher opportunity costs of time, more skilled parents choose to invest more time in their young children. This force significantly amplifies the intergenerational correlation of human capital. However, at the same time, I find that the parental time investment channel actually reduces the cross-sectional dispersion of human capital. This result is driven by dynastic smoothing of the marginal value of human capital; individuals insure their descendants' lifetime utilities through the parental investment channel by investing more time in less able children. Finally, policy experiments suggest that interventions targeted at the college decision have little effect on intergenerational mobility. By contrast, I find that those targeted at parental time investment decisions, such as a proportional subsidy for such investments, may be an effective way to increase intergenerational mobility as well as social welfare, since they disproportionately raise investment in the children from disadvantaged families.

Keywords: parental time investment, human capital, intergenerational mobility, inequality, borrowing constraints, welfare

JEL codes: E24, I24, J22

1 Introduction

A substantial body of empirical research has found that intergenerational mobility is low in the United States (e.g., Solon, 1999).¹ While earlier studies over the last few decades have focused on the statistical facts describing the degree of intergenerational mobility, there has been growing interest in understanding what mechanisms most affect intergenerational mobility.² The answer to this question is essential in guiding decisions on policy interventions designed to influence the persistence of inequality with help of the predictions on the effectiveness and consequential costs of different types of policies.

In this paper, I construct a quantitative general equilibrium model that explores parental time investment in preschool-aged and younger children as a channel through which economic status can be transmitted intergenerationally. The focus on parental time investment is motivated by empirical evidence on parents' time-use documenting that more educated parents spend more time with their children (e.g., Guryan, Hurst and Kearney, 2008; and Ramey and Ramey, 2010 for recent evidence). For instance, focusing on sub-categories such as educational and recreational activities, which have human capital investment aspects, for parents who have a child less than 5 years old, I find that parents with a college degree spend 36 percent more time (7.6 hours per week) than parents without a college degree (5.6 hours per week).³ As the same number of minutes spent by the more educated parents can be even more effective, this positive educational gradient in parental time, interacting with the quality of care, can theoretically amplify the intergenerational correlation of human capital that would arise solely from the exogenous transmission of human capital via mechanisms such as genetic transmission. Furthermore, noting that (i) parental time inputs are crucial in child development especially in early childhood (e.g., Del Boca, Flinn, and Wiswall, 2014), which in turn can have persistent effects on the child's outcomes (e.g., Blau and Currie, 2006; and Knudsen, Heckman, Cameron, and Shonkoff, 2006) and (ii) initial conditions of adult human capital in early 20's can largely account for economic success

¹On one hand, there is a poverty trap: 34 percent of children whose parent's income level belongs to the bottom 20 percent suffer from the same economic disadvantage (bottom quintile) whereas only 8 percent of them can successfully move up to the top quintile income status; on the other hand, just as striking, but less frequently noted, is the intergenerational persistence of the income-rich: 37 percent of children whose parent's income belongs to the top 20 percent keep the same economic privilege (top quintile). These figures are based on the administrative income data from the core sample (roughly 10 million parent-child pairs) in Chetty, Hendren, Kline and Saez (2014a).

²The current empirical studies that identify causal effects typically use (i) a subset of the population that has desirable properties (e.g., twins, adoptees or siblings); (ii) natural experiments; and (iii) instrumental variables. Black and Devereux (2011) provide a thorough literature review on the empirical studies using each empirical strategy.

³This positive relationship holds when I control for the parental gender as well (9.2 vs 6.6 for women; and 5.9 vs 4.5 for men). The statistics are obtained from the 2003-2012 waves of the American Time Use Survey (ATUS). See Appendix for more details.

in later life (e.g., Huggett, Ventura, and Yaron (2011); and Keane and Wolpin (1997)), this parental time investment channel could potentially be an important source that affects intergenerational economic mobility.

The analysis in this paper is conducted through the lens of a dynamic general equilibrium model in which overlapping-generations are linked by altruistic parents who care about their descendants' utility, in the spirit of Becker and Tomes (1986).⁴ Parents differ in their own human capital and assets, and in the human capital of their children. They each decide how to split their time across investment in their child's human capital, market work, and leisure. The productivity of parental time investment depends on the parent's human capital and the child's human capital endowment, as in Cunha and Heckman (2007), both of which drive differences in parental time investment. The next generation's initial adult human capital is thus shaped by their parent's decisions. I also allow young adults to make their own college decision to further accumulate their human capital. The college decision is affected by their human capital, formed early in life, as well as their initial asset holding, endogenously determined by the inter-vivos transfers from the previous generation. In equilibrium, the return on investment in college education is greater for those who have higher human capital, amplifying the effect of early childhood human capital investment. This positive selection in equilibrium arises despite the endogenous inter-vivos transfers acting against it by financing those who would otherwise choose not to go to college.

To assess the model as a quantitative theory of intergenerational mobility, I evaluate the model economy extensively using the various measures of intergenerational mobility. My calibrated model reproduces several measures of intergenerational income mobility such as the intergenerational income elasticity (IGE), the percentile rank correlation and the quintile transition matrix, as seen in U.S. data. The model successfully replicates the observed magnitude of intergenerational mobility, including measures not directly targeted in the calibration. In particular, the model simulation shows that the intergenerational persistence of income is considerably high not only at the bottom quintile but also at the top quintile, as observed in U.S. data. Since the model features risky adult human capital due to idiosyncratic shocks to human capital as well as endogenous labor supply, it is able to produce a hump-shaped age profile of mean earnings (and income) as well as an increasing age profile of dispersion in earnings and income, consistent with U.S. data.

I then use the model economy to investigate the role of the parental time investment channel as a source of intergenerational mobility and cross-sectional disparity. Decomposing the results from my model, I find that the parental time investment channel accounts

⁴Specifically, my model builds upon the heterogeneous-agent incomplete-markets framework, a work-horse model of income and wealth inequality in quantitative macroeconomics. Note that intergenerational mobility is essentially a change in cross-sectional income inequality over generations.

for nearly 50 percent of the observed persistence in intergenerational income. Despite their higher opportunity costs of time, more skilled parents choose to invest more time in their young children due to the higher marginal return to time investment. This force significantly amplifies the intergenerational correlation of human capital that would arise solely from the exogenous transmission of human capital via mechanisms such as genetic transmission.⁵ However, at the same time, I find that the parental time investment channel reduces the cross-sectional dispersion of human capital. This result may appear puzzling since the underlying human capital formation technology has the property that the marginal productivity of parental time investment increases with the child's initial human capital endowment. I show that there is a countervailing force driven by the dynastic motive to smooth the marginal value of human capital; individuals insure their descendants' lifetime utilities through the parental investment channel by investing more time in less able children.

Finally, I use the model economy to evaluate two sets of government policies that can be used to influence intergenerational mobility. The first set of policies is targeted at college decisions while the second set of policies is designed to influence parental time investment behavior. The quantitative experiment results suggest that interventions directly targeted at the college decision have little effect on intergenerational mobility. By contrast, I find that those targeted at parental time investment decisions, such as a proportional subsidy for such investments, may be an effective way to increase intergenerational mobility, as they disproportionately raise investment in the children of less skilled parents.⁶

This paper relates to the few quantitative studies that assess the sources of intergenerational mobility. An early example is Restuccia and Urrutia (2004), who focus on two human capital investment channels: (i) primary and secondary education; and (ii) college education. Their main finding is that the primary and secondary education channel largely accounts for the intergenerational earnings persistence while the college education channel largely explains the disparity in earnings. The current study extends Restuccia and Urrutia's analysis by shifting their early education channel to parental time investment in the human capital of the young children under five. Another key distinction between my

⁵In the model, the exogenous transmission of human capital that determines the initial endowment of human capital at birth may represent many factors, other than pure nature (genetic transmission), such as prenatal investment, culture, preferences, etc. It is important to note that this paper does not attempt to sharply distinguish between nature and nurture.

⁶For example, Cunha and Heckman (2010) note "The best documented market failure in the life cycle of skill formation in contemporary American society is the inability of children to buy their parents or the lifetime resources that parents provide and not the inability of families to secure loans for a child's education when the child is an adolescent." My results suggest that the parental time investment subsidy can partially resolve this market failure by prompting human capital investment in the children whose parents are less-skilled.

model and theirs is that I allow households to accumulate assets. Since lifetime income for many individuals includes not only labor earnings but also capital income, it is essential to include capital accumulation when examining questions involving intergenerational lifetime income mobility.

A recent paper by Lee and Seshadri (2014) also examines a quantitative general equilibrium model of intergenerational mobility. A fundamental difference between my model and theirs involves our differing assumptions on human capital. Lee and Seshadri sharply distinguish between learning ability and human capital. Specifically, their learning ability (nature) is assumed to be exogenously inherited and independent of investment over a lifetime while human capital is formed by investment (nurture). I adopt an approach more in keeping with the skill formation literature (Cunha and Heckman 2007, 2010) in that I do not attempt to distinguish between nature and nurture. Instead, I model heterogeneity in the initial endowment of human capital at birth, which turns out to be a key ingredient underlying the dynastic smoothing role of the parental time investment channel isolated in my results. Another notable difference is my inclusion of idiosyncratic shocks to human capital period-by-period over a working life, in contrast to the once-and-for-all earnings shocks in the Lee and Seshadri's model. This distinction allows my model to achieve greater consistency with the observed age profile of inequality in U.S. data. Moreover, as I will show, mobility within a life is an important component that contributes to observed intergenerational mobility. A third distinguishing feature in my study is incomplete markets. Whereas individuals in the Lee and Seshadri model face only a natural borrowing limit, individuals in my model face a tighter borrowing limit as in Aiyagari (1994). This aspect of my model allows me to study the interaction between financial constraints and intergenerational mobility through human capital investment behavior.

My paper also relates to quantitative studies of the sources of lifetime inequality. A recent quantitative study by Huggett et al. (2011) shows that the differences in initial conditions, especially human capital, at age 23 can account for a large fraction of lifetime inequality. However, because the periods before the age of 23 are not modeled, that paper leaves an open question: what are the prior factors that shape the differences at age 23?⁷ By contrast, in my model where overlapping generations are linked as a dynasty in a general equilibrium framework, the distribution of the initial condition in human capital, determined after parental time investment but prior to labor market entry, is endogenous. One of the main findings in my paper suggests that the parental time investment channel acts as a mechanism that reduces the cross-sectional dispersion of human capital whereas

⁷Using structural estimation, Keane and Wolpin (1997) also find that unobserved endowment heterogeneity at age 16 accounts for 90 percent of the variance in lifetime utility whereas exogenous shocks to skills over the lifetime accounts for the rest.

Table 1: Timeline of the life-cycle events

		Parent's age								
		20-24	25-29	...	45-49	50-54	...	65-69	70-74	
Parent	Model age	1	2	...	6	7	...	10	11	
	Market work	← - - - - Labor-leisure				- - - - →			Retired	- →
	Human capital investment	College	Child care							
Child	Model age	Birth		...	1	2	...	5	6	
	Market work					← - - - - Labor-leisure			- - - - →	
	Human capital investment					College	Child care			

the college channel amplifies cross-sectional inequality. Therefore, this paper contributes to the literature by providing evidence on the sources that may have different effects on the cross-sectional heterogeneity early in life not only quantitatively but also qualitatively.

The paper is organized as follows. Section 2 describes the model. Section 3 explains how the parameters of the baseline model economy are determined, and Section 4 evaluates the properties of the baseline model economy. Section 5 presents the main quantitative results on the role of the parental time investment channel, and Section 6 provides the household-level analysis on human capital investment to help understand the mechanism of the model. Section 7 presents computational experiments on policy related to intergenerational mobility. Section 8 concludes.

2 Model environment

2.1 General description

The economy is populated by overlapping generations of a continuum of households. A household is composed of an adult who lives with a child until the child grows up. An adult lives for eleven model periods (age 20-74) as an economic decision maker. One model period corresponds to five years. In Table 1, I summarize the timeline of the lifecycle events for a pair of overlapping generations for illustration. An adult supplies labor beginning at period $j = 1$ (age 20) until retirement at $j = 10$ (age 65). An adult lives for two periods after retirement and dies at the end of period $j = 11$. The next generation is born when parents reach period $j = 2$. After 20 years, a child becomes an adult head of a new household facing the same lifetime structure as described above.

Households may differ in their human capital and their asset holding.⁸ In the model, human capital can be endogenously formed in two ways: (i) by parent’s time investment during early childhood; and (ii) at age 20 when a child becomes an independent adult, she decides whether to go to college or not.⁹ While an individual works, her human capital is subject to idiosyncratic shocks, as in Huggett et al. (2011). The capital market is incomplete and the only available asset is physical capital on which the rate of return is r . Thus the idiosyncratic shocks over the life cycle cannot be fully insured. The household’s recursive problems over the life cycle are described in detail in the next subsection.

There is a representative firm which produces output with constant returns to scale technology. Its production function is assumed to be Cobb-Douglas $Y = F(K, L) = AK^{\alpha_K} L^{1-\alpha_K}$ where K denotes aggregate capital, α_K is the output elasticity of capital and L denotes aggregate efficiency units of labor in the economy. Capital depreciates at the exogenous rate of δ .

There is a government that taxes labor earnings at a fixed rate of τ_s to provide social security payments ω to retirees. Government is assumed to balance its budget each period.

This paper focuses on a stationary environment in which all prices and aggregate quantities are constant over time. Therefore, the time index for the variables is omitted and I write the household’s problem recursively.

2.2 Household’s recursive problems

I assume that all households have identical preferences over consumption c and leisure l , which is represented by a standard separable utility function

$$U(c, l) = \frac{c^{1-\sigma}}{1-\sigma} + B \frac{l^{1-\varepsilon}}{1-\varepsilon}. \quad (1)$$

College education stage:

A child becomes an independent economic decision maker in the model period $j = 1$ (20 years old) with a human capital stock of θ . As will become clear below, this level of human capital, formed during the early childhood, is influenced by the parent. An important decision to be made at $j = 1$ is whether to go to college or not. College education is lumpy and it requires both the random physical cost ξ , which is assumed to follow a log

⁸I use the term, human capital (or skills), interchangeably with ability, following Cunha and Heckman (2007, 2010) who argue that the traditional dichotomy between ability and skills is misleading since they both are affected by genetic, environmental factors and investment.

⁹As the focus of this paper is the parental time investment in young children under five, I abstract from the primary and secondary education channel, of which quantitative implications have been studied by e.g., Fernández and Rogerson (1998), and Restuccia and Urrutia (2004).

normal distribution $\log \xi \sim N(m_\xi, \sigma_\xi^2)$, and the fixed time input ψ .¹⁰ After observing the random fixed cost draw ξ , households make a discrete choice regarding college education. The household's value at $j = 1$ before the realization of ξ is given by

$$V_{j=1}(\theta, a) = E_\xi \max [\Gamma_0(\theta, a), \Gamma_1(\theta, a, \xi)]. \quad (2)$$

where $\Gamma_0(\theta, a)$ is the value without a college degree, and $\Gamma_1(\theta, a, \xi)$ is the value of completing the college education.

The household's value if the agent chooses not to go to college is given by

$$\Gamma_0(\theta, a) = \max_{\substack{c \geq 0; a' \geq a_1 \\ n, l \in [0, 1]}} \left\{ U(c, l) + \beta \int V_{j=2}(\theta', a') dG(z') \right\} \quad (3)$$

$$\begin{aligned} \text{subject to } c + a' &\leq (1 - \tau_s)w\theta n + (1 + r)a \\ n + l &\leq 1 \\ \theta' &= \exp(z')\gamma\theta \end{aligned}$$

where w is the rental price of human capital per unit hours of work (or market wage), a is the level of assets determined by the previous generation's inter-vivos transfer decision, and a variable with a prime denotes its value in the next period. The household's after-tax earnings depend on the individual-specific wage $w\theta$, hours of work n , and the social-security tax rate τ_s . The level of human capital exogenously increases at the gross growth rate of $\gamma > 1$ and is subject to the idiosyncratic shock (or market luck) z of which the cumulative distribution is $G(z)$. As in Huggett et al. (2011), I assume that the z follows an i.i.d. normal distribution. Note that although z is drawn from an i.i.d. distribution, its effect is persistent over the rest of the life since z is not a shock to earnings but rather a shock to human capital, which essentially follows a random walk with an age-dependent deterministic drift in logs.¹¹

Next, the value of completing the college education after the realization of a fixed cost ξ is given by

¹⁰In this model, the college education provides a four-year college degree, and the dropout decision is not modeled. Hence, the fixed costs (both resources and time) are incurred for four years.

¹¹For example, taking the log of the law of motion for the human capital in (2), we get

$$\log \theta' = \log \gamma + \log \theta + z'.$$

where a drift term, $\log \gamma$, acts as a deterministic trend.

$$\Gamma_1(\theta, a, \xi) = \max_{\substack{c \geq 0; a' \geq \underline{a}_1 \\ n, l \in [0, 1]}} \left\{ U(c, l) + \beta \int V_{j=2}(\theta', a') dG(z') \right\} \quad (4)$$

$$\begin{aligned} \text{subject to } c + a' + \xi &\leq (1 - \tau_s)w\theta n + (1 + r)a \\ n + \psi + l &\leq 1 \\ \theta' &= \exp(z')(\gamma + \Delta)\theta. \end{aligned}$$

In my model, the benefit of the college education is represented by $\Delta > 0$, an increment in the growth rate of human capital. An advantage of this way of modeling the benefit is that Δ can be easily mapped to the college wage premium in the data.¹² In Section 6, I show that the equilibrium college decision rule features self-selection; that is, an individual who is better prepared, measured by human capital at $j = 1$, θ , is more likely to choose college education. The borrowing limit, faced by an agent whose age is j , is denoted by \underline{a}_j . As in Aiyagari (1994), this limit is defined as the maximum of the economy-wide borrowing limit \underline{b} and the natural debt limit, which guarantees repayment over the lifecycle event histories with probability one. Note that an individual can still work while attending college since he can work part-time. Another possible interpretation of the hours of work in this period would be full-time work for a year within $j = 1$ since a model period of five years is longer than four years in college. As described above, the opportunity cost of college education includes foregone earnings due to the time spent in college ψ .

Parental investment stage:

At the beginning of period $j = 2$, each household is endowed with a child. I assume that the child shares the household consumption c and does not make time allocation decisions relevant to the household's economic status during childhood. The child's human capital endowment at birth $\theta_c = \phi(\theta, \zeta)$ depends on both the parental ability θ and the idiosyncratic component ζ , drawn from a log normal distribution: $\log \zeta \sim N(m_\zeta, \sigma_\zeta^2)$. The household's state variables also include a , the level of asset holding determined in the previous period. The value of the household at period $j = 2$ before the realization of ζ is given by

$$V_{j=2}(\theta, a) = E_\zeta W(\theta, a, \zeta) \quad (5)$$

The functional equation summarizing a parent's decision problem after observing the child's ability $\theta_c = \phi(\theta, \zeta)$ is given by

¹²Holding other things constant, the college wage premium in the data corresponds to $\frac{wage_{col}}{wage_{no\ col}} - 1 = \frac{(\gamma + \Delta)}{\gamma} - 1 = \frac{\Delta}{\gamma}$ in the model.

$$W(\theta, a, \zeta) = \max_{\substack{c \geq 0; a' \geq a_2 \\ n, l, h \in [0, 1] \\ i \in \{0, 1\}}} \left\{ U\left(\frac{c}{q}, l\right) + \beta \int V_{j=3}(\theta', a', i) dG(z') + \eta \beta^4 V_{j=1}(\theta'_c, a'_c) \right\} \quad (6)$$

$$\begin{aligned} \text{subject to } c + a' + s \mathbf{1}_{i=1} &\leq (1 - \tau_s) w \theta n + (1 + r) a \\ n + l + h &\leq 1 \\ \theta' &= \exp(z') \gamma \theta \\ \theta'_c &= f(\theta, h, \phi(\theta, \zeta)). \\ a'_c &= \mathbf{1}_{i=1} a_0 \end{aligned}$$

where q denotes the household equivalence scale, $\eta \geq 0$ measures the degree of altruism, s is the fixed amount to be saved while living with a child for the future inter-vivos transfers, and $\mathbf{1}_{i=1}$ is the indicator function. Note that, with a positive η , parents invest their time in their child and have an incentive to make the inter-vivos transfer since they care about the value of their child's life when she becomes an adult in 20 years, which is represented by the last term of the objective function: $\eta \beta^4 V_{j=1}(\theta'_c, a'_c)$. Optimal parental time investment h is determined based on equating the marginal value of time in investment, leisure, and market work. To keep the model tractable, I assume that inter-vivos transfers (the transfer of wealth while a person is still alive) is a discrete choice. Thus the amount of the transfer is assumed to be fixed exogenously. The state variables at periods $j = 3, 4, 5$ include a binary indicator variable $i \in \{0, 1\}$ where 1 indicates that the household saves the fixed amount for the inter-vivos transfer that will be actually delivered at the beginning of $j = 6$ when the child becomes the head of a new household.

The intergenerational link is modeled following the dynastic utility approach in the sense that parents care about their child's utility, which depends on his own consumption and leisure and his child's utility, and so on. This recursive structure linked by altruism combines successive generations as a single dynasty as in Becker and Tomes (1986). However, altruism in my model differs from the basic Becker-Tomes altruism in two ways. First, the degree of altruism η is defined separately from the discount factor β as in Cunha and Heckman (2007). This is due to the fact that I consider a more detailed lifecycle framework while, in Becker and Tomes (1986), each generation's utility is aggregated so that the distinction between the discount factor and the degree of altruism is unnecessary. Second, in this paper, what parents care about is the child's utility derived not only from consumption but also from leisure since the utility function is defined over both consumption and leisure.

Following Cunha and Heckman (2007), I adopt the production function $\theta'_c = f(\theta, h, \theta_c)$, which describes how the child's developed ability θ'_c is formed depending on parental ability θ , parental time investment h , and the child's initial ability at birth θ_c .¹³ The intergenerational human capital production function $f(\theta, h, \theta_c)$ is assumed to have the following properties: (i) $f_1, f_2, f_3 > 0$; (ii) $f_{22} < 0$, $f_{21} > 0$ and $f_{23} > 0$. Two notable properties are the last two; the marginal return on parental time investment increases with parent's ability ($f_{21} > 0$) and with child's ability ($f_{23} > 0$). The last property is also called dynamic complementarity in the skill formation literature (Cunha and Heckman, 2007).

Before moving on to the description of typical working stages, there are two notable simplifications that are worth discussing regarding intergenerational transmission mechanisms in my model. First, although I model the inter-vivos transfer decision endogenously, I do not model the decision on bequests. Abstracting from bequests actually provides a better mapping from the data to the model because most empirical studies on intergenerational income (earnings) mobility including the empirical benchmark of the current paper (Chetty et al., 2014a), use matched samples in which the children are relatively young (around or above 30 years old), who are unlikely to be affected by bequests. Second, I abstract from parental monetary investment. This not only keeps the model relatively tractable and focused but is also in line with recent empirical studies. For instance, Del Boca, Flinn, and Wiswall (2014) show that parental time inputs are more important than parental expenditures, especially in early years. Also, parental income, a key determinant of the parental monetary investment, is often found to be not as strongly related to children's outcomes as the family background characteristics such as parental education (Blau, 1999; Sacerdote, 2007).

Remaining working stages:

Households keep making a typical work-leisure decision and consumption-saving decision for periods $j = 3, \dots, 9$ (age 30-64) until they are retired at $j = 10$. The household is composed of a parent and a child until the end of $j = 5$ when the child forms a new household. Recall that a parent who committed to leave the inter-vivos transfers at $j = 2$ ($i = 1$) is assumed to save the constant amount of s while living with the child. The total amount of savings with the interest is then transferred to the child when the child becomes independent (i.e., $a = s [\sum_{t=0}^3 (1+r)^t]$). Once the child becomes an adult, the parent's decision to leave the inter-vivos transfers does not affect their choice. Hence, from period $j = 6$, the state variables do not include i . The household's problem in these periods can be described recursively as

¹³See Aiyagari, Greenwood and Seshadri (2002) for a similar formulation of the human capital production technology.

$$V_j(\theta, a, i) = \max_{\substack{c \geq 0; a' \geq a_j \\ n, l \in [0, 1]}} \left\{ U\left(\frac{c}{q}, l\right) + \beta \int V_{j+1}(\theta', a', i) dG(z') \right\} \quad \text{if } j = 3, 4, 5 \quad (7)$$

$$\begin{aligned} \text{subject to } c + a' + s\mathbf{1}_{i=1} &\leq (1 - \tau_s)w\theta n + (1 + r)a \\ n + l &\leq 1 \\ \theta' &= \exp(z')\gamma\theta \\ V_{j=6}(\theta, a, i) &= V_{j=6}(\theta, a) \quad \text{for } i \in \{0, 1\} \end{aligned}$$

and

$$V_j(\theta, a) = \max_{\substack{c \geq 0; a' \geq a_j \\ n, l \in [0, 1]}} \left\{ U(c, l) + \beta \int V_{j+1}(\theta', a') dG(z') \right\} \quad \text{if } j = 6, 7, 8, 9 \quad (8)$$

$$\begin{aligned} \text{subject to } c + a' &\leq (1 - \tau_s)w\theta n + (1 + r)a \\ n + l &\leq 1 \\ \theta' &= \exp(z')\theta \end{aligned}$$

Note that, although agents are not allowed to accumulate human capital endogenously after period $j = 1$, their human capital accumulates exogenously at the gross growth rate of $\gamma > 1$ until $j = 5$. Thereafter, the growth rate stays constant at one. This structure parsimoniously generates the shape of the empirical age-profile of wage that rises initially and stays flat near the retirement (see e.g., Rupert and Zanella, 2012; Casanova, 2013 for recent evidence). As will be shown later, the hump-shaped earnings profile observed in the data does not need to rely on the hump-shaped wage profile since hours of work, endogenously determined by households, fall near the retirement, as observed in U.S. data.

Retirement stage:

When households retire ($j = 10, 11$), they receive a constant amount of social security payments $\omega(\theta)$ each period. The amount depends on their human capital θ , which stays constant after retirement. The value at the retirement stage is given by

$$V_j(\theta, a) = \max_{c \geq 0; a' \geq a_j} \{U(c, 1) + \beta V_{j+1}(\theta, a')\} \quad (9)$$

$$\text{subject to } c + a' \leq \omega(\theta) + (1 + r)a$$

and $V_{j=12}(\theta, a) = 0$. Households are not allowed to leave debts in the final period: $a' \geq \underline{a}_{11} = 0$.

2.3 Equilibrium

Let $x_j \in X_j$ denote the age-specific state space defined according to the household's recursive problems in the previous subsection. A stationary recursive competitive equilibrium is a collection of factor prices w, r , the household's decision rules $a_{j+1}(x_j), n_j(x_j), l_j(x_j), h(x_2), i(x_2)$, value functions $V_j(x_j)$, government transfers $\omega(\theta)$, the social security tax rate τ_s , and age-specific measures π_j over x_j such that

1. given factor prices, $a_{j+1}(x_j), n_j(x_j), l_j(x_j), h(x_2), i(x_2)$ solve the household's optimization problems defined in the previous subsection, and $V(x_j, j)$ are the associated value functions,
2. factor prices are competitively determined:

$$\begin{aligned} w &= F_2(K, L) \\ r &= F_1(K, L) - \delta, \end{aligned}$$

3. markets clear:

$$\sum_{j=1}^{11} \mu_j \int_{X_j} a_{j+1}(x_j) d\pi_j + \sum_{j=2}^5 \mu_2 \int_{X_2} i(x_2) s d\pi_2 = K \quad (10)$$

$$\sum_{j=1}^{11} \mu_j \int_{X_j} \theta n_j(x_j) d\pi_j = L \quad (11)$$

where μ_j is the fraction of households living in period j ,

4. social security budget balances:

$$G + \sum_{j=10}^{11} \mu_j \int_{X_j} \omega(\theta) d\pi_j = \sum_{j=1}^9 \mu_j \int_{X_j} \tau_s w \theta n_j(x_j) d\pi_j \quad (12)$$

5. the vector of age-specific measures of households $\pi = (\pi_1, \pi_2, \dots, \pi_{11})$ is the fixed point of $\pi(X) = P(X, \pi)$ where $P(X, \cdot)$ is a transition function determined by the household decision rules and the exogenous probability distributions of z, ζ and ξ ; and X is the generic subset of the Borel σ -algebra B , defined over the state space $\mathbf{X} = \prod_{j=1}^{11} X_j$.

3 Parameterization

I calibrate parameter values of the baseline model economy to match some relevant statistics obtained from U.S. data in recent periods after 1990. There are two sets of parameters. The first set of parameters is chosen externally without using model-generated data while the second set of parameters is determined jointly by minimizing the distance between the statistics from the model and from the data.

The externally chosen parameters are summarized in Table 2. The first three parameters, β , σ and ε , govern the household's preference. The value of the discount factor is set to $\beta = 0.90$. This parameter largely affects the capital-output ratio and the equilibrium interest rate in the economy. Later I show that the equilibrium capital-output ratio and the interest rate, along with the other calibrated parameters, are close to the standard values in the literature. I set the value of σ equal to 1.5 so that the intertemporal elasticity of substitution for consumption is 0.67 and the value of ε equal to 3.0, which implies an intertemporal elasticity of substitution for leisure of 0.33. The implied Frisch elasticity of labor supply is roughly twice as large as this value at the steady state hours of work. These parameter values lie within a broad range of their empirical estimates. I set q to 1.3 according to the OECD-modified equivalence scale which assigns 1 to the first adult and 0.3 to a child.

The gross growth rate of human capital in young periods ($j = 1, \dots, 5$) is calibrated to match the average wage growth over the lifetime. The choice of $\gamma = 1.06$ implies that the peak average wage is 54 percent higher than the average wage at the beginning of the life when lifetime idiosyncratic shocks are set to mean zero. This slope is roughly consistent with the recent empirical evidence in Rupert and Zanella (2012).

The two parameters regarding college education, Δ and σ_ξ , have related empirical counterparts. I set $\Delta = 0.583$ so that the college wage premium is 55%, which is in line with the college wage premium estimates since 1990 as documented in Autor, Katz and Kearney (2008) and Hubbard (2011). The standard deviation of the college costs σ_ξ is chosen to be 0.41 to yield the ratio of the third quartile to the first quartile of the college costs equal to 1.6, consistent with its value in 2000-2001 academic year according to the Digest of Education Statistics.

The social security labor income tax rate τ_s is set to 0.0854, which is the average value of social security contributions relative to the aggregate labor income for the 1990-2010 period (Social Security Bulletin, Annual Statistical Supplement, 2013). The model incorporates inter-vivos transfers along the extensive margin. Hence, $a_0 = s \left[\sum_{t=1}^4 (1+r)^t \right]$ is set to be 2.4 percent of five-year output per capita to match the ratio of average inter-vivos transfers to annual GDP per capita of 0.12 according to the estimates from the nationally

Table 2: Parameters chosen externally

	Values	Note or target
β	0.90	Discount factor
σ	1.5	IES for consumption = 0.67
ε	3.0	IES for leisure = 0.33
q	1.3	OECD-modified equivalence scale
γ	1.06	$\bar{w}_{peak}/\bar{w}_{j=1} = 1.54$
Δ	0.583	College wage premium = 0.55
σ_ξ	0.41	3rd quartile/1st quartile of college costs = 1.6
τ_s	0.0854	Social security contributions to labor income
a_0	0.024Y	Average inter-vivos transfers
α_K	0.33	Capital share
δ	0.3	Five-year capital depreciation rate
A	5	Output unit
m_ζ	$\exp(0.2)$	Human capital unit

representative samples in the Health and Retirement Study (McGarry 1999).

The capital share in the aggregate U.S. data leads to the choice of $\alpha_K = 0.33$. The five-year capital depreciation rate is set to $\delta = 0.3$. These parameter choices are consistent with the equilibrium business cycle literature. I set the two parameters to $A = 5$ and $m_\zeta = \exp(0.2)$ where m_ζ is the mean of the idiosyncratic component of ability at birth ζ which is defined in the next paragraph. These parameters determine the unit of output and human capital and the rest of the parameters are calibrated accordingly. The maximum economy-wide borrowing limit \underline{b} is set to zero in the baseline specification.

Table 3 summarizes the remaining parameters that are jointly determined by simulating the model economy. These parameter values are estimated by minimizing the distance between the relevant statistics from the data and those from the model-generated data.¹⁴ The first two are preference parameters. B is the parameter which determines the relative weight of leisure compared to consumption and η captures the degree of altruism. The intergenerational human capital production function is assumed to be $f(\theta, h, \theta_c) = \theta_c + (\theta h)^{\alpha_1} \theta_c^{\alpha_2}$, which satisfies the assumptions discussed in the previous section. This adds the two parameters (α_1 and α_2) which govern curvatures with respect to each input. The newborn's ability θ_c is assumed to be the weighted average of their parent's ability θ and the idiosyncratic component ζ in logs: $\log \theta_c = \alpha_0 \log \theta + (1 - \alpha_0) \log \zeta$. Therefore, this setting adds a single weighting parameter α_0 . The idiosyncratic component ζ is assumed to follow a log normal distribution. The standard deviation σ_ζ is determined internally

¹⁴See Appendix D for the details.

Table 3: Parameters chosen internally

Category	Parameter values	Description
Preferences	$B = 0.734$ $\eta = 0.246$	$U(c, l) = \frac{c^{1-\sigma}}{1-\sigma} + B \frac{l^{1-\varepsilon}}{1-\varepsilon}$ Degree of altruism
Human capital formation technology	(α_1, α_2) $= (0.472, 0.387)$	$f(\theta, h, \theta_c)$ $= \theta_c + (\theta h)^{\alpha_1} \theta_c^{\alpha_2}$
Initial human capital at birth	$\alpha_0 = 0.175$	$\phi(\theta, \zeta) = \theta^{\alpha_0} \zeta^{1-\alpha_0}$
Idiosyncratic component of ability at birth	$\sigma_\zeta = 0.616$	$\log(\zeta) \sim N(m_\zeta, \sigma_\zeta^2)$
Idiosyncratic shocks to human capital	$\sigma_z = 0.171$	$z \sim N(0, \sigma_z^2)$
College net tuition and fees	$m_\xi = 0.201Y$	$\log(\xi) \sim N(m_\xi, \sigma_\xi^2)$
College time cost	$\psi = 0.346$	
Social security payment	$\omega_s = 0.093$	$\omega(\theta) = \omega_s \theta$

while the mean m_ζ is set externally in Table 2. The lifetime idiosyncratic shocks to human capital z , following a normal distribution, have mean zero with the standard deviation of σ_z . Finally, there are three parameters regarding college costs. The resource costs are assumed to follow a log normal distribution, thereby adding two parameters (i.e., m_ξ and σ_ξ^2). The final parameter is the time cost ψ spent in college.

Table 4 shows the relevant statistics both in U.S. data and in the model-generated data, which are used to obtain the estimates of these ten parameters reported in Table 3. The target statistics regarding time-use are obtained from the 2003-2012 waves of the American Time Use Survey (ATUS).¹⁵ The first target is the average weekly hours of work: $41.3/112 = 0.37$. Note that I assume that the weekly feasible time endowment is 16×7 hours, excluding time for sleeping and basic personal care. The degree of altruism largely governs the unconditional mean of the parental time investment and the fraction leaving inter-vivos transfers. Thus, two additional target statistics are the average of the parental time investment in U.S. data, which is 6.3 hours per week or 0.056 ($= 6.3/112$) in the model, and the fraction of parents making inter-vivos transfers in US data, which is 0.29 (McGarry, 1999). The coefficient parameters α_1, α_2 of the intergenerational human capital production function largely affect the degree of heterogeneity in parental time, affecting the gap between the parental time spent by college graduates and the one by non-college graduates.¹⁶ Thus, I include this ratio of the conditional mean of parental time for parents with a college degree to that for parents without a college degree in the data, which turns out to be 1.36, as another target statistic. I choose the four-year college graduation rate of

¹⁵See Appendix E for details.

¹⁶ α_2 also affects the average parental time investment quite significantly.

31 percent as another target statistic for the parameter of the fixed time cost ψ for college.

For the parameter that governs the mean of the college cost distribution, m_ξ , I use the data from the Digest of Education Statistics. The average of the ratio of annual college tuition and required fees (excluding room and board) for four-year institutions to the per capita real GDP for the recent periods 1990-2011 is 0.22, according to the Digest of Education Statistics (2011, Table 349) and the Bureau of Economic Analysis. In order to approximate actual costs faced by students, I also include the non-tuition expenses such as books, other supplies, commuting costs, and room and board expenses that would not have to be paid by a person who chooses not to go to college as in Abbott, Gallipoli, Meghir and Violante (2013). These non-tuition expenses amount to approximately 30 percent of the average tuition and fees. In 2000-2001, the average grants (federal, state/local, and institutional) received by full-time students in four-year colleges weighted by numbers enrolled was approximately 50 percent of the average tuition and fees. Based on the above information, the target statistic for the mean of the college fixed cost distribution m_ξ in the model is set to be the equilibrium ratio of average (tuition and non-tuition) expenses after financial aid to per capita GDP. This ratio equals 0.18, which is 0.14 in a model period of five years.

The cross-sectional variance of log wage is chosen as a statistic regarding variability of idiosyncratic shocks to human capital over the lifetime σ_z and that of the idiosyncratic component of ability at birth σ_ζ . Although the degree of wage inequality in the model is monotonically increasing in both σ_z and σ_ζ , their economic mechanism is very different. This is because σ_z affects households over the working life while σ_ζ affects the variability of the initial condition in human capital. For instance, in the case when σ_z is relatively larger, households would experience more volatile idiosyncratic shocks to human capital, the effect of which accumulates over the life cycle. As a result, the lifecycle profile of wage inequality would become steeper. Therefore, it is important to introduce a target which can pin down the relative contribution of each shock process to the overall wage inequality. For this reason, I choose the difference between the variance of log wage at age 50-54 and that of log wage at age 25-29 as an additional target. These statistics on wage inequality in U.S. data for recent periods, obtained from Heathcote et al. (2010), are reported in Table 4.

The weight α_0 largely affects the degree of association across generations. The relevant target for α_0 is thus chosen as the IGE of family income. As discussed in detail in the next section, the U.S. data benchmark in this paper is from Chetty et al. (2014a). Their baseline IGE estimate is 0.344. As a target, I use this statistic which is computed using the proxy income variable equivalently defined in the model to be consistent with the empirical

Table 4: Target statistics from U.S. data and the model-generated data

Target statistics	Data	Model
Average hours worked n	0.37	0.37
Average parental time investment h	0.056	0.056
Educational gradient in h ($\bar{h}_{col}/\bar{h}_{less}$)	1.36	1.36
Fraction with a college degree	0.31	0.31
Average college expenses/per-capita GDP	0.14	0.14
Variance of log wage	0.40	0.41
Slope of variance of log wage	0.18	0.18
Fraction leaving inter-vivos transfers	0.29	0.29
Intergenerational income elasticity	0.34	0.35
Average social security replacement rate	0.40	0.40

counterpart.¹⁷ Finally, the social security payments are connected to the human capital level at the moment of retirement in a simple way: $\omega(\theta) = \omega_s \theta$. A target statistics for ω_s is set as the average replacement rate of 40 percent.

The equilibrium interest rate under the set of calibrated parameters reported in Table 2 and Table 3 turns out to be 0.222, which translates to the annual interest rate of 4.09 percent. The capital to output ratio in equilibrium is 0.632 (or 3.16 if annualized) which is broadly consistent with U.S. data (Bureau of Economic Analysis and Bureau of Labor Statistics) when the capital is measured by the sum of fixed private capital, durables, inventories, and land.

4 Evaluating the model economy

4.1 Intergenerational mobility

Prior to the quantitative exercises, it is necessary to evaluate the performance of the baseline model as a quantitative theory of intergenerational mobility by looking at the statistics that measure the degree of intergenerational mobility from the model-generated data. To compute statistics for the model, I solve the model economy and simulate it to generate 50,000 parent-child pairs, which serve as the model-generated data. The intergenerational mobility estimates reported below are based on family income in order to be consistent with the U.S. data counterparts from Chetty et al. (2014a).¹⁸

¹⁷See the next section for the precise definition of the proxy income.

¹⁸Their family income is the five-year per parent average of the pre-tax income defined as either the sum of Adjusted Gross Income, tax-exempt interest income and the non-taxable portion of Social Security and Disability benefits (if a tax return is filed) or the sum of wage earnings, unemployment benefits, and

I consider three measures of intergenerational mobility. The first is the IGE, a conventional way to measure the degree of intergenerational mobility in the literature. The IGE is the slope coefficient obtained by running the following log-log regression equation:

$$\log y_{child} = \rho_0 + \rho_1 \log y_{parent} + \varepsilon \quad (13)$$

where y is the permanent income. The IGE provides a straightforward interpretation: a one percent increase in parental permanent income is associated with a ρ_1 percent increase in their children’s permanent income. Thus, a high ρ_1 implies low intergenerational mobility. In the literature estimating intergenerational mobility, the biggest challenge is the data requirement: we need a data set which contains career-long earnings histories (or permanent income) for at least two successive generations. In practice, researchers replace permanent income with proxy income measured at a point in the life cycle. Hence I present the statistics from the model by using both lifetime income and the proxy income which is defined similarly to Chetty et al. (2014a).¹⁹

The second way to measure intergenerational mobility is to use a rank-rank specification instead of a log-log specification, as proposed by Chetty et al. (2014a) and Chetty, Hendren, Kline, Saez, and Turner (2014b). In other words, I estimate the slope parameter after replacing log income with the percentile rank of income within one’s own generation in (13). The slope coefficient in a rank-rank specification (or the rank correlation) has a similar interpretation: a one percentage point increase in parent’s percentile rank is associated with a ρ_1 percentage point increase in their children’s percentile rank.²⁰ Unlike the IGE, the rank correlation is known to be less sensitive to the treatment of zero income observations and is relatively robust to the point of measurement in the income distribution (Chetty et al. 2014a, 2014b).

The third measure I consider is the quintile transition matrix. The (a, b) element of the matrix gives the conditional probability that a child’s lifetime income is in the a -th quintile of his generation’s distribution, given that his parent’s income is in the b -th quintile of her own generation’s distribution. This provides a richer description of how economic status is

gross social security and disability benefits (otherwise). In the model, family income is the five-year per parent sum of labor earnings, interest income, and social security benefits. Family income is the preferred variable for the studies of intergenerational mobility including both sons and daughters (Lee and Solon, 2009), which applies to the gender-neutral model in this paper.

¹⁹The model proxy income is chosen as follows: The age at which the parent’s income is measured is 40-44 ($j = 5$), and the age at which the child’s income is measured is 30-34 ($j = 3$). In Chetty et al. (2014a), the child’s income is measured by income when children are around 30 years old, averaged over two years. The parent’s income is averaged over five years when parents are, on average, 41-45 years old.

²⁰Note that the rank-rank slope estimate is simply equal to the correlation coefficient in percentile rank since the independent and dependent variables, both of which are normalized by transforming the income level to the percentile ranks, have the same variance.

Table 5: Intergenerational mobility estimates

	Baseline model		U.S. data
	lifetime income	proxy income	Chetty et al (2014a)
IGE: log-log slope	0.365	0.346	0.344
Rank corr: rank-rank slope	0.356	0.366	0.341

Notes: 50,000 pairs of a parent and a child are simulated for the model-generated data. The log-log slope estimate is obtained from a univariate regression equation where the dependent variable is the child’s log income and the independent variable is the parent’s log income. The rank-rank slope estimate is obtained from an equivalent regression equation replacing log transformation with the percentile rank.

transmitted across generations than do the first two measures.

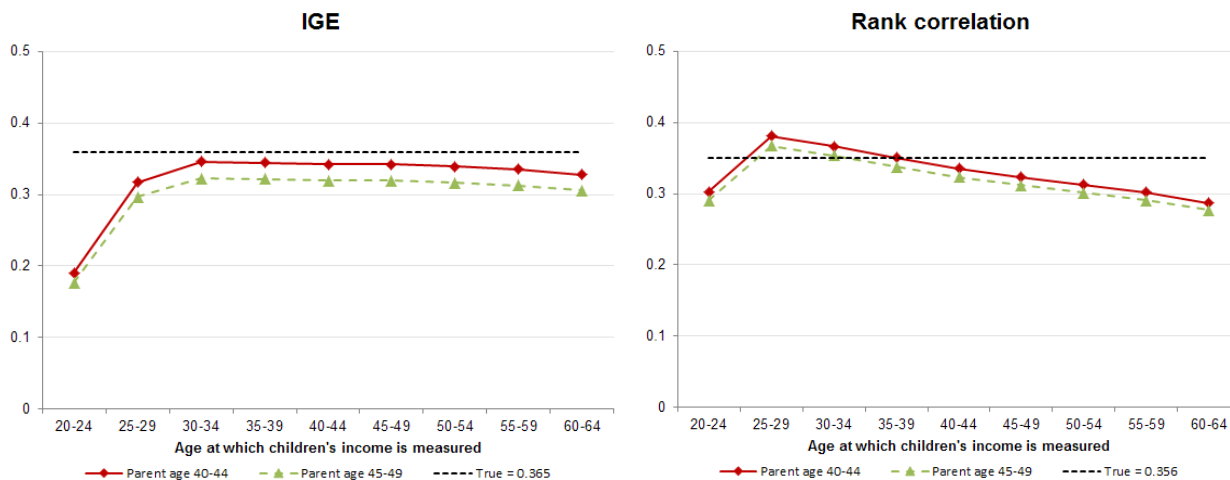
Table 5 reports the first two measures (i.e., slope estimates) from the model and the data. The first column uses the lifetime income. The estimate of the log-log slope (IGE) for lifetime income is 0.365, and the estimate of the rank-rank slope for lifetime income is 0.356. If the proxy income is used, the log-log slope estimate becomes 0.346 and the rank-rank slope estimate is 0.366. Recall that the IGE using the proxy income is the only statistic that has been used as a target in the calibration process. Note that the differences between the slope estimates using the lifetime income and the proxy income are not sizeable; that is, the intergenerational mobility estimates (in particular, the percentile rank correlation) using the five-year proxy income provide accurate approximation. This is in contrast to some empirical studies that note that the short-term income (even multi-year averages) may not represent the permanent income because of persistent transitory shocks.²¹

It is important to note that the degree of approximation using the proxy income depends on the point at which income is measured, as can be seen in Figure 1. In this figure, I plot the estimates of the IGE (left panel) and the rank correlation (right panel) by varying the age at which children’s income is measured while holding constant the age at which parents’ income is measured at 40-44 (red solid) or at 45-49 (green dashed). In line with the literature (Solon, 1999; Haider and Solon, 2006), I find that there is serious attenuation

²¹For instance, Mazumder (2005) shows that the IGE estimate could be as high as 0.6 when fifteen-year averages are used, compared to his IGE estimate of 0.388 when four-year averages are used. In contrast, Chetty et al. (2014a) find much less attenuation bias with five-year averages from their data, noting the possibility that Mazumder’s results are due to his imputation of parent’s top-coded income (which accounts for roughly 20 to 60 percent of parents in his sample). The IGE estimates in this paper show much less attenuation bias, which is in line with Chetty et al. (2014a).

Another issue, which I do not consider in this paper, is the classical measurement error leading to attenuation bias when parents’ income is measured with error. Solon (1992) suggests using the multi-year averages to mitigate the errors-in-variables bias. This widely known issue is not considered because the model-generated data are accurately measured without measurement error.

Figure 1: Lifecycle bias by age of child



Notes: In both panels, I vary the age at which children’s income is measured while holding the age at which parents’ income is measured constant at 40-44 (red solid line) or at 45-49 (green dashed line). The left panel shows the IGE estimates and the right panel shows the rank correlation estimates. The black dotted lines show the corresponding estimates using the lifetime income.

bias in the IGE estimates when children’s income is measured too early. For instance, the left panel shows that the IGE estimate when children’s income is measured in the early 20’s is less than half the true value using the lifetime income (black dotted line). The IGE estimates become stable once the children’s age is over 30. The rank correlation estimates show similar patterns with the two key differences. First, the absolute magnitude of the attenuation bias is smaller. Second, the rank correlation moderately declines with the age at which children’s income is measured.²² Regarding the lifecycle bias with respect to parents’ age (red solid line vs green dashed line), the rank correlation estimates are found to be more robust to the age at which parents’ income is measured than IGE estimates.²³

We now move on to the third measure: the quintile income transition matrix. Table 6 compares the transition matrices using the model-generated data to the transition matrix obtained from U.S. data. Overall, the model successfully reproduces the income quintile transition matrix constructed from U.S. data (Chetty et al. 2014). In particular, the model generates salient features in the quintile transition matrix from U.S. data: high probabilities of staying in the bottom quintile (0.35 in the model with lifetime income and 0.34 in the

²²This pattern is also present in Chetty et al. (2014a)’s data using the SOI (the Statistics of Income) sample in their Figure III.

²³In Appendix, I provide a more complete picture on this so-called lifecycle bias in the intergenerational mobility estimates for every combination of the parent/child ages.

Table 6: Quintile transition matrices

Parent quintile	Model										U.S. data Chetty et al (2014a)				
	Lifetime income					Proxy income					Child quintile				
	1st	2nd	3rd	4th	5th	1st	2nd	3rd	4th	5th	1st	2nd	3rd	4th	5th
1st	.35	.24	.18	.13	.10	.36	.23	.18	.13	.09	.34	.28	.18	.12	.08
2nd	.27	.23	.20	.18	.13	.26	.24	.20	.17	.13	.24	.24	.22	.18	.12
3rd	.19	.22	.21	.20	.17	.19	.22	.22	.20	.17	.18	.20	.22	.22	.18
4th	.13	.20	.22	.22	.24	.13	.19	.22	.23	.24	.13	.16	.21	.24	.25
5th	.06	.12	.20	.26	.37	.05	.12	.19	.27	.37	.11	.12	.17	.24	.37

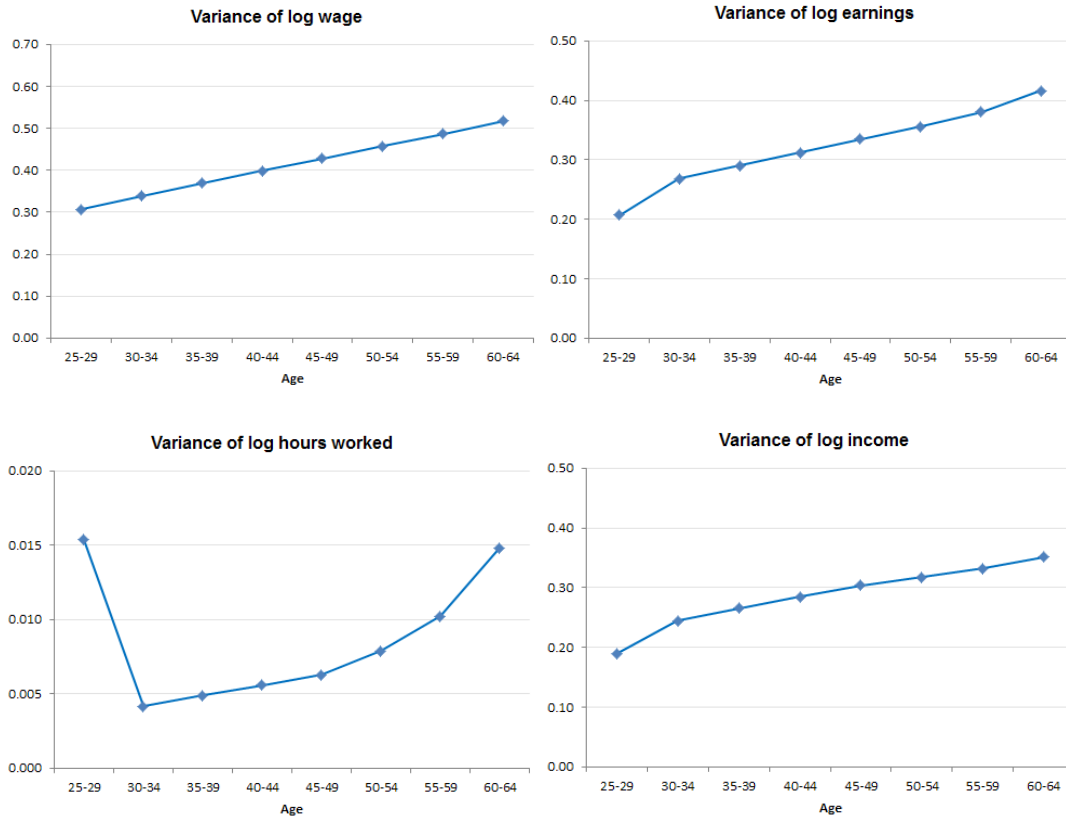
data) and in the top quintile (0.37 in the model with lifetime income and 0.37 in the data). Thus, the model is able to reproduce the fact that low intergenerational mobility in the U.S. is not simply due to the intergenerational poverty trap but is also due to the rich families that sustain their economic status intergenerationally.

4.2 Inequality over the life cycle

One of the notable features of the model economy in this paper, on top of the classical Becker-Tomes framework, is the inclusion of risky human capital in a more detailed life cycle structure. The idiosyncratic shocks to human capital over the working life move households' economic status up and down over the lifecycle, leading to mobility within a generation. This feature is important since intergenerational mobility of lifetime income not only depends on such determinants as genes and education but also mobility over the lifecycle. Thus a model that abstracts from mobility over the working life could incorrectly infer the contribution of pre-adulthood to intergenerational mobility.

To assess the behavior of the model along the dimension of lifetime inequality, Figure 2 shows the inequality statistics over the post-schooling life cycle from the model-generated data. The inequality in each variable is measured by the cross-sectional variance of log. The left top panel shows the rise in wage inequality over the life cycle. Recall that the post-schooling wage process is parsimoniously modeled and is used as a main calibration target. For instance, the average slope in the middle part was directly targeted to match the data since this information is crucial to disentangle the contribution of the two underlying sources of human capital (individual wage) dispersion in the economy (i.e., the volatility of lifetime idiosyncratic shocks to human capital σ_z and the volatility of the idiosyncratic component of human capital at birth σ_ζ).

Figure 2: Life-cycle inequality



Notes: Dispersion over the life cycle is measured using the variance of log of five year averages.

The top right panel shows the earnings inequality over the life cycle, measured by the variance of log. Note that inequality in the flow variables such as earnings, income, and work hours represent longer-term inequality aggregated over five years than those based on the annual data. In addition to its linearly rising pattern in the middle of the working lifetime driven by the linearly increasing wage dispersion over the life cycle, it is worth noting that earnings inequality profile becomes slightly convex later in life.²⁴ This is due to the disparity in hours worked which steeply rises near the retirement period as can be seen in the bottom left panel. The convexity in the earnings profile later in life and the U-shape of the variance of log hours worked are consistent with the empirical evidence reported in Heathcote et al. (2010). Finally, the bottom right panel shows the age profile of income inequality. Its shape is very similar to the earnings profile. The increasing patterns of the age profile of inequality in wage, earnings, and income are also present in U.S. data (see e.g., Díaz-Giménez et al., 2011), and the model successfully generates the pattern.

5 The role of the parental time investment channel

5.1 Parental time investment and intergenerational mobility

In this subsection, I assess the quantitative importance of the parental time investment channel in explaining intergenerational mobility. I consider several special cases alongside the baseline specification in order to decompose the relative contribution of the key elements. Table 7 summarizes the intergenerational income mobility estimates obtained from the alternative specifications as well as the baseline specification.²⁵ The first row reproduces the intergenerational mobility estimates from the baseline specification. Note that the baseline model has three direct intergenerational links: (i) the (endogenous) parental time investment; (ii) the (endogenous) inter-vivos transfers; and (iii) the exogenous transmission of human capital at birth.

The second row reports the intergenerational mobility estimates from the first special case ($\eta = 0$ and $h = \bar{h}$) in which I set the degree of altruism η equal to zero and impose that every parent in the second period invests the average parental time obtained in the baseline specification. Note that, since altruism is eliminated, parents have no incentive to save for inter-vivos transfers. The IGE estimate from this specification is 0.267 and the rank correlation estimate is 0.255. These are approximately four-fifths of their counterparts

²⁴The rising age profile of the earnings dispersion between age 25 to age 60 is quantitatively similar to the empirical counterparts following the time effects view in Huggett et al. (2011) after accounting for the fact that they use the annual frequency data.

²⁵Throughout the paper, the standard errors of the intergenerational mobility estimates are omitted since they are very small and the resulting p-values of the estimates are close to zero.

Table 7: Intergenerational mobility estimates using lifetime income

<i>Variable: lifetime income</i>	change relative		change relative	
	IGE	to baseline	rank corr	to baseline
Baseline specification	.365		.356	
Homogenous time investment + No IVT	.267	-.098	.255	-.101
No parental time investment + No IVT	.172	-.193	.164	-.192
No inter-vivos transfer (IVT) margin	.347	-.018	.338	-.018

Notes: The mobility estimates are obtained using lifetime income from 50,000 pairs of a parent and a child. The second row (homogenous time investment plus no inter-vivos transfers) imposes no altruism ($\eta = 0$) and the same time investment at the mean value from the baseline model ($h = \bar{h}$). The third row (no parental time investment and no inter-vivos transfers) sets $\eta = h = 0$. The last row (no inter-vivos transfers) imposes $s = a_0 = 0$. See the notes in Table 5 for the descriptions of the IGE and the rank correlation.

from the baseline specification where more skilled parents invest more time in their children. The third row reports the results from the second alternative specification where I close the parental education channel by setting η and h equal to zero for every parent. Therefore, in this counterfactual specification, generations are linked by the exogenous transmission of human capital only. Both of the intergenerational persistence estimates become much lower: the IGE estimate becomes 0.172 and the rank correlation estimate is 0.164. These estimates are roughly half of the baseline estimates, suggesting that the exogenous transmission of human capital alone can account for roughly half of the observed intergenerational mobility estimates.²⁶

It is important to recall that the above two specifications not only affect the parental time investment behavior but also inter-vivos transfer decisions. In order to separate out the effect of the inter-vivos transfer margin on intergenerational mobility, the last row reports the results from a specification in which I close only the inter-vivos transfer margin by setting $s = a_0 = 0$. The intergenerational mobility estimates fall but the magnitudes are quite small: the IGE and rank correlations fall roughly by 0.02. Therefore, most of the quantitative contributions found in the first and second alternative specifications are due to the quantity margin of the parental time investment and the overall parental time investment channel, respectively. The result that intergenerational mobility is significantly

²⁶This result is not surprising since nature, which could account for a significant portion of the exogenous transmission of human capital in the model, is often found to be very important in psychology, sociology and recent economics literature (e.g., see Sacerdote, 2010; Plug and Vijverberg, 2003). Note that the exogenous transmission of human capital captures not only genetic transmission (nature) but also any family factors that could indirectly affect the child's initial endowment of human capital (e.g., prenatal investment).

Table 8: Intergenerational quintile transition matrix of lifetime income

<i>Variable:</i>											
<i>Lifetime income</i>		Baseline					Homogenous time investment + No IVT				
		Child quintile					Child quintile				
		1st	2nd	3rd	4th	5th	1st	2nd	3rd	4th	5th
Parent quintile	1st	.35	.24	.17	.13	.10	.31	.23	.19	.16	.11
	2nd	.27	.23	.20	.18	.13	.24	.21	.21	.18	.16
	3rd	.19	.22	.21	.20	.17	.20	.21	.20	.20	.19
	4th	.13	.19	.22	.22	.24	.15	.19	.21	.22	.22
	5th	.06	.12	.19	.26	.37	.09	.16	.19	.24	.32
		No parental time investment + No IVT					No IVT				
		Child quintile					Child quintile				
		1st	2nd	3rd	4th	5th	1st	2nd	3rd	4th	5th
Parent quintile	1st	.26	.23	.20	.18	.14	.35	.23	.18	.14	.10
	2nd	.23	.21	.20	.19	.17	.26	.22	.21	.17	.14
	3rd	.20	.20	.20	.20	.20	.20	.22	.20	.19	.18
	4th	.18	.19	.20	.21	.22	.13	.19	.22	.23	.23
	5th	.13	.17	.20	.23	.27	.06	.14	.19	.26	.35

Notes: See the notes in Table 7 for the description of the alternative models.

affected by human capital transmission unlike financial asset transfers has important implications for policy as shown in Section 8.

Table 8 provides more detailed information on how intergenerational mobility changes as the parental time investment channel is altered. Compared to the baseline model, the transition matrix computed from the specification where the positive education gradient in parental time investment and the inter-vivos transfer channel are eliminated (top right) shows that 4.2 percentage points more of the children whose parent is from the bottom quintile are able to escape this poverty trap in the next generation and 1.2 percentage points more of them can move up to the top quintile in the next generation. A symmetric change occurs at the top quintile. For example, in the baseline model, 37 percent of children whose parent's income is at the top quintile stay in the top quintile in the next generation, whereas 32 percent of them stay there in the next generation when the educational gradient in parental time investment is assumed to be flat. In the transition matrix from the specification where both the parental time investment channel and the inter-vivos transfer is closed (bottom left), the persistence in both the top quintile and the bottom quintile decrease further, consistent with the intergenerational slope estimates. It is interesting to note that the transition matrix when only the inter-vivos transfer channel is closed (bottom right) shows that the persistence in the top quintile decreases by 1.4 percent while

Table 9: The effect of human capital formation channels on intergenerational persistence of human capital

<i>Variable: human capital</i>	rank corr	change
At birth	.233	
After parental time investment	.410	<i>+.177</i>
After college education	.425	<i>+.016</i>

Notes: The slope estimates are obtained from a univariate OLS regression where the independent variable is the log (or percentile rank) of the parent’s human capital and the dependent variable is the log (or percentile rank) of the child’s human capital at a point in the life cycle.

the persistence of the bottom quintile barely changes. This suggests that the effect of the inter-vivos transfers is disproportionately larger in the rich families. Therefore, the parental time investment channel per se, after separating out the effect of the inter-vivos transfers, appears to have a slightly larger effect on poor families than on rich families.

In Table 9, I present another way to assess the role of the parental time investment channel in explaining the intergenerational persistence of economic status. The basic idea is to see how the intergenerational association of human capital changes before and after the parental time investment stage in the baseline model.²⁷ To implement this idea, the first row presents the rank correlation of the initial human capital (θ_c) while the second row presents the rank correlation using the human capital after the parental investment stage ($\theta'_c = \theta_{j=1}$). For the purpose of comparison, I also present the rank correlation using the human capital after the college education stage ($\theta_{j=2}$). It is worth noting that the intergenerational persistence, measured by the rank correlation, increases significantly after the parental time investment stage: the rank correlation of human capital across generations increases by 0.177 after the parental time investment channel. The positive educational gradient in parental time investment interacting with the quality of care significantly amplifies the intergenerational correlation of human capital that would arise solely from the exogenous transmission via mechanisms such as genetic transmission. In sharp contrast, the college education channel contributes little to the intergenerational persistence of human capital as the rank correlation increases slightly after the college education channel.

²⁷Note that this exercise considers the intergenerational correlation of human capital. Thus, the issue regarding the effect of the inter-vivos transfers on intergenerational mobility is less relevant here.

Table 10: The effect of parental time investment channel on cross-sectional inequality

<i>Variable: human capital</i>	Gini	change	% change
Ability at birth	.287		
After parental time investment	.229	-.058	-20.3
After college education	.314	+.086	+37.6

5.2 Parental time investment and cross-sectional inequality early in life

The previous subsection shows that the parental time investment channel tightens the intergenerational economic association and its quantitative contribution is significant. In this subsection, we look at the effect of the parental time investment channel on the cross sectional variation of human capital early in life. Since the intergenerational mobility essentially measures how persistent cross-sectional lifetime inequality is over generations, this exercise is a natural choice to understand the role of the parental time investment channel as a source that shapes the intergenerational mobility as well. The cross-sectional variation in human capital is measured by the Gini index.

Table 10 summarizes the inequality estimates computed at different life stages using the baseline model specification. The inequality estimate in the first row is measured using children at birth, and the one in the second row is measured using children at the end of the parental time investment stage. Therefore, the change from the first row to the second row shows the contribution of the parental time investment channel to the cross-sectional inequality in human capital. Notice that the parental time investment channel reduces the cross-sectional inequality by 20 percent. This is in sharp contrast to a large increase in the Gini index from the second to the third row, which largely captures the role of the college education channel.²⁸

Why does the parental investment channel reduce the cross-sectional variation despite the fact that the marginal product of parental time investment increases with the child's initial human capital endowment? The key to understanding this result is that the equilibrium choice of parental time investment decreases with the child's human capital at birth, conditional on the parent's human capital, as shown below in Section 6. A key force that drives this tendency is the dynastic smoothing of the marginal value of human capital, analogous to the consumption smoothing of the infinitely-lived households in a standard

²⁸Another factor that contributes to the rise in inequality from the second to the third row is the idiosyncratic shocks to human capital z . Its contribution, which should be taken out to correctly measure the role of the college channel, is relatively small (approximately a 5 percentage point increase).

dynamic model. The difference here is that parents choose to invest in their child's human capital which in turn affects her future lifetime consumption and leisure since they care about the descendant's utility.²⁹ Hence, this force leads parents to invest more time in a less able child whose marginal utility from extra time investment is larger.

At first glance, one may find it incompatible that the parental time investment channel amplifies intergenerational persistence but reduces cross-sectional inequality. This is perhaps because the intergenerational persistence and inequality are typically observed to move together. For example, Corak (2013) documents a positive relationship between inequality and intergenerational persistence found in the cross-country evidence, which is called the Great Gatsby curve. However, it is important to note that the raw data we observe give us a reduced-form relationship. This means that the positive relationship between inequality and intergenerational persistence could be a function of various factors. Hence, what the above exercises imply is that the parental time investment channel may not be a promising candidate that drives the positive relationship between inequality and intergenerational persistence, if one looks for the single key factor.

6 Analysis of human capital investment at the micro level

This section takes a deeper look at the household's human capital investment behavior using the baseline model economy. I first examine the college education and then the parental time investment. Throughout the exercises in this section, I use the equilibrium market prices determined in the baseline model.

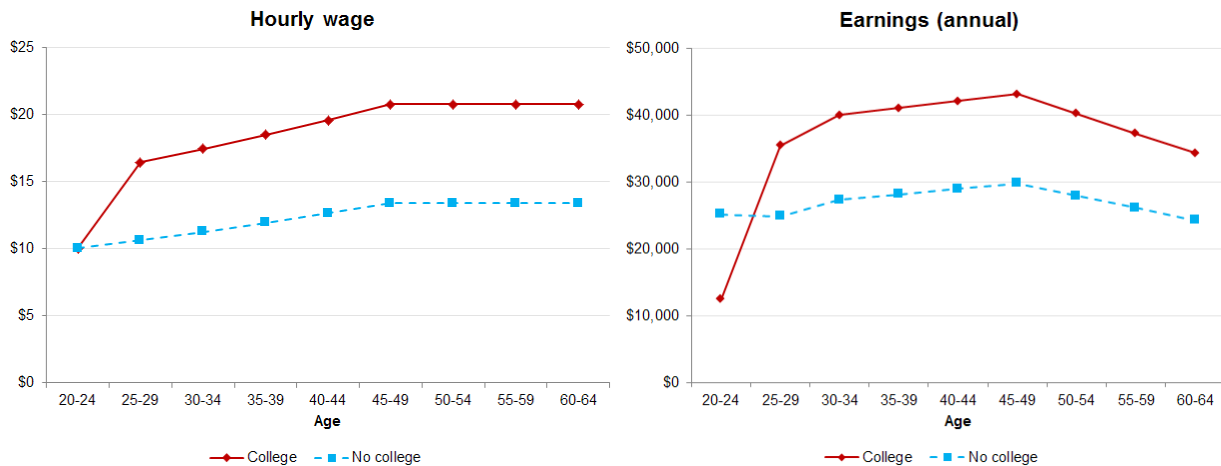
6.1 College education

The first exercise is a controlled experiment at the micro level that is designed to quantify the lifetime effects of college education. This can help understand the mechanism underlying the key finding that the college education channel amplifies the differences in human capital while it hardly changes the intergenerational mobility. Specifically, I compare two individuals who enter the labor market with the same level of human capital and wealth, but make a different college decision due to different fixed cost realizations.³⁰ That way,

²⁹Parent's asset transfers (inter-vivos transfers) also feature the same smoothing property. Note that the inter-vivos transfer decision is subject to a borrowing constraint because parents cannot borrow against their child's future income.

³⁰I impose the lowest fixed cost draw for the college-goer so that its wealth effect on labor supply is negligible. For the other person who does not go to college, the actual value of the fixed cost draw is irrelevant since it is not paid as long as it is greater than the threshold value.

Figure 3: Lifetime effects of college education



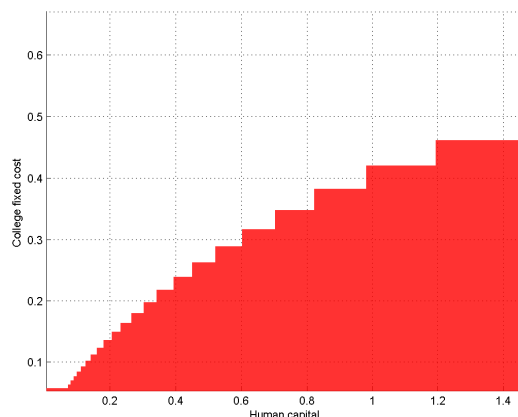
we can see the partial effect of college education while holding constant other conditions including the effect of early education. For illustration, the common level of initial human capital is normalized so that the wage in the initial period is \$10 per hour. I also control for initial assets by assuming both receive zero inter-vivos transfer, and I control for lifetime market luck by setting the idiosyncratic shocks to human capital over the lifetime to mean zero.

Figure 3 shows the lifecycle profiles of wage and earnings for the two individuals.³¹ The red solid line is for the person who goes to college and the blue dotted line is for the one who does not go to college. In the left panel, we can see that both of them experience wage increases until the middle of their lives that become flat later in life. The difference in the wage profiles is caused solely by the college education that is completed at the end of the first period (age 20-24). This gap in the lifecycle profile of wages leads to the differences in earnings, as can be seen in the right panel. Except for the first period when the one who chooses not to go to college earns more labor income by working longer hours, we can see that the individual who graduated from college earns significantly more over the remaining lifetime. If I sum up their earnings over the whole life, the one who graduated from college earns 34.5% more than the non-college graduate over the entire life.³² It is important to note that this lifetime monetary benefit of college education is a controlled figure since they started their economic life with the same human capital as well as the same wealth.

³¹For expositional purposes, wage and earnings from the model are transformed to the hourly wage and the annual earnings, respectively.

³²This is based on the simple average. If I use the present discounted value using the market interest rate, the difference becomes somewhat smaller (25 percent), but still sizeable.

Figure 4: college decision rule and $j = 1$ distribution

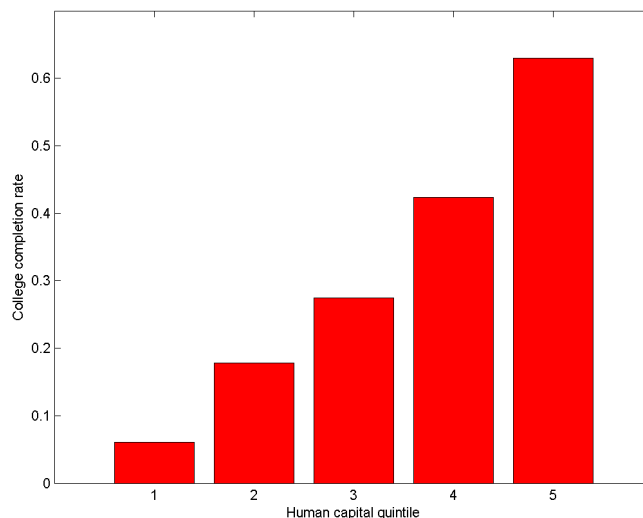


In Figure 4, I plot the equilibrium discrete decision rule of college education for those who receive zero inter-vivos transfers. The red area represents the state space where an individual would choose to go to college. Holding the college cost constant, the equilibrium decision rule shows that a person would go to college if his or her human capital is above the threshold level. Given the significant monetary benefit of college education, a natural question is why some people choose not to go to college even at the same college cost. The reason is that the return to college, which is accumulated over the life cycle, increases as the initial human capital is higher, while the direct college costs are independent of the human capital, and the marginal opportunity cost of going to the college (i.e., foregone earnings) is transitory and relatively small especially when households are young.³³ This property of the college decision rule leads to the self-selection in equilibrium, meaning that a better prepared student is more likely to complete the college education.

To visualize the importance of college readiness that exists in the model, in Figure 5, I present college completion rates by the quintiles of the equilibrium human capital distribution at the beginning of the college schooling period. It clearly shows that more able students are more likely to have a college degree, indicating positive selection into college. For instance, the college completion rate is greater than 60 percent if an individual's human capital is located in the top quintile while it is less than 10 percent if an individual's human capital is located in the bottom quintile. This pattern is consistent with the empirical evidence in Carneiro and Heckman (2003) who show a comparable positive relationship

³³In reality, merit-based scholarship could make the college cost smaller for the people with higher ability. This would strengthen the importance of human capital in deciding whether to go to college. The effect of need-based scholarship may work in the other direction; however, it is less clear since children's ability is not perfectly correlated with parental income, which is typically a criterion for such scholarship.

Figure 5: College completion rates by human capital quintiles



between college completion rates and the human capital measured by the Armed Forces Qualifying Test. Since heterogeneity in the level of human capital when making the college decision is endogenously determined by their parent’s characteristics and investment, we move on to the other key human capital investment channel: parental time investment in early childhood.

6.2 Parental time investment

I begin by characterizing the properties of the parental time investment decision rule. Table 11 shows the coefficient estimates from the regression equation:

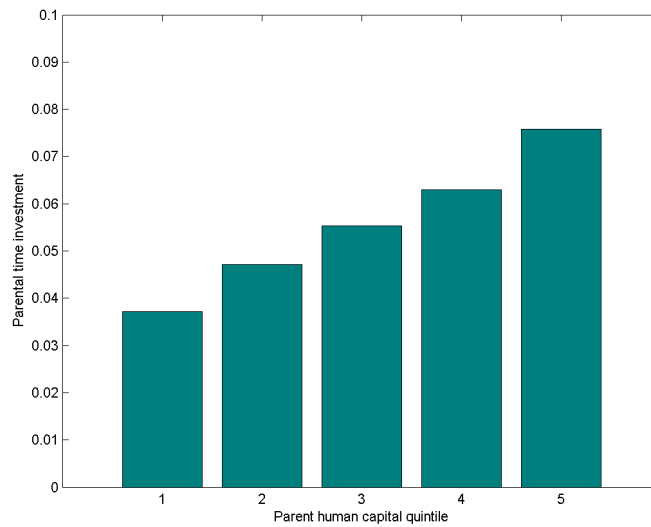
$$\log(h) = \beta_0 + \beta_1 \log(\theta) + \beta_2 \log(\theta_c) + \beta_3 a + \varepsilon. \quad (14)$$

Since the marginal return to parental time investment increases with the parent’s human capital under the assumed human capital production technology, the optimal parental time is increasing in parent’s human capital holding asset constant. This can be seen from the positive value of β_1 . In equilibrium, this decision rule leads to the positive educational gradient in parental time investment as can be seen in Figure 6, which plots the equilibrium parental time by the quintiles of parental human capital. As more skilled parents spend more time with their children, which in turn help their children to be better prepared for college, this property of the human capital investment behavior acts as a mechanism that could increase the intergenerational persistence of economic status.

Table 11: Parental time investment by the idiosyncratic component index of child's innate ability

Dependent variable:	Specification	
	(a)	(b)
log parental time investment $\log(h)$		
log parent's human capital $\log(\theta)$.70	.72
log child's human capital $\log(\theta_c)$	-1.04	-1.04
parent's assets a		.38
R^2	.96	.97

Figure 6: Parental time investment by parent's human capital quintiles



On the other hand, altruism, which integrates generations as a single dynasty, implies that the parental time investment channel may decrease cross-sectional dispersion. To see this, note that the optimal parental time investment (h) decision is characterized by

$$B(1 - n - h)^{-\varepsilon} = \beta^4 \eta D_1 V_{j=1} (\theta_c + (\theta h)^{\alpha_1} \theta_c^{\alpha_2}, a) \alpha_1 \theta^{\alpha_1} h^{\alpha_1 - 1} \theta_c^{\alpha_2}. \quad (15)$$

The left-hand side represents the marginal cost of h (which is the marginal utility of leisure) and the right-hand side summarizes the marginal benefit of h . The marginal benefit has two components. First, additional time would develop the child's ability further. This positive marginal product of h is captured by the second half terms ($\alpha_1 \theta^{\alpha_1} h^{\alpha_1 - 1} \theta_c^{\alpha_2}$). However, what is valued by parents is not the level of child's human capital per se, but the lifetime utility which the child enjoys. Therefore, the first half terms ($\beta^4 \eta D_1 V_{j=1} (\theta_c + (\theta h)^{\alpha_1} \theta_c^{\alpha_2}, a)$) translate the marginal product of human capital into the present-value marginal utility which parents actually care about. Note that, although the marginal cost of h (left-hand side) is independent of the child's ability at birth θ_c , the marginal benefit of h (right-hand side) may shift up or down depending on the relative size of the two effects: (i) it may go up since the human capital technology implies that marginal return to h increases with the child's human capital; and (ii) it may go down because of diminishing marginal utility.³⁴ The equilibrium behavior in the model economy shows that the second effect dominates the first effect as can be seen in Table 11. The estimate of β_2 is negative, meaning that parental time investment decreases with the child's human capital at birth, holding the parent's human capital constant. This number is robust when I control for the parent's asset level as well. This suggests that, even though additional time is more productive with a more able child, parents compensate a less able child for his lower endowment of human capital by spending more time with him.³⁵

I now move on to the controlled experiment that quantifies the lifetime effects of the early human capital investment by parents in terms of lifetime earnings.³⁶ Specifically, I consider two children who are born with the same level of the idiosyncratic component of

³⁴Consider an example of having two different kinds of children. Assume that one is born with a higher level of human capital than the other. Since the more able kid is expected to earn more over the lifetime holding parental inputs constant, the same one-dollar increase in wage, caused by parental human capital investment, would be more valued by the child born with a lower level of human capital.

³⁵This result is consistent with the results in an empirical micro study by Bernal (2008) that finds that mothers compensate less able children by spending more time with them despite lower returns, using the NLSY79 data. The compensating nature of the parental investment can be also seen in another theoretical setting where parents care about inequality among multiple children (Behrman, Pollak, and Taubman, 1982). Their setting is different from the current setting where parents compensate their child who is born with low ability endowment relative to themselves.

³⁶It is worth noting that the monetary benefit computed here does not account for a gap in leisure. The total benefit is even larger when I account for leisure differences, although it is hard to visualize the benefit in terms of money.

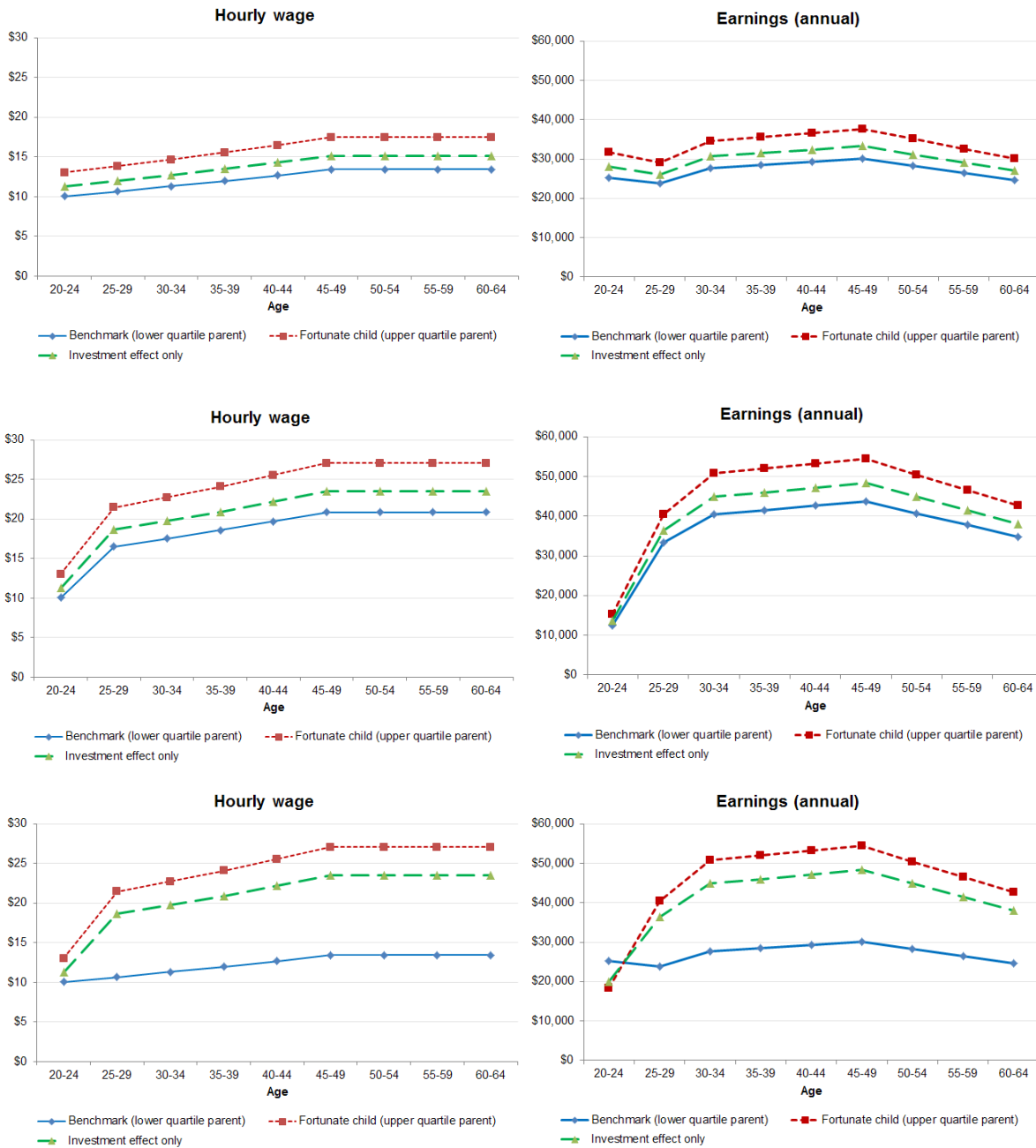
ability at birth ζ but with parents whose level of human capital differ; the benchmark child (blue solid line)'s parent is assumed to have a human capital level at the lower quartile (25th percentile), while the treatment child (red dotted line)'s parent has a higher human capital level at the upper quartile (75th percentile). As in the controlled experiment of the previous subsection, I control for lifetime market luck at mean zero. For the purpose of illustration, the units are transformed so that the benchmark child's initial wage is ten dollars per hour.

Figure 7 summarizes the results. To isolate the parent's effect aside from the college effects, I compare the two individuals' lives separately (i) in the case when neither chooses to go to college (by assuming that the realized college costs are very high) plotted in the top two figures and (ii) in the case when both graduate from college plotted in the middle figures. In both cases, the fortunate child who has a better parent (red dotted line) experiences 35 percent higher wage profiles over the entire life than the benchmark child (blue solid line), which leads to higher lifetime earnings. For instance, the fortunate child earns 24.5 percent more over the lifetime than the benchmark child in the first case when neither goes to college and earns 24.0 percent more in the second case when both go to college. It is important to note that these numbers reflect the total effects of parents. This is because the two children are born with different levels of ability at birth even though their idiosyncratic component ζ is the same.³⁷ To separate out the effect of parental time investment only, I adjust the treatment child's ability at birth so that both children are born with the same ability θ_c (green dashed line). Then the gap in the lifetime earnings between the adjusted treatment child and the benchmark child shrinks to 10.6 percent in the first case, and 10.3 percent in the second case. Though it may appear that these estimates are not sizeable, it is worth noting that these estimates do not account for the case when the early education channel interacts with the college education channel, which I now discuss below.

The bottom figures in Figure 7 illustrate the power of the interaction effect between the parental time investment channel and the college education channel. In this case where the college fixed cost realization is commonly set to a middle value, the benchmark child chooses not to go to college. However, if the benchmark child had a more skilled parent who would allocate more time in parental investment, this child would have been prepared enough so that going to college becomes optimal. The treatment child enjoys this benefit. The lifetime earnings gap between the two due to both non-investment and investment effects reaches 68.0 percent. The investment effect per se leads to a 50.9 percent increase in lifetime earnings, which is significantly larger than the previous two cases where I control

³⁷Recall that ability at birth $\phi(\theta, \zeta)$ depends positively on both parent's ability θ and the idiosyncratic component ζ .

Figure 7: Lifetime effects of parental time investment in early childhood



Notes: The top two figures are for the case when everyone chooses not to go to college while the middle two figures are for the case when everyone chooses to go to college. The bottom figures show the marginal case. The blue line represents the benchmark kid whose first period wage is normalized to be ten dollars. The red dotted line shows the total effects of the parent (including both non-investment and investment effects) while the green dashed line refers to the investment effect only.

for the college effect. The results thus show that the effect of the birth lottery can be much larger when the effect of parental investment is amplified by the college education decision. These exercises demonstrate that the magnitude of the market failure in the life cycle skill formation literature, noted by Cunha and Heckman (2007, 2010), could be quite significant when the endogenous human capital investment channels interact with each other.

7 Policy experiments

In this section, the calibrated model economy is used to study the implications of several government policies that might be considered as tools to influence intergenerational mobility. First, I consider a set of policies that are intended to affect the college decision. Second, I consider a set of hypothetical policies that are targeted to affect parental time investment behavior. Note that results for the policy experiments in this section represent long-run general equilibrium effects in which prices vary so that markets clear. For illustration, I show dollar values whenever necessary based on the assumption that the annual GDP per capita in the baseline model is \$50,000, a value close to nominal US GDP per capita in 2011-2012. In all exercises below, policies are designed to cost no more than the amount of social security budget surplus in the baseline specification.

7.1 Policies related to college decision

I consider three kinds of policies that can potentially affect households' college decision. The first is to relax borrowing limits at model age 1 (college education stage) to allow borrowing up to the 10 percent of annual GDP per-capita in the baseline specification.³⁸ The second and third exercise is to adjust the mean of the college cost distribution m_ε by 10 percent. Recall that the value of m_ε was calibrated to match the average college expenses after (federal, state/local, and institutional) grants. Therefore, this exercise can be interpreted as a policy that subsidizes college expenses more (if m_ε falls) or less (if m_ε rises). In all the above exercises, I explore the equilibrium effects on the human capital investment behavior (i.e., college completion rates and average parental time investment), the fraction making inter-vivos transfers, the intergenerational mobility estimates (the IGE and percentile rank correlation), aggregate output (Y), average labor productivity, and consumption-equivalent welfare changes.

Table 12 summarizes the policy experiments related to college decision. The second row shows the results when the borrowing limit at period $j = 1$ is relaxed. Note that its effect

³⁸Caucutt and Lochner (2012) study the role of borrowing constraints in parental investment in a partial equilibrium setting.

Table 12: Policy experiments: policies related to college

Policies at $j = 1$	Col (%)	\bar{h}	IVT (%)	IGE	Rank corr	Y (% Δ)	ALP (% Δ)	Welfare (% Δ)
Baseline	31.0	26.7	29.3	.365	.356			
Borrowing $_{j=1}$ relaxed (by \$5,000)	36.7	26.8	34.3	.363	.355	+2.4	+3.1	+1.8
Col cost down by 10% (by \$5,025)	37.3	27.1	35.6	.370	.363	+3.2	+4.1	+2.1
Col cost up by 10% (by \$5,025)	25.7	26.3	24.4	.355	.345	-3.0	-3.6	-1.3

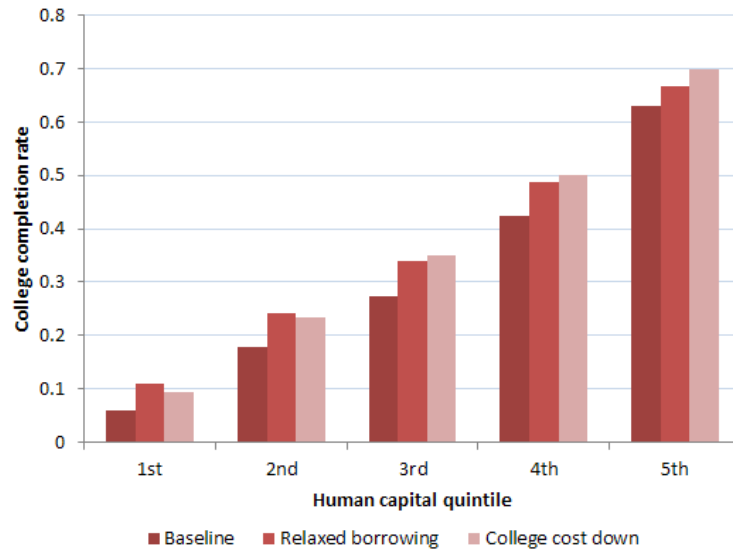
Notes: The average parental time investment is expressed as hours per month. GDP per capita (Y) and average labor productivity (ALP), defined as GDP per total hours worked are percentage changes relative to the baseline specification. Welfare changes are measured by the percentage change in consumption that is required for the households to be equivalent in terms of the value of the utilitarian social welfare function. For illustration, dollar values are computed using \$50,000 as a benchmark annual GDP per capita.

on the college completion rate is sizable; the fraction with a college degree increases by 5.7 percentage points since more people who are currently credit-constrained but would like to go to college and pay off their debt later can afford college costs.³⁹ This in turn increases the quality of aggregate labor input, thereby increasing output by 2.4 percent. Further, the change in social welfare, measured by consumption equivalent, is positive. However, it is interesting to note that quite a sizeable increase in college-educated households does not imply that intergenerational mobility increases significantly; the magnitude of the decreases in the IGE and rank correlation is quantitatively insignificant.

The next row presents the case in which the mean of four-year college costs are down by \$5,025, thereby making college education cheaper and more available to the general public. Note that its effect on college completion rate is quite similar to the effect of relaxing borrowing constraints by \$5,000; approximately 6 percentage points more of households have a college degree. Although the difference is quantitatively irrelevant, it is interesting to note that intergenerational mobility actually decreases when the college costs are reduced unlike the case when borrowing limits are relaxed. To increase intergenerational mobility via the amount of college expenses, the last row suggests that the college costs should go up instead.

³⁹ Although this change in the college completion rate may appear large, it should be noted that this is not inconsistent with the literature that finds the quantitative insignificance of the short-run credit constraints in college education (e.g., Keane and Wolpin, 2001; Carneiro and Heckman, 2002; and Cameron and Taber 2004). This is because the changes captured in this paper are long-run effects in the presence of intergenerational human capital transmission. Any change initiated by a change in the short-run credit in the college decision period may be amplified in the long run when the increase in parents' human capital is transmitted to a higher human capital of their descendants.

Figure 8: College completion rates by human capital quintiles: baseline vs policies related to college



Why do we have such a puzzling result that intergenerational mobility falls when more people go to college due to a lower college expenses while it increases when more people go to college due to a larger borrowing limit? Note that the key difference is the source of financial relief for young households. Specifically, when college costs are financed by loans, the households should pay back in the future. This adds burden to the young households whose consumption has to be sacrificed if they want to make inter-vivos transfers. However, when college costs are just reduced, this does not affect the college-educated parents' behavior, providing more room to save money for the inter-vivos transfers. Therefore, in the long run, we see a higher increase of the fraction making the inter-vivos transfers when college expenses are reduced. This tends to strengthen economic association across generations.

To understand this point further, Figure 8 shows the conditional mean of college completion rates by human capital quintiles across the above three specifications. Note that the average college completion rate of those whose human capital is located at the first and second quintiles increases noticeably more when borrowing limits are relaxed whereas the average college completion rate of those whose human capital is at the higher quintiles rise more sharply when college costs become lower. The difference in the fraction of the inter-vivos transfers is crucial for this result. This is because the parents who make inter-vivos transfers are not only richer and but also more skilled, and thus they are more likely to have a more-able child both due to genes and investment, as discussed in the previous section. Hence, in the case when college costs are lowered, the slope of college completion

rates as a function of human capital does not drop unlike the case when borrowing limits are relaxed. This explains why intergenerational mobility may fall in the case when college costs are lowered although both policies increases the college completion rate.

Nevertheless, it is worth noting that all of the policies that are designed to influence the college decision are shown to have limited effects on intergenerational mobility. This is consistent with the main result in Section 5 that the contribution of the college education channel (or the inter-vivos channel, which affects the college decision) to intergenerational mobility is relatively small. In the next subsection, I move on to the policies that are directly targeted to influence parental time investment behavior.

7.2 Policies targeted at parental time investment behavior

In this subsection, I consider three kinds of policies that intend to change parental time investment behavior in period $j = 2$. First, I consider a simple form of subsidy: a lump-sum transfer to the parents who live in period $j = 2$.⁴⁰ Second and more importantly, I consider a subsidy s_h that is paid proportional to parental time investment h . An important feature of this policy is that, unlike individual wage or the marginal return of parental time investment, the proportional parental time investment subsidy is independent of the parent's human capital. Therefore, the same amount s_h matters more for less skilled parents. Lastly, I consider an increase in labor tax which is only applied to the households living in period $j = 2$. Since this policy would decrease the marginal benefit of market work, it can indirectly promote parental time investment.

Table 13 summarizes the results. The second row presents the effects of a lump-sum transfer. Note that this policy, unlike the other policies, is not directly targeted at parental time investment behavior although this sort of policy could be considered as a tool to relieve the financial burden of young households, which in turn could have an indirect effect on parental time investment decisions. The results show that this indirect effect (via the income effect) is small and thus this policy barely changes any statistics at the expense of sizeable transfers.⁴¹

The third row shows the results regarding the parental time subsidy. A striking finding is that a very small amount of subsidy can alter the behavior of the parental time investment significantly. For instance, when s_h is set such that the average amount of subsidy paid to

⁴⁰For example, as of 2013, South Korea introduced a policy that provides a lump sum transfer to the parents who have a child of age 5 or less. The amount ranges roughly from \$100-\$400 per month in 2013 dollars depending on the age of the child and the use of day-care centers. Importantly, the subsidy is independent of the parents' income level.

⁴¹Del Boca et al. (2014) also conducts a similar exercise and finds that cash-transfers to households with children do not improve children's quality since parents consumes more and enjoy more leisure.

Table 13: Policy experiments: policies related to parental time investment

Policies at $j = 2$	Col (%)	\bar{h}	IVT (%)	IGE	Rank corr	Y (% Δ)	ALP (% Δ)	Welfare (% Δ)
Baseline	31.0	26.7	29.3	.365	.356			
Lump-sum transfer	30.8	26.7	28.9	.364	.356	-0.1	-0.1	+0.4
Time investment subsidy	30.6	31.2	28.6	.348	.339	+1.8	+2.5	+4.0
Labor tax	30.3	30.9	28.1	.355	.346	+1.1	+1.2	-0.4

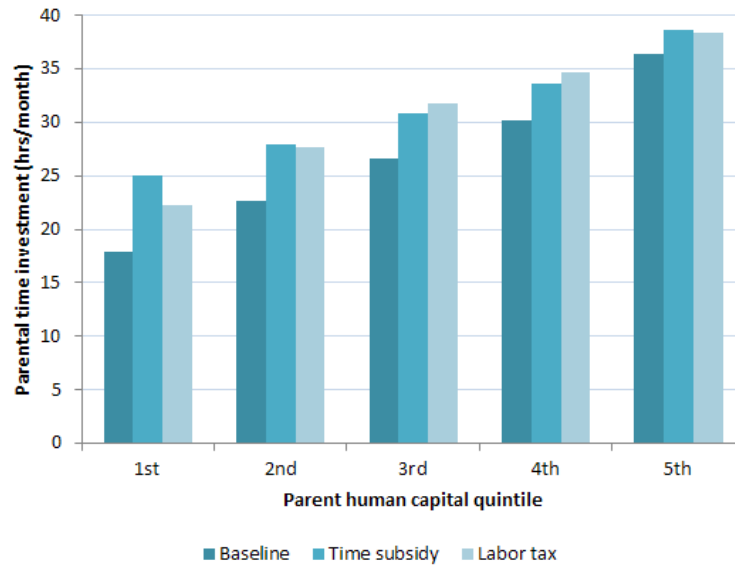
Notes: The amount of lump-sum transfer is chosen as the five percent of annual GDP per capita in the baseline specification. This amounts to \$208 per month. The labor tax $\tau_{n(j=2)}$ is set to 15 percent, which applies to people whose model age is 2. The proportional parental time investment subsidy s_h is chosen to have a similar increase in the mean parental time investment as in the case of labor tax. In equilibrium, this policy requires 0.08 percent of GDP. See Table 12 for the descriptions of the statistics considered.

the households of model age 2 is \$41 per month (which costs 0.08 percent of the GDP), the average parental time investment increases by 16.9 percent (from 26.7 to 31.2 hours per month). Furthermore, we can see that both the IGE and percentile rank correlation fall noticeably, in particular compared to the college-related policies we investigated in the last subsection. A beneficial by-product due to an increase in average parental time investment is the rise in output, average labor productivity, and social welfare, driven by higher quality of average human capital in the labor force.

In the fourth row, we can see that a similar increase in the average parental time investment is achieved by increasing the labor tax rate τ_n . Specifically, an increase in τ_n in period $j = 2$ by 15 percentage points can increase the mean of the parental time investment up to 31 hours per month (from 27 hours per month in the baseline specification), which is comparable to the case of time subsidy (second row). Despite the similarity in terms of their effect on the average parental time investment, there are quantitative differences. First, the labor tax has a weaker impact on intergenerational mobility. For instance, the magnitude of the fall in the rank correlation are much weaker in the case of the labor tax (-0.010) than in the case of the parental time subsidy (-0.017). Furthermore, their effects on output and labor productivity are both positive but different quantitatively; the parental time investment subsidy achieves a larger improvement in output and average labor productivity than the labor tax does.

We now look at why the parental time investment subsidy increases intergenerational mobility more than the labor tax, despite the fact that the two policies have similar quantitative effects on the average parental time investment. The key to understanding this

Figure 9: Parental time investment by parent’s human capital quintiles: baseline vs policies related to parental time investment



difference is the fact that the parental time investment subsidy is in effect progressive because it is independent of parent’s human capital whereas the labor tax is proportional to parent’s human capital. In Figure 9, I plot the conditional mean of parental time investment by parent’s human capital quintiles in the three cases: (i) the baseline model; (ii) parental time investment subsidy; and (iii) labor tax. A striking difference arises at the bottom quintile where the average parental time investment increases much more prominently in response to the parental time investment subsidy. Since the parents whose human capital is low have a relatively lower opportunity cost of parental time investment (i.e., lower wage), they increase time investment in their child’s human capital sharply with respect to the same amount of the incentive s_h . This helps those children who are born in less advantaged families that on average invest less in their children than parents with higher human capital. As some of them at the margin complete the college education, those children who are born in less advantaged families are able to move up. Therefore, this policy effectively compensates the disadvantage that arises due to the investment effect of the skill formation market failure discussed in the previous section, and, as a result, increases not only intergenerational mobility but also social welfare quite significantly.

In Table 14, I illustrate the effect of the two policies on the relationship between the parental characteristics, their child’s human capital endowment and the parental time investment in equilibrium. The coefficient estimates in Table 14 are obtained from a regression equation where the dependent variable is log parental time investment and the

Table 14: Equilibrium relationship between parental time investment and human capital

Dependent variable:	Specification		
log parental time investment $\log(h)$	(1)	(2)	(3)
log parent's human capital $\log(\theta)$.72	.52	.62
log child's human capital $\log(\theta_c)$	-1.04	-1.00	-.97
parent's assets a	.38	.23	.59
R^2	.97	.96	.96

Notes: Each column is obtained using the following specifications: (1) baseline; (2) parental time investment subsidy; and (3) labor income tax. A constant term is included in the regression equation. Standard errors are omitted since they are very small in all the estimates.

regressors are log parent's human capital and log child's human capital endowment. In column (1), the baseline specification implies that, holding the child's human capital and the parent's assets constant, the elasticity of parental time investment with respect to parent's human capital is roughly 0.7. This elasticity decreases under both policies. It becomes 0.5 under the parental time investment subsidy in column (2), and becomes 0.6 under the labor income tax in column (3). However, note that the magnitude of the fall in this elasticity is larger when parental time investment subsidy is used, thereby leading to a larger decline in the intergenerational persistence of income. It is also interesting to note that the partial effect of parent's wealth actually decreases in the case of the parental time subsidy (.38 to .23) while it increases in the case of the labor income tax (.38 to .59).

To sum up, the key lesson from the exercises in this section is clear. The policies that are related to the college decision are not very effective in reshaping intergenerational mobility. By contrast, the policies that closely affect the parental time investment behavior are shown to be more effective in influencing intergenerational mobility. A key feature of such policies is that the marginal incentive of parental time investment should be relatively higher for the parents with lower human capital. The constant amount of parental time subsidy, a simple form of such policy, is shown to be quite effective in increasing intergenerational mobility.

8 Concluding remarks

In this paper, I have investigated parental time investment in preschool-aged and younger children as an intergenerational human capital transmission channel in a general equilibrium incomplete markets framework. I have found that the parental time investment channel accounts for a sizable fraction of intergenerational persistence and reduces cross-sectional dispersion of human capital. The parental time investment channel acts as a mechanism

through which households insure their descendants, thereby strengthening intergenerational association and reducing cross-sectional inequality.

The policy experiments show that the proportional parental time investment subsidy would be more effective than the policies that change college expenses (either directly or indirectly) if policymakers want to influence intergenerational mobility. In particular, subsidy proportional to time invested in children is shown to help the disadvantaged children whose parents are less skilled by prompting their parents to increase the quantity of human capital investment. This partially resolves the market failure (the inability of children to buy their parents who can have long-lasting effects on themselves) along the quantity margin. Although not explicitly studied in this paper, a government policy which can encourage disadvantaged parents to bring their children to high-quality child care centers may be a good candidate to improve this market failure along the quality margin as well.

It should be noted that the key results in this paper are based on the model economy calibrated to recent U.S. data, which feature a high educational gradient in parental time with children, a high college wage premium, high intergenerational persistence, etc. Therefore, one should be careful in generalizing the main results internationally since the key predictions on the role of the parental time investment may change if the model is calibrated to other countries which do not necessarily have the same features as in the U.S. Section 6, which is intended to help understand the mechanism in more detail, can be useful for this end.

There is an extension that might be useful to consider. In this paper, human capital is broadly defined. However, the skill formation literature finds that different kinds of skill are formed in a different fashion. It would be interesting to see how intergenerational mobility is affected by different kinds of human capital, formed differently at different stages of early childhood. This would require a larger-scaled model with multi-dimensional human capital such as cognitive and socioemotional skills (Cunha and Heckman, 2007, 2010; Cunha et al., 2010). This is left for future work.

References

- [1] Abbott, Brant, Giovanni Gallipoli, Costas Meghir, and Giovanni L. Violante. 2013. "Education Policy and Intergenerational Transfers in Equilibrium" NBER Working Paper No. 18782
- [2] Aiyagari, S. Rao. 1994. "Uninsured Idiosyncratic Risk and Aggregate Saving." *The Quarterly Journal of Economics* 109 (3): 659-684.
- [3] Aiyagari, S. Rao, Jeremy Greenwood, and Ananth Seshadri. 2002. "Efficient Investment in Children." *Journal of Economic Theory* 102 (2): 290-321.
- [4] Autor, David H., Lawrence F. Katz, and Melissa S. Kearney. 2008. "Trends in US Wage Inequality: Revising the Revisionists." *The Review of Economics and Statistics* 90 (2): 300-323.
- [5] Becker, Gary S. and Nigel Tomes. 1986. "Human Capital and the Rise and Fall of Families." *Journal of Labor Economics* 4 (3): S1-39.
- [6] Behrman, Jere Richard, Robert A. Pollak, and Paul Taubman. 1982. "Parental Preferences and Provision for Progeny." *Journal of Political Economy* 90 (1): 52-73.
- [7] Bernal, Raquel. 2008. "The Effect of Maternal Employment and Child Care on Children's Cognitive Development*." *International Economic Review* 49 (4): 1173-1209.
- [8] Black, Sandra E. and Paul J. Devereux. 2011. "Recent Developments in Intergenerational Mobility." *Handbook of Labor Economics* 4: 1487-1541.
- [9] Blau, David M. 1999. "The Effect of Income on Child Development." *Review of Economics and Statistics* 81 (2): 261-276.
- [10] Blau, David M. and Janet Currie. 2006. "Pre-School, Day Care, and After-School Care: Who's Minding the Kids?" *Handbook of the Economics of Education* 2: 1163-1278.
- [11] Cagetti, Marco and Mariacristina De Nardi. 2006. "Entrepreneurship, Frictions, and Wealth." *Journal of Political Economy* 114 (5): 835-870.
- [12] Cameron, Stephen V. and Christopher Taber. 2004. "Estimation of Educational Borrowing Constraints using Returns to Schooling." *Journal of Political Economy* 112 (1): 132-182.
- [13] Carneiro, Pedro and James J. Heckman. 2002. "The Evidence on Credit Constraints in Post-Secondary Schooling." *The Economic Journal* 112 (482): 705-734.

- [14] ———. 2003. "Human Capital Policy." In *Inequality in America: What Role for Human Capital Policies?*, edited by James J. Heckman, Alan B. Krueger and Benjamin M. Friedman, 77. Cambridge, MA: MIT Press.
- [15] Casanova, Maria. 2013. "Revisiting the Hump-Shaped Wage Profile." Working paper
- [16] Castaneda, Ana, Javier Díaz-Giménez, and José-Víctor Ríos-Rull. 2003. "Accounting for the US Earnings and Wealth Inequality." *Journal of Political Economy* 111 (4): 818-857.
- [17] Caucutt, Elizabeth M. and Lance Lochner. 2012. "Early and Late Human Capital Investments, Borrowing Constraints, and the Family." NBER Working Paper No. 18493
- [18] Chetty, Raj, Nathaniel Hendren, Patrick Kline, and Emmanuel Saez. 2014a. "Where is the Land of Opportunity? the Geography of Intergenerational Mobility in the United States" NBER Working Paper No. 19843
- [19] Chetty, Raj, Nathaniel Hendren, Patrick Kline, Emmanuel Saez, and Nicholas Turner. 2014b. "Is the United States Still a Land of Opportunity? Recent Trends in Intergenerational Mobility." NBER Working Paper No. 19844
- [20] Corak, Miles. 2013. "Income Inequality, Equality of Opportunity, and Intergenerational Mobility." *Journal of Economic Perspectives* 27 (3): 79-102.
- [21] Cunha, Flavio and James J. Heckman. 2007. "The Technology of Skill Formation." *The American Economic Review* 97 (2): 31-47.
- [22] ———. 2010. "Investing in our Young People." NBER Working Paper No. 16201
- [23] Del Boca, Daniela, Christopher Flinn, and Matthew Wiswall. 2014. "Household Choices and Child Development." *The Review of Economic Studies* 81 (1): 137-185.
- [24] De Nardi, Mariacristina. 2004. "Wealth Inequality and Intergenerational Links." *The Review of Economic Studies* 71 (3): 743-768.
- [25] Díaz-Giménez, Javier, Andy Glover, and José-Víctor Ríos-Rull. 2011. "Facts on the Distributions of Earnings, Income, and Wealth in the United States: 2007 Update." *Federal Reserve Bank of Minneapolis Quarterly Review* 34 (1): 2-31.
- [26] Fernández, Raquel and Richard Rogerson. 1998. "Public Education and Income Distribution: A Dynamic Quantitative Evaluation of Education-Finance Reform." *American Economic Review* 88 (4): 813-833.

- [27] Guryan, Jonathan, Erik Hurst, and Melissa Kearney. 2008. "Parental Education and Parental Time with Children." *The Journal of Economic Perspectives* 22 (3): 23-16A.
- [28] Guvenen, Fatih. 2006. "Reconciling Conflicting Evidence on the Elasticity of Intertemporal Substitution: A Macroeconomic Perspective." *Journal of Monetary Economics* 53 (7): 1451-1472.
- [29] Haider, Steven J. and Gary Solon. 2006. "Life-Cycle Variation in the Association between Current and Lifetime Earnings." *American Economic Review* 96 (4): 1308-1320.
- [30] Heathcote, Jonathan, Fabrizio Perri, and Giovanni L. Violante. 2010. "Unequal we Stand: An Empirical Analysis of Economic Inequality in the United States, 1967–2006." *Review of Economic Dynamics* 13 (1): 15-51.
- [31] Hubbard, William HJ. 2011. "The Phantom Gender Difference in the College Wage Premium." *Journal of Human Resources* 46 (3): 568-586.
- [32] Huggett, Mark, Gustavo Ventura, and Amir Yaron. 2011. "Sources of Lifetime Inequality." *The American Economic Review* 101 (7): 2923-2954.
- [33] Keane, Michael P. and Kenneth I. Wolpin. 1997. "The Career Decisions of Young Men." *Journal of Political Economy* 105 (3): 473-522.
- [34] ———. 2001. "The Effect of Parental Transfers and Borrowing Constraints on Educational Attainment." *International Economic Review*: 1051-1103.
- [35] Knudsen, Eric I., James J. Heckman, Judy L. Cameron, and Jack P. Shonkoff. 2006. "Economic, Neurobiological, and Behavioral Perspectives on Building America's Future Workforce." *Proceedings of the National Academy of Sciences of the United States of America* 103 (27): 10155-10162
- [36] Kopczuk, Wojciech, Emmanuel Saez, and Jae Song. 2010. "Earnings Inequality and Mobility in the United States: Evidence from Social Security Data since 1937." *The Quarterly Journal of Economics* 125 (1): 91-128.
- [37] Krusell, Per and Jr Smith Anthony A. 1998. "Income and Wealth Heterogeneity in the Macroeconomy." *Journal of Political Economy* 106 (5): 867-896.
- [38] Lee, Chul-In and Gary Solon. 2009. "Trends in Intergenerational Income Mobility." *The Review of Economics and Statistics* 91 (4): 766-772.

- [39] Lee, Sang Yoon Tim and Ananth Seshadri. 2014. "On the Intergenerational Transmission of Economic Status." Working paper.
- [40] Mazumder, Bhashkar. 2005. "Fortunate Sons: New Estimates of Intergenerational Mobility in the United States using Social Security Earnings Data." *Review of Economics and Statistics* 87 (2): 235-255
- [41] McGarry, Kathleen. 1999. "Inter Vivos Transfers and Intended Bequests." *Journal of Public Economics* 73 (3): 321-351.
- [42] Plug, Erik and Wim Vijverberg. 2003. "Schooling, Family Background, and Adoption: Is it Nature Or is it Nurture?" *Journal of Political Economy* 111 (3): 611-641.
- [43] Quadrini, Vincenzo. 2000. "Entrepreneurship, Saving, and Social Mobility." *Review of Economic Dynamics* 3 (1): 1-40.
- [44] Ramey, Garey and Valerie A. Ramey. 2010. "The Rug Rat Race." *Brookings Papers on Economic Activity* 41 (1): 129-199.
- [45] Restuccia, Diego and Carlos Urrutia. 2004. "Intergenerational Persistence of Earnings: The Role of Early and College Education." *The American Economic Review* 94 (5): 1354-1378.
- [46] Rupert, Peter and Giulio Zanella. 2012. "Revisiting Wage, Earnings, and Hours Profiles." .
- [47] Sacerdote, Bruce. 2007. "How Large are the Effects from Changes in Family Environment? A Study of Korean American Adoptees." *The Quarterly Journal of Economics* 122 (1): 119-157.
- [48] ———. 2010. "Nature and Nurture Effects on children's Outcomes: What have we Learned from Studies of Twins and Adoptees." *Handbook of Social Economics* 1: 1-30.
- [49] Solon, Gary. 1992. "Intergenerational Income Mobility in the United States." *American Economic Review* 82 (3): 393-408.
- [50] Solon, Gary. 1999. "Intergenerational Mobility in the Labor Market." *Handbook of Labor Economics* 3: 1761-1800.
- [51] Tauchen, George. 1986. "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions." *Economics Letters* 20 (2): 177-181.

A Cross-sectional inequality

I assess the model along the closely related dimension: cross-sectional inequality. Table A1 reports the Gini indices for earnings, income and wealth from the model as well as U.S. data.^{42,43} Before we compare the cross-sectional inequality statistics between the model and the data, it should be noted that, since a model period is five years, inequality measures for the flow variables such as earnings and income represent the long-term inequality. The long-term inequality is less commonly studied due to the data limitations (similar to the reasons discussed above for intergenerational mobility), compared to the short-term inequality using annual data sets. To transform the inequality statistics obtained from annual flow variables, I multiply them by the Shorrocks mobility coefficient (one-year earnings Gini to five-year earnings Gini) computed in Kopczuk, Saez, and Song (2010).

The model economy generates a substantial amount of heterogeneity especially in wealth (Gini of 0.44). In the data, wealth is most concentrated, followed by income and earnings.⁴⁴ The income Gini is quite similar to the earnings Gini both in the data (0.37 vs 0.38) and in the model (0.31 vs 0.31). Note that the model cannot match the high degree of concentration, especially in wealth. This is not surprising since the model economy in this paper abstracts from any of key elements such as heterogeneity in discount factor, bequests, entrepreneurs, and return differentials that have been found to be essential to reproduce the top tail of the wealth distribution in the literature.⁴⁵

B Lifecycle bias in the intergenerational mobility estimates

Table A2 reports the IGE estimates by varying the point (for both parents and children) at which the proxy income is measured. Although the true IGE, estimated by using the

⁴²Since the U.S. inequality statistics are from Heathcote et al. (2010) whose sample criteria restricts age (25-60), the model statistics for cross-sectional earnings inequality are computed based on the equivalent samples ($j = 2 - 8$).

⁴³One may notice that wealth Gini from the data is lower than the values typically obtained using the Survey of Consumer Finances (SCF) (e.g., Díaz-Giménez et al. 2011), which oversamples rich households to better capture the highly concentrated wealth distribution. This discrepancy arises because Heathcote et al. (2010) truncate the top distribution (roughly top 1.5 percent) of their SCF samples to make the SCF statistics comparable to the statistics from other data sets including the Current Population Survey. See Heathcote et al. (2010) for more detailed discussion.

⁴⁴The income shown in the Table A1 are based on the sum of earnings, private transfers and asset income before the tax and government transfers.

⁴⁵Such quantitative studies of wealth inequality include Krusell and Smith (1998), Quadrini (2000), Castaneda et al. (2003), De Nardi (2004), Cagetti and De Nardi (2006), and Guvenen (2006) among others.

Table A1: Cross-sectional inequality: model and data

	Gini coefficient	
	Model	Data
Earnings	.31	.37
Income	.31	.38
Wealth	.44	.69

Notes: The statistics in U.S. data are obtained as follows. Five-year earnings Gini and income Gini reported in this table are computed by multiplying the Gini indices using annual household-level data (Current Population Survey) in Heathcote et al. (2010) by the Shorrocks mobility coefficient for earnings in Kopczuk et al. (2010). Wealth Gini is from Heathcote et al. (2010). For consistency, the model statistics are computed using the same age restrictions (25-60) as in the two empirical studies above.

Table A2: IGE estimates: life-cycle bias

		Child's age								
		20	25	30	35	40	45	50	55	60
Parent's age	20	.16	.25	.27	.27	.26	.26	.26	.26	.25
	25	.32	.54	.59	.58	.58	.58	.57	.57	.55
	30	.22	.37	.40	.40	.40	.40	.40	.39	.38
	35	.21	.34	.37	.37	.37	.37	.36	.36	.35
	40	.19	.32	.35	.35	.34	.34	.34	.34	.33
	45	.18	.30	.32	.32	.32	.32	.32	.31	.31
	50	.17	.28	.30	.30	.30	.30	.30	.30	.29
	55	.16	.26	.29	.28	.28	.28	.28	.28	.27
	60	.14	.24	.26	.26	.26	.26	.26	.25	.25

Notes: Ages denote the first age of the five year period at which lifetime income is measured.

Table A3: Percentile rank correlation estimates: life-cycle bias

		Child's age								
		20	25	30	35	40	45	50	55	60
Parent's age	20	.15	.17	.17	.16	.15	.15	.14	.14	.13
	25	.42	.52	.50	.48	.46	.44	.43	.41	.39
	30	.33	.41	.40	.38	.36	.35	.34	.32	.31
	35	.32	.39	.38	.36	.35	.34	.32	.31	.30
	40	.30	.38	.37	.35	.34	.32	.31	.30	.29
	45	.29	.37	.35	.34	.32	.31	.30	.29	.28
	50	.28	.36	.34	.33	.31	.30	.29	.28	.27
	55	.27	.34	.33	.31	.30	.29	.28	.27	.26
	60	.25	.32	.31	.30	.28	.27	.26	.26	.24

Notes: Ages denote the first age of the five year period at which lifetime income is measured.

lifetime income, is 0.366 as reported in Table 5, the estimates vary quite significantly from 0.14 to 0.59 depending on the timing at which income is measured. There are several systematic patterns regarding the lifecycle bias. First, regardless of when the parent's income is measured, the IGE estimates are seriously downward-biased if the child's income is measured early in their life. For instance, when the child's income is measured at 20-24, the IGE estimates are close to half the true IGE (0.366). This is consistent with prior empirical research that points out attenuation bias when children's income is measured at early ages (Solon, 1999; and Haider and Solon, 2006). On the other hand, holding the parent's age fixed, the IGE estimates become insensitive to a point at which child's income is measured as long as it is measured at the age of 30 or above.

Interestingly, similar yet different patterns of biases arise with respect to the parent's age at which the proxy income is taken. First, if the parent's income is measured before the age of 25, the IGE estimates are downward-biased.⁴⁶ Second and more importantly, the IGE estimates are significantly overstated if the parent's income is measured right after the college education stage, and then they decrease monotonically as the parent's income is measured at later ages. Therefore, holding the child's age fixed at an age greater than 30, the sign of the bias changes around the parent's age of 40. Although the same pattern is observed with respect to the timing at which the child's income is measured, it is important to note that the size of the bias in the IGE estimates is found to be much larger with respect to the timing at which parent's income is measured.

In Table A2, I perform the same exercise with a rank-rank specification instead of a

⁴⁶This finding may not be a relevant issue because, in practice, while the case in which child's income is measured at early ages is quite common (since data on two successive generations should be linked), such a case in which parent's income is measured at early ages is much less likely.

log-log specification. Overall, we can see the right-skewed inverse U shape in the rank-rank slope as a function of the timing at which child’s (or parent’s) income is measured: that is, (i) there is attenuation bias when either child’s income or parent’s income is measured at early ages; and (ii) the slope estimates rise sharply and then decrease gradually as the measured-age rises. There are three notable differences compared to the IGE estimates. First, the overall degree of bias in the rank correlation estimates is quite smaller than in the IGE estimates. For instance, except for the extreme cases when parent’s income is measured at age 20-24 or when child’s income is measured at age 20-24, the rank-rank estimates range from 0.27 to 0.52. In percentage terms, they range from 68% to 146% of the true slope. This is tighter than that of the log-log slope (IGE) estimates: 66%–160%. The second difference is that, holding the parent’s measured income fixed, the rank-rank slope estimates rise sharply in the early 20’s and then gradually decrease whereas the IGE estimates are virtually flat with respect to child’s age after they sharply rise in the 20’s. These findings are in line with Chetty et al. (2014a) who show that their rank-rank slope estimates rise steeply in the early 20’s and then steadily decrease after age 30. Third, holding the child’s measured income constant, the rank-rank slope is actually quite insensitive to parent’s age beyond age 30. This is in contrast to the IGE slope estimates which diminish quite rapidly as the parent’s age at which proxy income is taken.

C Determining parameters using simulation

A vector of the ten parameters $\hat{\Theta} = (B, \eta, \alpha_1, \alpha_2, \alpha_0, \sigma_\zeta, \sigma_z, m_\xi, \psi, \omega_s)$ in Table 3 is determined by simulating the model economy. Specifically, define $M_m(\Theta)$ as the m -th target statistic obtained from the model-generated data with the set of parameters Θ ; and D_m as the same i -th target statistics obtained from data, as defined in Table 4. Then $\hat{\Theta}$ is the minimizer of the objective function: $\sum_{m=1}^{10} [\log(M_m(\Theta)/D_m)]^2$. I use the downhill simplex method to solve this minimization problem. Each of the target statistics $\{M_m, D_m\}_{m=1}^{10}$ is chosen to be associated with a corresponding parameter(s) as described in Section 3.

D Data

Statistics regarding time-use are computed using the 2003-2012 waves of the American Time Use Survey (ATUS). To compute average hours worked and the fraction holds a college degree, I consider both men and women and include those whose age is greater than or equal to 20 and less than 65. To construct a variable of parental time investment in the child’s human capital, I focus on the educational activities that requires the existence

of both a parent and a child in a common space. Such categories include reading to/with children, playing with children, doing arts and crafts with children, playing sports with children, talking with/listening to children, looking after children as a primary activity, caring for and helping children, doing homework, doing home schooling, and other related educational activities. As the focus is the time investment in the young children's human capital, the time investment is computed using households whose youngest child is less than five years old. When computing the time investment variable, I also exclude households who are still enrolled in school and parents whose age is greater than 55. For all time-use statistics reported, the ATUS statistical weights are used.

Note that the parental time investment variable does not include the activity of physical care for children, which accounts for quite a large portion of time. However, it is interesting to note that, even with the definition of the parental time including the physical care activities, I also find a similar size of the positive educational gradient and it is robust to the parental gender as well. Furthermore, I also broaden the definition of the parental time investment to include some activities that have educational aspects but do not necessarily require the direct/active contact between a parent and a child. Such activities are organizing and planning for children, attending children's events, waiting for/with children, picking up/dropping off children, attending meetings and school conferences for children, waiting associated with children's education. The inclusion of such educational activities that could have indirect impacts on the children's human capital development increases the mean by 14 percent but barely changes the educational gradient. The time-diary survey also reports secondary activities and part of them may also include childcare. However, since the childcare time recorded as secondary activities is expected to be less active and the same hours may not be effective as an input to the human capital function, I do not consider the time of childcare recorded as secondary activities, and only focuses on childcare activities reported as a main activity.