Sales Talk*

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Abstract

How should a seller optimally sell his good to a buyer whose willingness to pay depends on his privately-known taste and on product characteristics privately known by the seller? The optimum is characterized by a mediated selling protocol and is sometimes implementable by bilateral face-to-face cheap talk after which the seller asks a price conditional on the conversation. Posted prices without cheap talk are suboptimal. The seller benefits ex-ante from private information and never benefits from committing to a disclosure or a certification technology. Ex-ante revenue-maximizing mechanisms are equilibria of this informed seller game and coincide with core mechanisms.

KEYWORDS: Informed seller; consumer heterogeneity; product information disclosure; mechanism design; value of information.

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The seminal papers of Myerson (1981) and Riley and Zeckhauser (1983) establish that a take-it-or-leave-it offer (a.k.a. posted price) is the profit-maximizing procedure for a seller who has an object to sell and faces a single buyer whose willingness to pay is private information. Yet, in practice, often we observe that sellers initiate the selling process by inviting non-binding offers and by engaging in information exchanges with the buyer before finalizing the asking price. These non-binding offers are also called "indications of interests" or "indicative bids." Formally, they are cheap talk claims about a buyer's willingness to pay for the asset. This process is particularly prevalent in the sale of very valuable assets such as private or public companies, and it is formally known as "book-building."[1]

Theoretically, it has proven difficult to justify the prevalence of such cheap talk communication: Posted prices are optimal when there is one buyer, while standard auctions with a reserve price are optimal when there are multiple buyers. Some theoretical justification is provided in Milgrom (2004), Ye (2007) and Quint and Hendricks (2013) who focus on the role of indicative bids as a way to select (pre-qualify) bidders in models where bidding in the actual auction is costly. Since these papers focus on the role of indicative bids as a bidder-selection tool, they do not justify the use of indication of interests for selling a good to a *single buyer*. Farrell and Gibbons (1989) and Matthews and Postlewaite (1989) show that cheap talk can expand the set of equilibrium payoffs in bargaining environments with private values.

Sellers often have private information about the asset for sale that is relevant to the buyer. At the same time the buyer's willingness to pay depends on idiosyncratic factors that are private information. In this paper we analyze such model where there is bilateral private information and the buyer's willingness to pay is described by a "match function" that depends on the type of the seller and on the type of the buyer. We show that bilateral cheap talk communication followed by price offers conditional on the communication strictly increases the seller's revenue compared to posted prices and, in certain cases, such a trading protocol turns out to implement the optimum. Thus, cheap communication followed by a take-it-or-leave-it offer is optimal or gets much closer to implementing the optimum compared to just posting price.

Formally, we are analyzing the problem of an informed principal (the seller) in which the valuation of the agent (the buyer) depends both on his type and on the seller's type and the seller cares only about revenue. The first set of questions we answer concerns the features and the implementation of revenue-maximizing procedures. The second set of questions concerns the value of private information for the seller. Under which conditions is the privacy of information about product characteristics valuable? Are there circumstances under which having private information is detrimental for the seller? Is it possible for the seller to leverage his private information and to extract the entire surplus of the buyer?

[1] Cornelli and Goldreich (2001) explain: "Under the formal book-building procedure, the investment banker solicits indications of interest from institutional investors. Such indications consist of a bid for a quantity of shares and might include a maximum price (i.e., a limit price) or other details." Boone and Mulherin (2007) show that indicative bids are prevalent both when multiple buyers are present and the seller employs an auction (which represents roughly half of company sales in their sample), as well as in cases where the seller negotiates with a single buyer (negotiations represent the other half of company sales in that sample). For a more recent example, see Graulich (2013).

The answer to these questions is of practical importance since in many real-world trading problems–the sale of experience goods (restaurant meals, vacation packages), credence goods (accounting services, legal or health advise) or other niche goods or services–sellers have information relevant to buyers. The literature on the design of revenue-maximizing selling procedures has been quite influential but has almost primarily focused on cases where the seller has *no* private information. In such a setup, Myerson (1981) and Riley and Zeckhauser (1983) show that when the seller faces one buyer the revenue-maximizing procedure is a posted price.[2] Yilankaya (1999) allows for bilateral private information but the seller's information does not affect the buyer's willingness to pay; that is, valuations are private. He shows that a posted price that depends on the seller's information is optimal and that revenue at the optimum is the same as when the seller's information is commonly known. This information-irrelevance result has been generalized in various ways by Tan (1996), Skreta (2011) and Mylovanov and Troeger (2013a,b).

At an abstract level, this paper contributes to the literature of mechanism design by an informed principal. Mechanism design by an informed principal is conceptually quite distinct from standard mechanism design where it is assumed that the principal has no private information. In that case the revelation principle reduces the search for the optimal mechanism to a constrained optimization problem. The problem is more difficult when the principal has private information since the selection of the mechanism itself may signal information about the principal. The mechanism-selection process can be viewed as a signaling game and it inherits all the intricacies of the bargaining literature where the informed party proposes, but is more complex since the proposals involve direct and indirect mechanisms and the consideration of off-path beliefs.

We now provide an overview of our approach and results.

Our methodology for informed principal. We are analyzing a game where an informed party (the seller) chooses mechanisms, so it is not a priori clear what an "optimal" mechanism is, since the mechanism-selection game can have multiple equilibria. To solve this problem we follow a somewhat indirect approach. We first ignore the issue of mechanism selection and focus on general mediated mechanisms that satisfy (interim) incentive and participation constraints for the seller and the buyer. Since the seller cares about maximizing revenue, incentive compatibility for the seller imposes that all seller types generate the same expected revenue. This simple, but important observation, implies that maximizing interim revenue for the seller coincides with ex-ante revenue maximization. We then show that for any mechanism that satisfies incentive and participation constraints only for the buyer, we can construct an equivalent mechanism in terms of interim payoffs for the buyer and ex-ante payoffs for the seller, that satisfies incentive compatibility for the seller. These two findings imply that maximizing the seller's revenue from the interim perspective subject to all the constraints for the seller and the buyer is equivalent to maximizing ex-ante revenue subject to the constraints only for the buyer.

[2]Posted prices remain optimal when the seller and the buyer interact repeatedly regardless of whether the seller can commit to the future sequence of mechanisms, write a long-term contact subject to renegotiation or even when he cannot commit (Hart and Tirole (1988) and Skreta (2006)).

The next set of results connects optimal mechanism with solution concepts of informed principal problems. We establish that in our model an ex-ante optimal mechanism is an expectational equilibrium of the mechanism-selection game. We also show that the set of ex-ante optimal mechanisms coincides with the set of core mechanisms as defined in Myerson (1983). Thus, in our model, mechanism-selection by an informed principal can be reduced to a constrained maximization problem.

Certification and Optimal Disclosure. The result that the seller's incentive compatibility constraints do not affect the seller's ex-ante expected revenue has another interesting implication: The seller never has any incentive to invest, ex-ante, in a certification technology (even if it is free), nor does he benefit from being able to commit ex-ante to *any* information disclosure policy. This result is not true if we restrict attention to posted prices (Koessler and Renault (2012)), but applies as long as the seller can use general selling mechanisms.

Characterization. The aforementioned results hold for very general environments provided that the seller cares only about revenue. They hold for general match functions and they do not depend on the structure nor on the dimensionality of the seller's and the buyer's type spaces. To provide an explicit analytical characterization of optimal mechanisms we impose a bit more structure: we assume that the buyer's type space is an interval of the real line and that the match function is convex in the buyer's type. The match function can depend in an arbitrary way on the seller's type and can be non-monotonic in the buyer's type. We find that, in contrast to the case of private values, posted prices are not optimal in general even if the match is strictly increasing in *both* the seller's and the buyer's type. The optimal mechanism typically conditions the payment and the probability of trade on both the seller's and the buyer's type, so it cannot take the form of a simple take-it-or-leave-it offer. The characterization of the optimal mechanism is illustrated in several examples.

Implementation. Despite the fact that the optimum cannot be implemented by a posted price in general, the optimum can sometimes be implemented by a selling protocol that involves dynamic and bilateral cheap talk communication between the buyer and the seller and different take-it-or-leave-it prices following the communication phase. In some cases, the seller sells the good for a low price but without information, while in other cases, the good bundled with information at a higher price. Buyers who are relatively indifferent about the specificities of the good choose the low price and no information. Buyers who do care significantly about the characteristics of the good are willing to pay the higher price for the good bundled with information. Such a selling procedure is "natural" in the sense that it neither requires a mediated mechanism nor any commitment. Its optimality and superiority over posted prices has been identified by Koessler and Renault (2012) in a discrete horizontal differentiation example, and recently generalized to a model with continuous buyers' types by Balestrieri and Izmalkov (2012). A similar example with both horizontal and vertical differentiation is used as an illustration of our general model.

Value of seller's private information. We establish that the seller is always (weakly) better off, ex-ante, when he is privately informed about product characteristics. Indeed, his maximal expected revenue is always weakly higher than his ex-ante expected revenue when his information is commonly known. This is not a priori obvious: it is not true if we restrict attention to posted prices, and it does not extend, in general, to interim revenues. We show that the situations where the seller's private information is irrelevant are actually quite special, thus, more often rather than not, the seller *strictly* benefits from having private information. We provide sufficient conditions on the match function and the distribution of the buyer' types for information irrelevance. Our conditions are different from those provided in the private-value setup of Mylovanov and Troeger (2013b). We also discuss how they relate to the conditions of Gershkov, Goeree, Kushnir, Moldovanu and Shi (2013) that deem Bayesian and dominant strategy implementation equivalent.

Related Literature. Despite the practical importance and the potential for numerous applications few papers have been written on mechanism-selection by informed principals following the seminal contributions of Myerson (1983), Maskin and Tirole (1990) and of Maskin and Tirole (1992). Myerson (1983) is completely general; it formulates the inscrutability principle and proposes various solution concepts for the mechanism-selection game, the notions of core, undominated, and safe mechanisms. In Maskin and Tirole (1992), Tisljar (2002, 2003), the agent has no private information. In Maskin and Tirole (1990) and Yilankaya (1999), players have private values (in particular, the agent's utility function does not depend on the principal's type). Maskin and Tirole (1990) show that, under some conditions, the informed principal neither gains nor loses if his private information is revealed before contracting takes place. Yilankaya (1999) considers a bilateral trading problem not covered by Maskin and Tirole (1990) and shows that the optimal mechanism is a posted price. In Mylovanov and Troeger (2013a,b), the agent's utility function does not depend on the principal's type. The most closely related paper is Balestrieri and Izmalkov (2012). They are the first to consider an informed seller problem in which the buyer's valuation depends on the buyer's and the seller's type. They solve a symmetric horizontal differentiation problem which belongs to our framework.

Apart from the literature on informed principal, this paper is related to the literature on signaling seller's information through the choice of the selling procedure. The seminal paper Milgrom and Roberts (1986), as well as the numerous papers that cite it, illustrates how the seller can signal information about an experience good through the choice of price. Cai, Riley and Ye (2007), Jullien and Mariotti (2006) and Kremer and Skrzypacz (2004) study how a revenue maximizing seller can signal information through the choice of the auction format and/or the reserve price. This paper is also related to the literature on seller's design of optimal information structures (Eső and Szentes (2007), Ottaviani and Prat (2001), Rayo and Segal (2010)) and disclosure of product information (Anderson and Renault (2006), Johnson and Myatt (2006), Sun (2011), Koessler and Renault (2012), Sun and Tyagi (2014)). The key difference of our work compared to these last papers is that the seller chooses general incentive compatible mechanisms rather than take-it-or-leave-it prices, so that the features of the optimal mechanisms specify the allocation (the price and the probability of trade) and the informativeness of the mechanisms as a function of both the buyer's and the seller's private information. In other words, we consider general selling procedures which include general communication channels, and take into account the incentive compatible conditions of the seller: the seller does not commit on a disclosure rule, what he discloses is a best response given his information.

1 Motivating Examples

In this section we characterize revenue-maximizing mechanisms in simple examples. The purpose of these examples is to illustrate the qualitative differences of our setup with the ones considered in the literature. In particular, the first example emphasizes the role of bilateral cheap talk in selling procedures. Except stated otherwise, all examples have uniformly distributed types.

Example 1 (Selling House Wine) [3] Consider the situation where a seller (for example, a restaurant owner) wants to sell a good (for example, a carafe of house wine) with characteristics s_L (for example, a Lirac) or s_R (for example, a Riesling). The characteristics of the good is private information to the seller. The buyer (for example, a customer of the restaurant) either only likes product s_L (and is willing to pay up to 30 for it), or he only likes product s_R (and is willing to pay up to 32 for it), or is indifferent about the product characteristics (and is willing to pay up to 20 for any variety). The taste of the buyer is private information to the buyer and is respectively denoted by t_l, t_r and t_i. The match function $v(s, t)$ describing the buyer's valuation as a function of the seller's information $s \in \{s_L, s_R\}$ and the buyer's taste $t \in \{t_l, t_r, t_i\}$ is described in Table 1.

$v(s,t) =$

	t_l	t_r	t_i
s_L	30	0	20
s_R	0	32	20

Table 1: The buyer's match function for house wine.

With a posted price and no information transmission between the seller and the buyer, the seller's highest feasible expected revenue is 15, in which case every buyer's type purchases the good at price 15. Another mechanism is to post a price of 20 after a fully revealing cheap talk message from the seller to the buyer, yielding expected revenue $20 \times 2/3 \simeq 13.3 < 15$. Posting a price equal to 20 is actually the revenue-maximizing feasible mechanism when the seller's type is common knowledge. Hence, this example already shows that, by restricting attention to simple take-it-or-leave-it posted prices, the seller is strictly better off when his type is privately known.

We now show that the seller can get an expected revenue even higher than 15 by posting different prices as a function of the outcome of a *bilateral cheap talk* communication game between the seller and

[3] A version of this example appeared in Koessler and Renault (2012) who study equilibrium information disclosure for a seller restricted to posting prices. It was inspired from a conversation with V. Bhaskar. The wine story was suggested by Marco Ottaviani.

the buyer. First, the seller asks the buyer whether he cares about product characteristics or not; that is, the buyer reveals whether his type is t_i or not (without telling whether his type is t_l or t_r). If the buyer reveals that his type is t_i, then the seller posts the price 20 without revealing any information about product characteristics, in which case the seller's revenue is 20. Otherwise, if the buyer reveals that his type is not t_i, then the seller posts the price 30 and reveals his information about product characteristics. In this case the seller's revenue is 30 if the buyer's taste and the product match, and is 0 otherwise. It is easy to check that this scenario constitutes an equilibrium of the communication game followed by a take-it-or-leave it offer by the seller and an acceptance decision by the buyer.

In the previous scenario, the seller of type s_R would like to sell his good at price 32 instead of 30 to the buyer's type t_r. However, if this was the case, the seller of type s_L would deviate and claim to be type s_R instead of s_L. Ideally, we would like to know what is the revenue-maximizing communication equilibrium followed by conditional prices in such a problem.[4] This is an interesting but extremely difficult problem is general.[5] Instead, in this paper we are looking at the revenue-maximizing mediated mechanism, which is incentive compatible for the buyer and the seller. In the example above, there is indeed an even better (mediated) selling procedure, which extracts all surplus from the buyer. To understand how to construct such a mechanism, assume that a mediator proposes a mechanism $\rho(s,t) \in [0,1] \times \mathbb{R}$ to the buyer and the seller, which specifies as a function of the report s from the seller and the report t from the buyer, the probability of trade as the first coordinate and the price paid by the buyer to the seller as the second coordinate.

Consider the following mechanism where in each cell of the matrix the first component is the probability of trade and the second one the expected payment:

$\rho(s,t) =$

	t_l	t_r	t_i
s_L	$1,30$	$0,0$	$1,20$
s_R	$0,0$	$1,32$	$1,20$

This mechanism is incentive-compatible for the buyer, satisfies his participation constraints, and it extracts all his surplus. However, it is not incentive-compatible for the seller since type s_L could get a higher expected revenue by claiming to be type s_R instead of s_L.

Consider now the mechanism $\hat{\rho}$ obtained from ρ by averaging payments across the seller's types, that is:[6]

$\hat{\rho}(s,t) =$

	t_l	t_r	t_i
s_L	$1,15$	$0,16$	$1,20$
s_R	$0,15$	$1,16$	$1,20$

[4] Notice that if $v(s_R,t_r) = 30$ instead of 32, then the previous selling procedure extracts all surplus from the buyer, and is therefore optimal among all possible selling procedures.

[5] In particular, equilibrium outcomes of dynamic unmediated cheap talk games are only well understood with one-sided incomplete information and finite action and type spaces (see, e.g., Aumann and Hart, 2003).

[6] This transformation is later used in Lemma 1 to show that the incentive compatibility constraints of the seller are irrelevant in terms of feasible ex-ante expected revenue.

This new mechanism is feasible, and it is clearly optimal since it extracts all surplus. The interim expected revenue of the seller is equal to 51/3 whatever his type, so it is strictly higher than his interim revenue obtained from any other selling procedure described above. In this example, it turns out that this interim revenue is also strictly higher than the highest feasible interim revenue of the seller when his type is commonly known ($51/3 > 20 \times 2/3$), whatever the type of the seller. This last property is however not general (see the next example).

One might wonder whether the optimal mechanism above could be selected in equilibrium by the seller himself, before the buyer decides to accept it or not. We show that the answer to this question is always positive: an ex-ante optimal mechanism is always an equilibrium of the mechanism selection game; however, the reverse is not true in general. In addition, the set of ex-ante optimal mechanisms exactly coincides with the set of core allocations as defined by Myerson (1983). ◇

In the house wine example the match function is non-monotonic in the seller's and in the buyer's type. In the following example we show that, even if the match function is both monotonic in the seller's and in the buyer's type, a posted price may still be sub-optimal:

Example 2 (Sub-optimality of posted prices with monotonic valuations) Consider the situation where both the seller and the buyer have two equally likely types, s_1 and s_2 for the seller and t_1 and t_2 for the buyer. The match function describing the buyer's valuation is given by:

$v(s,t) =$

	t_1	t_2
s_1	6	7
s_2	0	1

In this example, if the seller reveals his type to the buyer and posts a price, then, since the seller-incentive compatibility implies that both types must have the same expected revenue, the highest equilibrium revenue is 1/2, with a price of 1/2 for seller type s_1 (which is accepted by both buyer types) and a price of 1 for seller type s_2 (which is accepted only by buyer type s_2). Instead, posting a price of 3 without information revelation yields a revenue of 3 whatever the type of the seller (both buyer types buy at this price). Both these selling procedures are dominated by the following mechanism, which can be shown to be optimal and yields expected revenue of 6.5/2 for all types of the seller:

$\rho(s,t) =$

	t_1	t_2
s_1	$1, 6$	$1, 0.5$
s_2	$0, 0$	$1, 6.5$

Another optimal mechanism, obtained by averaging the price across the seller's types as in the previous example, is:

$$\hat{\rho}(s,t) = \begin{array}{|c|c|c|}\hline & t_1 & t_2 \\ \hline s_1 & 1,3 & 1,3.5 \\ \hline s_2 & 0,3 & 1,3.5 \\ \hline \end{array}$$

Contrary to the previous example, the optimal mechanism does not extract all the surplus and the seller is ex-ante indifferent between having and not having private information. To see this last point, note that when the seller's type is commonly known, it is optimal for s_1 to ask a price of 6 which is always accepted, and for s_2 to ask a price of 1 which is accepted with probability 1/2, yielding ex-ante revenue $\frac{6.5}{2}$ exactly as the optimal mechanism when the seller's information in private. Despite being ex-ante indifferent between having private information or not, at the interim stage seller type s_1 regrets that his type is private information, whereas type s_2 is glad. $\diamond$

These examples already illustrate several economic points that contrast some results of the earlier literature on informed seller problems and private values: First, the seller's type being private information may be ex-ante strictly beneficial to the seller. Second, bilateral cheap talk communication or more general mediated selling mechanisms are typically strictly better for the seller than simple take-it-or-leave it offers. Finally, posted prices are suboptimal even if the match function is strictly increasing in *both* the seller's and the buyer's type.

2 Model

Environment. We consider the following Bayesian incentive problem with a monopoly seller and a single buyer with unit demand.[7] The seller has perfect and private information about the *product's characteristics*, denoted by $s \in S$ and also called the *type of the seller*. The buyer has perfect and private information about his *taste*, denoted by $t \in T$ and also called the *type of the buyer*. Type spaces S and T are compact metric spaces. Types s and t are independently distributed according to full support probability distributions. Both the seller and the buyer are risk-neutral.

The *match function* $v(s,t) \in \mathbb{R}$ describes the buyer's valuation for the product as a function of his own type (his privately known taste t) and the type of the seller (the product's characteristics s). The value of the object for the seller and the outside option for the buyer are type-independent, and therefore normalized to 0.[8] We assume that $v(s,t)$ is absolutely continuous in t for every s.

A *mechanism*, denoted by $\rho = (p,x) : S \times T \to [0,1] \times \mathbb{R}$, is a mediated rule determining the probability $p(s,t)$ of sale (i.e., the probability that the buyer gets the good) and the expected transfer $x(s,t)$ from the buyer to the seller as a function of the seller's type s and the buyer's type t.[9] Given

[7] Equivalently, we could think about a continuum of anonymous buyers of mass one.

[8] Since the value for the seller is null, the efficient allocation is trivial: it is obtained when the good is sold with probability one whenever $v(s,t) > 0$. The case where seller has type-dependent valuations is discussed in Section 8.

[9] Hence, we assume implicitly that transfers are balanced ex-post. Without loss of generality, we can look at expected transfers because both players are assumed to be risk neutral.

risk-neutrality, the expected utility of the seller from ρ given s and t under truthful reporting is

$$x(s,t), \tag{1}$$

and the expected utility of the buyer from ρ given s and t is

$$u(s,t) = p(s,t)v(s,t) - x(s,t). \tag{2}$$

The interim expected payoffs from ρ for the seller and the buyer are respectively denoted by

$$X(s) \equiv E_T[x(s,t)], \tag{3}$$

and

$$U(t) \equiv E_S[u(s,t)] = E_S[p(s,t)v(s,t) - x(s,t)]. \tag{4}$$

Feasible Mechanisms. We say that a mechanism ρ is *feasible* if it satisfies (interim) incentive compatibility and participation constraints for the seller and for the buyer. That is, the seller and the buyer have an incentive, once they have privately learned their type, to reveal it truthfully to the mediator and to participate to the mechanism. For every s and s' in S, and for every t and t' in T:

$$X(s) = X(s'), \tag{S-IC}$$

$$X(s) \geq 0, \tag{S-PC}$$

$$U(t) \geq U(t'|t) \equiv E_S\left[p(s,t')v(s,t) - x(s,t')\right], \tag{B-IC}$$

$$U(t) \geq 0. \tag{B-PC}$$

Incentive compatibility for the seller is equivalent to

$$X(s) = X(s') = \bar{X} \equiv E_S[X(s)], \tag{5}$$

for all s and s'. In other words, incentive compatibility for the seller imposes that his interim expected revenue is the same whatever his type, so it is the same as his ex-ante expected revenue. Given the importance of this observation for the subsequent analysis, we state it as a proposition.

Proposition 1 *At every feasible mechanism the seller's interim payoff coincides for all types and is equal to his ex-ante payoff. Hence, under feasibility, maximizing the seller's revenue ex-ante is the same as maximizing his revenue whatever his type.*

Given this observation, we do not have to distinguish between ex-ante and interim optimality for the seller and we can define:

Definition 1 The mechanism (p, x) is *optimal* if it maximizes the ex-ante revenue $\bar{X} = E_S[X(s)]$ under the feasibility constraints, i.e., under the incentive compatibility and participation constraints (S-IC), (S-PC), (B-IC) and (B-PC).

We say that a mechanism is a full-information optimal mechanism[10] if it maximizes the seller's interim revenue whatever its type when the seller's type is commonly known. More precisely:

Definition 2 The mechanism (p, x) is a *full-information optimal mechanism* if for every $s \in S$ it maximizes $X(s)$ under the following ex-post incentive compatibility and participation constraints of the buyer:

$$u(s,t) \geq p(s,t')v(s,t) - x(s,t'), \text{ for every } t, t' \in T; \tag{6}$$

$$u(s,t) \geq 0, \text{ for every } t \in T. \tag{7}$$

Comparing the optimal ex-ante revenue with the full-information optimal ex-ante revenue enables us to evaluate the ex-ante value of private information for the seller.

3 Mechanism Selection Game: The Informed Principal Problem

So far we have talked about feasible mechanisms without being explicit about who chooses a mechanism. Following most of mechanism design literature, we give full bargaining power to one of the parties to choose the mechanism. We assume that the seller selects the mechanism after he has learned his type. Therefore, we are dealing with an *informed principal* problem, where the principal is the seller, and the agent is the buyer.

3.1 Foundations

We first summarize some fundamental results and solution concepts on informed-principal problems (see Myerson, 1983 and Mylovanov and Troeger, 2013a for more details). The two fundamental results are the inscrutability and the revelation principle.

Inscrutability Principle. Following Myerson (1983) we can assume, without loss of generality, that the seller proposes (along the equilibrium path) the same mechanism ρ whatever his type s, so that the choice of the mechanism conveys no information to the buyer. In that way, we get the weakest possible informational and participation constraints for the buyer since no information about the seller's type is revealed before the buyer makes his message and participation choices.

Revelation Principle. According to the revelation principle (e.g., Myerson, 1982) we can assume, without loss of generality, that the principal proposes (along the equilibrium path) a direct revelation mechanism, and that after the mechanism has been chosen all players (the principal and the agent)

[10] This terminology was first used in Maskin and Tirole (1990, 1992). Notice that it only refers to the seller's information about s being commonly known; the agent's information about t is still private.

reveal truthfully their types to the mediator implementing the mechanism and follow his recommendation to accept or reject the mechanism. In addition, since we have only one buyer we are in a particular case (described page 1772 in Myerson, 1983) in which, along the equilibrium path, we can consider without loss of generality mechanisms in which, whatever (s,t), no player is ever asked to reject, since $\rho(s,t) = (0,0)$ (no transaction) could be used instead.

We now turn to the main solution concepts used in the informed principal literature. A mechanism ρ is called *dominated* by another mechanism ν iff all principal's types are better-off under ν than under ρ and at least one principal type is strictly better-off under ν. The mechanism ρ is *undominated* iff it is feasible and not dominated by any other feasible mechanism. A mechanism is called *safe* if it is feasible and satisfies ex-post incentive compatibility and participation constraints for the buyer. That is, a safe mechanism is feasible regardless of the buyer's beliefs about the seller's type. A safe and undominated mechanism is called a *strong solution*. Notice that if the full-information optimal mechanism turns out to satisfy the incentive compatibility constraint for the seller then it is safe (but it is not necessary a strong solution because it may be dominated).

To define the next solution concept we need the following definition of feasibility introduced in Myerson (1983).

Definition 3 A mechanism ρ is *feasible given* $\bar{S} \subseteq S$ if it satisfies incentive compatibility and participation constraints for the seller (that is, (S-IC) and (S-PC)), and the following conditional incentive compatibility and participation constraints for the buyer:[11]

$$E_{\bar{S}}\big[p(s,t)v(s,t) - x(s,t)\big] \geq E_{\bar{S}}\big[p(s,t')v(s,t) - x(s,t')\big],$$

$$E_{\bar{S}}\big[p(s,t)v(s,t) - x(s,t)\big] \geq 0.$$

Definition 4 A mechanism ρ is a *core mechanism* if it is feasible, and if ν is a mechanism that is preferred by some types $s \in S$ of the seller, then ν should not be feasible given *at least some* $\bar{S} \supseteq S^*$, where S^* is the set of principal types that strictly prefer ν to ρ.[12]

To define an equilibrium of the mechanism selection game, Myerson (1983) considers the notion of *generalized mechanism* with arbitrary sets of reports from the players to the mediator, and arbitrary sets of instructions from the mediator. This is because off-the-equilibrium the seller may not necessarily use direct and truthful mechanisms. For each strategy profile that is a Nash Equilibrium of the generalized mechanism given some beliefs of the buyer, corresponds an equilibrium outcome which consists of a pair of allocation and payment rules $(\tilde{p}, \tilde{x})$. Roughly put (for full details see Myerson (1983) pages 1779–1780), a mechanism $\rho = (p,x)$ is an *expectational equilibrium* iff it is feasible, and for every generalized mechanism $\tilde{\rho}$, there exists a belief μ for the buyer, a reporting and participation

[11] $E_{\bar{S}}[\cdot]$ denotes the conditional expectation over S given $s \in \bar{S}$.

[12] Mylovanov and Troeger (2013a) introduced a stronger notion than the core mechanism, called (strong) *neologism-proof mechanism*. The definition is similar to a core mechanism, but now ν should not be incentive compatible for a restricted set of beliefs over S^* (see Mylovanov and Troeger, 2013a for more details on this restriction).

strategy profile that are a Nash Equilibrium of $\tilde{\rho}$ given μ, with associated outcome $(\tilde{p}, \tilde{x})$ such that for all $s \in S$:

$$X(s) \equiv E_T[x(s,t)] \geq E_T[\tilde{x}(s,t)] \equiv \tilde{X}(s).$$

Theorem 2 in Myerson (1983) shows that any strong solution is an expectational equilibrium (but a strong solution need not exist).

3.2 The Problem Resolved

The results of this subsection resolve the complications arising from the fact that the party choosing the mechanism–the seller–has private information. The first result, Proposition 2, shows that the set of ex-ante optimal mechanisms coincides with the set of core mechanisms as defined in Myerson (1983). The second result, Proposition 3, establishes that an ex-ante optimal mechanism is an expectational equilibrium of the mechanism-selection game. This result reduces mechanism-selection by an informed principal to a constrained maximization problem.

We first notice that the property of equal interim expected revenue for the seller whatever his type in a feasible mechanism also holds for a Nash equilibrium outcome $(\tilde{p}, \tilde{x})$ (given some belief μ) of a *generalized* mechanism off the equilibrium path. To see this, suppose that the equilibrium outcome $(\tilde{p}, \tilde{x})$ of a generalized mechanism is such that $\tilde{X}(s') > \tilde{X}(s)$. Then, given this generalized mechanism, seller type s can deviate and mimic the behavior of s', which is strictly profitable. This contradicts the fact that we are considering an equilibrium outcome of $(\tilde{p}, \tilde{x})$. Hence, the equilibrium condition for the seller implies that

$$\tilde{X}(s) = \tilde{X}(s') = E_S[\tilde{X}(s)], \text{ for all } s \text{ and } s'. \tag{8}$$

We also notice that an optimal mechanism is undominated, and any feasible mechanism which is not optimal is dominated by another feasible mechanism. However, an optimal mechanism may not be safe, as in the two introductory examples. We now establish that, in our framework, the set of optimal mechanisms exactly coincides with the set of core mechanisms.

Proposition 2 *The set of optimal mechanisms coincides with the set of core mechanisms.*

Proof. We first argue by contradiction that an optimal allocation is in the core. Consider an optimal mechanism (p, x) and suppose that it is not in the core. Then, there exists an alternative mechanism $(\tilde{p}, \tilde{x})$ that is strictly preferred by some types of the seller (it generates strictly higher interim expected revenue) and it is feasible given any superset of the set of types that strictly prefer it compared to (p, x). In particular, it is feasible given the whole set S, which implies that the associated interim expected revenue of the seller, $\tilde{X}(s) = \tilde{X}$, does not depend on s. Since the mechanism (p, x) is also feasible, $X(s) = X$ does not depend on s. Hence, $\tilde{X} > X$, which contradicts the optimality of (p, x).

We now argue that a core mechanism is optimal. Take a core mechanism (p, x) and suppose that it is not optimal, this means that there exists another mechanism $(\tilde{p}, \tilde{x})$ that is feasible and generates strictly higher revenue $\tilde{X} > X$. But, then $(\tilde{p}, \tilde{x})$ is strictly preferred by all seller types and is feasible given the entire set S so it blocks (p, x), contradicting the fact that (p, x) is in the core. ■

The previous proposition establishes the equivalence of ex-ante optimal mechanisms with core mechanisms, a cooperative solution concept. But are ex-ante optimal mechanisms part of an equilibrium of the informed seller mechanism-selection game? The next result establishes that an ex-ante optimal mechanism is always an expectational equilibrium:

Proposition 3 *An optimal mechanism is an expectational equilibrium.*

Proof. Let (p, x) be an optimal mechanism. We argue that the seller proposing this mechanism is part of an expectational equilibrium. Suppose that the buyer's beliefs when he observes any deviation to a mechanism (not necessarily direct) remain equal to the prior; that is he has passive beliefs off the equilibrium path. Consider any deviation to an alternative mechanism, and let $(\tilde{p}, \tilde{x})$ be a Nash equilibrium outcome generated by this alternative mechanism when the buyer has passive beliefs. As we established in (8), all types of the seller expect the same revenue $\tilde{X}$. Then, optimality of (p, x) implies $\tilde{X} = \tilde{X}(s) \leq X = X(s)$ for every s, which means that the deviation is not profitable for the seller whatever his type. Hence, (p, x) is an expectational equilibrium mechanism with passive beliefs off the equilibrium path. ■

Proposition 3 turns the informed seller into a maximization problem that can be solved using analogous techniques as the ones used to solve standard mechanism design problems, where an uninformed party is choosing the mechanism.

Before we close this section, we would like to note that the optimal mechanism may not be the unique expectational equilibrium. In particular, we have:

Proposition 4 *Every feasible mechanism in which the interim revenue is higher than the full-information optimal interim revenue is an expectational equilibrium.*

Proof. Let $X^*(s)$, $s \in S$, be a profile of feasible interim revenues for the seller. By (S-IC) we have $X^*(s) = X^*$, for every $s \in S$. Denote by $X^\#(s)$ the full-information optimal interim revenue of the seller when his type is s, and assume that $X^* \geq X^\#(s)$ for every $s \in S$. Assume by way of contradiction that X^* is not an equilibrium revenue. Then, there exists a (generalized) mechanism M such that, for any belief μ of the buyer after observing the deviation to M, there exists a seller's type $s \in S$ such that

$$X[M, \mu](s) > X^*,$$

where $X[M, \mu](s)$ is an interim Nash equilibrium revenue of type s induced by M and μ. By the seller's incentive compatibility constraint $X[M, \mu](s)$ does not depend on s. In particular, $X[M, \mu](\mu) > X^*$ for any degenerated belief $\mu \in S$. This yields $X[M, \mu](\mu) > X^\#(\mu)$, a contradiction with the assumption that $X^\#(\mu)$ is the full-information optimal interim revenue for the seller's type $\mu \in S$. ■

As an illustration, consider the house wine example of the introduction. The full-information optimal interim revenue of the seller is equal to $40/3$ whatever the seller's type, while the interim optimal revenue is $51/3 \geq 40/3$. Since in this example the full-information optimal mechanism is feasible, we know from Proposition 4 that it is an expectational equilibrium. The posted price mechanism with no information transmission is also an expectational equilibrium because it yields an expected revenue of $15 \geq 40/3$. Similarly, the mechanism implemented with bilateral cheap talk and contingent prices, which yields an expected revenue of $50/3 \geq 40/3$, is an expectational equilibrium. However, by Proposition 2 none of these mechanisms is a core mechanism.

4 Does Private Information Hurt or Benefit the Seller?

A natural question to ask is whether a privately informed seller fairs better compared to a seller whose information is common knowledge, or to a seller who can credibly commit to disclose his information truthfully to the buyer. More generally, we would like to assess whether the seller benefits from having access to any disclosure or certification technology.

A central step towards addressing these questions is to investigate the impact of the seller-incentive compatibility constraints on the set of feasible allocations. Interestingly, we establish that requiring a mechanism to be incentive compatible for the seller–that is, to satisfy (S-IC)–does not affect the set of feasible interim payoffs for the buyer and the set of feasible ex-ante revenues for the seller:

Lemma 1 *Take a direct mechanism (p, x) that gives the buyer interim payoff $U(t'|t)$, $t, t' \in T$, when his type is t and he reports t'. There exists a mechanism $(\tilde{p}, \tilde{x})$ that satisfies the seller's incentive constraint, generates the same ex-ante revenue for the seller, and gives the buyer the same interim payoff, that is $\tilde{U}(t'|t) = U(t'|t)$, for all $t, t' \in T$.*

Proof. Fix a mechanism (p, x), and let

$$\tilde{x}(s, t) = E_S[x(s, t)],$$

$$\tilde{p}(s, t) = p(s, t),$$

for all $s, t \in S \times T$. Clearly, the ex-ante expected revenue induced by $(\tilde{p}, \tilde{x})$ is the same as the ex-ante expected revenue induced by (p, x). Also, $(\tilde{p}, \tilde{x})$ is incentive compatible for the seller since $\tilde{x}(s, t)$ does not depend on s. In addition, the interim expected utility of the buyer of type t who behaves as t' under the mechanism $(\tilde{p}, \tilde{x})$ is the same as his interim expected utility under the mechanism (p, x), for all $t, t' \in T$. ■

Lemma 1 has several important consequences. First, it implies that the seller always (weakly) benefits in terms of ex-ante expected revenue from having private information.

Proposition 5 The seller benefits from private information. *The optimal mechanism generates weakly higher ex-ante expected revenue compared to the full-information optimal mechanism.*

Proof. First observe that the full-information optimal mechanism may not be incentive-compatible for the seller when his type is private information. However, following Lemma 1, we can construct an equivalent mechanism in terms of ex-ante expected revenue and interim payoff of the buyer that is incentive compatible for the seller. Hence an allocation that generates the same ex-ante revenue for the seller is feasible. The conclusion immediately follows. ■

We already know from the examples in Section 1 that the seller sometimes strictly benefits from private information. Proposition 5 establishes that he never regrets having private information, while in Section 7 we investigate the conditions that render the seller indifferent from having and not having private information. In Appendix A we also provide conditions under which the seller can leverage the fact that his type is unknown to the buyer to extract the entire surplus.[13]

A second implication Lemma 1 is that access to certification and disclosure rules does not benefit the seller:

Proposition 6 Access to Certification and Disclosure Rules does not benefit the seller. *The ability of the seller to certify his information or to commit to some information disclosure rule ex-ante does not lead to a higher ex-ante expected revenue.*

Indeed, any additional disclosure about the seller's type would just make the incentive compatibility and participation constraints for the buyer harder to satisfy. Hence, the seller would never have any incentive, ex-ante, to invest in any disclosure or certification technology.

We complete this section with an observation that allows us to consider a relaxed version of the program that characterizes optimal mechanisms.

Corollary 1 *If a mechanism satisfying (B-IC) and (B-PC) induces a positive ex-ante expected revenue, then there is a feasible mechanism that induces the same ex-ante expected revenue and interim payoff for the buyer.*

Hence, to find the optimal mechanism for the seller we can maximize his ex-ante expected revenue subject to (B-IC) and (B-PC) while ignoring (S-IC) and (S-PC).

5 The Seller's Mechanism-Selection Program

An implication of Propositions 2 and 3 is that we can obtain the set of core mechanisms, as well as mechanisms that are part of an equilibrium of the mechanism-selection game by solving for the ex-ante

[13] Our conditions encompass situations in which the buyer has no private information as is the case in Maskin and Tirole (1992) and Tisljar (2002, 2003). They also apply to the house wine example (Example 1) and to the horizontal and vertical differentiation example of Section 6 when $V_0 > 1$ and $V_1 > 1$.

optimal mechanism subject to (S-IC), (S-PC) and (B-IC), (B-PC). Lemma 1 and Corollary 1 imply, in turn, that we can ignore (S-IC) and (S-PC). Thus, the informed seller problem reduces to solving:

$$\text{Maximize } \bar{X} = X(s) = X(s')$$

$$\text{subject to (B-IC), (B-PC) and resource constraints.}$$

We now investigate the implications of (B-IC) and (B-PC) for feasible and optimal mechanisms and payoffs. We start with implications that hold for the abstract type spaces and match functions we have been considering so far, and then examine the case where the buyer's type belongs in compact subset of the real line (and is thus single-dimensional) and the match function is convex in the buyer's type.

5.1 General Implications and Properties

Lemma 2 *At an optimal mechanism $U(t^*) = 0$ for some $t^* \in T$.*

Proof. Take a feasible mechanism (p, x) that maximizes the seller's revenue and let $U(t)$ denote the corresponding interim expected payoff of the buyer. Let t^* be the type that minimizes $U(t)$, which exists by continuity of U and compactness of T.[14] We now show by contradiction that at a revenue-maximizing mechanism it holds that $U(t^*) = 0$. Suppose not, that is $U(t^*) \equiv \Delta > 0$. Then the seller can employ a mechanism $(\tilde{p}, \tilde{x})$, with $\tilde{p} = p$ and $\tilde{x}(s, t) = x(s, t) + \Delta$. It is immediate that the new mechanism is incentive-compatible since $\tilde{p} = p$, and the payment rule is modified by the *same* constant for all types. To see that it satisfies participation constraints for all types, note that for all $t \in T$, it holds that

$$U(t) \geq U(t^*),$$

which follows from the fact that t^* minimizes U. Since $(\tilde{p}, \tilde{x})$ sets $U(t^*) - \Delta = 0$, it satisfies participation constraints for all t. Also, since given $(\tilde{p}, \tilde{x})$ all types pay $\Delta > 0$ more compared to (p, x), it strictly dominates it in terms of revenue, contradicting the optimality of (p, x). ■

In general, the buyer's participation constraints bind at an endogenously determined type. Hence, finding the optimal allocation (p, x) may be quite complex because we do not a priori know where the participation constraints bind. For this reason, it is interesting to know when participation constraints bind at the same type at all feasible mechanisms (p, x). We now provide conditions for this to be the case.

For every $s \in S$, consider the set $\arg\min_t v(s, t)$ of minimizers of the buyer's valuation when the seller's type is s. This set is non-empty since T is compact and $v(s, t)$ is continuous in t for every

[14]There can actually be a set of minimizers but we can, without loss of generality, pick any one. Continuity of U follows from Theorem 2 of Milgrom and Segal (2002), since the buyer's type belongs to a compact metric space which is a normed vector space and since $pv(s, t) - x$ is absolutely continuous in t for every (p, x), which follows since we have assumed that $v(s, .)$ is absolutely continuous in t.

s. When this set is a singleton, or when the selection of the minimizer is irrelevant, we denote by $t^{min}(s) \in \arg\min_t v(s,t)$ a (the) minimizer of $v(s,t)$ given s.

Lemma 3 *The buyer's interim expected utility is minimized at the same type $t^{min} \in T$ at all mechanisms (p,x) that satisfy (B-IC) if and only if $t^{min} \in \arg\min_t v(s,t)$ for every s.*

Proof. We first show when t^{min} minimizes $v(s,t)$ in t for all s, then at every (B-IC) mechanism (and, therefore, at every feasible mechanism) $U(t) = E_S[u(s,t)]$ is minimized at $t = t^{min}$. To see this, note that

$$E_S[u(s,t)] \geq E_S\left[p(t^{min},s)v(t,s) - x(t^{min},s)\right] \geq E_S\left[p(t^{min},s)v(t^{min},s) - x(t^{min},s)\right] = E_S\left[u(s,t^{min})\right],$$

where the first inequality follows from (B-IC), and the second follows from the fact that t^{min} is a minimizer of v for each s.

Now we establish that if the buyer's interim expected utility is minimized at the same type $t^{min} \in T$ at all (B-IC) allocations (p,x) then it must be the case that $t^{min} \in \arg\min_t v(s,t)$ for every s. To see this, consider the following family of allocations: Fix any $\hat{s} \in S$ and let (p,x) be such that $p(s,t) = 1$ for all t if $s = \hat{s}$ and $p(s,t) = 0$ for all t if $s \neq \hat{s}$; and $x(s,t) = 0$ for all s and t. Such an allocation trivially satisfies (B-IC) since it does not depend on t. Since the buyer is only awarded the good at $\hat{s}$ for all t, we have $E_S[u(s,t)] = v(\hat{s},t)$, which is minimized at $t^{min} \in \arg\min_t v(\hat{s},t)$. Since, we supposed that the buyer's interim expected utility is minimized at the same type at all (B-IC) allocations (p,x), it must also satisfy this requirement at the particular family of (B-IC) allocations we chose. This implies $t^{min} \in \arg\min_t v(\hat{s},t)$ for all $\hat{s} \in S$. ■

The two previous lemmas imply the following corollary:

Corollary 2 *Assume that $t^{min} \in \arg\min_t v(s,t)$ for every s. At an optimal mechanism, the buyer's participation constraint binds at t^{min}, i.e., $U(t^{min}) = 0$.*

5.2 Convex Environments

From now on we impose more structure. We assume that $t \in T = [\underline{t}, \bar{t}]$ is distributed according to a continuous density function f with c.d.f. F, where $0 \leq \underline{t} < \bar{t} < +\infty$. No specific assumption is required on the set of types of the seller. We also assume in this section that for every $s \in S$, $v(s,t)$ is not only absolutely continuous but also convex in t and that it has a unique minimizer $t^{min}(s) \in \arg\min_t v(s,t)$ for every s.

Assumption C (Convexity). For every $s \in S$, $v(s,t)$ is convex in t and has a unique minimizer $t^{min}(s) \in \arg\min_t v(s,t)$ for every s.

5.2.1 Implications of the Buyer's Incentive Compatibility Constraints

Incentive compatibility for the buyer is equivalent to

$$t \in \arg\max_{t' \in T} E_S \left[p(s,t')v(s,t) - x(s,t') \right]. \tag{9}$$

Notice that $E_S [p(s,t')v(s,t) - x(s,t')]$ is convex in t since $v(s,t)$ is convex in t. Hence, $U(t)$ is also convex in t because it is a maximum of convex functions. Let[15]

$$P(t) \equiv E_S \left[p(s,t) \frac{\partial v(s,t)}{\partial t} \right]. \tag{10}$$

When $\frac{\partial v(s,t)}{\partial t} = 1$, $P(t)$ is the interim probability that the buyer of type t obtains the good given the mechanism (p,x). The incentive constraints of the buyer translate into the following well-known requirements (see, e.g., Figueroa and Skreta, 2011).

Lemma 4 *Let $t^* \in T$ be any type of the buyer. A mechanism (p,x) is incentive-compatible for the buyer iff*

$$P(t') \geq P(t) \qquad \text{for all } t' \geq t; \tag{11}$$

$$U(t) = U(t^*) + \int_{t^*}^{t} P(\tau) d\tau \quad \text{for all } t \in T. \tag{12}$$

The proofs of the results in this subsection are in the Appendix.

Corollary 3 *Let $t^* \in T$ be any type of the buyer. The incentive compatibility condition (12) for the buyer is satisfied if and only if, for every $t \in T$:*

$$\hat{X}(t) \equiv E_S [x(s,t)] = E_S \left[p(s,t)v(s,t) - \int_{t^*}^{t} p(s,\tau) \frac{\partial v(s,\tau)}{\partial \tau} d\tau \right] - U(t^*). \tag{13}$$

Let

$$J^L(s,t) \equiv v(s,t) + \frac{F(t)}{f(t)} \frac{\partial v(s,t)}{\partial t}, \; J^R(s,t) \equiv v(s,t) - \frac{1-F(t)}{f(t)} \frac{\partial v(s,t)}{\partial t}.$$

and for every $t^* \in T$,

$$J(s,t;t^*) \equiv \begin{cases} J^L(s,t) & \text{if } t < t^* \\ J^R(s,t) & \text{if } t > t^*. \end{cases}$$

With the help of this notation we can, as usual, express the seller's revenue at an incentive compatible mechanism as a function of the allocation rule:

[15] By absolute continuity of $v(s,t)$ in t, its derivative with respect to t exists almost everywhere but we omit the qualification henceforth.

Lemma 5 *At a mechanism satisfies incentive compatibility for the buyer, the ex-ante expected payoff of the seller can be expressed as:*

$$\bar{X} = \int_{\underline{t}}^{\bar{t}} E_S \left[p(s,t)J(s,t;t^*)\right] f(t)dt - U(t^*). \tag{14}$$

where $t^* \in T$.

5.2.2 Obtaining Optimal Mechanisms

Since we can ignore the seller's incentive constraint, the problem of the seller is to *find* $t^* \in T$ and $p(\cdot,\cdot)$ that maximize (14) with $U(t^*) = 0$ under the condition that $P(\cdot)$ is increasing and $\int_{t^*}^{t} P(\tau)d\tau \geq 0$ (i.e., $U(t) \geq 0$) for every $t \in T$.

Proposition 7 *Under Assumption C, the type* t^* *and assignment rule* p *that maximize*

$$\bar{X} = \int_{\underline{t}}^{\bar{t}} E_S \left[p(s,t)J(s,t;t^*)\right] f(t)dt, \tag{15}$$

subject to $P(\cdot)$ *is increasing and* $P(t) \leq 0$ *for* $t < t^*$ *and* $P(t) \geq 0$ *for* $t > t^*$ *fully characterize the ex-ante revenue-maximizing feasible mechanism for the seller.*

Proof. Feasibility follows from Lemma 4 and the following observations: The constraints that $P(\cdot)$ is increasing and $\int_{t^*}^{t} P(\tau)d\tau \geq 0$ together are equivalent to: $P(\cdot)$ is increasing and $P(t) \leq 0$ for $t < t^*$ and $P(t) \geq 0$ for $t > t^*$. That is, under (B-IC), the participation constraint for the buyer is simply $P(t) \leq 0$ for $t < t^*$ and $P(t) \geq 0$ for $t > t^*$. The payment rule x can be obtained from p via (13). ■

Given the observations in Proposition 7, a procedure that gives the optimal mechanism is: For every $t^* \in T$, choose the assignment rule that maximizes (15) subject to $P(\cdot)$ is increasing and $P(t) \leq 0$ for $t < t^*$ and $P(t) \geq 0$ for $t > t^*$. Denote the maximal revenue $\bar{X}[t^*]$. Then repeat the process for all $t^* \in T$. The optimal mechanism is obtained for the t^* that maximizes $\bar{X}[t^*]$. This procedure is illustrated in Section 6.[16]

5.2.3 Full-Information Optimal Mechanisms

As a benchmark, we characterize optimal mechanisms when the seller's type is common knowledge–the *full-information optimal* mechanism. In this case the incentive and participation constraints for the buyer are the hardest to satisfy, since they have to hold for every realization of the seller's type s, rather than in expectation.

[16] Ledyard and Palfrey (2007) characterize optimal mechanisms in a private-value environment with linear and hence monotonic payoffs. Mylovanov and Troeger (2013b) extend the analysis of private values by allowing non-monotonic payoffs. We differ in two respects: First, the match function is not only general and non-monotonic in the agent's type, but depends also on the principal's type. Second, we propose an alternative solution procedure.

Proposition 8 *Suppose that Assumption C holds. Consider the following mechanism:*

$$p^{\#}(s,t) \equiv \begin{cases} 1 & \text{if } J(s,t;t^{min}(s)) > 0, \\ 0 & \text{if } J(s,t;t^{min}(s)) \leq 0, \end{cases}$$

and

$$x^{\#}(s,t) = p(s,t)v(s,t) - \int_{t^{min}(s)}^{t} p(s,\tau)\frac{\partial v(s,\tau)}{\partial \tau}d\tau.$$

If $p(s,t)\frac{\partial v(s,t)}{\partial t}$*, is increasing in* t *for every* s*, then* $(p^{\#}, x^{\#})$ *is a full-information optimal mechanism.*

Proof. First, note that since $t^{min}(s)$ minimizes v, then at an inventive-compatible mechanism $u(s,t)$ is minimized in t at $t = t^{min}(s)$. Optimality follows from the fact that $p^{\#}$ maximizes $\bar{X}$ (described in (15)) pointwise, and the payment rule sets $u(t^{min}(s), s) = 0$, which is the lowest possible value given the ex-post participation constraints. Feasibility directly follows from the requirement that $p(s,t)\frac{\partial v(s,t)}{\partial t}$ is increasing in t for every s. ■

If the pointwise optimum fails to be incentive compatible, then we have to solve the problem taking explicitly into account the constraint that $p(s,t)\frac{\partial v(s,t)}{\partial t}$ be increasing in t for every s. As mentioned before, the incentive and the participation constraints are more costly to satisfy when the seller's type is known. This is because incentive compatibility requires that $p(s,t)\frac{\partial v(s,t)}{\partial t}$ is increasing in t for *each* s, rather than in expectation over s. This is illustrated in the next section.

6 Application: Horizontal and Vertical Differentiation

There are two possible types of products modeled as $S = \{0,1\}$. There is one buyer with a taste parameter in $T = [0,1]$. Both the seller's and the buyer's types are uniformly distributed. The match function describing the buyer's valuation as a function of his taste and the type of the product is:

$$v(0,t) = V_0 - t$$
$$v(1,t) = V_1 - 1 + t.$$

Without loss of generality we assume that ex-ante, type 1 product is more desirable for the buyer, that is $V_1 \geq V_0$. To exclude uninteresting extreme configurations we also assume that $-1 \leq V_0 \leq V_1 \leq 2$. First, note that $v(s,t)$ is linear and therefore convex in t. Since $t^{min}(s)$ is 1 for $s = 0$, while $t^{min}(s)$ is 0 for $s = 1$ it is not a priori obvious at which type t^* (B-PC) binds at the optimal mechanism when the seller's type is private information. For any $t^* \in [0,1]$ we can write:

$$J(0,t;t^*) = \begin{cases} V_0 - 2t & \text{if } t < t^* \\ V_0 - 2t + 1 & \text{if } t > t^*, \end{cases} \qquad J(1,t;t^*) = \begin{cases} V_1 + 2t - 1 & \text{if } t < t^* \\ V_1 + 2t - 2 & \text{if } t > t^*. \end{cases}$$

Notice that $J(0,t;t^*) + J(1,t;t^*) = V_0 + V_1 - 1$ for every t and t^*, and $2P(t) = p(1,t) - p(0,t)$ for every $t \in [0,1]$.

Full-Information Optimal Mechanism: We first analyze the case where the seller's type is common knowledge. Point-wise optimization yields:

$$p^{\#}(0,t) = \begin{cases} 1 & \text{if } t < \frac{V_0}{2} \\ 0 & \text{otherwise,} \end{cases} \qquad p^{\#}(1,t) = \begin{cases} 1 & \text{if } t > \frac{2-V_1}{2} \\ 0 & \text{otherwise.} \end{cases}$$

Since $p(s,t)\frac{\partial v(s,t)}{\partial t}$ is increasing in t for every s Proposition 8 holds, and the point-wise optimum is indeed the revenue-maximizing assignment rule. The optimal transfers when the buyer knows that $s=0$ and $s=1$ are respectively given by

$$x^{\#}(0,t) = p(0,t)v(0,t) - \int_1^t p(0,\tau)\frac{\partial v(0,\tau)}{\partial \tau}d\tau = \begin{cases} \frac{V_0}{2} & \text{if } t \leq \frac{V_0}{2} \\ 0 & \text{if } t > \frac{V_0}{2}, \end{cases}$$

and

$$x^{\#}(1,t) = p(1,t)v(1,t) - \int_0^t p(1,\tau)\frac{\partial v(1,\tau)}{\partial \tau}d\tau = \begin{cases} \frac{V_1}{2} & \text{if } t \geq \frac{2-V_1}{2} \\ 0 & \text{if } t < \frac{2-V_1}{2}. \end{cases}$$

When $V_i < 0$, the interim revenue of type i seller is zero. Otherwise, when $V_0, V_1 \geq 0$ the full-information interim revenue of the seller is

$$X^{\#}(s) = \int_T x^{\#}(s,t)dt = \begin{cases} \frac{V_0^2}{4} & \text{if } s = 0 \\ \frac{V_1^2}{4} & \text{if } s = 1, \end{cases}$$

and the full-information ex-ante expected revenue is $\bar{X}^{\#} = \frac{1}{8}(V_0^2 + V_1^2)$.

Optimal mechanism when the seller's type is private information: We now turn to find the optimal mechanism when the seller's type is private information. From Proposition 7 we have to find t^* and $p(s,t)$ that maximize

$$\bar{X} = \frac{1}{2}\int_0^1 \Big[p(0,t)J(0,t;t^*) + p(1,t)J(1,t;t^*)\Big] f(t)dt, \tag{16}$$

subject to $P(\cdot)$ is increasing and $P(t) \leq 0$ for $t < t^*$ and $P(t) \geq 0$ for $t > t^*$. The last equation can be rewritten as

$$\bar{X} = \int_0^1 \left[-P(t)J(0,t;t^*) + p(1,t)\left(\frac{V_0 + V_1 - 1}{2}\right)\right] f(t)dt \tag{17}$$

$$= \int_0^1 p(1,t)\frac{V_0 + V_1 - 1}{2} f(t)dt - \int_0^{t^*} P(t)(V_0 - 2t)f(t)dt - \int_{t^*}^1 P(t)(V_0 - 2t + 1)f(t)dt, \tag{18}$$

and incentive and participation constraints for the buyer reduce to the conditions that $2P(t) = p(1,t) - p(0,t)$ is increasing, negative for $t < t^*$, and positive for $t > t^*$. The allocation rule that maximizes the revenue point-wise for some presumed $t^* \in [0,1]$ is:

$$p^*(0,t) = \begin{cases} 1 & \text{if } t < \frac{V_0}{2} \text{ and } t < t^* \\ 1 & \text{if } t^* < t < \frac{V_0+1}{2} \\ 0 & \text{otherwise,} \end{cases} \qquad p^*(1,t) = \begin{cases} 1 & \text{if } \frac{1-V_1}{2} < t < t^* \\ 1 & \text{if } t > \frac{2-V_1}{2} \text{ and } t > t^* \\ 0 & \text{otherwise.} \end{cases}$$

We start with two preliminary observations: If V_0 and V_1 are below zero, then the buyer's valuation is always negative, so he is never assigned the good and the optimal and full-information optimal mechanisms coincide. If $V_0 \geq 1$ and $V_1 \geq 1$ the buyer is always assigned the good and the seller extracts the entire surplus from the buyer by selling the good at a price equal to $\bar{X} = \frac{V_0+V_1-1}{2}$. In this case, we have full pooling. The optimal ex-ante expected revenue is strictly higher than the full-information ex-ante revenue. The following proposition characterizes the optimal mechanism for more interesting intermediate values of V_i.

Proposition 9 *In the horizontal and vertical differentiation example, the optimal mechanism has the following properties:*

- *Partial Pooling: If $V_0 + V_1 > 1$, then*

$$(p^*(0,t), p^*(1,t)) = \begin{cases} (1,0) & \text{if } t < \frac{1-V_1}{2} \\ (1,1) & \text{if } \frac{1-V_1}{2} < t < \frac{V_0+1}{2} \\ (0,1) & \text{if } t > \frac{V_0+1}{2}. \end{cases} \qquad \hat{X}(t) = \begin{cases} \frac{2V_0+V_1-1}{4} & \text{if } t \leq \frac{1-V_1}{2} \\ \frac{V_0+V_1-1}{2} & \text{if } \frac{1-V_1}{2} \leq t \leq \frac{V_0+1}{2} \\ \frac{V_0+2V_1-1}{4} & \text{if } t \geq \frac{V_0+1}{2}, \end{cases}$$

  and the ex-ante expected revenue is

$$\bar{X}^* = \begin{cases} \frac{1}{8}(2V_0 + 2V_1 + V_0^2 + V_1^2 - 2) & \text{if } V_1 \leq 1 \\ \frac{1}{8}(4V_0 + 2V_1 + V_0^2 - 3) & \text{if } V_1 \geq 1, \end{cases}$$

  which is strictly higher than the full-information ex-ante revenue.

- *Full Separation: If $V_0 + V_1 \leq 1$, then the allocation rule and ex-ante expected revenue are the same as in the full-information optimal mechanism:*

$$(p^*(0,t), p^*(1,t)) = \begin{cases} (1,0) & \text{if } t < \frac{V_0}{2} \\ (0,0) & \text{if } \frac{V_0}{2} < t < \frac{2-V_1}{2} \\ (0,1) & \text{if } t > \frac{2-V_1}{2}. \end{cases} \qquad \hat{X}(t) = \begin{cases} \frac{V_0}{4} & \text{if } t \leq \frac{V_0}{2} \\ 0 & \text{if } \frac{V_0}{2} \leq t \leq \frac{2-V_1}{2} \\ \frac{V_1}{4} & \text{if } t \geq \frac{2-V_1}{2}, \end{cases}$$

$$\bar{X}^* = \bar{X}^{\#} = \frac{1}{8}\left((\max\{0, V_0\})^2 + (\max\{0, V_1\})^2\right).$$

Proof. See Appendix B ■

This proposition generalizes the observations made in the discrete house wine example from the introduction. In general, a simple posted price is suboptimal and the seller strictly gains from the fact that the buyer does not know the product characteristics. When $V_0 = V_1 = V \in [1/2, 1]$, the optimum can be implemented via bilateral cheap talk communication followed by contingent prices: the buyer first reveals whether his type t belongs to the interval $[\frac{1-V}{2}, \frac{V+1}{2}]$ or not. If t belongs to that interval then the seller posts the price $\frac{2V-1}{2}$ without revealing product information. Otherwise, the seller posts the price $\frac{3V-1}{4}$ and reveals product information to the buyer.

7 On the Irrelevance of the Seller's Information

So far we have seen that the seller benefits from having private information because, in general, uncertainty about the seller's type relaxes (B-IC) or (B-PC) or both. It is worth investigating the circumstances under which the seller is ex-ante indifferent between having private information and not (we have seen this possibility in Example 2). We do so gradually in order to illustrate the differences between the case examined in this paper and the private values case in Mylovanov and Troeger (2013b).

First we provide conditions under which uncertainty about the seller's type does not relax the participation constraints (B-PC), and establish that these are not enough to render the seller's information irrelevant.

Proposition 10 *Suppose that Assumption C holds and that $t^{min}(s) \equiv t^{min}$ is the same for every seller's type s. Consider the mechanism (p^*, x^*) where,*

$$p^*(s,t) \equiv \begin{cases} 1 & \text{if } J(s,t;t^{min}) > 0 \\ 0 & \text{if } J(s,t;t^{min}) \leq 0, \end{cases}$$

and

$$x^*(s,t) = E_S \underbrace{\left[p(s,t)v(s,t) - \int_{t^{min}}^{t} p(s,\tau)\frac{\partial v(s,\tau)}{\partial \tau} d\tau \right]}_{x^{\#}(s,t)}.$$

If

$$P(t) \equiv E_S \left[p(s,t)\frac{\partial v(s,t)}{\partial t} \right],$$

is increasing in t, then (p^*, x^*) is the optimal mechanism for the seller.

Proof. We first establish that the mechanism is optimal. This is clearly the case since p^* maximizes $\bar{X}$ (described in (15)) pointwise, and the payment rule sets $U(t^{min}) = 0$, which is the lowest possible value given the participation constraints. We now establish that the mechanism is feasible. Incentive

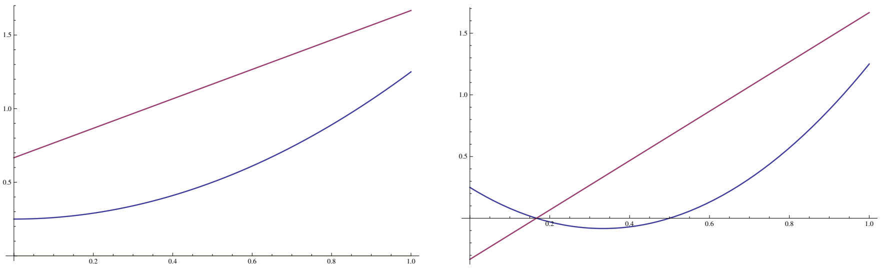

Figure 1: Left graph the match functions; right the virtual valuations.

compatibility constraints follow trivially since the proposition requires that $P(t) \equiv E_S\left[p(s,t)\frac{\partial v(s,t)}{\partial t}\right]$ is increasing in t. Participation constraints follow immediately by Lemma 3. The incentive and participation constraints for the seller are satisfied given Lemma 1. ■

The assumption that $t^{min}(s) \equiv t^{min}$ is the same for all seller's types implies that (B-PC) always binds at t^{min} at an optimal mechanism regardless how much the buyer knows about the seller. Then uncertainty about s does not relax (B-PC). We now show that this is not enough for the seller's information to be irrelevant. We do so in a particularly interesting example in which $v(s,t)$ satisfies Assumption C and is strictly increasing in *both* t and s.[17]

Example 3 Let $s \in \{s_1, s_2\}$, where the prior probability of s_1 is $\sigma \in (0,1)$, and let $t \in [0,1]$ with a uniform prior distribution. Assume that the match function is

$$v(s_1,t) = t^2 + 1/4,$$
$$v(s_2,t) = t + 2/3.$$

Note that it is strictly increasing in s and in t. The virtual valuations are:

$$J(s_1,t) = 3t^2 - 2t + 1/4, \quad \text{and} \quad J(s_2,t) = 2t - 1/3.$$

The match functions and the virtual valuations are depicted in Figure 1. Maximizing pointwise we obtain the following assignment rule:

$$p(s_1,t) = \begin{cases} 0 & \text{if } t \in (1/6, 1/2) \\ 1 & \text{otherwise,} \end{cases} \qquad p(s_2,t) = \begin{cases} 0 & \text{if } t < 1/6 \\ 1 & \text{if } t > 1/6. \end{cases}$$

This mechanism is not incentive compatible for the buyer when he knows the seller's type because $p(s_1,t)\frac{\partial v(s_1,t)}{\partial t} = \begin{cases} 0 & \text{if } t \in (1/6,1/2) \\ 2t & \text{otherwise,} \end{cases}$ is not monotonic in t. The full-information optimal mechanism

[17] See also Example 5 in the Appendix for an illustration with a finite number of buyer's types. On the contrary, Example 4 in the Appendix illustrates a leading case where the match function is $v(s,t) = s + t$ (and thus strictly monotone in both arguments) where the seller's information is irrelevant.

is given by a posted price equal to 1/4 or 1/2 for the seller type s_1, and a posted price equal to 5/6 for the seller type s_2. The full-information optimal interim revenue for each seller type is

$$X^{\#}(s_1) = 1/4 \quad \text{and} \quad X^{\#}(s_2) = 25/36,$$

whereas the corresponding ex-ante revenue is $\bar{X}^{\#} = \frac{25-16\sigma}{36}$. The solution of the pointwise maximization is, however, incentive compatible when the seller's type is private information if

$$P(t) = \sigma p(s_1,t)\frac{\partial v(s_1,t)}{\partial t} + (1-\sigma)p(s_2,t)\frac{\partial v(s_2,t)}{\partial t} = \begin{cases} \sigma 2t & \text{if } t < 1/6 \\ 1-\sigma & \text{if } t \in (1/6,1/2) \\ 1+\sigma(2t-1) & \text{if } t > 1/2, \end{cases}$$

is increasing in t, i.e., if $\sigma \leq 3/4$. The associated ex-ante expected revenue is given by

$$\bar{X}^* = \bar{X}^{\#} + \int_0^{1/6} J(s_1,t)\,dt = \frac{29}{108}\sigma + \frac{25}{36}(1-\sigma).$$

The corresponding interim revenue is $X^*(s_1) = X^*(s_2) = \bar{X}^*$. Information is therefore ex-ante valuable for the seller when $\sigma < 3/4$. It is however not interim valuable for type s_1 because $X^*(s_1) < X^{\#}(s_1)$ when $\sigma < 3/4$. $\diamond$

We now proceed to add conditions under which uncertainty about the seller's type does not relax the incentive constraints (B-IC), and establish that, then, the seller's information is irrelevant.

Definition 5 (*Single-Crossing*) A function $f : X \to \mathcal{R}$ is *single crossing*[18] if for every $x \leq x'$, $f(x) \geq (>)\ 0$ implies that $f(x') \geq (>)\ 0$.

Assumption SC (Single Crossing). Both J^R and $-J^L$ satisfy *single-crossing* in t for *every* s.

Observe that $J(s_1,t)$ in Example 3 violates Assumption SC.

Proposition 11 *Suppose that Assumptions C and SC hold and that $t^{min}(s) = t^{min}$ is the same for every seller's type s. The following mechanism (p^*, x^*) is optimal:*

$$p^*(s,t) \equiv \begin{cases} 1 & \text{if } J(s,t;t^{min}) > 0 \\ 0 & \text{if } J(s,t;t^{min}) \leq 0, \end{cases} \quad \text{and} \quad x^*(s,t) = E_S\left[x^{\#}(s,t)\right]. \tag{19}$$

Proof. Optimality and the feasibility constraints for the seller follow immediately from arguments used to establish Proposition 10. Since the proposed mechanism sets $E_S\left[u(s,t^{min})\right] = 0$, it satisfies participation constraints for all t. To establish (B-IC), note that the fact that $-J^L$ and J^R satisfy single-crossing implies that p^* is 1 for low t's, zero for middle t's and 1 again for high t's. This

[18] This terminology is also used by Milgrom (2004).

observation, together with the convexity of v for each s, implies (11), while (12) follows immediately, since

$$E_S\left[u(s,t)\right] = E_S\left[p(s,t)v(s,t) - x(s,t)\right] = E_S\left[\int\limits_{t^{min}}^{t} p(s,\tau)\frac{\partial v(s,\tau)}{\partial \tau}d\tau + U(s,t^{min})\right]. \qquad \blacksquare$$

An important corollary is that when Assumptions C and SC hold and $t^{min}(s)$ is the same for all s, the seller's maximal revenue is equal to the case where his information is public–so his private information is irrelevant:

Corollary 4 (Irrelevance of the Seller's Information) *Under the conditions of Proposition 10, the seller-maximal equilibrium revenue is equal to the case where his information is commonly known. In other words, the optimal mechanism delivers the same ex-ante expected revenue as the full-information optimal mechanism.*

The conditions that lead to the irrelevance of the seller's information are quite special. It is worth investigating what is the specific role of each of these assumptions. The assumption that $t^{min}(s)$ is same for all s implies that (B-PC) always binds at the same type. The implication is that averaging over s does not relax the *buyer's participation constraint*. Assumptions C and SC imply that the point wise optimum satisfies the property that $p(s,t)\frac{\partial v(s,t)}{\partial t}$ is increasing in t for every s. The implication is that averaging over s does not relax the *buyer-incentive compatibility*.

One may wonder whether it is possible to tighten the conditions and provide *necessary and sufficient* conditions for an informed principal's private information to be irrelevant. The following observation provides sufficient conditions for the irrelevance of the principal's private information for an abstract informed seller problem and shows that when one of these conditions fails, there are circumstances where the principal strictly benefits from private information. The latter part highlights that it appears impossible to find general necessary conditions for information irrelevance.

Proposition 12 Irrelevance of Principal's Information. *The seller's expected revenue at a revenue-maximizing mechanism is equal to his expected revenue at the full-information optimal mechanism if (i) at all feasible mechanisms $U(t)$ is minimal at the same type t^{min}, and (ii) for any optimal Bayesian incentive compatible mechanism, there exists an equivalent (in terms of interim payoffs for the buyer and the seller) dominant-strategy incentive compatible mechanism.*[19]

Proof. Proposition 5 established that the optimal mechanism (p^*, x^*) for the informed seller generates as least the ex-ante revenue of the full-information optimal mechanism $(p^\#, x^\#)$, that is $\bar{X}^* \geq \bar{X}^\#$. We establish by contradiction that under conditions (i) and (ii) we have that $\bar{X}^* = \bar{X}^\#$. Suppose that $\bar{X}^* > \bar{X}^\#$. Condition (ii) implies that there exists a mechanism $(\tilde{p}, \tilde{x})$ that is incentive compatible for the buyer when the seller's type is common knowledge and that generates the same

[19]See Gershkov et al. (2013) for sufficient conditions for condition (ii) to hold. Their conditions are different from ours in Proposition 11. They assume linear private values, however their results apply also when values are linear but interdependent when one cares only about interim payoffs and not about social surplus.

interim utilities as (p^*, x^*); in particular, $U^*(t^{min}) = \tilde{U}(t^{min}) = 0$. By condition (i) t^{min} is the worst type both at (p^*, x^*) and $(\tilde{p}, \tilde{x})$. Hence, $(\tilde{p}, \tilde{x})$ satisfies (interim) participation constraints but may not satisfy ex-post participation constraints. Now, consider the modified mechanism $(\tilde{p}, \hat{x})$ such that

$$\hat{x}(s,t) = \tilde{x}(s,t) + \tilde{u}(s, t^{min}), \text{ for every } s \text{ and } t.$$

This mechanism is still ex-post incentive compatible for the buyer since for every s, $\hat{u}(s,t)$ is obtained from $\tilde{u}(s,t)$ by adding a constant: $\hat{u}(s,t) = \tilde{u}(s,t) - \tilde{u}(s,t^{min})$. Hence, by condition (i) we have $\hat{u}(s,t) \geq \hat{u}(s,t^{min})$ for every s and t, i.e., $\hat{u}(s,t) \geq 0$ for every s and t. Thus, $(\tilde{p}, \hat{x})$ satisfies ex-post participation constraint. Since $E_S[\tilde{u}(s,t^{min})] \equiv \tilde{U}(t^{min}) = 0$ the ex-ante expected revenue at the mechanism $(\tilde{p}, \hat{x})$ is the same as with the mechanism $(\tilde{p}, \tilde{x})$, and is therefore $\bar{X}^*$. Now, since we assumed that $\bar{X}^* > \bar{X}^{\#}$ we conclude that there is some type s for the seller that obtains a strictly higher interim revenue at the mechanism $(\tilde{p}, \hat{x})$ than at the full-information optimal mechanism, a contradiction with the optimality of full-information optimal mechanism $(p^{\#}, x^{\#})$ when type s is commonly known. ■

If either (i) or (ii) fails than it is possible that the seller strictly benefits from having private information. To see that, consider the example in Section 6 when $1 < V_0 = V_1 = V < 2$. This example violates condition (i) but satisfies condition (ii), since when $V > 1$, at the optimal Bayesian-incentive compatible mechanism the seller posts a price of $\bar{X} = V - \frac{1}{2}$ and the buyer always accepts. This optimal mechanism is dominant strategy incentive-compatible. However, in this example, we have seen that the seller generates strictly more revenue when his information is private. Next, consider Example 3. This example satisfies condition (i) but fails condition (ii) since the optimal mechanism is not incentive compatible when the seller's type is common knowledge.

8 Discussion and Concluding Remarks

In this paper we considered an informed seller problem in which the buyer's valuation depends both on his type and on the seller's type. To the best of our knowledge this is the first paper that fully analyzes an informed principal problem with bilateral private information in which the buyer's willingness to pay depends both on the seller's type and on his own taste. Under the assumption that the seller wants to maximize revenue (or has type-independent valuation), we showed that we can obtain an equilibrium of the mechanism-selection game by solving for the ex-ante optimal mechanism–thus in our setup the informed principal problem reduces to a maximization problem. We also established that the set of ex-ante optimal mechanisms coincides with core mechanisms.

Even though we labeled the principal seller and the agent buyer, the model employed is abstract and minimal conditions are imposed on the match function. It thus lends itself to a number of other interpretations with a large number of applications. For instance, one can imagine the principal being a worker informed about his characteristics who cares about the wage, and the buyer being a firm whose willingness to pay the worker depends on the quality of the match that is itself a function of firm- or task-specific details privately known to the firm. Another set of applications consists of the principal being an expert (lawyer, doctor, teacher, consultant) whose expertise and abilities are private information. The expert provides services to the agent whose willingness to pay depends on the quality of the match that is also a function of the agent's idiosyncratic needs and characteristics. The expert just cares about the fee.

The fact that the principal cares only for monetary transfers is key for our analysis and results. When the seller/principal has type-dependent costs or valuations, then the seller-incentive constraints are more complex and generally restrict the set of feasible outcomes. More importantly, an ex-ante optimal mechanism may *not* be an expectational equilibrium for the mechanism-selection game. We illustrate this point, and provide some robustness checks of our results in Appendix D. A recent important paper by Mylovanov and Troeger (2013b) analyzes an informed principal problem assuming private values. They show that neologism-proof allocations are not only equilibria of the mechanism-election game, but also ex-ante optimal mechanisms. It may be worth investigating whether ex-ante optimal mechanisms are equilibria in other informed principal problems. We leave the investigation of this for future research.

Compared to earlier works that had considered situations where simple posted prices are optimal, our analysis has also highlighted the superiority of a selling protocol where the buyer and the seller engage in direct bilateral communication (exchange of cheap messages) first, and then the seller asks a price that is contingent on the outcome of the previous communication. Such a selling procedure is natural since it neither requires a mediator nor commitment power from the seller. We find that is it not only the case that this selling protocol dominates posted prices, but in some cases it implements the optimal mechanism. Another interesting direction for future research is the characterization of the set of outcomes that are achievable with unmediated procedures without commitment.

Appendices

A Full Surplus Extraction

Proposition 5 tells us that seller benefits (weakly) from having private information. In this section, we investigate when the seller can leverage the fact that his type is unknown to extract the entire surplus from the buyer.[20]

In order to extract the entire surplus the seller must employ a mechanism (p, x) that consists of an efficient assignment rule p and a payment rule that leaves zero expected surplus to all types of the buyer. In other words, the assignment rule must be $p(s,t) = 1$ if $v(s,t) \geq 0$, and zero otherwise. The payment rule must be such that $E_S[u(s,t)] = E_S[p(s,t)v(s,t) - x(s,t)] = 0$ for all $t \in T$. Full-surplus

[20] The forces that permit full surplus extraction in our model are distinct from the ones in Severinov (2008) who examines an informed principal model where the principal's and the agents' information is statistically correlated.

extraction satisfies (B-IC) if:

$$0 \geq E_S[p(s,t')(v(s,t) - v(s,t'))] \text{ for all } t, t' \in T. \tag{20}$$

Notice that the RHS of (20) is zero for all s such that $p(s,t') = 0$. Let $S^+(t')$ denote the set of seller types such that $p(s,t') = 1$, that is $s \in S$ such that $v(s,t') \geq 0$. Then, (20) can be equivalently expressed as follows:

$$0 \geq E_{S^+(t')}[v(s,t) - v(s,t')])] \text{ for all } t, t' \in T. \tag{21}$$

Writing the constraints that type t does not want to mimic t', and the reverse, we obtain the following conditions that must be true for each pair $t, t' \in T$ in order for full surplus extraction to satisfy (B-IC): $E_{S^+(t')}[v(s,t)] \leq E_{S^+(t')}[v(s,t')]$ and $E_{S^+(t)}[v(s,t')] \leq E_{S^+(t)}[v(s,t)]$.

Suppose that t, t' are opposite, that $v(s,t) = -v(s,t')$, then it is immediate to see that the IC conditions are satisfied. In the same vain, but more generally, if the intersection of $S^+(t)$ and $S^+(t')$ is empty then (20) holds. Another possibility is when expected valuation is constant: Suppose that for all $t \in T$, $E_{S^+(t)}[v(s,t)] = c$, where $c > 0$. Then, a revenue-maximizing mechanism for the seller is $x^*(s,t) = c$ and $p^*(s,t) = 1$ for all s and t. This mechanism extracts all the surplus from the buyer, $U(t) = 0$ for all $t \in T$.

B Proofs

Proof of Lemma 4. Since $U(t)$ is convex, it is differentiable almost everywhere. Hence, from equations (9) and (10), the mechanism (p, x) is incentive-compatible for the buyer iff

$$P(t) \in \partial U(t), \tag{22}$$

where $\partial U(t)$ is the set of subgradients of U at t ($P(t)$ is a subgradient of U at t iff $U(t') \geq U(t) + P(t)(t' - t)$ for every $t' \in T$).

($\Rightarrow$) The convexity of $U(t)$ implies that $P(t)$ is increasing, so (11) is satisfied. Since $v(s,t)$ is convex in t and T is convex, Hypothesis 1 in Krishna and Maenner (2001) is satisfied. Hence, from Proposition 1 in Krishna and Maenner (2001) we have, for every $t, t^* \in T$:

$$U(t) = U(t^*) + \int_{t^*}^{t} P(\tau) d\tau.$$

($\Leftarrow$) From (12) we have, for every $t, t' \in T$:

$$U(t') - U(t) = \int_{t^*}^{t'} P(\tau) d\tau - \int_{t^*}^{t} P(\tau) d\tau = \int_{t}^{t'} P(\tau) d\tau.$$

By (11) we have $\int_t^{t'} P(\tau) d\tau \geq P(t)(t' - t)$. Hence, $U(t') - U(t) \geq P(t)(t' - t)$, i.e., $P(t) \in \partial U(t)$. ■

Proof of Corollary 3. Condition (12) can be rewritten as

$$E_S\left[p(s,t)v(s,t) - x(s,t)\right] = U\left(t^*\right) + \int_{t^*}^{t} P(\tau)d\tau = U\left(t^*\right) + \int_{t^*}^{t} E_S\left[p(s,\tau)\frac{\partial v(s,\tau)}{\partial\tau}\right]d\tau,$$

i.e.,

$$\hat{X}(t) = E_S\left[p(s,t)v(s,t) - \int_{t^*}^{t} p(s,\tau)\frac{\partial v(s,\tau)}{\partial\tau}d\tau\right] - U\left(t^*\right).$$

■

Proof of Lemma 5. For every $t^* \in T$, the first term of (13) can be rewritten as follows:

$$\int_T E_S\left[p(s,t)v(s,t) - \int_{t^*}^{t} p(s,\tau)\frac{\partial v(s,\tau)}{\partial\tau}d\tau\right] f(t)dt =$$

$$\int_T E_S\left[p(s,t)v(s,t)\right] f(t)dt - \int_T E_S\left[\int_{t^*}^{t} p(s,\tau)\frac{\partial v(s,\tau)}{\partial\tau}d\tau\right] f(t)dt.$$

Let us focus on the second term of the above equality. Integrating by parts we get:

$$\begin{aligned}
&\int_T E_S\left[\int_{t^*}^{t} p(s,\tau)\frac{\partial v(s,\tau)}{\partial\tau}d\tau\right] f(t)dt \\
&= -\int_{\underline{t}}^{t^*} E_S\left[p(s,t)\frac{\partial v(s,t)}{\partial t}F(t)\right]dt + \int_{t^*}^{\bar{t}} E_S\left[p(s,t)\frac{\partial v(s,t)}{\partial t}\left[1-F(t)\right]\right]dt \\
&= -\int_{\underline{t}}^{t^*} E_S\left[p(s,t)\frac{\partial v(s,t)}{\partial t}\frac{F(t)}{f(t)}\right]f(t)dt + \int_{t^*}^{\bar{t}} E_S\left[p(s,t)\frac{\partial v(s,t)}{\partial t}\frac{1-F(t)}{f(t)}\right]f(t)dt.
\end{aligned}$$

■

Proof of Proposition 9.

Possibility 1: $V_0 \leq 1 \leq V_1$ *and* $V_0 + V_1 \geq 2$. In this case we have $\frac{1-V_1}{2} \leq 0 \leq \frac{2-V_1}{2} \leq \frac{V_0}{2} < \frac{V_0+1}{2} \leq 1$.

(1a) Let $\frac{2-V_1}{2} \leq t^* \leq \frac{V_0}{2}$. Pointwise maximization yields

$$(p^*(0,t), p^*(1,t)) = \begin{cases} (1,1) & \text{if } t < \frac{V_0+1}{2} \\ (0,1) & \text{if } t > \frac{V_0+1}{2}, \end{cases}$$

so $2P(t) = 0$ for $t < \frac{V_0+1}{2}$ and $2P(t) = 1$ for $t > \frac{V_0+1}{2}$, which is feasible. Equation (16) simplifies to

$\bar{X} = \frac{1}{8}(4V_1 + 2V_0 + V_0^2 - 3)$. The interim utilities and payments of the buyer are respectively given by

$$U(t) = \int_{t^*}^{t} P(\tau)d\tau = \begin{cases} 0 & \text{if } t < \frac{V_0+1}{2} \\ \frac{1}{2}(t - \frac{V_0+1}{2}) & \text{if } t > \frac{V_0+1}{2}, \end{cases}$$

and

$$\hat{X}(t) = E_S\left[p(s,t)v(s,t)\right] - U(t) = \begin{cases} \frac{V_0+V_1-1}{2} & \text{if } t < \frac{V_0+1}{2} \\ \frac{V_0+2V_1-1}{4} & \text{if } t > \frac{V_0+1}{2}, \end{cases} \tag{23}$$

and the solution is consistent for any t^* such that $\frac{2-V_1}{2} \leq t^* \leq \frac{V_0}{2}$.

(1b) Let $0 \leq t^* < \frac{2-V_1}{2} \leq \frac{V_0}{2}$. Pointwise maximization yields

$$(p^*(0,t), p^*(1,t)) = \begin{cases} (1,1) & \text{if } t < t^* \\ (1,0) & \text{if } t^* < t < \frac{2-V_1}{2} \\ (1,1) & \text{if } \frac{2-V_1}{2} < t < \frac{V_0+1}{2} \\ (0,1) & \text{if } t > \frac{V_0+1}{2}, \end{cases}$$

so $2P(t) = 0$ for $t < t^*$, $2P(t) = -1$ for $t^* < t < \frac{2-V_1}{2}$, $2P(t) = 0$ for $\frac{2-V_1}{2} < t < \frac{V_0+1}{2}$ and $2P(t) = 1$ for $t > \frac{V_0+1}{2}$, which is not feasible. Given the constraints, we have to set $p(1,t) = p(0,t) = 1$ for $t^* < t < \frac{2-V_1}{2}$ and therefore we get the same mechanism as in case (1a), and the solution is consistent for any $t^* < \frac{2-V_1}{2}$.

(1c) Let $\frac{V_0}{2} \leq t^* \leq \frac{V_0+1}{2}$. This gives exactly the same solution as in case (1a).

(1d) Let $\frac{V_0+1}{2} \leq t^* \leq 1$. Pointwise maximization yields

$$(p^*(0,t), p^*(1,t)) = \begin{cases} (1,1) & \text{if } t < \frac{V_0}{2} \\ (0,1) & \text{if } t > \frac{V_0}{2}, \end{cases}$$

which is not feasible because $2P(t) = 1 > 0$ for $\frac{V_0}{2} < t < t^*$, so we have to set $p(1,t) = p(0,t) = 1$ on this interval, so that

$$(p^*(0,t), p^*(1,t)) = \begin{cases} (1,1) & \text{if } t < t^* \\ (0,1) & \text{if } t > t^*. \end{cases}$$

The transfers are then obtained as in (23):

$$\hat{X}(t) = E_S\left[p(s,t)v(s,t)\right] - U(t) = \begin{cases} \frac{V_0+V_1-1}{2} & \text{if } t < t^* \\ \frac{V_0+2V_1-1}{4} & \text{if } t > t^*. \end{cases}$$

Since $\frac{V_0+2V_1-1}{4} > \frac{V_0+V_1-1}{2} \iff t > \frac{V_0+1}{2}$, it is optimal to set $t^* = \frac{V_0+1}{2}$ and we get again the same mechanism as in case (1a).

Possibility 2: $1 \le V_0 + V_1 \le 2$. In this case we have $\frac{1-V_1}{2} \le \frac{V_0}{2} \le \frac{2-V_1}{2} \le \frac{V_0+1}{2}$.

(2a) $\frac{V_0}{2} \le t^* \le \frac{2-V_1}{2}$. Pointwise maximization yields

$$(p^*(0,t), p^*(1,t)) = \begin{cases} (1,0) & \text{if } t < \frac{1-V_1}{2} \\ (1,1) & \text{if } \frac{1-V_1}{2} < t < \frac{V_0}{2} \\ (0,1) & \text{if } \frac{V_0}{2} < t < t^* \\ (1,0) & \text{if } t^* < t < \frac{2-V_1}{2} \\ (1,1) & \text{if } \frac{2-V_1}{2} < t < \frac{V_0+1}{2} \\ (0,1) & \text{if } t > \frac{V_0+1}{2}. \end{cases}$$

This is not feasible because $P(t) > 0$ for $t \in [\frac{V_0}{2}, t^*]$ and $P(t) < 0$ for $t \in [t^*, \frac{2-V_1}{2}]$. Hence, we have to set $P(t) = 0$ on these intervals. Since $\frac{V_0+V_1-1}{2} > 0$, to maximize $\bar{X}$ given by (18) we should set $p(1,t) = p(0,t) = 1$ on that range. Then, the optimal mechanism in this case is:

$$(p^*(0,t), p^*(1,t)) = \begin{cases} (1,0) & \text{if } t < \frac{1-V_1}{2} \\ (1,1) & \text{if } \frac{1-V_1}{2} < t < \frac{V_0+1}{2} \\ (0,1) & \text{if } t > \frac{V_0+1}{2}. \end{cases}$$

Equation (16) simplifies to

$$\bar{X} = \begin{cases} \frac{1}{8}(2V_0 + 2V_1 + V_0^2 + V_1^2 - 2) & \text{if } V_1 \le 1 \\ \frac{1}{8}(4V_0 + 2V_1 + V_0^2 - 3) & \text{if } V_1 \ge 1, \end{cases}$$

which is strictly higher compared to the full-information revenue $\frac{1}{8}(V_0^2 + V_1^2)$. The interim utilities and payments of the buyer are respectively given by

$$U(t) = \int_{t^*}^{t} P(\tau) d\tau = \begin{cases} -\frac{1}{2}(t - \frac{1-V_1}{2}) & \text{if } t < \frac{1-V_1}{2} \\ 0 & \text{if } \frac{1-V_1}{2} < t < \frac{V_0+1}{2} \\ \frac{1}{2}(t - \frac{V_0+1}{2}) & \text{if } t > \frac{V_0+1}{2}, \end{cases}$$

and

$$\hat{X}(t) = E_S\left[p(s,t)v(s,t)\right] - U(t) = \begin{cases} \frac{2V_0+V_1-1}{4} & \text{if } t < \frac{1-V_1}{2} \\ \frac{V_0+V_1-1}{2} & \text{if } \frac{1-V_1}{2} < t < \frac{V_0+1}{2} \\ \frac{V_0+2V_1-1}{4} & \text{if } t > \frac{V_0+1}{2}, \end{cases} \tag{24}$$

and the solution is consistent for any t^* such that $\frac{1-V_1}{2} < t^* < \frac{V_0+1}{2}$.

(2b) $0 \leq t^* \leq \frac{1-V_1}{2}$. Pointwise maximization yields

$$(p^*(0,t), p^*(1,t)) = \begin{cases} (1,0) & \text{if } t < \frac{2-V_1}{2} \\ (1,1) & \text{if } \frac{2-V_1}{2} < t < \frac{V_0+1}{2} \\ (0,1) & \text{if } t > \frac{V_0+1}{2}. \end{cases}$$

This is not feasible since $t^* < \frac{2-V_1}{2}$, so we should set $p(1,t) = p(0,t) = 1$ for $t^* < t < \frac{2-V_1}{2}$ and we get

$$(p^*(0,t), p^*(1,t)) = \begin{cases} (1,0) & \text{if } t < t^* \\ (1,1) & \text{if } t^* < t < \frac{V_0+1}{2} \\ (0,1) & \text{if } t > \frac{V_0+1}{2}. \end{cases}$$

Equation (16) yields $\bar{X} = \frac{1}{8}(2V_0 + V_0^2 - 3 + 4V_1 + 4t^* - 4(t^*)^2 - 4t^*V_1)$, which is maximized for $t^* = \frac{1-V_1}{2}$ and we get again the same mechanism as in case (2a).

(2c) $\frac{1-V_1}{2}, 0 \leq t^* \leq \frac{V_0}{2}$. Pointwise maximization yields

$$(p^*(0,t), p^*(1,t)) = \begin{cases} (1,0) & \text{if } t < \frac{1-V_1}{2} \\ (1,1) & \text{if } \frac{1-V_1}{2} < t < t^* \\ (1,0) & \text{if } t^* < t < \frac{2-V_1}{2} \\ (1,1) & \text{if } \frac{2-V_1}{2} < t < \frac{V_0+1}{2} \\ (0,1) & \text{if } t > \frac{V_0+1}{2}. \end{cases}$$

This is not feasible because $P(t) < 0$ for $t \in [t^*, \frac{2-V_1}{2}]$, so as before we should set $p(1,t) = p(0,t) = 1$ on that range and we get exactly the same mechanism as in case (2a).

(2d) $\frac{V_0+1}{2} \leq t^* \leq 1$. This situation is similar to situation (2b) and yields the same optimal mechanism as in (2a).

(2e) $\frac{2-V_1}{2} \leq t^* \leq \frac{V_0+1}{2}$. This situation is similar to situation (2c) and yields the same optimal mechanism as in (2a).

Possibility 3: $V_0 + V_1 \leq 1$. Let $\frac{1-V_1}{2} \leq t^* \leq \frac{V_0+1}{2}$. Pointwise maximization yields

$$(p^*(0,t), p^*(1,t)) = \begin{cases} (1,0) & \text{if } t < \frac{V_0}{2} \\ (0,0) & \text{if } \frac{V_0}{2} < t < \frac{1-V_1}{2} \\ (0,1) & \text{if } \frac{1-V_1}{2} < t < t^* \\ (1,0) & \text{if } t^* < t < \frac{V_0+1}{2} \\ (0,0) & \text{if } \frac{V_0+1}{2} < t < \frac{2-V_1}{2} \\ (0,1) & \text{if } t > \frac{2-V_1}{2}. \end{cases}$$

This is not feasible because $P(t) > 0$ for $t \in [\frac{1-V_1}{2}, t^*]$ and $P(t) < 0$ for $t \in [t^*, \frac{V_0+1}{2}]$. Hence, we have to set $P(t) = 0$ on these intervals. Since $\frac{V_0+V_1-1}{2} \leq 0$, to maximize $\bar{X}$ given by (18) we should set $p(1,t) = p(0,t) = 0$ on that range. Then, the allocation rule is the same as in the full-information optimal mechanism:

$$(p^*(0,t), p^*(1,t)) = \begin{cases} (1,0) & \text{if } t < \frac{V_0}{2} \\ (0,0) & \text{if } \frac{V_0}{2} < t < \frac{2-V_1}{2} \\ (0,1) & \text{if } t > \frac{2-V_1}{2}. \end{cases}$$

The same logic applies for $t^* \notin \left(\frac{1-V_1}{2}, \frac{V_0+1}{2}\right)$ and leads to the full-information optimal mechanism. ■

C Additional Examples

Example 4 (Uniform Additive Case: Illustration of Proposition 10) Suppose that the buyer's valuation is $v(s,t) = s + t$ and that s and t are uniformly distributed on $[0,1]$. Here $t^{min}(s) = 0$ for every s. The buyer's virtual valuation is $J^R(s,t) = s + 2t - 1$, so

$$p^*(s,t) = \begin{cases} 1 & \text{if } t \geq \frac{1-s}{2} \\ 0 & \text{if } t < \frac{1-s}{2}, \end{cases}$$

and (19) becomes

$$x^*(s,t) = \begin{cases} 3/4 & \text{if } t \geq 1/2 \\ 2t - t^2 & \text{if } t \leq 1/2. \end{cases} \tag{25}$$

The associated ex-ante and interim expected payoff of the seller is:

$$\bar{X}^* = X^*(s) = \int_T x^*(s,t)dt = \frac{7}{12}.$$

The associated ex-post and interim rent for the buyer is

$$u^*(s,t) = p^*(s,t)v(s,t) - x^*(s,t) = \begin{cases} t^2 - 2t & \text{if } t \leq \frac{1-s}{2} \\ s - t + t^2 & \text{if } \frac{1-s}{2} \leq t \leq 1/2 \\ s + t - 3/4 & \text{if } t \geq 1/2, \end{cases}$$

and

$$U^*(t) = E_S[u^*(s,t)] = \begin{cases} t^2 & \text{if } t \leq 1/2 \\ t - 1/4 & \text{if } t \geq 1/2. \end{cases}$$

Notice that ex-post participation is not satisfied since $u^*(s,t)$ may be negative. For this simple example $P(t)$ from (10) becomes $P(t) = E_S\left[p(s,t)\right]$ which is increasing in t. For *each* s, we also have that $P(s,t) = p(s,t)$ is increasing in t, so that when s is commonly known, (p^*, x^*) is also incentive compatible and gives the same ex-ante expected payoff as $(p^*, x^\#)$. Indeed, we have

$$x^\#(s,t) = p(s,t)v(s,t) - \int_{\underline{t}}^{t} p(s,\tau)\frac{\partial v(s,\tau)}{\partial \tau}d\tau = \begin{cases} \frac{1+s}{2} & \text{if } t \geq \frac{1-s}{2} \\ 0 & \text{otherwise.} \end{cases}$$

The interim expected revenue at s is

$$X^\#(s) = \int_0^1 x^\#(s,t)dt = \left(\frac{1+s}{2}\right)^2,$$

and the ex-ante expected revenue is $\int_0^1 X^\#(s)ds = \frac{7}{12} = \bar{X}^*$. The associated ex-post and interim rent for the buyer is

$$u^\#(s,t) = p^*(s,t)v(s,t) - x^\#(s,t) = \begin{cases} s/2 + t - 1/2 & \text{if } t \geq \frac{1-s}{2} \\ 0 & \text{otherwise} \end{cases},$$

and $U^\#(t) = U^*(t)$. Notice that when s is commonly known, the mechanism also satisfies ex-post participation for the buyer ($u^\#(s,t) \geq 0$ for every t). However, when s is privately known by the seller there is no feasible mechanism achieving $\bar{X}^* = \frac{7}{12}$ that satisfies ex-post participation for the buyer. $\diamond$

Example 5 Consider the following match function:

$v(s,t) =$

	t_l	t_m	t_r
s_L	1	1	2
s_R	0	4	5

The seller's types are equally likely, whereas $\Pr(t_l) = \Pr(t_r) = \frac{2}{5}$ and $\Pr(t_m) = \frac{1}{5}$. The full-information optimal mechanism for this example generates ex-ante expected revenue $\frac{17}{10}$ and it is:

$\rho^\#(s,t) =$

	t_l	t_m	t_r
s_L	$1,1$	$1,1$	$1,1$
s_R	$0,0$	$1,4$	$1,4$

The optimal mechanism generates ex-ante expected revenue $\frac{19}{10}$ and it is:

$\rho(s,t) =$

	t_l	t_m	t_r
s_L	$1,0.5$	$0,2$	$1,3$
s_R	$0,0.5$	$1,2$	$1,3$

Another alternative is a mechanism that never assigns the object to t_r and extracts all the surplus from t_m and t_r:

$\hat{\rho}(s,t) =$

	t_l	t_m	t_r
s_L	$0, 0$	$0, 2.5$	$1, 3.5$
s_R	$0, 0$	$1, 2.5$	$1, 3.5$

◇

D Robustness: Production Costs / Valuation for the Seller

Our analysis hinged upon Lemma 1 and Proposition 3. We now discuss the extend to which these results hold under alternative specifications. Clearly, Lemma 1 still holds when the seller has type-dependent production costs $c(s,t)$ that are sunk at the moment trade takes place. Then, at the interim stage, the seller's outside option is type-dependent and given by $-E_T[c(s,t)]$. But, given that costs are sunk, all types of the seller want to maximize the payment they receive, so still the seller's incentive compatibility constraints imply that expected transfer must be the same for all types. However, Lemma 1 ceases to hold when the seller has type-dependent costs that are incurred when transaction takes place or when the seller has type-dependent consumption value for the good for sale. To see this, consider the following example:

$v(s,t) =$

	t
s_1	1
s_0	-0.5

When the seller cares only about revenue, the optimal mechanism is

$\rho(s,t) =$

	t
s_1	$1, 0.5$
s_0	$0, 0.5$

so the optimal ex-ante expected revenue is 0.5. Assume now that there is a type-dependent selling cost $c(s_1) = c$ and $c(s_0) = 0$. Then, the optimal mechanism (for c small) is [21]

$\rho(s,t) =$

	t
s_1	$1, 0.25$
s_0	$1, 0.25$

so the optimal ex-ante revenue is $0.25 - c/2$. Clearly, there is a discontinuity of the optimal revenue in c at $c = 0$: Even for arbitrary small type-dependent costs, Lemma 1 does not apply: incentive compatibility of the seller is binding. In addition, with flow type-dependent costs, it is not necessarily

[21] Details available upon request.

true that the ex-ante optimal allocation is an expectational equilibrium, so Proposition 3 also fails.[22]

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[22]We are grateful to Timofyi Mylovanov for such an example.

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