DINKS, DEWKS & Co.
Marriage, Fertility and Childlessness in the US∗

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Abstract

Along with modern family lifestyles, came given caricaturing names: couples choosing to be childless have been denoted as DINKS, for “double incomes no kids”, and couples with children as DEWKS, for “dually employed with kids”. In our theory, we distinguish the decision to have children from the choice of the number of children, and explain marriage, childlessness, and fertility as a function of men’s and women’s wages. The deep parameters of the model are identified from the 1990 US Census. The quantitative model allows to measure voluntary and involuntary childlessness from the data, and to understand the following facts: first, single women are more childless than married but, when mothers, their fertility is close to the one of married mothers, second, childlessness exhibits a U-Shaped relationship with education for both single and married women, and third, there is a hump-shaped relationship between marriage rates and education levels.

Keywords: Completed Fertility, Childlessness, Marriage, Education.
JEL Classification Numbers: J13; O11; O40.

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1 Introduction

The twentieth century faced a radical change in American family patterns. Among these main transformations, the generalization of divorce has been widely studied (see Becker (1993)). In this paper, we focus on the consequences of another fundamental change in the United-States: the increasing discrepancy between marriage and motherhood. Indeed nowadays, neither marriage systematically implies parenthood nor singleness means childlessness. New types of families have become more common, such as the DEWKS (Dually Employed With Kids), the KOOPF (Kids Of One Parent Families) or the DINKS (Double Income No Kids).\footnote{Each of these different types of group represents a different target for the marketing literature; for example, toys and clothes for children will be offered to DEWKS while DINKS will be receiving information about travels or luxury goods.}

This paper answers two questions that have not been treated in previous endogenous fertility models and that become more and more a life-type decision in the U.S.: which single women become mothers and which married women remain childless? Because of the evolution in family patterns, analyzing average fertility in the US requires to explore the determinants of motherhood among married and singles as well as the reason why people choose to marry or not. In this perspective, we need to go further the unitary endogenous fertility benchmark model (Becker and Lewis (1973), Becker and Tomes (1976) and Galor (2005)). For this purpose, we construct a simple two sexes model of endogenous fertility where agents play a two-stage marriage game.

Our model allows to replicate three stylized facts drawn from the U.S. Census Bureau data for the year 1990: \(i\) single women are much more likely to be childless, however, when they choose to become mothers, their fertility is almost the same as the fertility of married mothers for all categories of education,\footnote{Since we use U.S. data taken from IPUMS, singles are represented by the category never-married women and married by the ever-married women. For comments about cohabitation, see Appendix C.} \(ii\) there is a U-shaped relationship between childlessness and education both for single and married women: childlessness rates are the highest for extreme levels of education. And \(iii\) the relationship between marriage rate and education is hump-shaped. All these facts are discussed more in detail in Section 2.

A growing literature is concerned both about family composition and fertility choices, however, it does not allow to explain these three facts altogether. Greenwood et al. (2003) and Regalia et al. (2008) analyze both the marriage and fertility decisions in a framework where individuals can divorce. We do not consider this outside option in order to simplify our theoretical results: it allows us to concentrate on the choice of becoming a parent or remaining...
childless.\textsuperscript{3} We want to go further in the fertility analysis and the already well known negative relationship between fertility and income in order to incorporate other components of family choices such as childlessness. Consequently, our work complements the preceding ones, still allowing to replicate marriage rates in the United States for women having completed their fertility life-cycle in 1990 (we only consider women between 45 and 70 years old). The way of modeling is also different from theirs due to our choice not to include divorce: we have a cooperative decision process inside the household while Greenwood et al. (2003) use a Nash bargaining framework and Regalia et al. (2008) use a unitary decision model where the woman chooses the amount of children that the couple has.\textsuperscript{4}

To the best of our knowledge, this paper constitutes the first attempt to study the determinants of marriage and childlessness in a unified framework. Gobbi (2011) is the only paper to model the choice to remain childless. She studies the determinants and the evolution of voluntary childlessness during the XX\textsuperscript{th} century. However, this paper does not look at marriage opportunities which is an important component of the decision to remain childless or not.

In our model, we assume that agents of both sexes play a two-stages marriage game. During the first step, they are randomly matched with a partner of the opposite sex and they decide whether to marry or not.\textsuperscript{5} If a man decides to remain single, then he will not be able to have children and will consume his life-cycle income. A single woman can either become mother or remain childless, so she faces a trade-off between consumption and fertility. When both man and woman decide to get married, they enter into a negotiation process in which they determine (i) if they have children or not, (ii) how many and (iii) how much each of the spouses will consume. Following a large literature initiated in the 90’s, we assume a collective cooperative negotiation process. This process is a special case of the general framework proposed by Chiappori and his co-authors (see Bourguignon et al. (1993), Bourguignon and Chiappori (1994), Browning and Chiappori (1998)) to modelize households’ behaviors. As shown by Chiappori, this framework is empirically relevant.

We specifically assume that the negotiation power of spouses is bounded and positively related to their relative wage. As education is the only determinant of wages in our model,

\textsuperscript{3}The models of Greenwood et al. (2003) and Regalia et al. (2008) allow to have childlessness but it is not the aim of their work to explain it neither theoretically nor empirically.

\textsuperscript{4}Notice also that the fertility measure that they use is a fertility per age measure while we use the completed fertility.

\textsuperscript{5}To simplify, there is no second round. Random matching is assumed as a simplifying hypothesis, a positive degree of assortative matching would give us one more degree of freedom to fit the data, which would improve the results. We do not consider the possibility of matching between agents of the same sex nor adoption, again for a simplification purpose.
negotiation power is in fact positively related to spouses’ relative education. The boundedness of the bargaining power function comes from the legal aspect of marriage, spouses have to help themselves both in sickness and poverty. In line with Iyigun and Walsh (2007), children are considered as a public good for the couple and there is no gender differences in preferences for children; while as in Echevarria and Merlo (1999), we assume that the time cost of rearing children is in part supported by men.

Men will want to marry because it gives them the opportunity to have children and eventually to increase their consumption. As a counterpart, they will have to give part of their time to childrearing. The advantage of marriage for women is that men alleviate the time cost of raising children and might also increase their consumption. Marriage also generates economies of scale since spouses share the expenses on the public good cost. Following this setup, there is always a surplus coming from marriage. By adopting our collective cooperative decision model rather than a Nash bargaining process where potential spouses share the marriage surplus, we avoid marriage rate equal to one.

The U-shaped relationship between childlessness and education of the mother is driven by the coexistence of involuntary and voluntary causes of childlessness and our model is able to reproduce this evidence. Other social sciences have largely discussed on the definition of involuntary and voluntary childlessness (see Poston and Trent (1982), Morgan (1991) and Toulemon (1996)). A woman will be involuntary childless if she cannot procreate because of biological constraints that lead to subfecundity; the diminished ability to reproduce (see McFalls (1979) for details about factors that can lead to subfecundity). In the model, women remain involuntarily childless when they do not have a minimum amount of commodities needed to be able to procreate. The existence of involuntary childlessness among disadvantaged groups in the United States is treated in McFalls (1979). About the existence of high levels of involuntary childlessness in very poor countries societies, Romaniuk (1980) provides

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6 This assumption comes from de la Croix and Vander Donckt (2010) and is also in line with empirical findings in Friedberg and Webb (2006) and Lührmann and Maurer (2007). An alternative consists in assuming a negotiation power that depends on the spouses’ relative labor income rather than on their relative wage/education as in Iyigun and Walsh (2007). However, Pollak (2005) shows that the latter option should be preferred when non labor incomes are endogenous which is the case in our paper. Indeed, despite wages are exogenous, labor income depends on fertility choices and are therefore endogenous.

7 Indeed, with a Nash bargaining process, if there exists a marriage surplus to be shared, nobody would want to remain single. In this type of framework, the only way to avoid marriage rates equal to one would consist of assuming some negative shocks on the quality of the matching.

8 McFalls (1979) argues that the lower-income groups are much more exposed to causes of subfecundity than the rest of the population. Subfecundity factor that might affect the poorest in developed countries are diseases, malnutrition, psychopathological problems (drug abuse, stress, psychoses) and also some environmental factors. Another argument is that the access to quality health care, and in particular, to fertilization services, is neither free nor reimbursed by the government. Consequently, poor individuals are more constraint with subfecundity because they do not have access to the same technologies.
a good example. The definition of voluntary childlessness is more discussed, a restrictive position defines as voluntarily childless, women who never wanted to become mothers, while a more broad way to define it includes women who postponed pregnancy until they were no longer able to child bear. We take the broad definition; in the model, educated women remain childless because of voluntary reasons. Indeed, in line with the literature, we assume that bearing and rearing a child takes time and this opportunity cost is high for more educated women. With this framework, we are able to explain the U-shaped relationship between childlessness and the level of education. The relationship between these two variables is closer to a J-shaped for married women because marriage works as a life-time instrument against extreme poverty.

The existence of a fixed time cost of becoming parents explains why single women choose less often than married to be mothers as well as the fact that when they become mothers, their fertility is almost as high as the one of married mothers. Indeed, for a given wage, because single women have to pay a higher cost in terms of time, since they get no help from a partner, they are less prone to pay this fixed costs. However, when they agree to pay it, they compensate by having a higher fertility, reducing in this way the average time cost per child.

To go beyond qualitative claims, we use the U.S. Census data for the year 1990 to identify the deep parameters of the model and analyze its relevance with simulations exercise. It is able to reproduce the three stylized facts enumerated at the beginning. In the quantitative model, the relationship between aggregate fertility and women’s wage is not monotonic as in benchmark endogenous fertility models, where the focus is the high opportunity cost of rearing children for more educated women and consequently are not able to account for childlessness among poor women. We also use the model to understand the changes that occurred over the period 1960-90 in marriage and fertility patterns. The model predicts that 50% of the changes in marriage rates and 30% of childlessness rates are explained by the rise in education levels. However, the rise in fertility for this period cannot be accounted for (see Doepke et al. (2007) for literature on the Baby Boom).

The rest of the paper is organized as follows. Section 2 describes our stylized facts in details. Our theoretical model is exposed in Section 3 while Section 4 displays the identification strategy for the parameters of the model and provides simulation results. Section 5 runs counter-factual experiments to understand the changes that occurred between 1960 and 1990. Section 6 concludes.
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Table 1: Education categories.

2 Three Facts from 1990 US Census

We use the 5% sample of the U.S. Census 1990\(^9\) and restrict our attention to women having completed their life-cycle fertility (we look at women aged between 45 and 70 years old, Appendix D shows the facts presented in this section for each five year cohort). We are only interested in the never married and ever married women, so we drop women who are separated, divorced widowed and married when their spouse is absent. This accounts for 30.5% of women. Note that these women have decided to be married or not and to have children or not during the second half of the XX\(^{th}\) century, so they have been fully concerned with the mutation of the American family pattern. We divide the population into 12 categories of education as reported in Table 1. The Table also reports the number of observations (sum of singles and ever married) per category. One should know that each of these observations has a weight given by the Census and represents between 2 and 186 individuals. Table 2 summarizes the stylized facts we focus on in this paper. For each moment, we report the mean and the standard error of the mean.

Fact 1: Single women are much more likely to be childless, however, when they chose to become mothers, their fertility is lower by no more than one child compared to married mothers for all categories of education.

As displayed in Table 2, there is a great differential in motherhood rates between single and married women. Among singles, the highest motherhood rate equals 47.7% for women with grade 11. The lowest rate of motherhood among married women only reaches 80.9% for PhDs. On average, 72.86% of single women remain childless while 91.89% of married women become mothers.

\(^9\)The data is taken from IPUMS.
<table>
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<th>education category</th>
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<th>childlessness rate of couples</th>
<th>completed fertility of married mothers</th>
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<table>
<thead>
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More surprisingly, we can observe that once single women decide to have children, they have almost the same fertility as married mothers. Around grade 11, there is almost no difference between the fertility of single and married mothers. The largest fertility differentials are observed for extremes levels of education but remain lower than one child. We can finally observe that, whatever the marital status, fertility is negatively related to education. This stylized fact is maybe the most famous in the field of population economics (see Becker (1993) and Galor (2005)).

**Fact 2**: Childlessness exhibits an U-Shaped relationship with education for both singles and married women.

The relationship between childlessness and education is not monotonic as the relationship that we observe between fertility and education, this implies that being childless is not a simple corner solution of the usual fertility choice problem. The reason of this U-shaped relationship is the existence of both involuntary and voluntary factors leading a women to remain childless. For married, the U-shaped relationship looks more to a J-shaped, because marriage is used as a way of insuring against involuntary childlessness for women with the lowest levels of education. The increasing side of the U-shaped relationship is easy to understand: education has a negative impact on women’s fertility; then highly educated women tend to be more childless than the others because the opportunity cost for them to be mothers is very high.\(^{11}\)

**Fact 3**: There is a hump-shaped relationship between marriage rates and education levels.

It is noticeable that marriage rates are very high for intermediate levels of education: from grade 5 to Bachelor degree, the marriage rate for women is above 90%. These rates are however much lower for extreme levels of education. Indeed, less than 70% of women without any education are married and around 75% of women having a PhD are married. Although these rates remain high, we guess that they would have probably been much lower if we had studied more recent census data. Interestingly, we will be able to replicate this stylized fact despite our model predicts that agents have a preference for partners having opposite earnings compared to their owns.

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\(^{10}\)Pages 150-151.

\(^{11}\)The negative relationship between involuntary childlessness and income is already documented in Wolowyna (1977), for 1971 canadian Census data. From a cross-country analysis, Poston and Trent (1982) show the existence of a U-shaped relationship between the development level of a country and the childlessness rate: the less developed countries have a high rate of childlessness because of high subfecundity leading to involuntary childlessness while more developed countries are concerned with the increase in voluntary childlessness.
In order to see the robustness of these stylized facts, we have also looked at whether facts 1, 2 and 3 were still present if we considered the population without blacks. For married women, including or excluding blacks does not change anything: the same values for completed fertility of mothers and the same proportions of childless and married are found for every education level. We see however some differences for single women, in particular for education levels between grade 5 and grade 12. The completed fertility of mothers of black single women is higher than the one of white single women for all education levels. The negative relationship between number of children and education is present for both races. Compared to whites, black single women are less childless for all the education levels and the U-Shaped relationship, described in fact 2, is more pronounced for them. However, without considering them, there still exists an U-Shaped relationship between childlessness and education among singles. Furthermore, we can find this U-Shaped relationship among other races like Natives, Asians and also Whites alone. To conclude briefly, black single women have more children and are less childless than white women but the relationship between fertility and education and childlessness and education is the same for both groups. More details are given in Appendix A.

3 The Model

3.1 Main Assumptions

We assume a two sex model, where sex is denoted by $i = \{m, f\}$. Adults have to decide whether they marry or not, how much they consume, $c^i$, and if any, how many children they have, $n$. Marriage is a two stage game: during the first stage, agents are on the marriage market, they are matched randomly with an agent of the opposite sex. Then, they decide to marry or to remain single. A match will end up in a marriage only if the two agents choose to marry. During the second stage of the game, agents decide how much they consume and potentially how many children they have. We will solve the problem backwards, considering first the consumption and procreation decisions conditionally on being married.

Each adult draws a non labor income $a^i > 0$ from a distribution $\mathcal{F}^i (\bar{m}^i, \sigma^i_a)$. We assume that the covariance between $a^i$ and $w^i$ is nil meaning that non labor income correspond to a domestic production. Contrary to wages, abilities in home production are not correlated with education attainment. The total non labor income for a couple equals $a^f + a^m$. Each individual also has a time endowment of 1 to be shared between working and child rearing.
On the labor market, the wage of the woman is denoted $w^f$ and the wage of the man is denoted $w^m$.

Each household also has to pay a good cost, $\mu$. This type of cost is present in Greenwood et al. (2011). The price of commodities is normalized to one. Childrearing costs time to parents: each child born needs $\phi \in ]0, 1[$ units of time to be raised.

The utility of an individual of sex $i$ is:

$$u (c^i, n) = \ln (c^i) + \ln (n + \nu)$$  (1)

We assume that preferences of men and women are the same.\(^{12}\) $\ln(\nu)$, with $\nu > 0$, is the reservation utility for childless individuals. $c^i$ is the individual’s consumption and $n$ the number of children that he or she has.\(^{13}\)

The presence of the parameter $\nu > 0$ makes children a luxury good. This is in line with the evidence that, in traditional societies, when children had low opportunity costs, the rich had more children than the poor.

We assume that single women can have children whereas single men cannot. Both married fathers and single or married mothers have to give a portion of their time for raising the children, this implies an opportunity cost for parents, the amount of time spent raising children is no longer available for work. We also assume that in order to be able to give birth, women have to consume at least $c_m$. The choice on fertility, both for single and married, is then conditioned on the constraint the woman consumes more than $c_m$:

$$c^f < c_m \Rightarrow n = 0$$  (2)

We also assume that individuals have to pay a fixed cost, $\eta \in (0, 1)$, in terms of time to become parents,

$$\eta(n) = \begin{cases} \frac{2}{n} & \text{if } n > 0 \\ 0 & \text{if } n = 0 \end{cases}$$  (3)

This is justified by the fact that the first child costs more in terms of time than the following

\(^{12}\)A recent and growing literature highlights the existence of gender differences in preferences with respect to risk aversion and competition (see Niederle and Vesterlund (2007) and Dargnies (2009)). Esraman and Kotwal (2004) provide a theory for the existence of such differences concerning fertility. We chose not assuming heterogeneity in preferences. Above the obvious analytical simplification, it is a way to measure the importance of pure economic mechanisms for our three stylized facts.

\(^{13}\)We also looked at a utility function such as: $u (c^i, n) = \ln (c^i) + \beta \ln (n + \nu)$ where $\beta$ reflects the taste for children, however, this parameter is not strongly identified as $\nu$ is also a fertility preference parameter. We then chose to fix $\beta = 1$. 

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3.2 Presentation of the Different Regimes [to be completed]

Individual choices on consumption, fertility and marital status lead to different “regimes”, corresponding to different life situations, depending on which constraint binds. In this subsection we list these different regimes, providing in each case the optimal consumption and fertility levels. These levels will be needed to identify the precise conditions under which each regime arises as a function of the partners’ wages. These will be provided in the next subsection.

We start with the simplest problem, the one of single men.

3.2.1 Single Men

A single man cannot have children. Therefore, he can only consume his total income. Let \( \bar{c}^m \) denote the consumption of a single adult man:

\[
\bar{c}^m = w^m + a^m - \mu
\]  

(4)

Then the indirect utility of a single man is: \( \bar{u}^m = \ln (w^m + a^m - \mu) + \ln \nu. \)

3.2.2 Single Women

Single women can have children, their budget constraint is:

\[
c^f + \phi (1 + \eta(n)) w^f n = w^f + a^f - \mu
\]  

(5)

Given her time constraint \( \phi (1 + \eta/n)n \leq 1 \) the maximum number of children a single woman can have is:

\[
\bar{n}_M = \frac{1 - \phi \eta}{\phi}.
\]

A single woman maximizes (1) subject to (5), (2), (3) and \( 0 \leq n \leq \bar{n}_M \). These women can be in different regimes, depending on which constraint is binding. To determine which regime binds, it is useful to define the following thresholds on the non-labor income.

\[\text{For evidence about this cost, see Turchi (1975). To fix ideas, the mean hours spent on childrearing for a one kid family is 9.274. For a two kids family it is 6.473. And for a three kids family, it is 6.130.}\]
Definition 1

\[ a = c_m \left( \frac{\phi(\nu + \eta) - 1}{\phi \nu} \right) + \mu \]  
\[ \overline{a} = c_m + \mu > a \]  

(6)  
(7)

We now list the possible regimes, before studying in the next section the precise conditions on wages and non-labor income under which each of them prevail.

Regime I. (Involuntary childlessness)  
If income is not sufficient to reach the consumption level required to have children, then:

\[ c_f^I = w_f + a_f - \mu < c_m \]  
\[ n_I = 0 \]  

(8)  
(9)

In this regime, women consume all their income and stand in a kind of poverty preventing them to freely choose their number of children.

Regime II. (Get fit to procreate)  
If consumption at the interior solution is below the level required to have children, it may be optimal to consume more in order to be able to procreate:

\[ c_f^{II} = c_m \]  
\[ n^{II} = \frac{w_f(1 - \phi \eta) + a_f - \overline{a}}{\phi w_f} \]  

(10)  
(11)

Notice that \( n^{II} \) is increasing in the wage as \( a_f < \overline{a} \) in this regime. Indeed, if \( a_f \geq \overline{a} \), then even if she does not work, the woman would be able to have children as she would consume more than \( c_m \).

Regime III. (Interior solution)  
If no constraint is binding and the solution with no children is dominated, the interior regime prevails:

\[ c_f^{III} = \frac{w_f(1 + \phi(\nu - \eta)) + a_f - \mu}{2} \]  
\[ n^{III} = \frac{1}{2} \left[ \frac{w_f(1 + \phi(\nu - \eta)) + a_f - \mu}{\phi w_f} \right] - \nu \]  

(12)  
(13)
In this interior regime, consumption is increasing in wages, while fertility is not always decreasing in wages. When the wage of a woman increases, her total income does so as well as the cost to rear her children. On the one hand, everything else equal, the rise in her total income makes her optimal fertility rate increase, this is an income effect. On the other hand, the increase of the time cost to rear children implies that reducing fertility is optimal, this is a substitution effect. Both effects go in the opposite direction and, as a result, if \( a^f > \mu \), the substitution effect dominates the income effect and \( n_{\text{III}} \) is decreasing with \( w^f \) while the reverse is true.

**Regime IV. (Voluntary childlessness)** When choosing to have no children yields the highest utility, the voluntary childlessness regime prevails:

\[
\begin{align*}
  c_{\text{IV}}^f &= w^f + a^f - \mu \\
  n_{\text{IV}} &= 0
\end{align*}
\]

In this regime, the substitution effect dominates the income effect and the wage of the woman is so high that the opportunity cost of having children is too expensive. Then, women living in this regime fully specialize in labor market activities.

**Regime V. (Hutterites’ fertility)** When it is optimal to devote all the time to having children, fertility is maximal:

\[
\begin{align*}
  c_{\text{V}}^f &= a^f - \mu \\
  n_{\text{V}} &= \bar{n}_M
\end{align*}
\]

This regime is the opposite to the previous one as in this case, the wage of the woman is so small that she decides to fully specialize in having children. The women who are involved in this regime have relatively high non labor income as \( a^f \geq \bar{a} \) is required to reach such a regime.

Conditionally on the being single, we can determine the regime in which a woman will be in as a function of her wage and her non labor income.

**Assumption 1**

\[ \nu(1 - c_m) - \eta > 0 \]
Definition 2 (Wage thresholds for singles)

\[ W_f^0 = \frac{c_m + \mu - a^f}{1 - \phi \eta}, \] (18)

\( W_f^1 \) is the lowest root of the quadratic equation:

\[ (w_f^f + \alpha^f - \mu) \nu = c_m \left( \frac{w_f^f(1 - \phi \eta) + \alpha^f - \mu - c_m}{\phi w_f^f} + \nu \right) , \] (19)

\[ W_f^2 = \frac{2c_m + \mu - a^f}{1 + \phi (\nu - \eta)}, \] (20)

\[ W_f^3 = \frac{a^f - \mu}{1 + \phi (\nu - \eta)}. \] (21)

\( W_f^4 \) is the lowest, or the only root of the quadratic equation:

\[ \ln (w_f^f + \alpha^f - \mu) \nu = 2 \ln \frac{w_f^f(1 + \phi (\nu - \eta)) + \alpha^f - \mu}{2} - \ln (\phi w_f^f), \] (22)

and

\[ W_f^7 = \frac{\alpha^f - \mu}{\phi (\nu + \eta) - 1}. \] (23)

Lemma 1 (Wage Thresholds for Singles) Under Assumption 1, there exist \( W_f^0, W_f^1, W_f^2, W_f^3, W_f^4 \) and \( W_f^7 \) such that:

1. when \( a^f < a^g \): \( W_f^0 < W_f^1 < W_f^2 < W_f^0 \) and,
   - \( \forall w_f^f < W_f^0 \), \( c^f < c_m \) and \( n_{II} < 0 \),
   - \( \forall w_f^f \geq W_f^0 \), \( c^f \geq c_m \), \( c_f^IV < c_m \) and \( n_{II} < 0 \)

2. when \( a^f \in [a^g, \pi] \): \( W_f^0 < W_f^1 < W_f^2 < W_f^4 < W_f^7 \) and,
   - \( \forall w_f^f < W_f^0 \), \( c^f < c_m \) and \( n_{II} < 0 \),
   - \( \forall w_f^f \in [W_f^0, W_f^1] \), \( u(c_f^IV, 0) > u(c_m, n_{II}) \),
   - \( \forall w_f^f \in [W_f^1, W_f^2] \), \( u(c_m, n_{II}) \geq u(c_f^IV, 0) \),
   - \( \forall w_f^f \in [W_f^2, W_f^4] \), \( u(c_m, n_{II}) \geq u(c_f^IV, 0) \),
   - \( \forall w_f^f \in [W_f^4, W_f^7] \), \( u(c_f^IV, 0) \geq u(c_m, n_{II}) \),
   - \( \forall w_f^f \geq W_f^7 \), \( n_{III} \leq 0 \), and Regime III is not defined,
3. when \( a^f \geq \alpha \); \( \mathcal{W}^f_3 < \mathcal{W}^f_4 < \mathcal{W}^f_7 \) and,

- \( \forall w^f < \mathcal{W}^f_3, u(c^f_v, \bar{w}_M) > u(c^f_v, 0) \) and \( n_{III} > \bar{n}_M \)
- \( \forall w^f \in [\mathcal{W}^f_3, \mathcal{W}^f_4], u(c^f_{II}, n_{III}) \geq u(c^f_v, \bar{n}_M) \) and \( u(c^f_{III}, n_{III}) > u(c^f_v, 0), \)
- \( \forall w^f \in [\mathcal{W}^f_4, \mathcal{W}^f_7], u(c^f_v, 0) \geq u(c^f_{III}, n_{III}) > u(c^f_v, \bar{n}_M), \)
- \( \forall w^f > \mathcal{W}^f_7, u(c^f_v, 0) > u(c^f_v, \bar{n}_M) \) and \( n_{III} < 0. \)

**Proof.**

When \( a^f < \underline{a} \), we can check that \( \mathcal{W}^f_7 < \mathcal{W}^f_4 < \mathcal{W}^f_0 \). The value of \( \mathcal{W}^f_0 \) as defined in Equation (18) solves \( n_{II} = 0 \) and is the wage that allows to consume at least \( c_m \) in order to have children. \( \mathcal{W}^f_7 \) defined in Equation (23) solves \( n_{III} = 0 \), as \( \partial n_{III}/\partial w^f < 0 \), for a value higher than \( \mathcal{W}^f_7 \), a women will be childless. In this case, neither Regime II is reachable because \( \mathcal{W}^f_7 < \mathcal{W}^f_0 \) nor Regime III because \( \mathcal{W}^f_7 < \mathcal{W}^f_0 \). Regime V is not reachable because consumption in Regime V is \( a^f - \mu < c_m \) for \( a^f < \underline{a} \). This means if \( a^f < \underline{a} \), women are either involuntarily childless or voluntarily childless: once women can afford to have children, the time constraint becomes too high.

When \( a^f \in [\underline{a}, \bar{a}[: \) Regimes I, II and IV can arise for low \( w^f \). From Equation (11), we know that \( n_{II} < 0 \Leftrightarrow w^f < \mathcal{W}^f_0 \) and that \( \partial n_{III}/\partial w^f > 0 \) so that Regime II is not defined for \( w^f < \mathcal{W}^f_0 \). For \( w^f > \mathcal{W}^f_0 \) women can be mothers in Regime II, so that if they choose not to have children, they are voluntarily childless (Regime I does not exist anymore).

Comparing utility in Regime IV with utility in Regime II, \( u(c^f_{IV}, 0) < u(c^f_{m}, n_{II}) \) if and only if,

\[
(w^f + a^f - \mu) \nu < c_m \left( \frac{w^f(1 - \phi \eta) + a^f - \mu - c_m}{\phi w^f} + \nu \right).
\]

The LHS is linear and increasing in \( w^f \) and the RHS is increasing and concave in \( w^f \) (since \( a^f - \mu < c_m \)), so that Equation (19) has at most two solutions. At \( \mathcal{W}^f_7 \), the LHS of inequation (24) is higher than the RHS, implying that the \( u(c^f_{IV}, 0) > u(c^f_{m}, n_{II}) \).

Now, we show that, for \( a^f \in [\underline{a}, \bar{a}[: \) \( \mathcal{W}^f_0 < \mathcal{W}^f_1 < \mathcal{W}^f_2 \). At \( w^f = \mathcal{W}^f_2 \), we have that,

\[
LHS(\mathcal{W}^f_2) = \frac{2c_m + (a^f - \mu) \phi (\nu - \eta)}{1 + \phi (\nu - \eta)} \mu
\]

\[
RHS(\mathcal{W}^f_2) = \frac{c_m}{2c_m - (a^f - \mu) \phi} \frac{1 + \phi (\nu - \eta)}{\phi}.
\]

When \( a^f \in [\underline{a}, \bar{a}[: \) \( RHS(\mathcal{W}^f_2) > LHS(\mathcal{W}^f_2) \), is satisfied under Assumption 1. This ensures that \( \mathcal{W}^f_2 \) is in between the two roots of Equation (19). As \( \mathcal{W}^f_1 \) is the smallest root of
Equation (19), then $W_1^f < W_2^f$. If $a^f > g$, $W_0^f < W_2^f$ and since the inequation (24) is not satisfied for $W_0^f$, then $W_0^f < W_1^f < W_2^f$. Then, we can conclude that, under Assumption 1, $\forall w^f \in [W_0^f, W_1^f[,$ $u(c_{IV}, 0) > u(c_{m}, n_{II})$ and that $\forall w^f \in [W_1^f, W_2^f[,$ $u(c_{m}, n_{II}) \geq u(c_{IV}, 0)$.

Now, we aim to show that $W_2^f < W_4^f$.

The value of $W_2^f$ as defined in Equation (20) solves $n_{II} = n_{III}$. At $W_2^f$, $u(c_{m}, n_{II}) = u(c_{III}, n_{III})$ and regime III can be reached. This means that, $\forall w^f > W_2^f$, $u(c_{III}, n_{III}) > u(c_{m}, n_{II})$. At $W_2^f$, $u(c_{m}, n_{II}) \geq u(c_{IV}, 0)$. So, we can conclude that $u(c_{m}, n_{II}) \geq u(c_{IV}, 0)$ at $W_2^f$.

Regime III exists for $w^f \in [W_2^f, W_4^f[]$ where the value of $W_2^f$ as defined in (23) solves $n_{III} = 0$. For all $w^f > W_7^f$, $u(c_{III}, n_{III})$ is not defined since $\partial n_{III}/\partial w^f < 0$ so that as $n_{III}$ would be negative. Then, for $w^f > W_7^f$, Regime IV prevails if single. Let us compare utility in Regime IV with utility in Regime III when $w^f \leq W_4^f$. $u(c_{IV}, 0) > u(c_{IV}, n_{III})$ if and only if,

$$\ln \frac{w^f + a^f - \mu}{w^f(1 + \phi(\nu - \eta)) + a^f - \mu} + \ln \nu > \ln \frac{w^f(1 + \phi(\nu - \eta)) + a^f - \mu}{\phi w^f} - 2 \ln 2. \quad (25)$$

Considering Equation (25), at $w^f = 0$, the LHS is equal to $\ln \nu$ and the RHS goes to $+\infty$. At $+\infty$, the limits of the LHS and the RHS are respectively,

$$\ln \frac{1}{1 + \phi(\nu - \eta)} + \ln \nu \quad (26)$$

and

$$\ln \frac{1 + \phi(\nu - \eta)}{\phi} - 2 \ln 2 \quad (27)$$

so that the RHS is above the LHS for low values of $w^f$. For large values of $w^f$, we cannot rank the two limits, and the RHS can be above or below the LHS. As both sides of the inequality are strictly decreasing and convex with $w^f$, the LHS can be equal to the RHS for either one or two values of $w^f$.

At $w^f = W_4^f$, the LHS is larger than the RHS (LHS - RHS = $\ln(1 + \eta/\nu)$), implying that $u(c_{IV}, 0) > u(c_{III}, n_{III})$. This implies that $W_4^f$ is either in between the roots of equation (22) or at the right of the only root. Since $u(c_{III}, n_{III})$ is not defined for $w^f \geq W_4^f$, the relevant root $W_4^f$ of LHS=RHS is therefore for a value of $w^f$ lower than $W_4^f$ and we have $W_4^f < W_4^f$. This proves that $u(c_{IV}, 0) < u(c_{III}, n_{III})$ for $w^f < W_4^f$, and that $u(c_{IV}, 0) \geq u(c_{III}, n_{III})$ for $w^f \in [W_4^f, W_4^f]$. Intuitively, because there exists a fix cost to become parent, the optimal fertility is not continuous in $w^f$. At the point $W_4^f$ utility with a positive fertility is equal to utility with zero fertility.

We showed that $\forall w^f < W_4^f$, $u(c_{III}, n_{III}) > u(c_{IV}, 0)$ and that at $W_2^f$, Regime III exists and
and the RHS are strictly decreasing and convex in \( w \).

Until now, we proved that \( W_0^f < W_1^f < W_2^f < W_3^f < W_4^f \) under Assumption 1.

Regime V is not reachable when \( a^f < \bar{a} \) since the consumption in Regime V is equal to \( a^f - \mu \), which does not allow the woman to reach the minimal consumption level allowing her to procreate.

We now turn to the case \( a^f \geq \bar{a} \), where Regime V is accessible for all \( w^f \geq 0 \). Regimes I and II do not exist since Regime III is reachable, meaning that even with no wage income, a woman can have a consumption higher than \( c_m \) allowing her to procreate. We then just need to compare the utilities in Regimes III, IV and V. From equation (13), \( n_{III} > \bar{n}_M \iff w^f < W_3^f \), with \( W_3^f \) defined in (21) solves \( n_{III} = \bar{n}_M \). For all \( w^f < W_3^f \), \( u(c_{III}, n_{III}) \) is not defined as \( n_{III} \) would be above the maximum possible (more time in life would be needed).

Let's now show that \( W_3^f < W_4^f < W_5^f \). We know from inequality (25) that both the LHS and the RHS are strictly decreasing and convex in \( w^f \). We can check that at \( W_3^f \) the LHS is lower than the RHS. It follows from the definitions of \( W_3^f \) and \( W_5^f \) (Equations (21) and (23)) that \( W_3^f < W_5^f \). Since at \( W_4^f \) the LHS is equal to the RHS, then \( W_3^f < W_4^f < W_5^f \).

At \( W_3^f \) we can show that \( u(c'_f, \bar{n}_M) = u(c'_{III}, n_{III}) > u(c'_{IV}, 0) \). As \( u(c'_{III}, n_{III}) \) is increasing in \( w^f \) and \( u(c'_f, \bar{n}_M) \) is unaffected by \( w^f \), we have that \( u(c'_{III}, n_{III}) \geq u(c'_f, \bar{n}_M) \) for \( w^f \in [W_3^f, W_4^f] \). As \( u(c'_{IV}, 0) \) is increasing in \( w^f \) and \( u(c'_f, \bar{n}_M) \) is unaffected by \( w^f \), we also have that \( u(c'_f, \bar{n}_M) > u(c'_{IV}, 0) \forall w^f < W_3^f \). As \( W_3^f < W_4^f < W_5^f \), that for \( w^f \in [W_3^f, W_4^f] \), \( u(c'_{IV}, 0) \geq u(c'_{III}, n_{III}) > u(c'_f, \bar{n}_M) \) and that \( u(c'_{IV}, 0) \) is increasing in \( w^f \) while \( u(c'_f, \bar{n}_M) \) is constant, then \( \forall w^f > W_3^f; u(c'_f, \bar{n}_M) > u(c'_{IV}, 0) \).

In Figures 1, 2 and 3, we represent the relationship between fertility choices of a single woman and her wage, depending on her non labor income. Figures 2 and 3 suppose that the substitution effect in Regime III dominates the income effect (\( a^f - \mu > 0 \)), we restrict the analysis to this hypothesis because it would be counterfactual to assume the reverse.

In Figure 1, women are childless no matter their wage. This is the case when their non labor income is too small. In this case, the wage allowing them to procreate is higher than the wage for which they would choose to be voluntary childless. This means that once women can afford a child, the time spent with the kid becomes too expensive.

Figures 2 and 3, show that there is not a monotonic relationship between the woman’s wage and fertility. In Figure 2, which corresponds to the case where the net non labor income (\( a^f - \mu \)) is not enough to cover the minimal amount of consumption needed to procreate, an increase in the wage of the woman can increase fertility. This is the case in Regime II: when
Figure 1: Fertility conditionally on being single when $a^f < a$

Figure 2: Fertility conditionally on being single when $a^f \in [a, \bar{a}]$

Figure 3: Fertility conditionally on being single when $a^f \geq \bar{a}$
the interior solution gives a consumption level lower than the minimum amount needed to have children, a woman can decide to consume $c_m$ and allocate all her remaining income to childrearing. As the woman’s consumption is fixed, fertility will be positively related to her wage until her revenue is high enough to reach the interior regime. This regime exhibits properties already present in the literature dealing with Malthusian regimes (see Galor (2005)).\footnote{In our simulation exercise, Regime II disappears because we simulate assuming that the fertility of a woman is discrete, and this does not provides a consumption level that is equal to $c_m$.} Women who are endowed with a high non-labor income are not concerned with this regime. Indeed, when $a^f \geq \overline{\pi}$, a woman can have the maximal number of children, get no labor income and consume more than $c_m$. In this case, Regime V is a corner solution of the interior regime. In this regime, the wage of the woman is so small that it is optimal for her to specialize in childrearing and consume her net non labor income $a^f - \mu$. Figure 3 is the benchmark relationship between the woman’s wage and fertility, as in de la Croix and Doepke (2003), extended to account for voluntary childlessness. Rich single women will mainly be voluntarily childless because the opportunity cost to have children (non participation to the workforce) is so high that they entirely specialize themselves. On the whole, uneducated women will either be involuntarily childless or in the maximal fertility case, this will depend on their non labor income.

These figures show another important feature; the existence of a fixed cost of becoming parent, implies that the passage from childless to parent cannot be represented by a continuous function. The fixed cost reduces the utility of mothers, for whom is never optimal to have a very low number of children.

3.2.3 Couples

To model couples decision making, we assume a cooperative collective decision model as in de la Croix and Vander Donckt (2010). Spouses negotiate on $c^m$, $c^f$ and $n$. Their objective function is:

$$U(c^f, c^m, n) = \theta(w^f, w^m) u(c^f, n) + (1 - \theta(w^f, w^m)) u(c^m, n)$$

where $\theta(w^f, w^m)$ is the wife’s bargaining power given by:\footnote{An alternative to this formulation would consist in including non labor income in the bargaining power, as advised by Pollak (2005). However, this would make our model much heavier without changing our main results as $a^*$ are exogenously drawn from a log-normal distribution.}

$$\theta(w^f, w^m) \equiv \frac{1}{2} \frac{w^f}{w^f + w^m}$$

$$\frac{1}{2} \frac{w^f}{w^f + w^m}$$
The negotiation power of both the wife and the husband admits a lower bound equal to $\frac{1}{2} \theta$. This assumption relies on the obligation imposed to spouses to respect a minimal level of solidarity inside marriage.

There exist some advantages to be married. Being married is the only way for men to have children while it is an opportunity for women to reduce the cost of children. Indeed, we assume that the husband bears a part $(1 - \alpha) < \frac{1}{2}$ of childrearing time. Furthermore, the expenses on the good cost $\mu$ are shared by both spouse. Finally, marriage gives the possibility for at least one of the partners to increase his/her consumption compared to the situation where he or she remains single.

Total non labor income of the household net of cost is $a = a^m + a^f - \mu$. Its budget constraint is:

$$c^f + c^m + \phi (1 + \eta(n)) (\alpha w^f + (1 - \alpha) w^m) n = w^m + w^f + a \quad (30)$$

Notice that the maximal fertility rate of a married woman equals

$$n_M = \frac{1 - \alpha \phi \eta}{\alpha \phi}$$

which is greater than the maximal fertility of a single woman $n_M > \bar{n}_M$. Married women can give birth to more children than single women only because men help them to raise children. From a biological perspective, the maximal number of children a woman can have equals $(2 - \phi \eta)/\phi$ which can be reached only if spouses shares childrearing cost equally.

The maximization problem of the couple is the following one:

$$\max_{c^f, c^m, n} \theta(w^f, w^m) \ln c^f + (1 - \theta(w^f, w^m)) \ln c^m + \ln(\nu + n)$$

subject to (30), (2), (3) and $0 \leq n \leq n_M$.\(^{17}\) Couples can then be in one of the following six regimes:

\(^{17}\)An alternative to this maximization program would consist of adding the two following participation constraints: $u^f(married) \geq u^f(single)$ and $u^m(married) \geq u^m(single)$. Adding these two constraints mean that a potential spouse with a very high negotiation power can have an interest in reducing his/her welfare in order to incite the potential partner to accept marriage. However, because rationality is common knowledge, each potential partner knows that such a marriage contract is not credible. Indeed, as there is no divorce in our set-up, the partner with the highest negotiation power can hold up and not respect the contract. Then, if the sharing rule induced by our maximization program does not lead to a situation where the two partners are better off being married than remaining single, the disadvantaged partner will not accept to marry.
Regime VI. (Involuntary childlessness) When incomes are too low to guarantee a sufficient consumption level to procreate:

\[ c^m_{VI} = (1 - \theta(w^f, w^m))(w^m + w^f + a) \]  
\[ c^f_{VI} = \theta(w^f, w^m)(w^m + w^f + a) < c_m \]  
\[ n_{VI} = 0 \]

Regime VII. (Eat and procreate) When wife’s bargaining share in income is too low to guarantee a sufficient consumption level allowing her to procreate, but it is optimal for the spouse to give up some consumption in order to have children:

\[ c^m_{VII} = \frac{(1 - \theta(w^f, w^m))[w^f(1 + \phi \alpha (\nu - \eta)) + w^m(1 + \phi (1 - \alpha)(\nu - \eta)) + a - c_m]}{2 - \theta(w^f, w^m)} \]  
\[ c^f_{VII} = c_m \]  
\[ n_{VII} = \frac{1}{2 - \theta(w^f, w^m)} \left[ \frac{w^f(1 + \phi \alpha (\nu - \eta)) + w^m(1 + \phi (1 - \alpha)(\nu - \eta)) + a - c_m}{\phi (\alpha w^f + (1 - \alpha)w^m)} \right] - \nu \]

This is the only regime in which fertility depends on the woman’s negotiation power. Indeed, we have assumed that there is no gender differences in preferences. It implies that given their wages and their non labor incomes, men and women have the same fertility desire ($\nu$). However, in the present regime, because the wife cannot increase her consumption, the only way to increase her own well-being is to increase fertility and, to do so, men have to reduce their own consumption, which has a negative impact on the husband’s well-being. Then, fertility is increasing with the woman’s negotiation power and decreasing with the negotiation power of the man. This last result does not necessarily mean that an increase of $w^m$ reduces fertility in this regime. Indeed, despite an increase in $w^m$ makes $\theta(w^f, w^m)$ higher, it also creates both substitution and income effects. Indeed, when the husband’s wage increases, the time cost of a child increases what incite parents to reduce their number of births (substitution effect). However, when $w^m$ rises, the total income of the family does the same what makes optimal fertility higher (the income effect). The net effect of an increase in $w^m$ is the result of the three effects. In more details, both substitution and income effects are contained in the term on the left hand side while the negotiation power

\[^{18}\text{Jones and Schoonbroodt (2009) provide an enlightening discussion on the possible impact of woman’s wage on fertility.}\]
effect is contained in the term on the right hand side of the following formula:

\[
\frac{\partial n_{\text{VIII}}}{\partial w^m} \geq \frac{(2\alpha - 1)w^f - (1 - \alpha)(a - c_m)}{\alpha w^f + (1 - \alpha w^m)} \geq \frac{(1 - \theta)w^f \left([1 + \phi\alpha(\nu - \eta)w^f + [1 + \phi(1 - \alpha)(\nu - \eta)]w^m + a - c_m\right)}{(w^f + w^m)((2 - \frac{1}{2})w^m + (1 + \frac{1}{2}\theta)w^f)}
\]

**Regime VIII. (Interior solution)** If no constraint is binding and the solution with no children is dominated we have the interior solution with \( c_f > c_m \) and \( n > 0 \):

\[
\begin{align*}
c^m_{\text{VIII}} &= \frac{1}{2}(1 - \theta(w^f, w^m)) \left[w^f(1 + \phi\alpha(\nu - \eta)) + w^m(1 + \phi(1 - \alpha)(\nu - \eta)) + a\right] \\
c^f_{\text{VIII}} &= \frac{\theta(w^f, w^m)}{2} \left[w^f(1 + \phi\alpha(\nu - \eta)) + w^m(1 + \phi(1 - \alpha)(\nu - \eta)) + a\right] \\
n_{\text{VIII}} &= \frac{w^f(1 + \phi\alpha(\nu - \eta)) + w^m(1 + \phi(1 - \alpha)(\nu - \eta)) + a}{2\phi(\alpha w^f + (1 - \alpha)w^m)} - \nu
\end{align*}
\]

An increase in the wage of the husband has a positive impact on the fertility only if the substitution effect dominates the income effect:

\[
\frac{\partial n_{\text{VIII}}}{\partial w^m} = \frac{\alpha w^f - (1 - \alpha)(w^f + a)}{2\phi(\alpha w^f + (1 - \alpha)w^m)^2} > 0 \iff w^f > \frac{1 - \alpha}{2\alpha - 1}a
\]  

(34)

Notice that if \( \alpha = 1 \), the substitution effect disappears and then \( \partial n_{\text{VIII}}/\partial w^m > 0 \). In this case, as in many papers (Galor and Weil (1996), de la Croix and Vander Donckt (2010), etc.), an increase in the wage of the man has a pure income effect on fertility. When \( \alpha < 1 \), the income effect always dominates the substitution effect in families with very low non labor income (\( a < \mu \)). In families with higher non labor incomes (\( a > \mu \)), the substitution effect dominates when the wage of the woman is very low relatively to non labor incomes.

In the case of an increase in the wife’s wage, we find:

\[
\frac{\partial n_{\text{VIII}}}{\partial w^f} < 0 \iff w^m > \frac{\alpha}{1 - 2\alpha}a
\]  

(35)

As \( \alpha > \frac{1}{2} \), this condition is always satisfied. Then, an increase in the wage of the woman always reduces fertility; indeed, the substitution effect always dominates the income effect.

Finally, in all families, the wage of the wife has a negative impact on the number of children while the wage of the husband has a positive impact only if \( w^f \) is sufficiently high.

Note that in the interior regime, fertility does not depend on the negotiation power \( \theta^f \) as there are no gender differences in preferences for children. In other words, in this regime, men
and women have the same preference for consumption relative to the quantity of children while in the eat and procreate regime, women have a stronger preference for children than men.

**Regime IX. (Voluntary childlessness)** When choosing to be childless yields the highest utility, we have:

\[
\begin{align*}
\ c_{ix}^m &= \left(1 - \theta(w^f, w^m)\right) \left(w^m + w^f + a\right) \\
\ c_{ix}^f &= \theta(w^f, w^m) \left(w^m + w^f + a\right) \\
\ n_{ix} &= 0
\end{align*}
\]

In this case, the wage of the wife is so high that rearing children is too expensive. Then, both spouses fully specialize in labor market activities. They are part of the DINKS of our model, the other DINKS being the couples involuntarily childless. The DEWKS are all the other married people in the model.

**Regime X. (Eat and procreate a maximum)** When it is optimal for the spouse to give up some consumption for his wife to specialize entirely in procreation:

\[
\begin{align*}
\ c_{x}^m &= \left(\frac{2\alpha - 1}{\alpha}\right) w^m + a - c_m \\
\ c_{x}^f &= c_m \\
\ n_{x} &= n_M
\end{align*}
\]

Compared to regime VII, the optimal fertility rate of the couple does not depend on the negotiation power of the wife. Indeed, as she has already reached her maximal fertility rate, she is no more able to give birth to an additional child to increase her utility.

**Regime XI. (Maximum fertility)** When it is optimal to specialize the bride entirely to the production of children:

\[
\begin{align*}
\ c_{x1}^m &= \left(1 - \theta(w^f, w^m)\right) \left(\left(\frac{2\alpha - 1}{\alpha}\right) w^m + a\right) \\
\ c_{x1}^f &= \theta(w^f, w^m) \left(\left(\frac{2\alpha - 1}{\alpha}\right) w^m + a\right) \\
\ n_{x1} &= n_M
\end{align*}
\]
When the wage of the husband has a positive impact on wife’s fertility in the interior regime, it is intuitive that single women become voluntarily childless for smaller value of \( w^f \). As for given values of \( w^f \) and \( a^f \), the total income of a couple is higher than the total income of a single woman, single women are more concerned with involuntary causes of childlessness.

**Definition 3 (Wage thresholds for marrieds when \( w^m = 0 \))** Wage such that we can afford positive fertility (\( n \to 0 \), \( c^f = c^m \) and \( c^m = 0 \)):

\[
W^f_A = \frac{c_m - a}{1 - \alpha \phi \eta}, \tag{36}
\]

Wage such that \( n_{VII} = 0 \):

\[
W^f_B = \frac{c_m - a}{1 - \alpha \phi (\eta + \frac{\nu \phi}{2})}, \tag{37}
\]

Wage such that \( n_{VII} = n_{VIII} \):

\[
W^f_D = \frac{2c_m - a (1 - \frac{1}{2} \theta)}{(1 - \frac{1}{2} \theta) (1 + \alpha \phi (\nu - \eta))}, \tag{38}
\]

Wage such that \( n_{VII} = n_M \):

\[
W^f_E = \frac{a - c_m}{\frac{\nu \phi}{2} (1 + \alpha \phi (\nu - \eta))}, \tag{39}
\]

Wage such that \( n_{VIII} = n_M \):

\[
W^f_F = \frac{a}{1 + \alpha \phi (\nu - \eta)}, \tag{40}
\]

Wage such that \( n_{VIII} = 0 \):

\[
W^f_G = \frac{a}{\alpha \phi (\nu + \eta) - 1}. \tag{41}
\]

\( W^f_C \) and \( \tilde{W}^f_C \) are respectively the lowest and the highest roots of equation \( U_{VII} = U_{IX} \) which is satisfied

\[
\Leftrightarrow \ln (\alpha \phi w^f (w^f + a)) = \left( 1 - \frac{1}{2} \theta \right) \ln \frac{c_m}{1 - \frac{1}{2} \theta} + \left( 1 + \frac{1}{2} \theta \right) \ln \frac{w^f (1 + \alpha \phi (\nu - \eta)) + a - c_m}{1 + \frac{1}{2} \theta} \tag{42}
\]

\( W^f_H \) and \( \tilde{W}^f_H \) are respectively the lowest and the highest root of equation \( U_{VIII} = U_{IX} \) which is satisfied

\[
\Leftrightarrow \frac{w^f + a}{w^f (1 + \alpha \phi (\nu - \eta)) + a} > \frac{w^f (1 + \alpha \phi (\nu - \eta)) + a}{4 \nu \alpha \phi w^f} \tag{43}
\]
Definition 4 (Non labor income thresholds for marrieds when $w^m = 0$)

\[
A_0 \equiv \frac{\alpha \phi (\nu + \eta) - 1 - \frac{1}{2} \theta (1 + \alpha \phi (\nu - \eta))}{(1 - \frac{1}{2} \theta \alpha \phi \nu)} c_m \tag{44}
\]

\[
A_1 \equiv \frac{\alpha \phi (\nu + \eta) - 1}{\alpha \phi \nu} c_m \tag{45}
\]

Note: $A_0 < A_1 < c_m$.

$A_4 = c_m$, $A_5 = \frac{c_m}{1 - \frac{1}{2} \theta}$

Lemma 2 (Wage Thresholds for Married) There exist $A_2 < A_3$ such that:

- $\forall a \in ]A_2, A_3]$, $W^f_C < \tilde{W}^f_C < W^f_D$
- $\forall a > A_3$, $W^f_C < W^f_D < \tilde{W}^f_C$

It implies that:

1. When $a \leq A_2$:
   - if $w^f < W^f_A$, a married couple cannot have children
   - if $w^f \geq W^f_A$, a married couple can have children but will decide to remain childless

2. When $a \in ]A_2, A_3]$:
   - if $w^f < W^f_A$, a married couple cannot have children
   - if $w^f \in ]W^f_A, W^f_C]$, a married couple can have children but will decide to remain childless
   - if $w^f \in ]W^f_C, \tilde{W}^f_C]$, a married couple will decide to have children in the eat and procreate regime
   - if $w^f \geq \tilde{W}^f_C$, a married couple will decide to remain childless

3. When $a \in ]A_3, A_4]$:
   - if $w^f < W^f_A$, a married couple cannot have children
   - if $w^f \in ]W^f_A, W^f_C]$, a married couple can have children but will decide to remain childless
• if $w^f \in [W_C^f, W_D^f]$, a married couple will decide to have children in the eat and procreate regime
• if $w^f \in [W_D^f, W_H^f]$, a married couple will decide to have children in the interior regime
• if $w^f \geq W_H^f$, a married couple will remain voluntarily childless

4. When $a \in ]A_4, A_5]$:
• if $w^f \leq W_E^f$, a married couple has the maximal number of children in the eat and procreate regime
• if $w^f \in [W_E^f, W_D^f]$, a married couple has a positive but not maximal number of children in the eat and procreate regime
• if $w^f \in [W_D^f, W_H^f]$, a married couple has children in the interior regime
• if $w^f \geq W_H^f$, a married couple remains voluntarily childless

5. When $a > A_5$:
• if $w^f \leq W_E^f$, a married couple has the maximal number of children in the interior regime
• if $w^f \in [W_E^f, W_D^f]$, a married couple has a positive but not maximal number of children in the eat and procreate regime
• if $w^f \geq W_H^f$, a married couple remains voluntarily childless

**Proof.** Before turning to the proof of each of the lemma’s part, we need to highlight some properties of regimes:

- Regimes VI and IX are continuous: indeed, $\forall w^f > 0, U_{v_1} = U_{ix}$. A couple is able to have children once its total income is greater than $c_m$ what is satisfied when $w^f > W_A^f$. It implies that $\forall w^f < W_A^f$, childlessness is involuntary while it is voluntary upon this threshold.

- Regimes VII and VIII are continuous: indeed, regime VII exists once $n_{VII} \geq 0$ what is satisfied $\forall w^f \geq W_B^f$ while regime VIII exists once $c_{VIII}^f \geq c_m$ what is satisfied $\forall w^f \geq W_D^f$. We can verify that when $w^f = W_D^f$, $c_{VIII}^f = c_m$ and $n_{VIII} = n_{VII}$, it induces that $U_{VII} = U_{VIII}$ for $w^f = W_D^f$. It also implies that regime VII is defined for all $w^f \in [W_B^f, W_D^f]$ while regime VIII is defined for all $w^f \in [W_D^f, W_H^f]$ (for $w^f \geq W_H^f$, $n_{VIII} \leq 0$). Both regimes are continuous.
• Equation $U(c^f_{\text{viii}}, c^m_{\text{viii}}, n_{\text{viii}}) = U(c^f_{\text{ix}}, c^m_{\text{ix}}, 0)$ always admits two positive solutions in function of $w^f$ in $\mathbb{R}$. Indeed, $U(c^f_{\text{viii}}, c^m_{\text{viii}}, n_{\text{viii}}) = U(c^f_{\text{ix}}, c^m_{\text{ix}}, 0)$

$$\Rightarrow [4\nu\alpha \phi w^f(w^f + a)]^\frac{1}{2} = w^f(1 + \alpha \phi(\nu - \eta)) + a$$ (46)

Let’s denote $LHS(w^f)$ the left-hand side of equation 46 and $RHS(w^f)$ its right-hand side. We have that $LHS(0) = 0$ while $RHS(0) = a > 0$. Furthermore, $RHS(w^f)$ is increasing and linear while $LHS(w^f)$ is increasing and concave. Indeed:

$$\frac{\partial RHS(w^f)}{\partial w^f} = 2\nu\alpha\phi(2w^f + a)(w^f[w^f + a])^{-\frac{1}{2}} > 0$$

$$\frac{\partial^2 RHS(w^f)}{\partial w^{f^2}} = -\nu\alpha\phi[w^f(w^f + a)]^{-\frac{3}{2}}a^2 < 0$$

Because of the respective curvatures of $LHS$ and $RHS$ and because $LHS(0) < RHS(0)$, equation 46 admits either none or two positive solutions in function of $w^f$. These solutions can be found by solving the following quadratic equation:

$$[4\phi\alpha\nu - (1 + \phi\alpha(\nu - \eta))]w^f' + 2[\phi\alpha(\nu - \eta) - 1]w^f - a^2 = 0$$ (47)

The discriminant of this equation equals $R \equiv 4[(\phi\alpha(\nu + \eta) - 1)^2 + 4\phi\alpha\nu - (1 + \phi\alpha(\nu - \eta))^2]a^2$. It is a linear function of $a^2 \geq 0 \forall a \in \mathbb{R}$. In order to ensure that our problem admits interior solutions for at least some values of $(w^f, a)$, we have to assume that $R > 0$. We can now determine the two solutions that we denote respectively $W^f_H$ and $W^f_M$:

$$W^f_H = \frac{-2([\phi\alpha(\nu + \eta) - 1]) + a\sqrt{R}}{2[4\phi\alpha\nu - (1 + \phi\alpha(\nu - \eta))^2]}$$,  \quad W^f_M = \frac{-2([\phi\alpha(\nu + \eta) - 1]) - a\sqrt{R}}{2[4\phi\alpha\nu - (1 + \phi\alpha(\nu - \eta))^2]}$$

We have already shown that if equation $U(c^f_{\text{viii}}, c^m_{\text{viii}}, n_{\text{viii}}) = U(c^f_{\text{ix}}, c^m_{\text{ix}}, 0)$ admits solutions in $\mathbb{R}^+$, it admits two solutions. Then, we know that $(W^f_H, W^f_M) > (0, 0)$. As $\phi\alpha(\nu + \eta) - 1 > 0$, $(4\phi\alpha\nu - (1 + \phi\alpha(\nu - \eta))^2)$ has to be positive in order to ensure that $W^f_M > 0$. We can then deduce that $W^f_M > W^f_H > 0$.

Importantly, we notice that both $W^f_H$ and $W^f_M$ are necessarily linearly increasing with $a$ and that $W^f_H = W^f_M = 0$ when $a = 0$. □

Figures 5 and 6 show the relationship between the wage of the wife and the fertility of the couple. They are similar to Figures 2 and 3. Since we assumed in last subsection that $a^f > \mu$, we have that $n_{\text{viii}}$ decreases with $w^f$.  

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Figure 4: Fertility conditionally on being married when $a \leq A_2$

Figure 5: Fertility conditionally on being married when $a \in ]A_2, A_3]$}

Figure 6: Fertility conditionally on being married when $a \in ]A_3, A_4]$
Figure 7: Fertility conditionally on being married when $a \in [A_4, A_5]$

Figure 8: Fertility conditionally on being married when $a > A_5$
The condition $c_m \leq \frac{1}{2\alpha} \left( \frac{2\alpha - 1}{\alpha} \right) w^m + a$ implies that $c_m$ does not bind for any value of $w^f$.

Regime VI applies to very poor couples who do not have enough income to ensure that the wife can consume enough to have children. In Regime VII, an increase in $w^f$ implies higher fertility rates: when $w^f$ increases, the wife’s negotiation power does so and increasing fertility is the only way to increase her indirect utility (since her consumption remains constant). It does not mean that increasing $w^f$ necessarily reduces the husband’s consumption since the total income of the couple has increased. Then, for sufficiently high woman’s wages, a couple will switch from this regime to the Regime X, where the wife procreates as much as the time allows her to do so. This is new with respect to Figures 2 and 3: being constrained to consume $c_m$ and having the highest fertility is not possible for singles because when their fertility is maximal, they consume all their non labor income, and if this is higher or equal to $c_m$, it means that the constraint cases do not exist for them. For marrieds, Regime X is simply a corner solution of Regime VII that can be reached because woman’s wage is higher than in the previous regime.

Most of the very poor couples will be in the eat and procreate regime, Regime VII. This highlights the role of marriage as way out of involuntary childlessness. Rich couples will either be parents in the interior regime or voluntarily childless.

### 3.3 Conditions for each Regime to Arise [to be reorganized]

We now consider formally the choice of accepting a marriage offer from a randomly drawn person of the opposite sex. The conditions under which each regime arises are expressed as conditions on the woman’s wage. We first consider the conditions under which the marriage offer will be rejected.

In order to make expressions clearer, we introduce the following notations,

**Definition 5**

\[
\begin{align*}
R(w^f, w^m) &\equiv w^m + w^f + a \\
J(w^f, w^m) &\equiv w^f \left( 1 + \phi \alpha (\nu - \eta) \right) + w^m \left( 1 + \phi (1 - \alpha) (\nu - \eta) \right) + a \\
P(w^m) &\equiv \frac{2\alpha - 1}{\alpha} w^m + a \\
\end{align*}
\]

$R(w^f, w^m)$ is the total income of the family. $P(w^f, w^m)$ denotes the net income of the family when women have the maximal number of children.
3.3.1 Would you Marry me? No

Let’s define the indirect utility of a man earning a salary $w^m$ married with a woman earning $w^f$ by $U(w^f, w^m)$:

$$U(w^f, w^m) = \max \left[ U(c_{V1}^f, c_{V1}^m, 0), U(c_m, c_{VII}^m, n_{VII}), U(c_{VIII}^f, c_{VIII}^m, 0), U(c_m, c_{IX}^m, n_M), U(c_{XI}^f, c_{XI}^m, n_M) \right]$$

where the consumption and fertility levels of each regime have been defined in the previous subsection as a function of wages.

**Proposition 1 (Regime I)** A woman is single and involuntarily childless if

$$a^f < \bar{a} \quad \text{and} \quad w^f < W_0^f$$

and if at least one of the following conditions holds:

$$w^f + a^f - \mu > \theta(w^f, w^m)R(w^f, w^m)$$

$$\bar{u}^m = \ln(w^m + a^m - \mu) + \ln(\nu) > U(w^f, w^m)$$

**Proof.** Condition (48) implies, by Lemma 1, $n_{II} < 0$ if $w^f < W_0^f$ and $c^f < c_m$, this ensures involuntary childlessness if single. Condition (49) implies that $u(c_f^I, 0) > u(c_{VIII}^I, 0)$. As women always prefer to be married with a husband allowing them to consume at least $c_{m}^{\min}$ rather than being an involuntary childless single, i.e.

$$u(c_f^I, 0) < \min\{u(c_m, n_{VII}), u(c_{VIII}^I, n_{VIII}), u(c_{IX}^I, 0), u(c_m, n_M), u(c_{XI}^I, n_M)\}$$

(49) is enough to guarantee that the women does reject the marriage offer. If she does not, (50) implies that the man rejects the offer. ■

**Proposition 2 (Regime II)** A woman is a single mother and consumes $c_m$ if

$$w^f > c_m + \mu - a^f,$$

$$w^f \in [W_1^f, W_2^f]$$
and if either (50) or the following condition holds:

\[
\ln (c_m) + \ln \left( \frac{w^f (1 + \phi (\nu - \eta)) + \alpha^f - \mu - c_m}{\phi w^f} \right) > \max \{ u(c_m, n_{VII}), u(c_{VIII}^f, n_{VIII}), u(c_{IX}^f, 0) \}
\]  

(53)

**Proof.** Condition (51) implies that she has enough income to consume at least \(c_m\). Condition (52) implies, by Lemma 1, that \(u(c_m, n_{II}) \geq u(c_{I}^f, 0)\) and that \(c_{III}^f < c_{\text{min}}\) which means that regime II brings to the woman the highest utility she could enjoy as single, as the only alternative is to remain childless.\(^{19}\) Inequality (53) implies that \(u(c_m, n_{II})\) is larger than \(\max\{u(c_m, n_{VII}), u(c_{VIII}^f, n_{VIII}), u(c_{IX}^f, 0)\}\), which is enough to ensure that the woman rejects the marriage offer. Indeed, a woman always prefers to be a single mother who consumes \(c_m\) rather than a childless married woman who consumes less than \(c_m\), i.e. \(u(c_m, n_{II}) > u(c_{I}^f, 0)\). Moreover, \(u(c_m, n_{III}) < \min\{u(c_m, n_{M}), u(c_{XI}^f, n_{M})\}\) since in regimes X and XI, the woman would at least consume \(c_m\) and would have the maximal number of children. If the woman does not reject the offer, (50) implies that the man rejects the offer. \(\blacksquare\)

**Proposition 3 (Regime III)** A woman is a single mother and makes an unconstrained choice (interior regime) if

\[
w^f \in ]W^f_2, W^f_4[ \quad \text{and} \quad c_m > a^f - \mu
\]  

(54)

\[
w^f \in ]W^f_3, W^f_4[ \quad \text{and} \quad c_m \leq a^f - \mu
\]  

(55)

and if either (50) or the following condition holds:

\[
u(c_{III}^f, n_{III}) > \max\{u(c_m, n_{VII}), u(c_{VIII}^f, n_{VIII}), u(c_{IX}^f, 0), u(c_m, n_{M}), u(c_{XI}^f, n_{M})\}
\]  

(56)

**Proof.** (54) implies that the interior regime can be reached. Condition (55) implies, by Lemma 1 that it brings her more utility than having either no child or a maximal number of children. Inequality (56) implies that she prefers to remain single than marrying the man she has been matched with. If the woman does not reject the offer, (50) implies that the man rejects the offer. \(\blacksquare\)

\(^{19}\)Notice that we never need to compare indirect utilities in the interior regimes (regimes III, IV, V) to indirect utilities in constrained regime since by definition, non constrained solutions are better than the others. Interior regimes are always accessible once \(a^f > \mu + c_m\), for any \(w^f\).
Proposition 4 (Regime IV) A woman is single and voluntary childless if condition (54) does not hold, if \( w^f > W^f_4 \), where \( W^f_4 \) is defined in Equation (22), and if either (50) or the following condition holds:

\[
  u(c^f_{iv}, 0) > \max\{c_m, n_{vii}, u(c^f_{viii}, n_{viii}), u(c^f_{ix}, 0), u(c_m, n_X), u(c^f_{xi}, n_{xi})\}
\]

Proof. Condition (54) implies that the interior regime is accessible. \( w^f > W^f_4 \) implies, by 1, that \( u(c^f_{iv}, 0) > \max\{u(c^f_{iii}, n_{iii}), u(c^f_{iv}, n_{M})\} \), hence she prefers to be single childless rather than single mother. Inequality (57) implies that she prefers to remain single than marrying the man she has been matched with. If the woman does not reject the offer, (50) implies that the man rejects the offer.

Proposition 5 (Regime V) A woman is single and has the maximum number of children \( n_M \) if \( a^f - \mu \geq c_m \), if \( w^f < W^f_3 \), where \( W^f_3 \) is defined in Equation (21), and if either (50) or the following condition holds:

\[
  u(c^f_{iv}, n_M) > \max\{u(c^f_{viii}, n_{viii}), u(c^f_{ix}, 0), u(c_m, n_M), u(c^f_{xi}, n_{M})\}
\]

Proof. Condition \( n_M \) if \( a^f - \mu \geq c_m \) implies that the single women is able to procreate without working. By Lemma 1, \( w^f < W^f_3 \) implies that \( u(c^f_{iv}, n_M) > \max\{u(c^f_{iii}, n_{iii}), u(c^f_{iv}, 0)\} \), i.e. having the maximal number of children maximizes the utility of the single woman. Inequality (58) implies that she prefers to remain single than marrying the man she has been matched with. If the woman does not reject the offer, (50) implies that the man rejects the offer.

From these first five propositions we can take the following insights about when will women want to remain single.

Very poor women will accept almost any match on the marriage market. This highlights the role of marriage as an institution protecting women against poverty and involuntary childlessness. We should also notice that a woman can accept to marry a very poor men (condition (49) does not hold) if this man’s wage is not much greater than her own wage. In this case, negotiation powers of each spouse would be sufficiently close to one half and potential spouses would be able to share the marriage surplus \( \frac{\lambda}{2} \) (the scale economy in housing, etc...). As shown on Figures 2 and 3, single women with high non labor income are neither concerned with involuntary childlessness nor with the eat and procreate regime. When Regime V is accessible, a poor women will always prefer to be single with the maximum
number of children rather than being an involuntary childless married woman or a married woman who has to be fed in order to procreate.

Regime IV is accessible for rich women. These women will accept to marry the man they have been matched with in only two circumstances. The first case is when the additional income coming from the man is not high enough to incite the woman to become a mother and sufficiently close to her own salary in order to share the marriage surplus which equals $\mu/2$. The second case is when the man’s wage is high enough to decrease the relative wage of the woman, as well as her bargaining power inside the household, and incite her not to fully specialize in the workforce participation. In other words, rich and highly educated women will accept to become married mothers only if they are matched with a rich man.

It becomes clear that the indirect utility of a poor woman is increasing with her potential husband wage while this is not necessarily the case for rich women. Indeed, we know that $\frac{\partial U(c^f, c^m, n)}{\partial w^m} > 0$, however, nothing ensures that $\frac{\partial u(c^f, n)}{\partial w^m} > 0$. Indeed:

$$\frac{\partial U(c^f, c^m, n)}{\partial w^m} = \frac{\partial \theta(w^f, w^m)}{\partial w^m} (u(c^f, n) - u(c^m, n)) + \theta(w^f, w^m) \frac{\partial u(c^f, n)}{\partial w^m}$$

$$+(1 - \theta(w^f, w^m)) \frac{\partial u(c^m, n)}{\partial w^m} > 0$$

$$\Leftrightarrow \theta(w^f, w^m) \frac{\partial u(c^f, n)}{\partial w^m} > -(1 - \theta(w^f, w^m)) \frac{\partial u(c^m, n)}{\partial w^m} - \frac{\partial \theta(w^f, w^m)}{\partial w^m} (u(c^f, n) - u(c^m, n))$$

Because an increase in $w^m$ can lead to a decrease in $c^f$, the LHS on this last equation can be negative especially when $w^m > w^f$, implying that $u(c^f, n) - u(c^m, n) < 0$.

It is also important to notice that men are not always better off when they are matched with richer women. In fact, they would like to be matched with very rich women since,

$$\lim_{w^f \to +\infty} c^m - \frac{1}{2}\theta(1 + \phi\alpha(\nu - \eta))w^f - \frac{1}{2}\theta(1 + \phi(1 - \alpha)(\nu - \eta)) + a - \mu] = 0$$

However, things are less clear-cut when woman’s wages are lower. Indeed, an increase in $w^f$ reduces the fertility of the couple and the negotiation power of the husband, which could finally reduce his consumption. Then, we can conclude that the model could predict higher marriage rate for rich men than for rich women while the reverse is true for poor agents; women will marry more often. Indeed, marrying a poor woman is an opportunity for a richer man to have children without reducing a lot his consumption.

The conclusion we can take from these propositions is that poor women tend to accept to marry more easily than rich women since marriage can play a role of protection against involuntary childlessness and even against living in a get fit to procreate regime. However,
high \( w^m \) also prevents married women from becoming voluntary childless as men’s wage increases their wife’s fertility. An increase in woman’s wage also protects against involuntary childlessness and eat to procreate regimes. Nevertheless, it incites women to become childless whether they remain single or decide to marry. In other words, richer women do not need to be protected against involuntary childlessness by marriage. On the contrary, for a rich woman, being matched with a rich man is the occasion to have children rather than being a voluntarily childless single because marriage reduces the opportunity cost to become mother.

Obviously, being single is not always a personal choice. Indeed, despite a woman would like to marry the man she has been matched with, she can remain single if this man does not accept to marry her. This is the case when condition (50) is verified.

### 3.3.2 Would you Marry me ? Yes

The utility of being single is given by,

\[
\Omega(w^f) \equiv \max \left\{ u(c_{f, 0}^I), u(c_m, n_{II}), u(c_{f, 0}^III), u(c_{f, \bar{n}_M}) \right\} = \max \left\{ u(c_{I, 0}^f), u(c_m, n_{II}), u(c_{f, n_{III}}), u(c_{f, \bar{n}_M}) \right\}
\]

as \( c_{IV}^f = c_{I}^f \) at given wage. Replacing the consumptions and fertility by their values leads to,

\[
\Omega(w^f) = \max \left\{ \ln (w^f + a^f - \mu), \ln \frac{c_m((1 - \phi(\eta - \nu))w^f + a^f - \mu - c_m)}{\phi w^f}, 2 \ln \frac{w^f(1 - \phi(\eta - \nu)) + a^f - \mu}{4} - \ln \phi w^f, \ln(a^f - \mu) \left( \nu + \frac{1 - \eta \phi}{\phi} \right) \right\}
\]

which is increasing in \( w^f \).

**Proposition 6 (Regime VI)** A woman is married and involuntarily childless if

\[
w^f \leq c_m + \mu - w^m - a \tag{59}
\]

and if the two following conditions hold:

\[
w^m + a^m - \mu < (1 - \theta(w^f, w^m)) R(w^f, w^m) \tag{60}
\]

\[
\ln \left( \nu \theta(w^f, w^m) R(w^f, w^m) \right) \geq \Omega(w^f) \tag{61}
\]
Proof. Condition (59) implies that \( R(w^f, w^m) \leq c_m \), and the couple’s total income is lower than \( c_m \). (60) implies that \( \bar{w}^m < u(c^m_{vI}, 0) \), while (61) implies \( u(c^f_{vI}, 0) \geq \Omega(w^f) \). Hence, husband’s and wife’s utility are higher when married than when single.

It is important to notice that the only motive for two very poor agents to marry while they won’t be able to procreate lies in the scale economies allowed by marriage: instead of paying a cost \( \mu \) to live as a single, they have to pay the same cost to live as a couple, this implies an average cost of \( \mu/2 \). Then, only the agents having close wages will live together in these conditions because close wages ensure that negotiation powers remain close to one half what allows to share the surplus \( \mu/2 \).

Proposition 7 (Regime VII) A woman is married and consumes \( c_m \) to procreate if (59) does not hold, if

\[
\frac{1}{2} \theta(w^f, w^m) J(w^f, w^m) < c_m, \quad (62)
\]

\[
\theta(w^m, w^f) \ln c_m + (1 - \theta(w^m, w^f)) \ln(1 - \theta(w^m, w^f)) + (2 - \theta(w^m, w^f)) \ln \frac{J(w^m, w^f)}{2 - \theta(w^m, w^f)} - \ln(\phi(\alpha w^f + (1 - \alpha)w^m)) > \max\{U(c^f_{vI}, c^m_{vI}, 0), U(c_m, c^m_{X}, n_M)\} \quad (63)
\]

and if

\[
c_m \leq J(w^m, w^f) - (2 - \theta(w^f, w^m)) \left[ (w^m + a^m - \mu) \nu \phi(\alpha w^f + (1 - \alpha)w^m) \right]^{1/2} \quad (64)
\]

\[
\ln c_m + \ln \left( \frac{1}{2 - \theta(w^f, w^m)} \left[ \frac{J(w^f, w^m) - c_m}{\phi(\alpha w^f + (1 - \alpha)w^m)} \right] \right) \geq \Omega(w^f) \quad (65)
\]

Proof. If (59) does not hold, the total income of the family is high enough to reach this regime. Condition (62) implies that \( c^f_{vI} < c_m \), and the interior regime is not accessible. Condition (63) implies that being childless or having the maximum fertility with \( c_m \) yields lower utility to the couple. (64) implies that \( u(c^m_{vI}, n_{vI}) \geq \bar{w}^m \), while (65) implies that \( u(c_m, n_{vI}) \geq \Omega(w^f) \), so that both spouses prefer to be married in this regime than to be single.

This regime highlights the role of marriage as a protection against involuntary childlessness. Indeed, two partners whose individual earnings are smaller than \( c_m \) could have children by pooling their income while they would have to remain childless and singles otherwise. Men who live in this regime agree to reduce their consumption in order to make their wife able to
have children. Compared to a childless regime, they have to transfer \( c_m - \frac{\theta(w^f, w^m)}{2} \mathcal{R}(w^f, w^m) \) to their wife.

Very rich men never live in this regime as they ensure their wife a consumption level higher than \( c_m \) whatever \( w_f \). Indeed,

\[
\lim_{w^m \to +\infty} c_f - \frac{1}{2}\theta[(1+\phi(1-\alpha)(\nu-\eta))w_m - \frac{1}{2}\theta[w_f(1+\phi\alpha(\nu-\eta))] + a^m + a^f - \mu] = 0
\]

It means that marriage can also be a protection against the eat and procreate regime as living with a rich man ensure to consume at least \( c_m \).

In this regime, an increase in \( w_f \) implies higher fertility rates. Indeed, when \( w_f \) increases, the wife’s negotiation power does so and increasing fertility is the only way to increase her indirect utility (since her consumption remains constant). It does not mean that increasing \( w_f \) necessarily reduces the husband’s consumption since the total income of the couple has increased. Then, for sufficiently high woman’s wages, a couple will switch from this regime to the ”Eat and Procreate a Maximum” regime.

**Proposition 8 (Regime X)** A woman is married and consumes \( c_m \) to procreate the maximum possible \( n_M \) if (59) does not hold, if (62) holds, if

\[
\theta(w^m, w^f) \ln c_m + (1 - \theta(w^m, w^f)) \ln (\mathcal{P}(w^f, w^m) - c_m)
\]

\[
+ \ln \left( \frac{1 + \alpha \phi (\nu - \eta)}{\alpha \phi} \right) > \max \{ U(c_{VI}^f, c_{VI}^m, 0), U(c_m, c_{VII}^m, n_{VII}) \} \tag{66}
\]

and if

\[
c_m \leq \left[ \frac{2 - \alpha}{\alpha} + \frac{\alpha \phi \nu}{1 + \alpha \phi (\nu - \eta)} \right] w^m + \frac{1 - \alpha \phi \eta}{1 + \alpha \phi (\nu - \eta)} (a^m - \mu) + a^f \tag{67}
\]

\[
\ln c_m + \ln \left( \frac{1 - \alpha \phi \eta}{\alpha \phi} + \nu \right) \geq \Omega(w^f) \tag{68}
\]

**Proof.** If (59) does not hold, the total income of the family is high enough to reach this regime. Condition (62) implies that \( c_{VIII}^f < c_m \), and the interior regime is not accessible. (66) implies that \( U(c_m, c_{X}^m, n_M) > \max \{ U(c_{VIII}^f, c_{VIII}^m, 0), U(c_m, c_{VII}^m, n_{VII}) \} \), i.e. being childless or not having the maximum fertility with \( c_m \) yields lower utility to the couple. (67) implies that \( u(c_{X}^m, n_M) \geq \bar{u}^m \), (68) implies \( u(c_m, n_M) \geq \Omega(w^f) \), and hence, both spouses prefer to be married in this regime than to be single. ■

It is obvious that a woman will always prefer to be married in this regime than being either a childless single or living in the get fit to procreate regime as a single.
We now introduce two wage thresholds surrounding the interior regime VIII.

**Proposition 9 (Regime XI)** A woman is married, consumes more than $c_m$ and the couple has $n_M$ children, if condition (62) is violated, if $w_f < W_5^F$ and if

$$w_m \leq \frac{(1 - \theta(w^m, w^f))(a - \mu) \left(1 + \frac{\alpha \phi (\nu - \eta)}{\alpha \phi}ight) - a^m + \mu}{1 - (1 - \theta(w^m, w^f)) \left(\frac{2a - 1}{\alpha} \left(\frac{1 + \alpha \phi (\nu - \eta)}{\alpha \phi}\right)\right)}$$

(69)

$$\ln \left(\theta(w^f, w^m) \mathcal{P}(w^f, w^m)\right) + \ln \left(\frac{1 + \alpha \phi (\nu - \eta)}{\alpha \phi}\right) \geq \Omega(w^f)$$

(70)

**Proof.** If condition (62) is violated, woman’s consumption is greater than $c_m$. Condition $w_f < W_5^F$ implies by Lemma 2 that the couple prefers to have the maximal fertility than letting the woman work. (67) implies that $u(c^m_{XI}, n_M) \geq \bar{u}^m$, (68) implies $u(c^f_{XI}, n_M) \geq \Omega(w^f)$, and hence, both spouses prefer to be married in this regime than to be single. ■

Women always prefer to be married in this regime than being either involuntarily childless single or single who has to eat in order to procreate. Indeed, in this last case, they would consume $c_m < c^f_{XI}$ and give birth to less children since in regime XI, their fertility is maximal. A couple can live in this regime as soon as condition (16) is violated; which can be the case when the husband’s income is high.

**Proposition 10 (Regime VIII)** A woman is married, and no constraint other than the budget constraint is binding for the couple (interior regime) if condition (62) is violated, if $w_f \in ]W_5^F, W_6^F[$, and if,

$$2 \ln \frac{R(w^f, w^m)}{2} + \ln \frac{\theta(w^f, w^m)}{\phi(\alpha w^f + (1 - \alpha)w^m)} > \Omega(w^f)$$

(71)

$$2 \ln \frac{R(w^f, w^m)}{2} + \ln \frac{1 - \theta(w^f, w^m)}{\phi(\alpha w^f + (1 - \alpha)w^m)} > \bar{u}^m$$

(72)

**Proof.** When condition (62) is violated, woman’s consumption is greater than $c_m$. $w_f \in ]W_5^F, W_6^F[$ implies by Lemma 2 that living in the interior regime is more enjoyable than having either no child or the maximal number of children. (71) implies that $u(c^f_{VIII}, n_{VIII}) > \Omega(w^f)$, i.e. the wife prefers to be married in this regime than to be single. (72) implies that $u(c^m_{VIII}, n_{VIII}) > \bar{u}^m$, and the husband prefers to be married in this regime than to be single. ■
Following equation (71), we can show that a woman always prefers to be married in this regime than being an involuntary childless single or single in the eat and procreate regime. The switch from regime XI to regime VIII could either come from an increase in \( w^f \) or a decrease in \( w^m \), if it is \( w^f \) that increases, then the wife’s indirect utility increases as well, implying that this regime is also preferred to regime II. If this switch comes from a reduction in \( w^m \), then it is not sure that her utility increases.

**Proposition 11 (Regime IX)** A woman is married, consumes more than \( c_m \) and the couple decides to remain voluntarily childless when condition (62) is violated, if \( w^f > \mathcal{W}_9^f \) and if,

\[
\begin{align*}
    u(c_{IX}^f, 0) &= \ln \left[ \nu \theta(w^f, w^m) \mathcal{R}(w^f, w^m) \right] > \Omega(w^f) \quad (73) \\
    u(c_{IX}^m, n_{IX}) &\geq \bar{u}^m \iff [1 - \theta(w^f, w^m)] \mathcal{R}(w^f, w^m) > w^m + a^m - \mu \quad (74)
\end{align*}
\]

**Proof.** When condition (62) is violated, woman’s consumption is greater than \( c_m \). When condition \( w^f > \mathcal{W}_9^f \) holds, it implies, by Lemma 2, that the couple prefers to be childless than having children. (73) implies that the woman prefers to be married than single and (74) implies that the man prefers to be married than single. ■

Remembering that \( n_{vIII} \) is decreasing with respect to \( w^f \), we know that \( n_{vIII} \) becomes negative for \( w^f > \frac{[1-2\theta(1-\alpha)(\nu+\eta)]w^m + a^m - \mu}{\partial^2 \alpha(\nu+\eta) - 1} \). As there exists a fix cost to have children, the true threshold for which married people decide not having children is smaller than this value. In fact, couples become voluntarily childless if the relative wage of the wife becomes so high that she has an interest to fully specialize in job market activities. This is true if \( w^f \geq \mathcal{W}_9^F \). The value of \( \mathcal{W}_9^F \) depends on the husband’s wage. Indeed, the higher \( w^m \), the higher \( w^f \) has to be to live in this regime: the higher the mans’ wage, the lower will be the relative wage of the wife, so that this will reduce the relative time cost of childrearing activities for the wife.

In line with condition (74), men do not necessarily accept to marry rich women if it implies not having children and even reducing their consumption. In the same way, rich women who would remain childless whenever they stay single or become married, will accept to marry only if it increases their consumption. As for very poor couples, very rich agents will accept to marry only if their wages are sufficiently close such that they will be able to share the marriage surplus equal to \( \frac{\mu}{2} \). So, everything else being equal, the model is able to predict lower marriage rates for extreme social classes and especially for rich women.
4 Identification of Parameters and Simulations

4.1 Identification

We identify the parameters of the model using the Simulated Method of Moments (SMM). The 9 parameters, listed in Table 3, are identified by minimizing the gap between empirical and simulated moments. These moments are the completed fertility of mothers and childlessness rates among both singles and married women, for the 12 education categories listed in Table 1. The objective function we minimize is given by:

\[ f(p) = [d - s(p)] [W] [d - s(p)]' \]

where \( p \) are the model’s parameters. \( d \) denotes the vector of empirical moments and \( s \) the vector of simulated moments, depending on the parameters. In order to construct the vector of simulated moments, we need some assumptions. First, we assume that, for each category of education, the non-labor income of women follows a log-normal distribution of mean \( \kappa_a \) and variance \( \sigma_a^2 \). Then we draw \( T \) observations from this distribution such that the non-labor income of woman \( j \) equals:

\[ a^j = e^{k^j \sigma_a + \kappa_a} \quad (75) \]

where \( k^j \) is randomly taken from a normal distribution with zero mean and unit variance. Writing

\[ \kappa_a = \ln (\bar{m}_a \bar{w}) - \frac{\sigma_a^2}{2} \]

where \( \bar{w} \) denotes women’s average wage, the parameter \( \bar{m}_a \) can be interpreted as the average ratio of non-labor income to labor income.

Wages are exogenous and computed as follows:

\[ w_e = \gamma \exp\{0.1e\} \quad (76) \]

where \( e \) denotes the average number of years of education in each category (Table 1). We assume that the gender wage gap \( \gamma \) is equal to 0.9 (Erosa et al. (2005)) and we also assume a Mincer coefficient of 0.1.\(^{20}\) With the previous assumption, the maximal wage among men equals 1.111 while the minimal wage among women equals 0.135.

Then, for each woman in each category of education, we draw a potential husband from the

\(^{20}\)In line with the literature, see the survey of Krueger and Mikael (2001).
empirical distribution of education levels among men.\textsuperscript{21} For each level of men’s education, non-labor income is drawn from the same distribution as women. Then, for each category of education for women, we obtain $T$ decisions about marriage and fertility and we are able to calculate the simulated moments.

$W$ is the optimal weighting matrix, i.e. the inverse of the variance-covariance matrix of the empirical moments (Duffie and Singleton (1993)). In our case, this matrix simplifies into a diagonal matrix composed by the inverse of the statistic standard errors of the empirical moments. Indeed, the covariances across various education categories are zero, as these categories are independent (notice also that they contain different number of observations). Moreover, the covariances between moments for married persons and moments for singles are also necessarily zero. Finally, the covariance between childlessness rates and fertility is also zero as childlessness is nil for all individuals for which $n > 0$.

Intuitively, the use of the optimal weighting matrix implies in our case that moments with the lowest standard errors will have a higher weight. Consequently, for these moments, a higher distance between the data and the simulated moments will be more penalized in the objective function.

The minimization of the objective function was first run using PIKAIA and the results used as initial values in UOBYQA. PIKAIA is a genetic algorithm developed by Charbonneau (2002), it allows to find global extrema in highly non linear optimization problems where there exists a high number of local extrema, which is possibly the case in our model. We used PIKAIA in a first step to identify the region in the parameter space where the global maximum lies. Once this region has been identified, we used a faster algorithm designed to find a maximum of a well behaved problem, called UOBYQA (Powell (2002)). It was developed for optimization when first derivatives of the objective function were not available and takes account of the curvature of the objective function, by forming quadratic models by interpolation. We ran these two algorithms in FORTRAN 90 to gain speed. We also assume that the number of births is an integer rather than a continuous variable as in our theoretical model. This simplification does not alter the main mechanisms at play but simplifies computational exercises (and is also realistic).

\textsuperscript{21}We rescaled the empirical marriage rate of men in order to have an equal number of men and women in the population.
<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>variance of the log normal distribution</td>
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</tr>
<tr>
<td>average ratio of non labor income to labor income</td>
<td>$m_a$</td>
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</tr>
<tr>
<td>preference parameter</td>
<td>$\nu$</td>
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<tr>
<td>min consumption level to be able to procreate</td>
<td>$c_m$</td>
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<td>good cost to be supported by a household</td>
<td>$\mu$</td>
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<td>bargaining parameter</td>
<td>$\theta$</td>
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<td>fraction of childrearing to be supported by women</td>
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<td>time cost of having children</td>
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</tr>
<tr>
<td>fixed cost of children</td>
<td>$\eta$</td>
<td>0.222</td>
</tr>
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</table>

Table 3: Identified parameters, $T = 100000$

4.2 Results

The identified parameters are listed in Table 3. Non labor income amounts on average to 84% of labor income. This number seems quite high, unless we interpret the non labor income as including the part of bequests, capital income and transfers (incl. social security) that are not correlated with the education level of the recipient. To have an idea of the magnitude of $\sigma_a$, we computed a Gini coefficient on women’s life-cycle income $w^f + a^f$. It is equal to 0.17. Hence, the estimated $\sigma_a$ is relatively on the low end, but it is not surprising as some dimensions of inequality are absent from the model, such as wage dispersion for similar education level.

To interpret the value of $c_m$ and $\mu$, remember that wages vary on a scale from 0 to 1 for women. The wage of a women having completed Grade 9 equals 0.333 so that women with a degree lower than Grade 10 are not able to pay $\mu$ just with their labor income. A single woman with the lowest wage (which is 0.135) will need a non labor income higher than 0.575 not to be involuntarily childless.

Parameter $\theta = 0.553$ means that the minimal negotiation power of a spouse equals $\theta/2 = 0.28$. The childrearing time is shared between the wife and the husband, we estimate that men are involved for 40% of this time. $\eta$ is the extra time recored to raise the first child, we find that this is 22% higher than the second child. Following the values of $\alpha$, $\eta$ and $\phi$, the maximal number of children that a married woman can raise, $n_M = 7$, while $\bar{n}_M = 4$. This is coherent with the literature on natural fertility such as Tietze (1957) for Hutterite women, Smith (1960) for the Coco Islands’ Malay women or Henripin (1954) for the first generations in Quebec.
The simulated moments that we obtain are represented in Figures 9, 10 and 11. These figures show that we are able to reproduce the three stylized facts presented in the introduction. Of course, we fit married data much better than single data because we have more observations for married women (1055171 observations representing 19882890 individuals for married women and 71832 observations representing 1498555 individuals for single woman). We are able to reproduce the U-shaped relationship between childlessness and education for both married and single women, although the U is slightly more pronounced in the simulation than in the data for married women. The fact that we are able to reproduce the U-shaped could mean that involuntary childlessness among the poorest is quantitatively plausible. We are also able to replicate that the difference between completed fertility of single mothers and married mothers is not larger than one child (Fact 1).
Figure 11: Marriage rates of women and men, by education categories. Data (black) and Simulation (grey)

As a test of our theory, we can compare the simulated marriage rates with their empirical counterpart. Remember that we did not use marriage rates to identify the parameters. Figure 11 shows that the model reproduces the hump-shaped relationship between marriage rates and education levels. However, the model does not replicate well the percentage of marriages for extreme categories of education among men: in the model, 87% of no educated men and 73% of men with a PhD will marry while in the data the proportions are respectively, 72% and 93%.

Table 4 shows the proportion of women in each regime. We see that in the uneducated categories, most single women are involuntarily childless (I) or with the maximal amount of children (V), depending on their non labor income (as it was shown in Figures 2 and 3). Poor married women are involuntary childless (VI), or in the eat and procreate regime (VII), or with the maximal amount of children (XI). This is consistent with Figures 5 and 6). Most of women with average education are married with children (around 80% of women are in the interior regime VIII). Women with the highest education are either voluntarily childless (single in IV or married in IX), or married mothers in the interior regime VII. Regime IX is the regime that we have in mind for DINKS while Regimes VII, VIII, X and XI are the corresponding regimes for DEWKS.

Our model provides a framework to interpret childlessness for both singles and married, allowing for involuntary childlessness for uneducated women and voluntary childlessness for highly educated ones. Table 4 shows that those regimes are not “empty”, and concern a significant fraction of the population. In particular, considering all education categories, involuntary childlessness concerns 5.8% of American women (5.3% singles and 0.5% mar-

22 Notice that Regime II (eat and procreate for single women) is not present in the simulations, this comes from the discrete choice of fertility in the quantitative analysis.
Table 4: Marital and fertility regimes as a function of women’s education

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
<th>IX</th>
<th>X</th>
<th>XI</th>
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<td>10.6%</td>
<td>0.0%</td>
<td>0.8%</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>1.0%</td>
<td>0.7%</td>
<td>13.7%</td>
<td>0.1%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>71.7%</td>
<td>12.3%</td>
<td>0.0%</td>
<td>0.5%</td>
<td></td>
</tr>
<tr>
<td>12</td>
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<td>0.3%</td>
<td>28.4%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>55.7%</td>
<td>15.5%</td>
<td>0.0%</td>
<td>0.1%</td>
<td></td>
</tr>
<tr>
<td>all</td>
<td>5.3%</td>
<td>1.5%</td>
<td>3.1%</td>
<td>0.9%</td>
<td>0.5%</td>
<td>2.5%</td>
<td>76.7%</td>
<td>5.9%</td>
<td>0.1%</td>
<td>3.6%</td>
<td></td>
</tr>
</tbody>
</table>

ried), while voluntary childlessness concerns 9% of American women (3.1% singles and 5.9% married). In our model, childlessness concerns either the very poor or the very rich women, either single or married (see Figures 2, 3, 5 and 6). The relatively high percentage of the population in these regimes allows to understand and give an explanation to the U shaped relationship between education and childlessness highlighted in the stylized facts.

The hump shaped relationship between marriage and education is related to the high childlessness rate among both uneducated and highly educated single women: marrying women who are either not fit to procreate (or would require a massive help from their husband to do so) or have low incentives to have children (high opportunity cost) is less attractive.

Maximum fertility regimes concern 4.5% of American women (0.9% singles and 3.6% married in regimes X and XI); those women, with low education and consequently, low opportunity cost to rear children, would like to have more children but are constraint by their time endowment. Still, the largest fraction of the population is married, in the interior regime (76.7% of American women), which allows to replicate the usual downward sloping relationship between fertility and mother’s education.

Altogether, our simple static model is able to provide an explanation for the relationship between fertility and education, childlessness and education, and finally, marriage and edu-
cation. Despite its static nature, the model also predicts some level of assortative mating: an individual is more likely to marry someone with a similar level of education. Since our model is static, and individuals have only one chance in their life to get married with a random person, this assortativeness level is lower than in the data. In reality, the assortativeness is higher because, first, people meet several possible matches (this would need a dynamic model), and, second, individuals are more likely to meet others who have similar level of education than the one they have.

A way to measure the degree of assortativeness is the one used in Table 5. Each cell gives, for each married man, his increased chances to marry a woman of a given category of education, compared with a pure random matching framework. If there was no assortativeness, all the cells would be equal to 1. The first cell means that a man of education 1 has 57 times more chances to marry a woman of education category 1 than in the case of pure random matching. Appendix E details how the matrix is computed. We are able to compute similar statistics with the simulated data, at the bottom of the same table. In the simulation we also have some assortative matching, although lower than in the data. Regressing each cell of the top part of the table on the corresponding cell of the bottom part leads to an $R^2$ of 0.24 meaning that we are able to account for 24% of the assortativeness mating.

If one wants to increase the rate of assortative matching in our model, one has to modify the bargaining rule (29) by making it more sensitive to the wage ratio. This will increase the rejection rate of matches with very different people. However, achieving more assortativeness will be at the expense of matching a reasonable marriage rate, as people only meet once in our static set-up. This is a limitation of our approach.

In general, with a model able to match the level of assortative mating, in particular among poor, we expect a lower $c_m$. In the facts, we see that among women with no education, 53% of them will marry a man with no education, this is much higher than in our model. Consequently, the $c_m$ given with a model that does not account for assortative mating, generates a lower amount of these couples, so that in general, couples where women have no education are richer, and the constraint on $c_m$ needs to be higher in order to have involuntary childlessness.

5 Counterfactual experiments

We have seen that our model allows to understand rather well the three facts of the introduction for the year 1990. In particular, we characterize well the relationship between
<table>
<thead>
<tr>
<th>Men education category</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
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<td>0.087</td>
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<tr>
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<td>2</td>
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<td>21.421</td>
<td>3.043</td>
<td>1.149</td>
<td>0.580</td>
<td>0.548</td>
<td>0.319</td>
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<td>0.194</td>
<td>0.070</td>
<td>0.051</td>
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<td>3</td>
<td>2.499</td>
<td>4.950</td>
<td>4.332</td>
<td>1.893</td>
<td>1.329</td>
<td>1.069</td>
<td>0.636</td>
<td>0.315</td>
<td>0.332</td>
<td>0.105</td>
<td>0.081</td>
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<td>1.989</td>
<td>1.681</td>
<td>0.852</td>
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<td>0.378</td>
<td>0.777</td>
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<td>0.241</td>
<td>0.364</td>
<td>0.770</td>
<td>0.861</td>
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<td>0.059</td>
<td>0.097</td>
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<td>0.962</td>
<td>0.969</td>
<td>0.982</td>
<td>0.992</td>
<td>1.002</td>
<td>1.017</td>
<td>1.045</td>
<td>1.063</td>
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<td>0.873</td>
<td>0.912</td>
<td>0.944</td>
<td>0.961</td>
<td>0.969</td>
<td>0.990</td>
<td>1.003</td>
<td>1.021</td>
<td>1.064</td>
<td>1.093</td>
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<td>12</td>
<td>0.715</td>
<td>0.732</td>
<td>0.810</td>
<td>0.851</td>
<td>0.887</td>
<td>0.915</td>
<td>0.969</td>
<td>1.009</td>
<td>1.054</td>
<td>1.156</td>
<td>1.232</td>
</tr>
</tbody>
</table>

**Table 5: Assortative Mating**
women education levels, childlessness and fertility. Now, we would like to use these identified relationships over a cross-section of individuals to understand the changes in marriage rates, childlessness and fertility over the period 1960-1990.

The shares of individuals in each education category in 1960 and 1990 for men and women are presented in Table 5. The main feature of this table is that the number of persons who completed grade 12 (category 7 in our table), significantly increased from 1960 to 1990 while persons with an education lower than grade 11 were fewer in 1990 compared to 1960. This implies an overall increase in the education level of both men and women.

### 5.1 Counterfactuals: men’s education shares in 1960

Let us first consider the effect of the rise in men’s education. Keeping the simulated parameters constant, we compute the simulated moments using the shares provided in Table 5 in the year 1960 and compare them with the simulated moments using the shares of 1990. We then compare the change in the simulated moments with the change in actual data over the period 1960-1990. In the data, we observe on the left panel of Figure 12 that the marriage rate decreased for low skilled women but increased for high skilled women between 1960 and 1990. This qualitative change in behavior is well accounted for by the model, as shown on the right panel of Figure 12: the rise in men’s education explains well the shift in the

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23Education categories 11 and 12 are merged in 1960.
relationship between marriage and education level: low educated women marry less often in 1990 because they are more often turned down by more educated men.

In 1960, completed fertility for singles is not given in the Census. We are thus limited to analyze changes in childlessness and fertility for married women only. The left panel of Figure 13 shows that childlessness rates of married women decreased over this period. On the right panel of Figure 13 we compare the simulated childlessness rates in 1990 with the hypothetical childlessness rates of 1960. As childlessness rates also decrease in the simulations, we conclude that the rise in education explains part of the overall decrease, for all education categories. For women with low education, the drop is achieved through marrying a richer husband, which allows to escape the minimum consumption constraint. The drop in childlessness for the highly educated women is strong in the data but low in the simulation; to account for that fact, one would need a model where the fraction of childrearing supported by women \(\alpha\) is covered by babysitters (see Hazan and Zoabi (2011)). Quantitatively, the rise in education explains one third of the drop in childlessness.

Fertility of married women increased a little over the period 1960-1990, for all education categories, as shown in the left panel of Figure 14. The rise in men’s education, on the contrary, predicts a drop in fertility rates, since men participate to child rearing (\(1 - \alpha = 0.74\)).

---

24 The relationship between childlessness and education in 1960 was not U-shaped. One possible explanation for this is that during the first half of the twentieth century, there existed no efficient medical way to reduce involuntary childlessness. Consequently, involuntary childlessness hit both rich and poor in the same way. The second half of the century has been the theater of scientific research on ways to reduce infecundity (i.e. in vitro fertilization in 1978). In the United States, most of the couples who ask for fertility treatments pay it from their pockets so that poor have limited access to these modern and expensive technics (Hughes et al. (2001)). Other than that, rich women protect themselves from involuntary childlessness much more efficiently than poor women because of better living conditions such as better nutrition, sanitation, possibility to give births at the hospital. The introduction of junk food and the increase of drugs abuse are important factors affecting the poor’s fecundity in more recent cohorts (see McFalls (1979)).
0.398), then higher education implies a higher opportunity cost to rear children. The right panel of Figure 14 shows that this drop is very slight though. Our model therefore fails to reproduce the rise in fertility over 1960-1990, but this is not surprising given that this period corresponds to the exceptional event of the baby boom. Since childlessness rates decreased and fertility of mothers increased, total fertility during the period has increased. The simulations are not able to reproduce this either, even though we can reproduce the decrease in childlessness. In order to be able to reproduce this increase, we would need a larger decrease in childlessness rate, as well as being able to match the fertility of mothers.

5.2 Counterfactuals: women’s education shares in 1960

To evaluate the effect of the rise in women’s education, we compute aggregate indicators for fertility from the above simulations using the education shares of 1990 and of 1960.
Table 7 decomposes the change in fertility into two components: the shift effect is obtained by comparing the aggregate fertility in 1990 with the fertility we would have had in 1960 if the education shares of women were the ones of 1990 and only the education shares of men changed. This effect is positive in the data, but negative in the simulation, as explained in the previous subsection. The second effect captures the change in the education shares of women. It is obtained by comparing aggregate fertility in 1960 computed with the shares of 1990 with aggregate fertility in 1960 computed with the shares of 1960. Here, both data and simulation indicate a drop of the same magnitude, related to the overall increase in the education level of women, who have less children because their opportunity cost is higher.

Table 7: Decomposition of the change in aggregate fertility

<table>
<thead>
<tr>
<th></th>
<th>1990</th>
<th>1960 (shares 90)</th>
<th>1960 shift</th>
<th>shares</th>
<th>total</th>
</tr>
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<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>data</td>
<td>2.902</td>
<td>2.052</td>
<td>2.525</td>
<td>0.850</td>
<td>-0.472</td>
</tr>
<tr>
<td>simulation</td>
<td>2.978</td>
<td>3.090</td>
<td>3.528</td>
<td>-0.112</td>
<td>-0.438</td>
</tr>
</tbody>
</table>

We are not able to reproduce the increase in mothers’ completed fertility because some of the women in the 1990 sub-sample are the mothers of the Baby Boom generations. During this Baby-Boom, some shocks have happened that are well above the scope of this paper. Our counterfactual experiment with education shares show that without this Baby Boom, childlessness should have decreased in a smaller proportion while completed fertility of mothers should have decreased. The distance between our simulation and data constitutes a measure of the impact of the Baby Boom on fertility: $0.112+0.850=0.962$.

5.3 Changes in the wage gap $\gamma$

Beyond the rise in overall education levels over 1960-1990, another first order change is the closing of the gender wage gap (Goldin (1990) or Jones et al. (2003)). Our theory predicts that a rise in women’s wage has different effects on fertility and childlessness, depending on which constraint binds the most on the household. To assess the aggregate effect of the closing of the wage gap, we simulate the model for various values of the parameter $\gamma$ of Equation (76).

Figure 15 shows the effect for the singles. As predicted by Proposition 1 there will be less poor woman in the involuntarily childlessness regime. But the educated woman will be more voluntarily childless (Proposition 4). A change in the wage gap has the usual properties on the fertility of mothers who are in the interior regime (Proposition 3): a decrease in
the wage gap decreases fertility since it increases the opportunity cost of rearing children. This is however not true for single women in the corner regime with maximum fertility (Proposition 5) for whom fertility does not depend on wage, meaning that the effect of a higher relative wage is not large enough for them to exit this regime.

The same negative effects on fertility are observed for the married women, see Figure 16. To get an idea of the magnitude of the effect of wage $w_f$ on fertility, we compute the elasticity of total fertility to the wage gap for the larger group (education category 7, married women). When the wage gap closes from 0.8 to 0.9, fertility drops from 3.38 to 3.21, which gives an elasticity of -0.051. [compare to literature, mc grattan, jones and manuelli 2005?]

The wage gap also has an effect on marriage rates. Poor women will marry more often, while highly educated women will marry less often. This effect is particularly large for Ph.D’s: if
the wage gap goes from 0.8 to 1, their marriage rate drops from 0.778 to 0.634.

5.4 Changes in the time allocation parameter $\alpha$

In our model we have assumed that the share mothers ($\alpha$) and fathers ($1 - \alpha$) devote to childrearing is constant across education groups and over time. Bianchi et al. (2004) show that the ratio of married mothers to married fathers time in child care declined between the mid-1960s and the late 1990s, which could be an indication that either social norms changed for all education categories, or that the increase in the education of women makes it optimal for fathers to spend more time with their children.

To have some insights on the role of $\alpha$ we did the following experiments.

First, fixing $\alpha = 1$ instead of identifying it from the cross sectional data, and estimating again the rest of the parameters, we find that the quality of the match is lower: the model becomes unable to reproduce (a) a reasonable marriage rate (especially for highly educated women who have lost their incentive to marry), (b) the U-shaped relationship between education and childlessness for married women, (c) the gap between fertility of the married mothers and fertility of the single mothers, who now face the same opportunity cost. Hence, allowing $\alpha < 1$ is pretty important to generate the nice features of the model.

Second, let us consider that couples set $\alpha$ optimally. A full fledged model with a proper bargaining on $\alpha$ would be the topic of another paper; however, a simple benchmark would be to assume the following rule:

$$\alpha = \begin{cases} 
1 & \text{if } w^f < w^m \\
1/2 & \text{if } w^f \geq w^m 
\end{cases}$$

With this specification the marriage rates are reasonable, the U-shaped relationship between childlessness and education of married women is preserved, but the model fails in reproducing the high fertility of low educated married mothers (by about one child for the two lower education categories), as poor married mothers face almost the same incentives as poor single mothers. Otherwise, the simple ad-hoc rule above does rather well, which indicates that bargaining over $\alpha$ could be a promising extension. Finally, we also tried to make $\alpha$ depend on the education of the mother independently of the education of the father: $\alpha = 1 - w^f / 2$. The properties of the simulation are similar to the previous case.
6 Conclusion

To analyze fertility behavior we distinguished explicitly the decision to have children or not from the choice of the number of children. This distinction turned out to be highlighting, both in terms of data and theory.

Data shows the following three facts. First, single women are much more childless but, when mothers, their fertility is lower by no more than one child compared to married mothers. Second, childlessness exhibits an U-Shaped relationship with education for both singles and married women. Third, there is a hump-shaped relationship between marriage rates and education levels. These facts are robust for different races and age cohorts. The first two facts have never been documented for both single and married before and nobody has ever provided a theory accounting for the three facts jointly.

We have developed a simple model that allows to analyze the effect of men and women wages on fertility going beyond the usual distinction between income and substitution effects. Wages play a complex role, shaping the incentives to accept a marriage or not, and affecting the allocation of resources in the couple.

The main highlight from the theory is to identify several “regimes” and the conditions under which these regimes will prevail. Some of these regimes are new compared to the literature, and turn out to be quantitatively important. The two involuntary childlessness regimes appear for women with low education and low non labor income, either single or married; we estimate that they account for 5.8% of American women. In the “eat and procreate” regime, the income of the woman is not high enough for her to be fit to procreate, but it is optimal for her husband to abandon part of his consumption in order to be able to produce children within the couple. In the voluntary childlessness regime, highly educated women do not have children because of their high opportunity cost.

Our theory also provides a framework to interpret childlessness for both singles and married, allowing for involuntary childlessness for uneducated women and voluntary childlessness for highly educated ones. Simulations show that those regimes are not “empty”, and concern a significant fraction of the population. The relatively high percentage of the population in these regimes allows to understand the U shaped relationship between education and childlessness highlighted in the stylized facts. Still, the largest fraction of the population is married, in the interior regimes, which allows to replicate the usual downward sloping relation between fertility and mother’s education.

Marriage interact with childlessness in two ways; for poor woman, marriage is an opportunity to get enough resources to be able to have children. Hence, marriage reduces involuntary
childlessness. For rich women, marriage reduces the opportunity cost of having children, as husbands also help to raise the children. Marriage therefore also reduces voluntary childlessness.

Identifying the nine structural parameters of the model using a simulated method of moments technique shows that the features of the data on fertility and childlessness are well captured. On the whole, our model provides a way of understanding altogether the relationship between fertility and education, childlessness and education, and finally, marriage and education.

Using the model to understand the changes that occurred over the period 1960-1990 we have learned that an increase in the education of the men leads to a decrease in involuntary childlessness and an increase in the marriage rate of educated persons. As the model fits very well the relationship between mother’s education and fertility, it also accounts well for the effect of the increase of the average education of women on fertility.

A more detailed study in the allocation of time between men and women could give further insights and predictions for the changes in today’s fertility trends. Allowing for a more complex marriage market structure would also improve the predictions of the model in terms of assortative matching.

References


Regalia, F., Ros-Rull, J.-V., and Short, J. (2008). What accounts for the increase in the number of single households?


Figure 17: Childlessness rate and completed fertility of mothers, by education categories, married women.

Figure 18: Childlessness rate and completed fertility of mothers, by education categories, single women.

A Races

In this appendix, we split the population in 4 races, Whites, Blacks, Natives and Asians. Figures 17 and 18 give the childlessness rate and completed fertility of mothers, by education categories, for married women and single women respectively.

In the main text we have assumed an homogeneous marriage market. Here we assume instead that there exists fragmented markets for each race. We therefore reestimate the parameters for each race independently. Results are provided in Table 8. Under each race, the number in parentheses is the total number of observations (unweighted). Blacks are characterized by a lower involvement of black fathers in childrearing and a lower fixed cost of having children. Non labor income is higher and less dispersed for Whites.
Parameter | all | blacks | whites | natives | asians
---|---|---|---|---|---
\( \sigma_a \) | 0.316 | 0.448 | 0.246 | 0.397 | 0.234
\( m_a \) | 0.860 | 0.937 | 1.034 | 1.017 | 0.872
\( \nu \) | 6.839 | 7.172 | 8.304 | 8.125 | 6.579
\( c_m \) | 0.339 | 0.364 | 0.469 | 0.298 | 0.344
\( \mu \) | 0.334 | 0.383 | 0.228 | 0.375 | 0.318
\( \theta \) | 0.553 | 0.399 | 0.752 | 0.646 | 0.572
\( \alpha \) | 0.598 | 0.835 | 0.425 | 0.714 | 0.681
\( \phi \) | 0.224 | 0.197 | 0.244 | 0.198 | 0.232
\( \eta \) | 0.222 | 0.083 | 0.129 | 0.059 | 0.264

Table 8: Identified parameters by race

B Disables

The U-shaped relationship between childlessness and education is highly affected when we consider the data without the individuals that have any lasting physical or mental health condition that prevents or causes difficulty working, living independently or taking care of their own personal needs (respectively, variables DISABWRK, DIFFMOB and DIFFCARE of ipums). Note that these variables say nothing about the ability to reproduce. For married the relationship between childlessness and education is not affected but for singles the relationship becomes flat for the first education levels and increases from grade 11 on.

Our position on this remark, is that disable women are, de facto, lowly educated and the constraint \( c^f > c_m \) reflects their incapacity to have children when they do not have a husband investing on them. Moreover, the Census does not distinguish in 1990 the disabilities carried since birth from the disabilities acquired later in life.\(^25\) Poor working conditions for lowly educated individuals are likely to affect their health when 45-70 years old. There is then a clear endogenous problem in the relationship between disability and education: women who lived in the worst conditions (the ones with the lowest education) when adults are the most likely to suffer health problems when older. In other words, the poor are more likely to have health problems when old than the rich. Consequently, we argue in favor to keep the disables in the data.

However, since 83.5% of single childless women with no school are disable, we also simulate

\(^{25}\)To see percentages about disability types: http://www.census.gov/hhes/www/disability/sipp/disab02/ds02t2.html
<table>
<thead>
<tr>
<th>Parameter</th>
<th>all</th>
<th>without “disables”</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_a$</td>
<td>0.317</td>
<td>0.295</td>
</tr>
<tr>
<td>$m_a$</td>
<td>0.860</td>
<td>0.873</td>
</tr>
<tr>
<td>$\nu$</td>
<td>6.812</td>
<td>6.948</td>
</tr>
<tr>
<td>$c_m$</td>
<td>0.343</td>
<td>0.307</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.334</td>
<td>0.310</td>
</tr>
<tr>
<td>$\dot{\theta}$</td>
<td>0.564</td>
<td>0.530</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.597</td>
<td>0.583</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.225</td>
<td>0.227</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.221</td>
<td>0.221</td>
</tr>
</tbody>
</table>

Table 9: Identified parameters without the “disables”

<table>
<thead>
<tr>
<th>Cat.</th>
<th>%</th>
<th>Cat.</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.8</td>
<td>7</td>
<td>3.6</td>
</tr>
<tr>
<td>2</td>
<td>4.8</td>
<td>8</td>
<td>2.8</td>
</tr>
<tr>
<td>3</td>
<td>4.0</td>
<td>9</td>
<td>3.8</td>
</tr>
<tr>
<td>4</td>
<td>3.6</td>
<td>10</td>
<td>3.0</td>
</tr>
<tr>
<td>5</td>
<td>3.8</td>
<td>11</td>
<td>2.8</td>
</tr>
<tr>
<td>6</td>
<td>4.0</td>
<td>12</td>
<td>8.2</td>
</tr>
</tbody>
</table>

Table 10: Percentage of single mothers, aged 45-70, with an unmarried partner

the model taking out the disables from the data. We can check that the $c_m$ parameter still plays a role, even though its estimated value is lower. The results without the disables are in Table 9.

## C  Cohabitation and Uneducated Single Mothers

When dealing with never married individuals we might be skeptical about whether these individuals are single or just unmarried with a partner (specially for poor women who have many children). In the Census 1990, among never married women aged 45-70, only 1.8% declared themselves as being with a partner. In particular, the percentages for mothers and childless are respectively 3.7% and 1.3%. The percentages vary, however, for different education levels: Table 10 provides the proportion of single mothers, aged 45-70, declaring to be with an unmarried partner for each education level.

This table confirms that very few women who have not married are living with a partner.
The highest percentages of cohabitation are seen for women aged 36-40, having achieved grade 1-4, or with a doctoral degree: respectively 21.5% and 21.7% of never married women in these education levels unmarried but have partner.

Bramlett and Mosher (2002) show that 42.71% of never married women, ever cohabited. Their definition for cohabitation is being unmarried but having a sexual relationship while sharing the same usual address. We then address the question about the duration of cohabitation. The probability of transition to marriage after 1 year of cohabitation is 30%, after 5 years, 70%, and after 10 years, 84%. Consequently, if cohabitation lasts, marriage is very likely to happen.

Rindfuss and VandenHeuvel (1990) compare cohabitants to both married and single individuals and conclude that cohabitants attitudes were in many aspects closer to singles than to married. In particular, cohabitants’ position relative to fertility expectations, is that cohabitants are more similar to singles than to married. Indeed, comparing the percentage of childless individuals who expect children, cohabitants are much closer to never-married than to married (among women, 11% of cohabiting, 4% of singles and 40% of married expect to have a child within two years). Consequently, although cohabitation has many of the marriage characteristics (sharing a dwelling unit), cohabitants also share some of the characteristics of singles (fertility expectations). This puts cohabitants in a middle position between singles and married.

A second question raised by our facts concerns the identity of those uneducated mothers who have sometimes so many children. Table 11 gives some information. The column \((1 - \phi n)w^f + a^f\) reports the total income of the person. Earnings are in column \((1 - \phi n)w^f\). Not surprisingly, earnings are decreasing with the number of children, probably because hours worked drop with number of children. More interestingly, the column \(a^f\) reports the difference between total income and earnings. We observe that fertility is increasing with \(a^f\), which is a prediction of our model. Now, who are these women? Very few of them report to be with an unmarried partner (see above). Less than half of them are black. A majority is head of the household.

### D 5 years cohorts

We show in this appendix that the stylized facts highlighted in Section 2 can also be found if we consider each 5-year cohort separately. The only remarkable difference between cohorts is in Figure 20 were we see that single women were more likely to be childless for older cohorts.
Table 11: Single Mothers with no Education

<table>
<thead>
<tr>
<th>n</th>
<th>nobs.</th>
<th>((1 - \phi_n)w_f + a_f)</th>
<th>((1 - \phi_n)w_f)</th>
<th>(a_f)</th>
<th>% with partner</th>
<th>% black</th>
<th>% Head of households</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17531</td>
<td>7302.867</td>
<td>5014.918</td>
<td>2287.949</td>
<td>6.7%</td>
<td>33.5%</td>
<td>35.0%</td>
</tr>
<tr>
<td>2</td>
<td>11797</td>
<td>7889.392</td>
<td>5817.365</td>
<td>2072.027</td>
<td>9.5%</td>
<td>39.1%</td>
<td>45.3%</td>
</tr>
<tr>
<td>3</td>
<td>7108</td>
<td>6923.743</td>
<td>4553.754</td>
<td>2369.989</td>
<td>11.1%</td>
<td>39.4%</td>
<td>52.5%</td>
</tr>
<tr>
<td>4</td>
<td>4636</td>
<td>6712.247</td>
<td>3998.476</td>
<td>2713.771</td>
<td>6.4%</td>
<td>32.1%</td>
<td>56.8%</td>
</tr>
<tr>
<td>5</td>
<td>2882</td>
<td>6239.593</td>
<td>3556.023</td>
<td>2683.57</td>
<td>5.0%</td>
<td>41.8%</td>
<td>61.1%</td>
</tr>
<tr>
<td>6+</td>
<td>5446</td>
<td>5844.518</td>
<td>2576.979</td>
<td>3267.539</td>
<td>6.1%</td>
<td>36.4%</td>
<td>64.5%</td>
</tr>
</tbody>
</table>

Figure 19: Childlessness rate and completed fertility of mothers, by education categories, married women.

E Assortative Mating

Counting the number of marriages by education categories, we find the following matrix:

This table is constructed in the following way: first, we drop from the data all the individuals who are not married (MARST > 1) or who do not have an identified partner in the Census (SPLOC= 0). Then, we sort observations, first, by their serial number, corresponding to the household, and then, their sex, so that the man of the household is before his wife in the data. We then generate a variable saying that the husband has a corresponding wife after him (the serial number for both has to be the same). The last step is to generate a variable with the education of the husband and another variable with the education of his wife.

The ratio

\[
\frac{z(i, j)}{\sum_j z(i, j)}
\]

gives the share of men of type \(j\) have married a woman of type \(i\). Dividing this number by
Figure 20: Childlessness rate and completed fertility of mothers, by education categories, single women.

Table 12: Marriages per education category. Men in columns, Women in Rows.

the share of women of type $i$ in the total population,

$$\frac{\sum_j z(i, j)}{\sum_{i,j} z(i, j)}$$

we obtain the elements of the matrix in the main text.