

On goods and premises

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Abstract

All goods and services are sold on premises, e.g. in a shop or on a website. Since premises differ across sellers, even homogeneous goods become somehow differentiated through the purchasing experience. Differentiation affects all the goods sold on given premises, and it is costly for buyers to get used to a different shop. I study the impact of such differentiation on the equilibrium level of horizontal diversity (between premises) and of prices in an otherwise competitive market. Sellers choose between two categories of premises. There are two types of buyers with different tastes on premises. I show that the market outcome never exhausts all the gains from trade. The market failure is independent of search costs, even when they become arbitrarily small. When buyers care sufficiently about the premises, the market may appear efficient: there is search, product diversity, price dispersion and all buyers accept an offer. However it is not efficient, since only the buyers of the minority type search until they find a good match, while majority types accept any offer.

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Most goods, even if perfectly homogeneous, are sold in what I here denote under the general term of ‘premises’. When you buy a particular brand of shoes, you buy it in a shop. The utility you derive from your purchase also depends on the shop itself. Search costs within a shop are negligible. However, changing for another shop with different rules, procedures, or organization has a cost.¹ One needs to get used to the shop, its prices and check whether it sells the brand of shoes you want. While trying to remain as close as possible to the pure competition case, I

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¹Even when you buy on the Internet, the website - its graphical organization and presentation - affects your experience. If you are already signed in, you can buy more easily - and feel more secure - than if you have to register to a new one.

consider search costs that are arbitrarily small² and model product diversity as follows. Sellers choose to sell a good or service on premises of either category A or B , at no cost. There are buyers of type a and type b in the market, in a proportion that is common knowledge, with a strict majority of type a . A buyer awards a higher valuation to a good match ($A&a$ or $B&b$) than to a mismatch ($A&b$ or $B&a$). Buyers are price takers but have the option of not buying, and of searching for other premises selling a different category or at a lower price.

In the absence of search costs, all buyers would be correctly matched and firms would make zero profit. However, any strictly positive search cost makes the market inefficient: some buyers are not correctly matched. I find that when search costs are arbitrarily small and buyers care sufficiently about the premises, the typical³ equilibrium involves full extraction of the surplus of the majority type a by the sellers. The buyers of the minority type b keep searching until they find a good match. As long as search costs are not exactly zero, the inefficiency remains and is independent of the size of these costs. The mechanism behind this equilibrium is the following: (i) sellers of either category set a price that corresponds to the participation constraint of type a buyers. Therefore category A sellers do not meet the participation constraint of type b buyers, (ii) sellers of category B therefore leave some surplus to buyers of type b . As a consequence (iii) buyers of type a accept any offer as they get exactly their reservation utility from both categories of firms, (iv) buyers of type b keep searching until they find a seller of category B , where they receive some surplus, (v) the profit of firms of category B is decreasing in their number (since they share the buyers of type b that search) while the profit of category A firms is independent of their number (since there is no search component in their demand). Hence in equilibrium (vi) the share of category B firms is such that their profit is exactly equal to that of category A firms.

The looks as if it were efficient: there is product diversity, search and price dispersion. However it is not: a share of buyers of type a is not correctly matched. Forcing the market to produce the category desired by the majority always increases aggregate consumer welfare. It also increases total welfare when the market does not supply enough of category A . In any case, a monopolist owning all the premises would increase total welfare by extracting the entire surplus. Hence, competition increases buyer surplus at the cost of creating inefficiencies in the matching process. This model applies to a variety of monopolistically competitive markets:

²The idea of the existence of a potential tradeoff between the taste for diversity of buyers and the minimization of transaction costs has been recently studied by [Woodruff (2002)].

³The equilibrium concept used here is Coalition-Proof Nash Equilibrium, as defined by [Bernheim, Peleg and Whinston (1987)]. Others Nash Equilibria exist, but are not robust to self-enforcing deviations of a mass of sellers.

clothing, theaters, bars, restaurants, bookstores, retail stores, shoe stores... Even to service providers as solicitors, physicians, etc. as long as there is a choice to be made by sellers and that it is not possible for a seller to satisfy all types of buyers. Here are two very simple examples.

Example 1 *The market for hair salons.* Each hairdresser can provide a large choice of haircuts, colors, at various prices. But when she must decide whether to opt for a ‘first come, first served’ policy or accept appointments, the two decisions are mutually exclusive (at least for a given time slot). By choosing a system of appointments, the hairdresser satisfies a share of the customers (those who prefer to plan ahead), but disappoints the other share that prefers to come in unexpectedly. The production costs are roughly the same but it is impossible to please both types of customers at the same time of the day.

Example 2 *A TV screen in a bar.* Bars can serve a variety of drinks, but when they have to decide whether or not to install a TV, it affects all consumers in the room. The barman can offer you any drink you want, your drinking experience is always influenced - positively or negatively, depending on your type - by the television. Consider that a majority of buyers prefer to go to bars that have a television, and that this preference is sufficiently important. The model predicts that bars with television will be more expensive, and that most buyers will therefore be indifferent between bars that have a television and those that do not, while the buyers who do not like having a TV in a bar will only go to places that cater specifically to their taste.

I briefly relate this paper to the relevant literature in the next Section. I present the setup of the model in Section 2. Section 3 characterizes the equilibrium result: I first show that none of the Nash Equilibria are efficient. Even if search costs are arbitrarily small, not all buyers are properly matched. Two cases may arise:⁴ when buyers do not care sufficiently about the premises, only one category is produced and when buyers do care sufficiently about the premises, there is product diversity and search in the only Coalition Proof Nash Equilibrium. In Section 4, I discuss the policy implications of these results. I extend the results to a continuum of types and larger search costs in Section 5. Section 6 concludes.

1 Literature

Product diversity is a well-documented topic. The idea that the characteristics of goods can be valued differently by different types of individuals has been formalized by [Lancaster (1966)]. The novelty of my results comes from a combination of the following 3 elements:

⁴I show a necessary and sufficient condition to be in either case.

First, the level of horizontal diversity is endogenously determined by the sellers, as opposed to random utility models ([Perloff and Salop (1985)], [Deneckere and Rotschild (1992)], [Anderson and Renault (1999)]). This implies that a continuum of firms may produce homogeneous goods in equilibrium if it is a best response for them.

Second, search costs are independent of the localization of the sellers, as opposed to models à la Hotelling ([Salop (1979)], [Stahl (1982)], [Dudey (1990)], [Gabszewicz and Thisse (1986)]). That is, in my model, sellers do not become more accessible to some buyers by changing their good category (position in a Hotelling/Salop model).

Third, I model a market, as opposed to the many models using a representative agent (as in [Dixit and Stiglitz (1977)]). Hence, the inefficiency is not related to the cost structure, but to the failure of the market mechanism itself.

My model is also related to the following contributions:

Price dispersion with homogeneous goods has been studied in several papers, mostly in the context of the Internet. See [Baye, Morgan and Scholten (2005)] for a survey. The price structure in horizontal matching models has been studied by [Besley and Ghatak (2005)], [Clark (2007)] and [Klumpp (2009)], but all assume diversity to be exogenously given. Endogenous diversity in oligopoly is studied among others by [Chen and Riordan (2006)], [Kuksov (2004)] and [Bar-Isaac, Caruana and Cunat (2008)].

The market failure in my model comes from the presence of nontransferabilities in the matching process (buyers are price takers and there is no bargaining). [Legros and Newman (2007)] study how non-transferable utility affects matching when differentiation is vertical. I study endogenous horizontal differentiation and price setting.

Also related is [Nocke, Peitz and Stahl (2007)] who show in a two-sided market that monopoly ownership can extract more gains from trade than a competitive ownership when network effects are strong.

The result that firms can extract consumer surplus in my model is close to [Diamond (1971)]. In the classical formulation of price competition, firms set a price equal to the marginal cost in equilibrium. Introducing search costs in the specification yields the so-called ‘Diamond Paradox’: a model of search with a large number of buyers and a large number of sellers does not converge to a competitive equilibrium à la Bertrand. In finite time, the price becomes the one that maximizes joint profit. The logic behind [Diamond (1971)] is the following. Consider that a time period is the time it takes for a buyer to visit a store. At the beginning of each period, the seller sets the price for the whole period. The only way for a consumer to learn the price set by a specific

store is to enter it. The commodity is bought once. Consumers know the distribution of prices today, and are aware that the price might change tomorrow. There is no product differentiation. Search costs take a very general form. The utility of a buyer is given by $U(p, z)$ with p the price and z the number of periods needed to buy. The condition is that U is decreasing in both arguments. For a given price level in the market, sellers always have an incentive to slightly increase their price until they reach the monopoly level. Indeed, by charging a little more than their competitors, sellers make sure that a buyer who enters their shop will not keep searching for a lower price elsewhere.

In this paper, since there is no capacity constraint for the sellers, and as buyers buy either zero or exactly one unit, a Diamond Paradox would be an efficient outcome (with the entire surplus extracted by the sellers). However, product differentiation changes the picture and, in equilibrium, profit maximizing sellers are not maximizing joint profit. Hence, while the price mechanism is closely related to the one of [Diamond (1971)], my results do not imply a similar paradox. Once horizontal differentiation is feasible, there is search and price dispersion in equilibrium, but another inefficiency arises, due to the mismatching of a share of buyers.

2 Setup

The economy is composed of two groups of mass 1. The first group is composed of buyers, with an exogenous fraction α of type a and $1 - \alpha$ of type b . In this presentation of the model, I consider $\alpha \in (\frac{1}{2}, 1)$.⁵ The second group is composed of sellers, who endogenously choose a category A or B . A ‘good’ match ($a\&A$ or $b\&B$) generates surplus V and a ‘bad’ match ($a\&B$ or $b\&A$) generates surplus $v < V$.⁶ The surplus is received by the buyer if she accepts the price set by the seller. The outside option is set to $r \in (0, v)$.⁷ Both categories are produced with no fixed cost and marginal cost normalized to 0. A buyer can buy either 0 or exactly 1 unit of either good. Parameters α, V, v, r and discount factor δ are common knowledge. The stages of the game are as follows:

1. Sellers simultaneously choose a category of premises (either A or B) and price offer ;
2. Buyers learn the share γ of sellers of category A , and the distribution of prices ;

⁵The results for $\alpha \in (0, \frac{1}{2})$ are symmetric. I voluntarily exclude the non-generic possibility of having exactly $\alpha = \frac{1}{2}$. Heterogeneous buyers implies $\alpha < 1$.

⁶The fact that only two values exist for the surplus is not crucial for my results. I show in section 5.2 that the necessary condition is to have a sufficiently high density of buyers sharing close preferences.

⁷The outside option cannot be normalized to zero, as it would imply that any positive outcome, even if discounted a large number of times, is always higher than r .

3. Each buyer is randomly matched with a seller. She observes the price and the chosen category of the seller she is matched with. Each buyer decides whether to **Accept** the offer, **Leave** the market and receive the outside option r or to **Search** for another seller. If a buyer searches, she is randomly matched with another seller, but her payoffs are discounted with a parameter $\delta < 1$. There is no limit for search, but the cumulated discount factor decreases to δ^s after s searches.⁸

3 Equilibria

In this Section, I assume arbitrarily small search costs ($\delta \rightarrow 1$).⁹ The results are presented as follows: (i) I show that there are only two potential prices in equilibrium and that the market outcome is never efficient (ii) I present and solve the case where buyers care sufficiently about premises, show there are multiple Nash Equilibria and that only one is Coalition Proof (iii) I briefly present the other case.

3.1 Main characteristics of the Nash Equilibria

The equilibrium price only takes two values that I denote by ‘high price’ $p = V - r$ and ‘low price’ $p = v - r$. The results of this subsection are mainly driven by a mechanism that can be related to the one used by [Diamond (1971)] while introducing search costs in a homogeneous market.

The difference here comes from the heterogeneous tastes of the buyers. The low price corresponds to the participation constraint of mismatched buyers. A seller that has positive demand from those buyers when the price is exactly $v - r$ certainly loses the demand from the whole group by slightly increasing the price. Therefore, there can be an incentive for sellers not to increase the price above this threshold. Similarly, the high price corresponds to the participation constraint of buyers with a good match, and any price above this value implies zero demand for the seller.

Lemma 1 *There are only two possible prices in a Nash Equilibrium: $p = V - r$ and $p = v - r$*

Sketch of the Proof. The formal proof is given in Appendix A.1. Sellers are free to choose their category at no cost. Therefore, if the expected profit of a seller of category i is higher than

⁸I model discounting as a search cost supported by the buyer (wasting time a given day) and not as postponed sales. Therefore, only the surplus of buyers is discounted. In the case with arbitrarily small search costs, none of the results is affected by this assumption. When search costs increase, discounting payoffs of sellers would decrease the social benefits of search.

⁹This assumption is relaxed in section 5.1. The formal proofs of the next propositions (in appendix) give the necessary conditions on δ for the considered equilibria. When $\delta \rightarrow 1$ all those conditions are fulfilled.

the expected profit of a seller of category j , this is not an equilibrium. While deciding whether to accept an offer or to search for another, a buyer considers the distribution of prices in the market \hat{p} . As there is a continuum of sellers, a single seller has no influence on \hat{p} . However, any seller knows \hat{p} , and can set her price in order to make buyers of a given type accept her offer. It is always a best response for a seller to slightly increase her price as long as she does not lose consumers by doing so. This can only happen for two levels of price: $v - r$ and $V - r$. At those levels, any increase in the price implies that one of the participation constraints is no longer fulfilled. ■

The impossibility of having an equilibrium price different from those two values is the key factor that drives the inefficiency of any Nash Equilibrium of this model. Indeed, either sellers sell to both types of buyers - this implies that some buyers are not correctly matched - or they specialize in only one type and set a high price, such that search never occurs - this implies either a share of mismatches or some buyers leaving the market.

Proposition 1 *The market outcome never exhausts all gains from trade*

Proof. *(by contradiction)* Assume both types of buyers search until they find a good match. This implies that sellers are specialized in one type. Hence, as shown in Lemma 1, it is a Best Response for every buyer to set a price slightly above the market level, even when it is exactly $p = v - r$. The only price in a Nash Equilibrium is therefore $p' = V - r$. At this level of price, the expected surplus on the market is at most r and buyers never search. This is a contradiction. ■

In any equilibrium, a share of the buyers is not correctly matched. This implies that the total gains from trade are strictly lower than $V - r$. If there is a mass of sellers of each type and if the price is strictly lower than $V - r$, all buyers search and the total gains from trade tend to $V - r$ (as δ goes to 1). This Proposition shows the existence of a market failure. Indeed, consider instead that a monopoly owns all the sellers. It is easy to show that, by setting a price slightly below $V - r$ and producing both categories, all buyers search and all the potential surplus of the economy is extracted. However, this monopoly would eventually let the buyers with zero surplus. As will be made clear below, competition leaves some buyers with surplus, at the cost of an inefficiency in the matching process. Whether regulation can help increase welfare is discussed in Section 4.

3.2 When buyers care sufficiently about the premises

When buyers do not care sufficiently about the premises, the equilibrium corresponds to a classical result of standard setting. The market provides only one of the two categories, at a low

price, there is no search and no one is excluded from the market (I develop this result in Section 3.3).

When the importance of the premises increases sufficiently, or when the majority is sufficiently large, the incentive for sellers to extract the surplus of a good match also increases and product diversity starts to become a Nash Equilibrium. Condition 1 is necessary and sufficient to be in this case.

Condition 1 $\alpha > \frac{v-r}{V-r}$

In this Section, I consider that condition 1 is true. The interpretation is twofold. If the ratio on the right hand side of the equation is sufficiently low, it means that sellers can make large surplus by setting the high price and selling only to the majority type. If the left hand side is sufficiently high, it means that the demand from the majority is large enough to compensate the loss from not attracting minority buyers.

In this subsection, I show that there are potentially four Nash Equilibria in the economy. Three of them coexist, depending on the values of the parameters. I list them below. Then, I explain why I consider a more restrictive concept of equilibrium: Coalition-Proof Nash Equilibrium (CPNE). I show that only one equilibrium, AS_{\min} , is a CPNE.

Definition 1 *Tyranny of the majority (high price):* TM_H . All sellers choose the category desired by the majority and sell it at the high price. The buyers of the minority are excluded from the market. There is no search.

Definition 2 *Tyranny of the majority (low price):* TM_L . All sellers chose the category desired by the majority and sell it at the low price. All buyers accept the offer. There is no search.

Definition 3 *Asymmetric supply - Some surplus left to the majority* AS_{maj} . There are sellers of both categories in the market. The sellers of category A (corresponding to the majority type a) sell at the low price and the sellers of category B sell at the high price. The buyers of the minority type accept any offer, while the buyers of the majority type search until they find a good match.

Definition 4 *Asymmetric supply - Some surplus left to the minority* AS_{\min} . There are sellers of both types in the market. The sellers of category B (corresponding to the minority

type b) sell at the low price and the sellers of category A sell at the high price. The buyers of the majority type accept any offer, while the buyers of the minority type search until they find a good match. This equilibrium is the only CPNE.

Lemma 2 *If condition 1 is true, there are exactly four potential Nash Equilibria in this game: TM_H , TM_L , AS_{maj} and AS_{min} . Two of them TM_L and AS_{maj} never coexist, and occur according to the value of the parameters.*

Sketch of the Proof. The formal proof is given in Appendix A.2. Here is the intuition for each of the equilibria.

- TM_H : it is not a best response for sellers to lower the price unless it is at most $p' = v - r$. If the price is exactly p' , the expected profit is $\pi' = v - r$. This is lower than the equilibrium profit $\pi = \alpha(V - r)$ by condition 1. It is not a best response for a seller to sell the other category, as the profit of the deviating seller is at most $\pi'' = (1 - \alpha)(V - r)$. Which is lower than π , since $\alpha > \frac{1}{2}$.
- TM_L : a seller that slightly increases the price never increases her profit, as she automatically loses a large share of the buyers. The profit of each firm is $\pi' = v - r$. The only possibility to increase profit is to sell the category desired by the minority at price $p = V - r$. Therefore, TM_L is a Nash Equilibrium if and only if $\pi'' < \pi'$.
- AS_{maj} : the share γ^* of firms of category A is such that their profit is exactly the same as the firms of category B , $\pi'' = (1 - \alpha)(V - r)$. A firm can increase its profit by deviating and selling category A at the low price if and only if $\pi'' < \pi'$. Therefore, if TM_L is a Nash Equilibrium, AS_{maj} is not.
- AS_{min} : the share $(1 - \gamma^*)$ of firms of category B is such that their profit is exactly the same as the firms of type A , $\pi = \alpha(V - r)$. Therefore, a deviating firm can at most receive profit π' or π'' which are lower by definition.

■

Considering the general definition of Nash Equilibrium, this economy displays a multiplicity of equilibria, and there is no way to predict which one is expected to be realized in practice. In the next Proposition, I use an alternative concept of equilibrium: Coalition-Proof Nash Equilibrium, as defined by [Bernheim, Peleg and Whinston (1987)]. This more restrictive definition implies that there is no *self-enforcing* deviation by a coalition of sellers that can gain from deviating.

Take for instance the first equilibrium, TM_H , which is a Nash Equilibrium because no single seller can make buyers search for it. However, there is a coalition of mass $(1 - \gamma')$ that would benefit from selling the category desired by the minority at a low price. If $(1 - \gamma')$ is not too high, those sellers can attract a sufficiently large number of buyers of type b to increase their profit. This deviation is *self-enforcing*, as none of those deviating sellers has any incentive to change her price or category. And there is no *self-enforcing* deviation by a sub-coalition that can increase her profit by doing so.

This concept is much more realistic in the context of this paper. Indeed, firms can communicate and exchange ideas, even if they do not explicitly coordinate. Some sellers can for reasons unrelated to profit maximization try to sell the other category. This can even be created from the demand side: a coalition of buyers of the minority type could start its own business, or simply give a certification or a label to firms who accept to sell their preferred category. All of those effects would make the equilibrium TM_H disappear.

Proposition 2 *When condition 1 is true, the only Coalition-Proof Nash Equilibrium of this game is AS_{\min} : the sellers of category B (corresponding to the minority type b) sell at a low price and the sellers of category A sell at a high price. The buyers of the majority type accept any offer, while the buyers of the minority type search until they find a good match.*

Sketch of the Proof. The formal proof is provided in Appendix A.3. In TM_H there exists a *self-enforcing* coalition that can increase its profit by selling the type desired by the minority at the low price. In TM_L and AS_{maj} , the profit of sellers is strictly lower than in AS_{\min} . Therefore, a *self-enforcing* deviation of a coalition of mass 1 increases its profit by playing AS_{\min} . ■

A key factor to understand the equilibrium outcome is that, as sellers are free to choose their category at no cost, the expected profit of all sellers is equivalent. The profit of sellers of category A is independent of their number and is $\pi = \alpha(V - r)$. Hence, the number of firms of category B is determined by the difference between high and low price (how much extra surplus a seller can extract by specializing in the majority type) and the share of buyers of the minority (how many buyers will search to reach a seller of category B). This equilibrium value is given by

$$\gamma^* = 1 - \frac{1 - \alpha}{\alpha} \frac{v - r}{V - v}. \quad (1)$$

It must be noted that the category desired by the majority of buyers can be produced by a minority of sellers in equilibrium.

Coming back to the TV screen example provided in the introduction, this result implies that if a majority of bar goers prefer TV screen, they are indifferent between high price bars with

a TV screen and low price bars without. Buyers who dislike TV screens however, would never consume in a high-price bar with a TV screen. They keep searching until they find a low-price bar corresponding to their preferences, i.e. without a TV. If high profit can be made by sellers specializing in those customers that like TV screen, the model predicts the market to reflect the preferences of buyers. But this is not always true, and the market outcome can perfectly display a large majority of bars without TV screen, patronized by all types of buyers, and only a few bars with a TV screen, only patronized by TV lovers.

Bars without a TV have no incentive to increase their price, as they would lose TV lovers. Bars with a TV have no incentive to decrease their price, as it would not be sufficient to attract consumers with other preferences.

3.3 When buyers care less about the premises

Assume now that condition 1 is not fulfilled, i.e.

$$\alpha < \frac{v - r}{V - r}$$

This corresponds to assuming that the relative surplus generated by a good match with respect to a mismatch is quite small, given the size of the majority. The intuition is that of a standard setting. There is only one category sold in equilibrium, at the low price.

Proposition 3 *When condition 1 is false, there is only one category provided in equilibrium. It is sold at low price, there is no search and no buyers are excluded from the market.*

Sketch of the Proof. The formal proof is given in Appendix A.4. The profit of each firm is $\pi' = v - r$. There is no incentive for any firm to increase the price, as the profit would be at most $\pi = \alpha(V - r)$. This is lower than π' as condition 1 is false. As the potential surplus from specializing in one category is not high enough, all sellers attract both types. Product diversity is not a Nash Equilibrium. If sellers set the low price but attract only one type of buyers, each seller has an incentive to slightly increase the price. And if one category is sold at the high price and the other at the low price, the profit of the latter sellers is at least $\pi' = v - r$, higher than what she can get by specialization. ■

As in the previous case, the market alone fails to provide an efficient level of product diversity. Note that a market where only category B , preferred by the minority, is produced is also a Nash Equilibrium.

Back to the TV example again, this depicts a world where most buyers like TV screen and no bar provides this service. This is not a sufficiently important issue for buyers, and a bar would not increase its profit by installing the screen and specialize in TV lovers.

4 Welfare and regulation

The objective of this Section is to discuss the welfare implications of the results presented above. I mainly focus on the case where condition 1 is true. First, I discuss the welfare impact of the simplest possible regulation: to force the sellers to choose the majority type. I show that this always increases aggregate consumers' surplus. This regulation also increases total welfare when the market does not provide enough of the type desired by the majority. Then, I explain how setting a market for licenses can increase total surplus close to the first best. I close this Section by presenting the result for the case when condition 1 is false.

Lemma 3 *If firms are only allowed to sell category A, the only Coalition-Proof Nash Equilibrium is TM_L : all firms sell category A, at the low price. All buyers accept the offer. There is no search.*

Proof. The two potential Nash Equilibria are TM_L , with $p = v - r$ and TM_H , with $p'' = V - r$.

(i) TM_H is not a CPNE.¹⁰ There exists a coalition ξ of sellers that can increase its profit by setting $p = v - r$, as buyers will search until they find a seller at price $v - r$. Slightly increasing the price decreases profit, so the deviation is *self-enforcing*.

(ii) It follows that TM_L is a CPNE. A mass 1 of sellers can increase its profit by setting p'' , but it is not *self-enforcing* since a subcoalition ξ of sellers can increase its profit by setting $p = v - r$.

■

The question is to find out whether a prohibition can be welfare improving. A social planner can broadly have two main objectives: (i) maximize the total surplus (ii) maximize the surplus of consumers.¹¹

While considering the aggregate surplus of consumers, it is easy to show that this regulation is always enhancing. Indeed, in the unregulated equilibrium AS_{\min} , the minority buyers receive surplus $S = V - (v - r)$, corresponding to the difference between their valuation for the good match and the low price, while the majority buyers receive a surplus equivalent to the outside option. In the regulated equilibrium TM_L , the majority buyers receive S while the minority buyers receive a surplus equivalent to the outside option.

While considering total surplus, there is a trade-off between the profit of sellers, which is lower in the regulated equilibrium, and the surplus of buyers. The key factor is to know if the

¹⁰Even if there is no product diversity in this game, this result differs from the paradox raised by [Diamond (1971)]. TM_H is the equilibrium that maximizes joint profit.

¹¹For instance, the latter is the official statement of the European Commission for market regulation policies.

loss in profit due to specialization is compensated by the gain in consumer surplus. Still, this is not Pareto Improving, as the expected utility of the buyers of type b is reduced.

Proposition 4 *If the gains from specialization are sufficiently high, the unregulated market supplies a large share of the category desired by the majority, and regulation decreases total welfare. Otherwise, regulation increases total welfare.*

Sketch of the Proof. The formal proof is provided in Appendix A.5. Regulation increases welfare if the total surplus is higher in TM_L than in AS_{\min} . This can be written as

$$\begin{aligned} \alpha V + (1 - \alpha)v &> V - (1 - \alpha)(v - r) \\ \Leftrightarrow V - v &< v - r \end{aligned} \tag{2}$$

■

This relates to the value of γ^* presented in equation 1. If the gain from specialization is high, the share of sellers of the majority type is also high, and regulation is not welfare improving. But when the gain is lower, the market does not supply enough of the majority type and regulation increases total welfare.

While price regulation seems to be mostly a theoretical object,¹² a more realistic alternative policy is to implement a market for licenses. Assume that a license is the only legal way for sellers to choose the category desired by the minority. The social planner sets a number of licenses sufficiently high for the minority type to search when sellers set the low price. In our case with search cost arbitrarily small, the number of licenses is also arbitrarily small. In this case, one can expect almost every buyer to benefit from a good match, and therefore the equilibrium to be close to the first best. More generally, for a share $1 - \gamma^*$ of licenses, the market price of a licence, L , is the solution to

$$\alpha(V - r) = \alpha(v - r) + \frac{1 - \alpha}{1 - \gamma^*}(v - r) - L$$

Depending on the level of search costs δ , the optimal share of licenses $1 - \gamma^*$ is the smallest that ensures search from buyers of type b . One can show (see appendix A.2) that this is the solution to

$$1 - \gamma^* = \frac{r}{V - v} \frac{1 - \delta}{\delta}$$

¹²Among other because $v - r$ is not constant through time and the market outcome is the only way to measure it. Also because this model only reflects the specific profits of the platform that manages the premises but not the price of the good itself.

The sellers of the minority type still set the low price.¹³ Those sellers set the lowest price but are also the ones that pay for the license. The profit of sellers and the surplus of buyers is exactly the same as in AS_{\min} . The difference in total welfare comes from the revenue of the licenses.

When condition 1 is false, it is straightforward that if firms are only allowed to sell category A , this weakly increases total welfare. If a system of licenses is introduced as described before, the equilibrium changes to AS_{\min} (with a constrained share of sellers of category A), therefore increasing the price of the majority category for which no license is paid. Thus, licensing decreases aggregate consumers' welfare - but increases total welfare as more consumers obtain a good match.

5 Extensions

5.1 Increasing search costs

The objective of this extension is to present the additional conditions on discount factor δ for the existence of the various equilibria when search costs are not arbitrarily small. The computations are provided in the formal proof of each of the equilibria in Appendix A.2.

5.1.1 When buyers care sufficiently about the premises

When condition 1 is true:

- TM_H is always an equilibrium.
- TM_L is an equilibrium if $(1 - \alpha) < \frac{(v-r)}{(V-r)}$ and $\delta > \frac{\alpha V - (v-r)}{\alpha(V - (v-r))}$. The first condition excludes specialization in the minority type, the second condition excludes the possibility for a seller of category A to increase the price, still have demand from majority buyers and increase its profit.
- AS_{\min} is an equilibrium if $\delta > \frac{\alpha r}{(1-\alpha)(v-r) + \alpha r}$. This condition ensures that there is a sufficiently large number of sellers of category B for buyers of type b to actually search.
- AS_{maj} is an equilibrium if $1 - \alpha > \frac{v-r}{V-r}$ and $\delta > \frac{(1-\alpha)r}{\alpha(v-r) + (1-\alpha)r}$. The first condition ensures there are enough buyers of the minority type and that enough surplus can be extracted from them. The second condition ensures that there is a sufficiently large number of sellers of category A for buyers of type a to actually search.

¹³Slightly increasing the price is not a best response since it implies losing the demand from buyers of the majority type. And there is no self-enforcing deviation of a coalition towards a higher price.

The main result is that for values of α not too close to 1, the equilibria are robust to increases in search costs. When search costs increase too much, the number of equilibria decreases. One can show that if TM_L is a Nash Equilibrium, AS_{\min} is also a Nash Equilibrium (the reverse is not true).

Example 3 Consider values of r, v, V such that TM_L exists, and therefore AS_{maj} does not: $r = 1, v = 2, V = 3$. TM_H is always an equilibrium, TM_L is an equilibrium for any $\delta > \frac{3\alpha-1}{2\alpha}$ (the right-hand side is always lower than 1 and increasing in α), AS_{\min} is an equilibrium for any $\delta > \alpha$.

Example 4 Consider values of r, v, V such that AS_{maj} exists, and therefore TM_L does not: $r = 1, v = 2, V = 4$. TM_H is always an equilibrium, AS_{maj} is an equilibrium (iff $\alpha < \frac{2}{3}$) for any $\delta > 1 - \alpha$ and AS_{\min} is an equilibrium for any $\delta > \alpha$.

5.1.2 When buyers care less about the premises

When condition 1 is false, both Nash Equilibria presented in Section 3.3 hold for any value of δ . Equilibria with product diversity can arise when δ decreases. One can show that, as long as $\delta < \frac{2r}{2r+V-v}$, there exists values of $\gamma^- < \gamma^+$ such that for any $\gamma \in (\gamma^-, \gamma^+)$ buyers buy any type without search. If buyers accept the offer regardless of the premises, sellers are indifferent between both categories. The higher the search cost, the broader the interval in which those equilibria exist.

5.2 A continuum of types

The specification of my model relies on assuming two discrete types, and two different values of consumer surplus. The objective of this extension is to give a continuous interpretation of the main equilibrium of my model, AS_{\min} .

As in the basic specification, there is a continuum of buyers and sellers of mass 1 and a fraction $\alpha > \frac{1}{2}$ of buyers of type a . A good match yields surplus V and buyers have an outside option of value r . Assume now that the valuation of a mismatch is drawn, for each buyer, from a continuous distribution f with support $[r, V]$. We try to characterize an equilibrium where sellers of categories A and B extract all the surplus of buyers of type a , and where buyers of type b search for sellers of type B that leave them some surplus.

First, I assume this equilibrium exists. I define the total profit for a firm of category A , and the total profit (and the maximization problem) for a firm of category B . Then, I derive the new value of γ . Finally, I show under which conditions it is actually an equilibrium.

The total profit for a firm of category A is

$$\pi_A = \alpha(V - r)$$

as in the discrete case.

To compute **the total profit of a firm of category B** , with price $p_B \in (r, V - r)$, one has to distinguish:

- *The demand from buyers of type a who pick up the seller first.* In this equilibrium, firms of type A give no more surplus than the outside option. This demand can be rewritten as $D_a(p_B)$, with $D_a(V - r) = 0$ and $D_a(r) = \alpha$
- *The demand from buyers of type b who pick up the seller first, plus the demand from buyers of type b who actually search (as $p_B^* < V - r$):* $\frac{1-\alpha}{1-\gamma}$,

$$\pi_B = (D_a(p_B) + \frac{1-\alpha}{1-\gamma})p_B.$$

Hence, the maximization problem yields

$$\frac{D'_a(p_B^*)p_B^*}{D_a(p_B^*)} = \frac{1-\alpha}{1-\gamma}.$$

A first comment is that, for such an equilibrium to exist, one needs a point with sufficiently high density (so the elasticity of the demand is high enough), and a share of buyers of type a sufficiently high (so the relative importance of this part of the demand is high enough).

Moreover, one needs a sufficiently small value of γ , such that the ‘search’ part of the demand, which is constant as long as $p_B < V - r$, is not too high. Other things being equal, the smaller γ , the highest the elasticity of total demand for a seller of category B .

However, even if γ is taken as exogenous in the maximization problem of an individual seller, it is still determined by the expected profit of a seller of category B . The expected profit of both categories of sellers being equal, γ must satisfy.

$$\gamma^* = 1 - \frac{(1-\alpha)p_B}{\alpha(V-r) - D_a(p_B)p_B}$$

which, for any $p < p_B^*$ is strictly decreasing in p_B . This is quite intuitive, as for any $p < p_B^*$ increasing the price increases the profit of a firm of category B , it also increases the number of firms of category B , and decreases the ‘search’ component of the demand.

Those prices correspond to a Nash Equilibrium if there exists a solution $\gamma^* \in (0, 1)$. This is true if there is a sufficiently high concentration of types, at a point that yields a sufficiently

high surplus of a mismatch, and with a sufficiently large majority of buyers of type a . This corresponds to the same intuition as the conditions of existence of this equilibrium in the discrete case.

If those conditions are fulfilled, setting another price is not a best response for a firm of category A (a price higher than $V - r$ yields zero demand, a price lower does not increase demand but decreases profit). The price of firms of category B is an equilibrium by definition, as it is the result of individual profit maximization.

6 Conclusion

This model relies heavily on two assumptions: the sellers have to make a choice - they cannot serve both types - and no seller has sufficient market power to attract buyers by changing her price. The first assumption is the reason why this model applies to premises - which affect all the goods and services - and not to the goods and services themselves. The second assumption is the key reason for the differences between the results of this model - with competitive markets - and the recent oligopoly models cited in the review of the literature.

I have shown that when buyers care sufficiently about the premises, the market does not exhaust all the gains from trade: a share of the buyers is not correctly matched. As long as search costs are not too high, the efficiency is independent of search costs, and so is the average price on the market. Therefore, the perfect competitive price is a limit case, since any arbitrarily small search costs make it disappear.

When the market outcome does not reflect the preferences of buyers, a social planner can increase aggregate welfare by forcing firms to sell only the type desired by the majority. This can be a rationale to consider prohibition laws, assuming they reflect the tastes of the majority, as something potentially more effective than a simple transfer of utility from the minority to the majority type. However, this is not Pareto Improving among consumers, and this is also expected to lower the profit of firms, which may therefore oppose such a regulation. Another policy enabling to increase total welfare without hurting sellers is to create a market for licenses, with as few licenses of the minority type as needed to make the buyers of this type actually search. Under this policy, all sellers and buyers get exactly the same surplus as in the unregulated market, and the extra surplus from a higher matching rate is extracted by the social planner, or by whoever owns the property rights on the licenses.

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A Technical Appendixes

A.1 Proof of Lemma 1

To prove Lemma 1, I use the two following Lemma.

Lemma 4 *Both categories of sellers have the same expected profit in equilibrium.*

Proof. Sellers are free to choose their type at no cost. Therefore, if the expected profit of a seller selling a service of category i is higher than the expected profit of a seller of category j , this is not an equilibrium. It is a best response for a seller of category j to sell category i . ■

Lemma 5 *For a given level of price, a seller sets her price in a way to decide what kind of buyers accept her offer, but has no influence on who searches for her.*

Proof. This Lemma is close to [Diamond (1971)]. While deciding whether to accept an offer or to search for another, a buyer considers the distribution of prices in the market \hat{p} . As there is a continuum of sellers, a single seller has no influence on \hat{p} . However, a seller knows \hat{p} , and can set her price in order to make buyers of a given type accept her offer. ■

Consider the following definitions:

Definition 5 *The participation constraint for a seller of category j is fulfilled for a buyer of type i if the seller offers a price leaving the buyer of type i utility higher than the reservation utility r . I denote this by PC_i^j , with $i \in \{a, b\}$, $j \in \{A, B\}$.*

Definition 6 *The incentive compatibility constraint for a seller of type j is fulfilled for a buyer of type i if the seller offers a utility higher than her (discounted) expected utility if she stays in the market. I denote this by IC_i^j , with $i \in \{a, b\}$, $j \in \{A, B\}$.*

Definition 7 *A buyer of type i accepts the offer of a seller of category j if and only if PC_i^j and IC_i^j are fulfilled.*

This is the proof of Proposition 1:

Proof. PART 1: $p > V - r$ is never a NE.

(i) Consider a pair i, j with $i \neq j$. For a firm i setting price p , PC_i^i is given by $V - p \geq r$, $\Leftrightarrow p \leq V - r$ and PC_j^i is given by $v - p \geq r \Leftrightarrow p \leq v - r$.

(ii) A seller always makes a positive profit. Assume all firms sell at price zero and make zero profit. Slightly increasing the price is a profitable deviation for a firm of type i , as there exist $p' > 0$ such that IC_i^i is fulfilled, i.e. $V - p' \geq \delta V$.

(iii) As a corollary of (ii), it is never a best response for a firm to have zero demand. Hence, $p > V - r$ is never a best response, because no buyer ever accepts the offer.

PART 2: $v - r < p < V - r$ is never a NE.

Consider a firm of category i . Denote the expected surplus proposed by a firm of category i to a buyer of type j . S_j^i . As $p > v - r$, PC_j^i is not fulfilled. The firm only consider buyers of type i , hence fulfilling IC_i^i and PC_i^i . As $p < V - r$, PC_i^i is already fulfilled.

IC_i^i is always fulfilled when the firm sets a price p' such that

$$S_i^i(p') \geq \max(\delta S_i^i(\hat{p}_i), \delta S_i^j(\hat{p}_j)).$$

(i) If $\max(\delta S_i^i(\hat{p}_i), \delta S_i^j(\hat{p}_j)) = \delta S_i^i(\hat{p}_i)$, there exists a price $p' > \hat{p}_i$ such that IC_i^i is fulfilled. Indeed, $S_i^i(p') \geq \delta S_i^i(\hat{p}_i)$ for some $p' > \hat{p}_i$. Hence, it is always a best response for a firm to increase the price as long as PC_i^i is fulfilled.

(ii) If $\max(\delta S_i^i(\hat{p}_i), \delta S_i^j(\hat{p}_j)) = \delta S_i^j(\hat{p}_j)$, then it is a BR for the firm to change and sell category j at price $p' > p_j$. Indeed:

a) By Lemma 4, in equilibrium, $\pi_i = \pi_j$.

- b) As $\max(\delta S_i^i(\hat{p}_i), \delta S_i^j(\hat{p}_j)) = \delta S_i^j(\hat{p}_j)$, IC_j^j is fulfilled and from PART 1, $p < V - r$.
- c) For the same reason, if there exist some firms selling at price \hat{p}_i in equilibrium, they must have non negative demand. Hence, IC_i^i is fulfilled. Then, as $S_i^i(\hat{p}_i) < S_i^j(\hat{p}_j)$, IC_i^j is also fulfilled, with $S_i^j(\hat{p}_j) > \max(\delta S_i^i(\hat{p}_i), \delta S_i^j(\hat{p}_j))$.
- d) Hence, there exists some $p' > p_j$ such that IC_i^j is still fulfilled.

PART 3: $p < v - r$ is never a NE.

Here, for both categories of firms and both types of buyers, PC is fulfilled.

- (i) For both categories of sellers to sell to both types of buyers to be a Nash Equilibrium, profit must be the same. As demand is 1 for any firm, it is only possible if p is the same for any firm. Also, IC_B^a and IC_a^B have to be binding. Indeed, if it is not fulfilled, a type of buyer searches. And if it is not exactly binding, slightly increasing the price is a best response, i.e. for IC_B^a

$$\begin{aligned} v - p &= \delta\gamma(V - p) \sum_{i=0}^{\infty} \delta^i (1 - \gamma)^i \\ v - p &= \frac{\delta(1 - \gamma)(V - p)}{1 - \delta(1 - \gamma)}. \end{aligned}$$

And similarly, IC_a^B

$$v - p = \frac{\delta(1 - \gamma)(V - p)}{1 - \delta\gamma}.$$

This is only possible if $\gamma = \frac{1}{2}$. But then, both equalities yield

$$\begin{aligned} (1 - \delta\gamma)(V - p) &= \delta\gamma(V - p) \\ (1 - \delta\gamma) &= \delta\gamma \\ 1 - \frac{\delta}{2} &= \frac{\delta}{2} \\ \delta &= 1 \end{aligned}$$

this is, no search cost at all.

- (ii) If a firm is interested in only one type of seller, the reasoning becomes the same as in PART 2, there is always an incentive to increase the price. ■

A.2 Proof of Lemma 2

TM_H is a Nash Equilibrium:

Proof. (i) It is not a best response for a seller to sell category B and set price $v - r < p' \leq V - r$, because the profit will be at most $(1 - \alpha)(V - r) < \alpha(V - r)$.

(ii) It is not a best response for a seller to sell category B and set price $p' \leq v - r$. By Lemma 5 a seller cannot make people search for it. So, the profit will be at most $v - r < \alpha(V - r)$ (by condition 1).

(iii) It is not a best response for a seller to sell category A at price $p' \leq v - r$. On her own, a seller can't make buyer search for her. So, the profit will be at most $v - r < \alpha(V - r)$ (by condition 1).

(iv) It is not a best response for a seller to sell category A at price $v - r < p' < V - r$. By condition 5 this yields demand α and therefore profit strictly lower than $\alpha(V - r)$. ■

TM_L is a Nash Equilibrium iff $(1 - \alpha)(V - r) < v - r$:

Proof. Profit at the equilibrium is $\pi_A = v - r$.

(i) It is not a best response for a firm to sell category B and set price $p = v - r$ as it will lose all buyers of type a .

(ii) It is not a best response for a firm to sell category B and set price $p' = V - r$, because the profit will be $(1 - \alpha)(V - r) < v - r$.

(iii) It is not a best response for a firm of category A to increase the price. Consider \tilde{p} , the threshold price such that for any $p'' > \tilde{p}$ buyers of type a start searching. There is no incentive to set $p > \tilde{p}$ as it leads to zero profit. Neither is it an incentive to set $V - r < p < \tilde{p}$. If $\tilde{p} < V - r$, it is not a BR to set $p''' < \tilde{p}$ as this yields profit $\alpha p''' < \alpha \tilde{p}$. Therefore, we only consider an increase of price to exactly \tilde{p} . Define $\tilde{p} = v - r + \varepsilon$. Buyers of type a do not search as long as

$$\begin{aligned} V - (v - r + \varepsilon) &\geq \delta(V - (v - r)) \\ \Leftrightarrow \varepsilon &= (1 - \delta)(V - (v - r)) \end{aligned}$$

since the condition is binding. Therefore, it is a best response for a firm to increase the price iff

$$\begin{aligned} \alpha(v - r + \varepsilon) &\geq v - r \\ \Leftrightarrow \delta &> \frac{\alpha V - (v - r)}{\alpha(V - (v - r))} \end{aligned}$$

■

AS_{min} is a Nash Equilibrium:

Proof. A buyer waits if her expected surplus is higher than the reservation utility, i.e.

$$\begin{aligned} \Leftrightarrow r &< \delta(1-\gamma) \sum_{i=0}^{\infty} \gamma^i \delta^i (V-v+r) \\ \Leftrightarrow r &< \frac{\delta(1-\gamma)}{1-\gamma\delta} (V-v+r). \end{aligned}$$

this simplifies to

$$V-v > \frac{r}{1-\gamma} \frac{1-\delta}{\delta}.$$

The profit in equilibrium is given by:

For a firm of category A : $\pi_A = \alpha(V-r)$.

For a firm of category B : $\pi_B = \alpha(v-r) + \frac{1-\alpha}{1-\gamma}(v-r)$.

I want to find γ such that $\pi_A = \pi_B$. Write:

$$\begin{aligned} \alpha(V-r) &= \alpha(v-r) + \frac{1-\alpha}{1-\gamma}(v-r) \\ \Leftrightarrow (1-\gamma)\alpha(V-v) &= (1-\alpha)(v-r) \\ \Leftrightarrow \gamma^* &= 1 - \frac{1-\alpha}{\alpha} \frac{v-r}{V-v}. \end{aligned}$$

It is actually an equilibrium: (i) At isoprofit, consumers of type b actually search. We know $\gamma^* = 1 - \frac{1-\alpha}{\alpha} \frac{v-r}{V-v}$. I want $V-v > \frac{r}{1-\gamma} \frac{1-\delta}{\delta}$ for buyers of type b to wait. Replacing γ by γ^* yields $\frac{1-\alpha}{\alpha} > \frac{r}{v-r} \frac{1-\delta}{\delta}$. This is always true when $\delta \rightarrow 1$. The condition on δ can be conveniently rewritten as $\delta > \frac{\alpha r}{(1-\alpha)(v-r) + \alpha r}$.

(ii) In equilibrium, it is not a best response to sell category A at price $p'_A \leq v-r$. This yields at most profit $\pi_A = v-r$ which is lowering the profit by condition 1.

(iii) In equilibrium, no one wants to produce category B at price $v-r < p'_B \leq V-r$. Increasing the price make consumers of type a lose, and yields at most profit $\pi'_B = (1-\alpha)(V-r)$ which is lower than π_B as I have assumed $\alpha \geq \frac{1}{2}$.

(iv) In equilibrium, it is not a best response to sell category A at price $v-r < p_A < V-r$. Demand is at most α and the firm therefore makes lower profits.

(v) It is not a best response to change of category. There is isoprofit at equilibrium and, for any $\gamma > \gamma^*$ the best response of any firm is to supply category B (as π_B is an increasing function of γ). ■

AS_{major} is a Nash Equilibrium iff $(1-\alpha)(V-r) > v-r$:

Proof. (i) Selling category A at price $p' > V-r$ yields lower profit as, by definition buyers of type a reject the offer and search.

(ii) Selling category A at price $p' = V - r$ is not a best response as long as buyers of type a reject the offer and search.

(iii) Selling category A at price $v - r < p' < V - r$ is not a best response, as by Lemma 5 it does not increase the demand, but, by lowering the price it lowers the profit.

(iv) Selling category A at price $p' < v - r$ is not a best response as by Lemma 5 it does not increase the demand, but, by lowering the price it lowers the profit.

(v) Selling category B at price $p' < v - r$ is not a best response as by Lemma 5 no one specifically search for the firm and therefore profit is at most $v - r$, which is lower than $(1 - \alpha)(V - r)$.

(vi) Selling category B at price $v - r \leq p' < V - r$ is not a best response as by Lemma 5. it does not increase the demand from buyers of type b and as long as buyers of type a reject the offer and search.

(vii) Selling category B at price $p' > V - r$ yields lower profit as, by definition buyers of type a reject the offer and search.

(viii) Using the same reasoning as for AS_{\min} , (ii) and (iv) are true when $\delta > \frac{(1-\alpha)r}{\alpha(v-r)+(1-\alpha)r}$.

■

No other equilibria exist under condition 1

Proof. I want to show that the previous equilibria are the only existing ones when condition 1 is true. Therefore, I still have to get rid of the following alternatives.

(1) All sellers sell category B at price $p_B = V - r$. Selling category A at price $p_A = V - r$ is a profitable deviation as it yields profit $\alpha(V - r) > (1 - \alpha)(V - r)$, by $\alpha > \frac{1}{2}$.

(2) All sellers sell category B at price $p_B = v - r$. Selling category A at price $p_A = V - r$ is a profitable deviation as it yields profit $\alpha(V - r) > v - r$, by condition 1.

(4) A fraction γ of sellers sell category A at price $p_A = v - r$ while a fraction $(1 - \gamma)$ sells category B at price $p_B = v - r$. (i) If buyers don't search for the seller of their category then, by setting $p_A = V - r$ a seller of category A does not lose buyers of type a and therefore increases profit (ii) If buyers do search then, a seller only serves buyers of its category, and there must exist a price $p' > v - r$ such that buyers still accept the offer. Thus setting p' is a profitable deviation.

(5) A fraction γ of sellers sell category A at price $p_A = V - r$ while a fraction $(1 - \gamma)$ sells category B at price $p_B = V - r$. As $\alpha \geq \frac{1}{2}$ one can never have the same profit when $\gamma \neq 1$.

(6) Given Lemma 1, I have exhausted all the potential Nash Equilibria. ■

A.3 Proof of Proposition 2

Proof. AS_{\min} is coalition-proof. By Lemma 1, any Nash-Equilibrium implies either $p = v - r$ or $p = V - r$. Selling category B at $p = V - r$ is not a profitable deviation, as demand would

be zero. Selling category A at $p = v - r$ can increase joint profit of a coalition of sellers, but is not *self-enforcing*. Indeed, as the demand for such firms only comes from buyers of type A , each seller has an incentive to slightly increase the price - the participation constraint of buyers of type b is non-binding.

TM_H is not coalition-proof. There exist a coalition of size $(1 - \gamma') < (1 - \gamma^*)$ that would increase her profit by selling a good of category B at price $p = v - r$. Such a deviation is *self-enforcing*, as this price is the best response of any member of the coalition given that all the other members play the same strategy. Hence, Tyranny of the majority at high price is not a coalition-proof Nash Equilibrium.

TM_L and AS_{maj} are not coalition-proof. As the profit of each firm is higher in AS_{min}, and as AS_{min} is coalition-proof, a coalition of mass 1 always has an incentive to choose the equilibrium AS_{min}, and this strategy is *self-enforcing*. ■

A.4 Proof of Proposition 3

There is a Nash Equilibrium where all sellers sell the good desired by the majority at the high price:

- Proof.** (i) Buyers of type a buy without search (surplus $V - v + r \geq r$).
(ii) Buyers of type b buy without search (surplus r).
(iii) It is not a BR to sell category A at price $v - r < p'_A \leq V - r$. This decreases the profit to at most $\pi'_A = \alpha(V - r) < v - r$
(iv) It is not a BR for a firm to sell category B at price $p_B \leq v - r$. This yields at most profit $\pi_B = (1 - \alpha)(v - r)$. Why? Because buyers of type a wait¹⁴ to match a buyer of their type, while buyers of type b do not search for the deviating firm.
(v) It is not a BR for a firm to sell category B at price $v - r < p'_B \leq V - r$. This yields at most profit $\pi'_B = (1 - \alpha)(V - r)$. This is smaller because condition 1 is false and $\alpha \geq \frac{1}{2}$.

■

There is a Nash Equilibrium where all sellers sell the good desired by the minority at high price:

- Proof.** (i) Buyers of type b buy without search (surplus $V - v + r \geq r$).
(ii) Buyers of type a buy without search (surplus r).
(iii) It is not a BR for a firm to sell category A at price $v - r < p'_A < V - r$. This decreases the profit to at most $\pi_A = \alpha(V - r)$ [profit loss by the fact that condition 1 is false].
(iv) It is not a BR for a firm to sell category A at price $p'_A \leq v - r$. This yields profit at most

¹⁴If condition 2 is false, sellers are indifferent.

$\pi'_A = \alpha(v - r)$. Why? Because consumers of type 2 wait¹⁵ to match a buyer of their type, while buyers of type a do not search for the deviating firm.

(v) It is not a BR for a firm to sell category B at price $v - r < p'_B \leq V - r$. This yields profit $\pi'_B = (1 - \alpha)(V - r)$. This is smaller because condition 1 is false and $\alpha \geq \frac{1}{2}$.

■

Consider the following condition:

Condition 2 $\delta < \frac{2r}{V-v+2r}$

If condition 2 is true, there exist threshold values (γ^-, γ^+) such that, for any $\gamma^- < \gamma < \gamma^+$, buyers accept any offer without search and sellers are indifferent between both types. Any $\gamma \in (\gamma^-, \gamma^+)$, with $\gamma^- = \frac{V-v+r}{V-v} - \frac{r}{\delta(V-v)}$ and $\gamma^+ = 1 - \gamma^-$, with price $p_B = p_A = v - r$ is a Nash Equilibrium.

Proof. (i) Matched buyers buy without search (surplus $V - v + r \geq r$).

(ii) Mismatched buyers buy without search as long as $\gamma \in (\gamma^-, \gamma^+)$.

(iii) It is not a BR for a firm to sell category A at price $v - r < p'_A \leq V - r$. This decreases the profit to at most $\pi'_A = \alpha(V - r)$ [profit loss by the fact that condition 1 is false].

(iv) It is not a BR for a firm to sell category B at price $v - r < p'_B \leq V - r$. This yields at most profit $\pi'_B = (1 - \alpha)(V - r)$. This is smaller because condition 1 is false and $\alpha \geq \frac{1}{2}$.

(iv) Firms are indifferent between producing category A or category B at price $p_A = p_B = v - r$, as, for any value of γ , we have $\pi_A = \pi_B = v - r$, by (i) and (ii), which satisfy isoprofit.

■

Proof that no other equilibria exist when condition 1 is false

Proof. I want to show that the previous equilibria are the only existing ones when condition 1 is false. Therefore, I still have to get rid of the following alternatives:

(1) All sellers sell category B at price $p_B = V - r$. Selling category A at price $p_A = V - r$ always yields higher profit since $\alpha \geq \frac{1}{2}$.

(2) All sellers sell category A at price $p_A = V - r$. As condition 1 is false, reducing price to $p'_A = v - r$ increases profit.

(4) A fraction γ of sellers sell category A at price $p_A = V - r$ while a fraction $(1 - \gamma)$ sells category B at price $p_B = v - r$. Sellers selling category A do not make buyers of type a search (yields surplus 'r'). Then, profit is at most $\alpha(V - r) < v - r$ since condition 1 is false.

(5) A fraction γ of sellers sell category A at price $p_A = V - r$ while a fraction $(1 - \gamma)$ sells category B at price $p_B = V - r$. Such a price is too high to attract. Hence, as $\alpha \geq \frac{1}{2}$ one can never have the same expected profit for both categories.

¹⁵If condition 2 is false, sellers are indifferent.

(6) A fraction γ of sellers produces category A at price $p_A = v - r$ and a fraction of sellers $1 - \gamma$ produce category B at price $p_B = V - r$. This means sellers of category B make profit $\pi_B = (1 - \alpha)(V - r)$. Then, if isoprofit is fulfilled, it is a BR for any seller to produce category A at price $p_A = V - r$ and get profit $\pi_A = \alpha(V - r) > \pi_B$.

(7) Given Lemma 1, I have exhausted all the potential Nash Equilibria. ■

A.5 Proof of Proposition 4

Total surplus in TM_L :

$$\begin{aligned} \text{Sellers:} & \quad v - r \\ \text{Buyers of type } a : & \quad V - v + r \\ \text{Buyers of type } b : & \quad r \\ \text{Total:} & \quad (v - r) + \alpha(V - v + r) + (1 - \alpha)r = \alpha V + (1 - \alpha)v \end{aligned}$$

Total surplus in AS_{\min}

$$\begin{aligned} \text{Sellers:} & \quad \alpha(V - r) \\ \text{Buyers of type } a : & \quad r \\ \text{Buyers of type } b : & \quad V - v + r \text{ (as } (1 - \gamma)(V - v + r) \sum_{i=0}^{\infty} \gamma^i \delta^i = \frac{(1 - \gamma)(V - v + r)}{1 - \gamma\delta}, \text{ and } \delta \rightarrow 1) \\ \text{Total:} & \quad \alpha(V - r) + \alpha r + (1 - \alpha)(V - v + r) = V - (1 - \alpha)(v - r) \end{aligned}$$

TM_L is preferred to AS_{\min} iff

$$\begin{aligned} \alpha V + (1 - \alpha)v & > V - (1 - \alpha)(v - r) \\ \Leftrightarrow V - v & < v - r. \end{aligned}$$