Money as a Store of Value: Nominal Assets and Real Interest Rates *

David Domeij and Tore Ellingsen†

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Abstract

The paper offers a new formulation of the old hypothesis that money exists due to shortage of other financial assets. The theory admits various monetary regimes, ranging from a strict gold standard to interest-bearing fiat money in the form of nominal government bonds. A central assumption is that claims on currently non-existing real assets, which will be productive in the future, are not tradable today. Due to the resulting scarcity of saving instruments, money serves as a useful store of value. Although all prices are flexible and all assets are liquid, monetary and fiscal authorities have considerable control over the long-run real interest rate and hence over real investment and output. Money tends to be particularly valuable when either the rate of growth is low or the turnover of tradable productive assets is high, and in these regimes optimal monetary policy is relatively loose.

JEL classification:

Keywords: Money, Inflation, Bubbles, Incomplete markets

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†Department of Economics, Stockholm School of Economics, Box 6501, SE-113 83, Stockholm, Sweden. david.domeij@hhs.se, tore.ellingsen@hhs.se.
1 Introduction

We propose a micro-founded and quantitative model of money and monetary policy. The model is rich enough to encompass several monetary regimes, ranging from a pure gold standard to a modern monetary regime in which a central bank effectively controls the nominal interest rate on public debt. Even though we focus on the long run, so all prices are perfectly flexible, the analysis identifies a close link between money growth and the nominal interest rate on the one hand and the economy’s real rate of interest on the other. In stark contrast to the conventional wisdom that long-run real interest rates are exogenous to policy, we claim that monetary and fiscal authorities face a trade-off between encouraging financial saving through a high interest rate, and increasing real investment through a low interest rate.

The model is populated by a continuum of infinitely lived heterogeneous agents facing rare but persistent idiosyncratic income shocks. Private instruments for precautionary saving include physical capital (or corporate debt backed by such capital) and shares (backed by corporate profit), as well as debt originating from individuals who borrow for consumption purposes. A government provides some insurance against the low-income states and also distributes uniform lump-sum transfers. These expenditures are funded through income taxation and through expansion of the public debt.

Despite the rich array of financial securities, the asset market is incomplete. The reason is that agents cannot trade securities based on future projects that have not yet materialized. This shortage of private assets creates a useful role for additional stores of value, such as fiat money or interest-bearing public debt.

In several respects, the model resembles that of Ljunqvist and Sargent (2004, Chapter 17), which in turn synthesizes Bewley (1980, 1983, 1986), Huggett (1993), and Aiyagari (1994). However, existing work along these lines focuses on the role of money in meeting short-run fluctuations in liquidity demand, assuming either that money is the only durable asset or that other durable assets are highly illiquid. By contrast, we admit durable physical capital as well as a well-developed market for financial assets, and we assume that all assets are perfectly liquid. The role of money in our model is not to fulfill short-run liquidity needs, but to expand the total value of financial assets and thereby helping agents smooth consumption in the face of long-run income fluctuations. In other words, our model emphasizes the same long-term savings motives as do Samuelson (1958) and Tirole (1985), but without invoking intergenerational trading frictions.

Moreover, inspired by Tirole (1985), Woodford (1990), Holmström and Tirole (1998), and Kiyotaki and Moore (2002, 2003), we contribute to the
Bewley-Aiyagari-Huggett framework by identifying a fundamental source of the market incompleteness that creates a role for money. The two crucial assumptions concern agents’ ability to contract on the entry and exit of projects (or firms). First, and uncontroversially, we assume that current projects will eventually deteriorate, at least in comparison to the average rate of growth. Second, we assume that there will always be new projects in the future, but claims on them cannot be traded today.\footnote{This assumption is particularly reminiscent of Tirole (1985, p 1508), who considers the case of successive rent creation, and of Kiyotaki and Moore (2003) who consider successive generations of projects.} As a case in point, consider Facebook, an internet company started by novices in 2003. When it became publicly traded in 2010, the company was worth around 40 billion US dollars. By contrast, in a complete asset market, well diversified investors would already have held financial claims on Facebook, and any other venture that the founder Mark Zuckerberg may have initiated, long before 2003. Due to these missing financial assets, the equilibrium interest rate under laissez faire may fall below the economy’s rate of growth, and as in Tirole (1985) there is then room for additional stores of value, or “bubbles”.

We show that for a broad range of parameters, there is an attractive equilibrium in which public debt is precisely such a bubble; that is, public debt may be positive, yet serving the debt does not require real resources at any time. In other words, the government never needs to run a surplus. This is true independently of whether the debt is in the form of currency or in the form of interest-bearing securities, such as bills or bonds. In the case that public debt is a bubble, we say that it constitutes outside (or fiat) money.

The emergence and obsolescence of real assets is not merely a convenient modeling device, but a substantive assumption with well-defined empirical counterparts (e.g., Hobijn and Jovanovic, 2001). For the conceptually simple case in which all dividends grow at the same rate as overall productivity and the rate of asset destruction is the same as the rate of asset creation, we find that a conservative yearly rate of asset turnover of about two percent comfortably suffices to justify bubbles corresponding to the public debt levels that we observe today.

Indeed, the model suggests that monetary policy has substantial power over real interest rates in the long run. Authorities affect the value of all assets both through the nominal interest rate on the public debt and through the rate of debt creation. Since there are two instruments at the authorities’ disposal, the real interest rate is decoupled from the rate of inflation. Hence, the notion of an inflation tax is somewhat misleading. The effective taxation of savers can be every bit as high with low interest rates and small money
growth as with higher interest rates and high money growth. However, the inflation level can still matter for the practical implementation of policy, as a low inflation level (target) would leave most of the policy making to the fiscal authorities rather than the monetary authorities.

Other things equal, we argue that high public debt is an indication that the government has opted for a relatively high real interest rate, much as in Allais (1947) and Diamond (1965). However, a large public debt can also come about due to low productivity growth or because of a high rate of turnover of real assets, for example due to technological shifts. In either case, the government has additional scope for reducing the real interest rate to promote investment and output. Lower real interest rates are desirable for agents who are currently relatively poor, and undesirable for agents who are currently relatively rich.

Our analysis comes in two parts. First, we investigate the the impact of asset creation and destruction in a simple endowment economy. Second, we incorporate analogous features into a standard calibrated model with capital, labor, and profits, to probe their quantitative implications. Throughout, we assume that agents are infinitely lived. We employ this dynastic framework both for tractability and in order to demonstrate that generational trading frictions are not needed for our results. However, the quantitative results do depend on strong savings motives, and for that purpose the model includes a counterpart to retirement; our labor income process is designed to mimic an average work-life of 40 years followed by 15 years of no labor income.

We perform several experiments to elicit the model’s quantitative response to changes in asset turnover and growth rates. Here are two of them. (i) Suppose the asset turnover rate goes up permanently from two to five percent. Then, if monetary policy targets the same real interest rate, the value of the money stock will double. However, the optimal response is to lower the real interest rate significantly – the new optimal rate is a full 75 basis points below the old optimal rate. Thus, extended periods of high asset turnover, like the electrification age or the IT-revolution, invite loose monetary policy. (ii) Suppose instead that the productivity growth rate decreases from two to one percent per year, with the central bank lowering the real rate by the same amount. Again, this experiment entails more than a doubling of the money stock. Also, the optimal monetary policy is to reduce the real rate further; that is, extended periods of low growth also invite loose monetary policy.

Of course, if growth is low, the nominal interest rate may be stuck at the zero lower bound. In that case, policy can only be loosened by issuing more debt. Observe that since the debt is nominal, a faster rate of debt issuance entails higher inflation and is always associated with a lower (steady-state)
level of real debt to output. For example, according to this model, a simple way for Japan to reduce the real value of its large public debt may be to constantly issue more of it.

Although the model lacks some potentially important features, such as temporary aggregate shocks and opportunities for stock-market bubbles, we also use it to assess whether it can justify US monetary policies of recent decades. When we calibrate the model to fit corresponding levels of US macroeconomic variables, we find the following: (a) Monetary policy in the 1970’s had unsustainably low real interest rates; the public debt would have imploded if the policy had continued. (b) The regime shift conducted by Volcker around 1980 put real interest rates at a significantly higher level than that preferred by a utilitarian social planner; the loss in output and wages was not compensated sufficiently by improved opportunities for private saving. (c) Interest rates in the Greenspan years were better in line with social welfare maximization, but still on the high side. Lower rates would have entailed lower public debt and more private investment.

Before proceeding, let us preemptively address the objection that the real interest rate has historically been above the growth rate, at least in developed countries such as the US. We disagree. First, the basis of this claim is usually the work of Abel et al (1989), who measure real rates of return to capital investment from statistics on physical investments and returns. This exercise requires careful assessment of the returns to land and entrepreneurship so as not to exaggerate the returns to capital. When Geerolf (2013) conducts the same computation with better data on land rents, his conclusion differs markedly from that of Abel et al. Second, the real interest rate on safe financial assets, as measured for example by the treasury bill rate, or even the constant maturity ten year bond rate, is below the rate of growth for extended periods of time. Third, the fact that some countries sometimes run budget surpluses does not imply that they could not have been running budget deficits those years instead. It is obviously wrong to infer from the fact that the interest rate is sometimes larger than the rate of growth, that it could not have been lower with a different policy.

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2 Both issues are high on our to-do list.
3 According to the model, there is no discontinuity in any of the endogenous variables as the real rate of interest move across the threshold provided by the growth rate. Thus, neither the normative trade-offs nor the positive analysis jump at this point.
2 Literature review

At an aggregate level, monetary theories can be distinguished according to their views on the variables constituting the Fisher identity

\[ r = i - \pi, \]

where \( r \) is the real rate of interest, \( i \) is the nominal rate of interest, and \( \pi \) is the inflation rate. According to Wicksell (1898, 1907), the real interest rate is essentially unaffected by nominal variables, so from the monetary authorities' point of view, \( r \) is a constant. This view is known variously as "the natural rate hypothesis," "the classical dichotomy" and "the Fisher hypothesis."

Mundell (1963) and Tobin (1965) argue that higher inflation will not necessarily entail a one-for-one increase in nominal interest rates, since higher inflation will lead people to hold portfolios with less money and more real assets, thus lowering the real interest rate and raising investment. Perhaps as a result of the stagflation years in the 1970s, monetary macroeconomics has nonetheless gradually come to embrace the natural rate hypothesis, and the "vertical long-run Phillips curve" famously predicted by Phelps (1967) and Friedman (1968). The only caveat is that monetary authorities may be able to affect real interest rates in the short run due to price and wage rigidities.

At least since Taylor (1993), this New Keynesian synthesis has dominated monetary policy analysis; see e.g., Bernanke, Gertler and Gilchrist (1999) and Woodford (2003).

Our model contradicts the Wicksellian part of the New Keynesian view. The long-run real interest rate is not constant. Instead it is influenced by nominal interest rates, usually under the control of the central bank, as well as the public debt, usually under the control of fiscal authorities. In this sense, we are substantially more critical of the classical dichotomy than was Keynes (1936, p 202-204), who concede to Wicksell the point that it is impossible in the to reduce the real interest rate below the rate at which there would be full employment, but fear that there exist monetary equilibria sustaining a higher real rate and persistent unemployment. But unlike the models of Mundell and Tobin, our model allows many partial relationships between output and inflation. While higher inflation will entail larger output if nominal interest rates fail to go up proportionally, nothing prevents nominal interest rates

\[ \text{Perhaps it's the Stockholm air. Following an intensive debate among Swedish economists, described in Bertil Ohlin's preface to Wicksell (1936), Myrdal (1939) formulated a more radical critique of the natural rate hypothesis. According to Myrdal, there is a range of equilibria that are consistent with full employment, and equilibria with a lower real rate are associated with a higher capital stock. With hindsight, our work can be seen as providing a formal justification of Myrdal's view.} \]
from going up as much or more than the inflation rate. Stagflations are simply the outcome of poorly coordinated monetary and fiscal policies, which leave both inflation and nominal interest rates relatively high.

At the more foundational level, monetary theory has debated why money comes into existence in the first place. Why do people end up wanting to own an intrinsically useless nominal asset?

Most of the literature centers around two answers. The first answer is that money serves as a preferred medium of exchange, overcoming a lack of double coincidence of wants in the presence of search frictions; see Nosal and Rocheteau (2011) and Williamson and Wright (2011) for detailed surveys. Such search frictions have recently also been incorporated into macro-models; e.g., Arouba, Waller, and Wright (2011). Among other things, the money-search literature offers a credible explanation for the interest rate differential between currency and bonds, a distinction that our model neglects. In some versions of search models there is also asymmetric information about underlying real asset values, in which case the models can also shed light on rate-of-return differentials between stocks and bonds, which we also neglect here; e.g., Lester, Postlewaite, and Wright (2012).

The second answer, which we invoke here, is that money serves as a useful store of value. According to this view, money is valued so highly because the aggregate supply of claims on real assets is low relative to aggregate desired savings. This literature in turn has two branches. One relies on intergenerational trading frictions, building on the overlapping generations model of Allais (1947) and Samuelson (1958); see for example Diamond (1965), Wallace (1980), Sargent and Wallace (1982), and Tirole (1985). Tirole’s model is closest to ours, as it admits production and investment in capital; it also has positive growth, which is essential for maintaining bubbles in the presence of infinitely lived “rents.” As agents need to save for their retirement, and cannot explicitly trade with future generations, equilibrium investment in the absence of bubbles may drive the interest rate below the economy’s growth rate. Tirole shows that there is then a unique “asymptotically bubbly” equilibrium in which the interest rate converges to the growth rate.

The second branch builds on the dynastic model of Bewley (1980, 1983). In the original version of this model, agents do not hold any tradable long-lived assets apart from money, which may then be used to smooth transitory income shocks. Imrohoroglu (1992) and Aykol (2004) study the cost of inflation in endowment economies where money is the only liquid store of value.

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5 Needless to say, these interest rate wedges pose worthy challenges for generalizations of our model.

6 Tirole considers the case of steady population growth. We instead keep population constant and assume steady productivity growth.
Recently, Wen (2014) calibrates a Bewley model for an economy with production, fitting the asset holdings that we observe in less developed countries with poorly functioning intermediation. He finds that the economic costs of inflation can be large in such an environment. Since inflation makes it costly to insure against liquidity needs, many people will lack the necessary insurance.\footnote{Clearly, we use the Bewley models for a different purpose. As nominal interest rates are flexible and all assets are liquid, inflation does not by itself distort saving in our model. This is by construction. If we were to introduce a liquidity motive in addition to our long-run savings motive, inflation would matter.}

But while these arguments are venerable, there is a long-standing concern that models of money as a store of value rely on artificial restrictions on the availability of other stores of value, especially claims on long-lived productive assets (e.g., Scheinkman, 1980; Santos and Woodford, 1997). Thus, while these models might provide credible descriptions of poorly developed economies, they are not considered to offer convincing explanations for the role of monetary assets in developed modern economies. As, Santos and Woodford (1997, p 48) summarize their analysis: “Our main results show the nonexistence of asset pricing bubbles under fairly general assumptions.”\footnote{Similarly, in his survey of the theory of bubbles, Le Roy (2004, p 801) regrets to report that “Within the neoclassical paradigm there is no obvious way to derail the chain of reasoning that excludes bubbles.”}

Recent work has responded to this criticism by pointing out that the sequential completeness of markets is a strong assumption. Kiyotaki and Moore (2003, Theorem 1) providing natural conditions under which financial markets are sequentially incomplete and there is a role for outside money.\footnote{Another response is that many stores of value are temporarily illiquid (e.g., Holmström and Tirole, 1998; Kiyotaki and Moore, 2002, 2003; Farhi and Tirole, 2012). Such illiquid assets are less desirable for people seeking instruments for short-term saving. As a result, there may be a role for purely nominal assets to serve the combined function of storing value and acting as a medium of exchange.}

As described above, this is the direction that we also take. As far as we know, we are the first to develop a fully-fledged calibrated model of this sort. While our contribution is theoretical, we think that it is empirically plausible. For example, influential studies of US time-series favor the hypothesis that shifts in monetary and fiscal policy impact the long-run real interest rate. For example, Garcia and Perron (1996) find a large upward jump in real interest rates associated with a regime-shift in US monetary and fiscal policies around 1980.\footnote{For a recent contribution and further references, see Bulkley and Giordani (2011).}

Sims and Zha (2006) characterize the nature of this and three other monetary policy regime-shifts.\footnote{Most other empirical studies of monetary policy focus on temporary policy shocks}
cal fit is not too good, as would be often the case even with an accurately specified regime-shift model, the results are quite striking. For example, Volcker’s regime gives rise to substantially lower output than Burns’ regime and even Greenspan’s. This finding squares poorly with the view that monetary policy merely affects the stabilization of interest rates and output around exogenously given trends.

3 Simple model

We first clarify the workings of our main mechanism within a simple endowment economy.

Time is discrete, and the horizon is infinite.

Preferences: Infinitely-lived risk averse agents, consuming a homogeneous good and having identical preferences, are distributed along a continuum of length 1. Their utility function is of the form

\[ U_i^t \equiv \mathbb{E} \sum_{\tau=0}^{\infty} \beta^\tau u(c_{i,t+\tau}), \quad (1) \]

where \( u(c) \) is the momentary utility function and \( \beta \in (0, 1) \) is the subjective discount factor.

Technology: There are two kinds of productive assets. First, there is a continuum of length \( A \) of productive and tradable Lucas trees. In each period that a tree is alive, it yields an amount \( d \) of non-storable fruit. Survival is an i.i.d. process; each tree survives to the next period with probability \( q < 1 \). Thus, \((1 - q)A\) trees die each period. Likewise, \((1 - q)A\) trees are born each period. Each new tree is paired with a random agent, who initially has full ownership of this tree; that is, in each period a fraction \( 1 - q \) of the agents receive a new tree. Second, there is a continuum of length \( N \) of infinitely lived non-tradable trees each yielding \( y \) units of fruit per period. Each agent owns one non-tradable tree; to distinguish them from the tradable trees, we refer to them as bushes from now on. (The role of the bushes is to keep consumption bounded strictly above zero and utilities bounded above negative infinity, which is convenient for computational purposes.)

There also exists a continuum of length \( M \) of an unproductive asset that has no intrinsic value but which is perfectly durable and can be traded.

and thus do not directly address the issue that concerns us here. Moreover, that literature studies narrow monetary measures such as M1 and M2. However, it is remarkable that even this analysis sometimes finds evidence of long-run non-neutrality (e.g., Forni and Gambetti, 2010; see also Bernanke and Mihov, 1998).
Whenever it turns out to have value in equilibrium, we will think of this asset as (fiat) money. For convenience, we normalize $M = A = 1$.

Let $p^a$ denote the price of trees and $p^m$ the price of fiat money, both denominated in terms of fruit. We assume that there is no aggregate uncertainty.

**Trade:** At any time, agents can trade fruit and shares in existing trees in a frictionless market, although borrowing is prohibited (for the time being). Claims on trees that has not yet emerged cannot be traded.

While agents could in principle also write contracts contingent on future ownership of trees that have not yet emerged, our central assumption is that such contracts will not be worth writing. We find the assumption realistic, and the ensuing analysis does not hinge on its precise justification, but let us nonetheless briefly indicate one reason why it is difficult to contract on non-existing assets: Suppose that the paired agent privately observes the tree’s emergence one period before other agents do so. The agent can choose whether to keep the emerging tree or covertly display and sell it to some other agents – potential buyers – before it becomes publicly visible. Suppose the seller has sold shares in own emerging trees, but some other agent has not. That agent can then buy the tree, pretend it emerged with him, and cash in the full market value of the tree in the next period. By contrast, the seller by waiting until the next period can only cash in the value of any retained shares. Thus, a coalition of agents will always find it profitable to deviate from any plan involving trade of claims on future trees. More precisely, it can be shown that any outcome in which a positive measure of agents share risk in a market for futures would fail to be coalition-proof, in the sense defined for this kind of private-information environment by Lacker and Weinberg (1993).

**Equilibrium:** We focus on stationary equilibria throughout. The solution to the agents’ problem is thus characterized by time-invariant decision rules for consumption, $c_t = \phi_c(a_t, m_t, \varepsilon_t)$, trees, $\bar{a}_{t+1} = \phi_a(a_t, m_t, \varepsilon_t)$, and money, $m_{t+1} = \phi_m(a_t, m_t, \varepsilon_t)$, as functions of agent-specific asset holdings.

Although the distribution of agents in wealth space is constant in a stationary equilibrium, there are fluctuations and uncertainty at the agent level.

### 3.1 Analyzing the simple model

At each date $t$, agent $i$ maximizes (1) via the choice of consumption $c_i^t$, trees, $\bar{a}_{i+1}^t$, and money, $m_{i+1}^t$, subject to the budget constraint (2)

$$c_i^t = y + (p^a + d)(a_i^t + \varepsilon_i^t) - p^a \bar{a}_{i+1}^t + p^m (m_i^t - m_{i+1}^t),$$

(2)
and short-sale constraints

\[
\tilde{a}_{t+1} \geq 0, \\
m_{t+1} \geq 0,
\]

where

\[ a_t^i = q\tilde{a}_t^i, \]

and where \( \varepsilon_t^i \) denote new trees and

\[
\varepsilon_{t+1}^i = \begin{cases} 
1 & \text{with probability } (1 - q); \\
0 & \text{with probability } q.
\end{cases}
\]

The timing convention is that \( \varepsilon_t^i \) is observed before decisions are made in period \( t \). The components of initial wealth, \( a_0^i, m_0^i, \) and \( \varepsilon_0^i \), and market prices are taken as given.

We seek a stationary distribution over agents’s holdings of trees and money, and values for the prices, \( p^a \) and \( p^m \), such that markets for trees and fiat money clears. For an agent whose borrowing constraint is not binding, the first-order necessary conditions from the agent’s optimization problem imply that for all \( i \) and \( t \),

\[
\begin{align*}
    u_c(c_t^i)p^a &= \beta q(p^a + d)E_t u_c(c_{t+1}^i), \\
    u_c(c_t^i)p^m &= \beta p^m E_t u_c(c_{t+1}^i),
\end{align*}
\]

where \( u_c(\cdot) \equiv \frac{\partial u(\cdot)}{\partial c} \), and \( E_t \) is the expectations operator conditional on time \( t \) information. Let \( R^a = \frac{q(p^a + d)}{p^a} \) denote the return to holding trees and \( R^m = \frac{p^m}{p^m} = 1 \) denote the return to holding money. Denote \( \Phi_t : \chi \rightarrow [0, 1] \) as the joint distribution over money and (old and new) trees, where \( \chi = M \times A \times \{0, 1\} \).

It follows that there are two types of stationary equilibrium: (i) a non-monetary equilibrium where \( p^m = 0, \phi_m(a, m, \varepsilon) = 0 \) for all \( (a, m, \varepsilon) \in \chi \), and \( p^a > 0 \) such that \( R^a > 1 \) and \( \int_\chi \phi_a(a, m, \varepsilon)d\Phi_t(a, m, \varepsilon) = A \), and a monetary equilibrium where \( p^a = \frac{qd}{1-q} \) and \( p^m > 0 \) such that \( R^a = R^m = 1, \int_\chi \phi_a(a, m, \varepsilon)d\Phi_t(a, m, \varepsilon) = A \) and \( \int_\chi \phi_m(a, m, \varepsilon)d\Phi_t(a, m, \varepsilon) = M \) for all \( t \).

In order to illustrate the conditions under which there will be a monetary equilibrium and to study its welfare properties, let us impose further parameters.