Private Money and Banking Regulation*

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Abstract

We show that the regulation of lending practices is desirable for the optimal provision of private money. In an environment in which bankers cannot commit to repay their creditors, neither an unregulated banking system nor a form of narrow banking in which banks hold 100% in reserves can provide the socially efficient amount of money. If bankers provided such an amount, then they would prefer to default on their liabilities. We show that a regulation that increases the value of the banking sector's assets (e.g., by limiting competition in credit markets) will mitigate the commitment problem. If the return on the banking sector's assets is made sufficiently large, then it is possible to implement an efficient allocation.

Keywords: Private money; banking regulation; limited commitment.
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1 Introduction

The institutions composing the banking system do many things, but one of their main functions is to create liquidity. Among many forms of liquidity creation, banks issue tradeable securities that can be used to facilitate payments and settlement. This is private money. For example, Gorton (1999) highlights the free banking era as a period in American monetary history in which privately issued monies circulated as competing media of exchange. More contemporarily, it has been argued by many observers of the recent financial crisis that repurchase agreements are the private monies of our time (e.g., see Gorton and Metrick, 2010 and the explanations therein). Therefore, a primary concern of monetary economists should be to know whether, putting stability issues aside, a private banking system is capable of creating enough of this kind of liquidity to allow society to achieve an efficient allocation of resources. In other words, can a private banking system provide the socially efficient amount of money? And if so, what are the characteristics of such a system? Should we leave the job to the invisible hand or should we regulate the banking system? Can narrow banking – whereby the business of lending is separated from the business of deposit-taking – provide the efficient amount of money?

To investigate these questions, we construct a general equilibrium model in which some private agents have the ability to issue liabilities that circulate as a medium of exchange. These agents then use the proceeds to make loans in the credit market, obtaining a profit from these activities after repaying their creditors. We refer to these agents as private bankers and to their liabilities as private money, as they will serve as means of payment. The frictions explaining the essential role of bankers as providers of the means of payment are traders' anonymity and lack of commitment. If the price of bank money is too high, then households will not be able to trade the socially efficient amount in the goods market. This is the mechanism through which the price of private money affects the equilibrium allocation.

What prevents the efficient provision of private money here? The answer is simple: Bankers cannot commit to repay creditors, and the threat of terminating their business may not be strong enough to induce them to always redeem their liabilities at par, whenever their franchise value, i.e. the present value of future profits, is too low.[1] To ensure that bankers do not over-borrow (i.e. overissue) and strategically default on their liabilities, we consider a mechanism that imposes

[1]This is very much in the spirit of the hypotheses made in Gu, Mattesini, Monnet, and Wright (forthcoming), Hart and Zingales (2011), Gertler and Karadi (2011), Boissay (2011), Cavalcanti and Wallace (1999a, 1999b), and Cavalcanti, Erosa and Temzelides (1999).

individual debt limits on each banker, as in Alvarez and Jermann (2000).[2] These individual debt limits constrain the banker's portfolio choices and discipline private money creation. While these limits guarantee the solvency of each banker, they also constrain the amount of liabilities that each banker can issue.

Our contribution to the literature is to show how the degree of competition in credit markets affects the bankers' ability to provide the efficient level of private money. We initially characterize a free banking regime as an equilibrium where any agents can engage in lending or money creation. As agents compete for making loans in the credit market, the return on the banking sector's assets is relatively low. That is, competition drives down the return on banks' assets. As a consequence, the return that bankers can pay on their liabilities cannot be too high: promises to pay a high return would erode their already low franchise value, and they would renege. However, from a social standpoint, we want them to pay a sufficiently high return on (private) money to eliminate the opportunity cost of holding it (the Friedman rule). In this case, any equilibrium is necessarily inefficient. Thus, competition on the lending side restricts the supply of money below the socially efficient amount.

This result indicates that raising the return on the banking sector's assets is efficient, as it is socially desirable to induce bankers to pay a high return on their liquid liabilities. In particular, in the absence of any subsidies, it rules out some forms of banking regulation such as the proposal of requiring banks to hold 100% in reserves. In and of itself, this form of narrow banking will not be able to provide an efficient amount of private money, as there is no clear way to increase the return on the assets of a bank that keeps 100% in reserves or low-return government bonds.

We discuss two ways to increase the return on the banking sector's assets. First, we investigate how restricting entry in the credit market generates some rents to the banking sector. However, in addition to restricting entry, the regulator also needs to impose a floor on the deposit rate. In this way, bankers obtain a higher return on their assets, which allows them to pay a higher return on money. In particular, it is possible to implement an efficient allocation with private money. Thus, an optimal regulatory framework is to combine the role of bankers as liquidity providers with their role as financial intermediaries. Second, we investigate how a subsidy to the banking sector can implement the first best.

The role of regulation in guaranteeing a high franchise value for banks has been recognized by many experts, and in this respect, our paper is closest to Hellman, Murdock and Stiglitz (2000). Although they consider a model of banks with moral hazard, they argue that the best way to guarantee a high franchise value is to put

[2] Their analysis builds on work by Kehoe and Levine (1993) and Kocherlakota (1996).

a cap on the interest rate paid on deposits. As they write it, by limiting the degree of competition in the deposit market, a deposit rate control will increase the per-period profits captured by each bank, increasing their franchise value. However, their analysis does not consider the role of certificates of deposits as liquid assets. While our analysis agrees with the general findings that a high bank franchise value is necessary, we show that this value should not originate from the liability side of bank's balance sheet.

There are several theories explaining why combining intermediation and liquidity provision under the same roof is a good idea. Andolfatto and Nosal (2009) show in the context of a costly state verification model that it is efficient to combine these two activities within the same institution whenever the monitoring cost is sufficiently high. In a similar spirit, Sun (2007) shows that combining liquidity provision and intermediation can be desirable. Kashyap, Rajan, and Stein (2002) have also argued that providing liquidity on both sides of their balance sheets (e.g., through lines of credit) may give banks a competitive advantage. Finally, Williamson (1999) argues that private money creation by intermediaries is optimal because it allows them to undertake productive investment opportunities in states of the world in which they do not have their own funds available. Our theory also predicts that the activities of liquidity creation and financial intermediation should be essentially combined, but the reasons for doing so are very different from the previous literature.

Our paper is clearly related to the large literature on the optimal creation of private liquidity. However, in this literature, the effects of competition in bank lending are usually excluded from the analysis. There are two strands in this literature. The first strand focuses on the role of liquidity as a means of payment. Cavalcanti, Erosa, and Temzelides (1999) and Cavalcanti and Wallace (1999a, 1999b) study private money creation in the context of a random matching model. Azariadis, Bullard, and Smith (2001) study private and public money creation using an overlapping generations model; Kiyotaki and Moore (2001, 2002) propose a theory of inside money based on the possibility of collateralization of part of a debtor's assets; and Monnet (2006) studies the characteristics of the agent that is most able to issue money. The second strand focuses on the role of liquidity as a means of funding investment opportunities. For example, Holmstrom and Tirole (1998, 2011) show that a moral hazard problem may limit the ability of firms to refinance their ongoing projects when there is aggregate uncertainty. They argue that this inefficiency can be resolved by the government issuing bonds to firms.

Other authors have focused exclusively on the study of competition in bank lending without explicitly accounting for the role of bankers as liquidity providers.

These include Yanelle (1997) and Winton (1995, 1997). Our results show that the degree of competition in bank lending crucially influences the bankers' willingness to create private money. Thus, it is important to characterize the interplay between these two activities.

One the paper that is close to ours is Hart and Zingales (2011), who show that an unregulated private banking system creates too much money. They present an environment similar to Gu, et. al. (forthcoming), to which our paper also bears a resemblance, where a lack of double coincidence of wants, a lack of commitment, and a limited pledgeability of collateral give rise to an essential role for a medium of exchange. A bank acts as a safe-keeping institution for the collateral and issues receipts that can circulate as a means of payment because the bank is able to commit to pay the bearer of a receipt on demand. Hart and Zingales uncover an interesting externality: A bank that issues more money to its customers increases the price level for all other customers as well. As a result, too much collateral is stored, and banks create too much money. We depart from their analysis in a fundamental way: While they assume that banks can commit to pay back the bearer of the receipts they issued, we assume they cannot. This suffices to overturn their result: We show that a poorly regulated banking system creates too little money.

Empirical work on bank liquidity creation is scant, and the Berger and Bouwman (2009) paper is, to the best of our knowledge, the only one that measures the amount of liquidity created by the banking system. The authors construct a measure of liquidity creation by comparing how liquid the entries on both sides of a bank's balance sheet are. According to this measure, a bank creates more liquidity the more its liabilities are liquid relative to its assets. Among other interesting things, they find that banks that create more liquidity are valued more highly by investors, as measured by the market-to-book and the price-earnings ratios.

To be clear, we are not concerned in this paper with the stability of the banking sector. This is clearly an important issue that also relates to liquidity creation. In particular, the business of liquidity transformation and the risks it entails have been highlighted most forcefully in the seminal paper by Diamond and Dybvig (1983). Their notion of liquidity is one of immediacy: Bank deposits are useful because they can be redeemed on demand when depositors have an urgency to consume. So the banking system is fragile whenever the bank cannot fulfill the demand for immediate redemption. This is the well-known problem of a bank being illiquid but solvent. However, Jacklin (1987) considers a solution to banks' inherent fragility, namely that banks issue tradeable securities. If depositors have an urge to consume, they can sell these securities instead of running to the bank. This notion of liquidity (namely the ease with which bank liabilities can be traded) is clearly related to ours.

The paper is structured as follows. In Section 2, we present the basic framework, and we discuss the role of its main ingredients in Section 3. In Section 4, we formulate and solve the planner's problem. In Section 5, we characterize equilibrium allocations in the case of an unregulated banking system. In Section 6, we discuss the role of a regulator, where we formulate the regulator's intervention and characterize the equilibrium allocations in the case of a regulated banking system. Section 7 concludes.

2 Model

Time $t = 0, 1, 2, ...$ is discrete, and the horizon is infinite. Each period is divided into two subperiods: day and night. There are three physical commodities: a day good (the numeraire), a night good (the consumption good), and a capital good. The consumption good is the most fragile good as it cannot be stored. The capital good can be stored from the day to the night, but cannot be stored across periods. Finally, the numeraire is storable across periods, from day to day, with a return $\beta^{-1} > 1$.

There are four types of agents: buyers, sellers, entrepreneurs, and bankers, with a $[0, 1]$ continuum of each type. Buyers, sellers, and bankers are infinitely lived. Entrepreneurs live for two periods only.[3] They are born in the day and live through the night of the next period.

Buyers and sellers consume and produce the numeraire in the day using a linear technology that requires one unit of effort for each unit they produce. At night, sellers produce $F(k, n)$ units of the consumption good when they use k units of capital and n units of effort. We assume that $F : \mathbb{R}_+^2 \to \mathbb{R}_+$ is twice continuously differentiable, increasing in both arguments, and strictly concave, with $F(0, n) = 0$ for all $n \geq 0$ and $F(k, 0) = 0$ for all $k \geq 0$. Only entrepreneurs can produce capital. They need *exactly* e units of the numeraire at date t to produce $\gamma\hat{k}$ units of capital in the night at date $t+1$, where $\hat{k} > 0$ is a constant. Entrepreneurs are heterogeneous with respect to their productivity levels γ, which is distributed according to the density function $g(\gamma)$ defined over $[0, \bar{\gamma}]$. We denote the cumulative distribution function of productivity levels by $G(\gamma)$. Notice that the numeraire can either be immediately consumed, or it can be converted into capital to be used in the next period, or it can be stored.

[3] This implies that we do not have to worry about capital accumulation and dynamic contracts. Alternatively, we could assume that entrepreneurs are infinitely-lived but we would have to assume that they cannot store capital from one period to the next.

Bankers are endowed with a technology that allows them to make their actions publicly observable at no cost. This means that if a banker decides to make his actions publicly observable, other agents can keep track of his balance sheet and income statement at each date.

We now describe preferences. Let $x_t^b \in \mathbb{R}$ denote a buyer's net consumption of the numeraire ($x_t^b < 0$ means that the buyer is a net producer), and let $q_t^b \in \mathbb{R}_+$ denote his consumption at night. His preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[x_t^b + u\left(q_t^b\right)\right],$$

where $\beta \in (0,1)$. The function $u : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable, increasing, and strictly concave, with $u'(0) = \infty$. Let $x_t^s \in \mathbb{R}$ denote a seller's numeraire consumption, and let $n_t^s \in \mathbb{R}_+$ denote his nighttime effort level. His preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[x_t^s - c\left(n_t^s\right)\right],$$

where $c : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable, increasing, and convex. Let $x_t \in \mathbb{R}_+$ denote a banker's consumption of the numeraire. Each banker has preferences given by

$$\sum_{t=0}^{\infty} \beta^t x_t.$$

Finally, an entrepreneur born at date t wants to consume only at date $t+1$. Specifically, each entrepreneur born at date t derives utility x_{t+1}^e if his consumption of the numeraire at date $t+1$ is $x_{t+1}^e \in \mathbb{R}_+$.

Buyers, sellers, and bankers lack any commitment, whereas entrepreneurs can fully commit to their promises.[4] We also assume that buyers and sellers are anonymous: There is no technology to verify their identities and their trading histories are not observable. Finally, we assume that a buyer cannot carry capital to a seller's location. Hence capital cannot be used as a means of payment in the night market, as in Aruoba, Waller, and Wright (2011).

In the day there is a perfectly competitive (Walrasian) market in which agents trade the numeraire and capital. In the night subperiod, only buyers and sellers trade. Following the literature, we refer to this night market as the decentralized market. For simplicity, we will use competitive pricing to determine the terms of trade in this market. Still, a medium of exchange remains essential as long as

[4] Instead of assuming that entrepreneurs can fully commit to repay their debt, we could assume that they are infinitely-lived and that they would lose access to credit if they were to renege.

we maintain the (intertemporal) double coincidence problem and anonymity; see Rocheteau and Wright (2005) for a discussion.

Finally, since buyers are anonymous and lack commitment, they cannot credibly use goods invested in the storage technology as a means of payment in the decentralized market. Thus, the storage technology corresponds to the concept of illiquid capital in Lagos and Rocheteau (2008).

3 Discussion of the Model

In this section, we explain how the pieces of the model fit together. To generate a demand for a medium of exchange, we build on Lagos and Wright (2005).[5] In the decentralized market, the absence of commitment and record-keeping implies that a buyer and a seller can trade only if a medium of exchange is available. As bankers can make their actions publicly observable, they are able to issue the means of payment as long as people trust that they will redeem them.

However, bankers also lack commitment and we need some punishment for default to guarantee that they make good on their promises. As in Gu et. al. (forthcoming), Cavalcanti, Erosa, and Temzelides (1999) and Cavalcanti and Wallace (1999a, 1999b), we assume that a renegade banker loses his assets as well as his ability to make his actions publicly observable. This means that a defaulter will lose future profit as he will not be able to finance his business by issuing money.

In the decentralized market, a seller may accept a banker's IOU because, contrary to any buyer, a banker is punished if he fails to redeem his IOUs. Thus, if the banker's future profits are high, everybody knows he has a lot to lose and they trust that the banker will redeem the notes. Figure 1 shows how a banker's note will circulate in the economy.

We make clear in the Appendix that lack of commitment is the only friction in our framework. With full commitment, we show that the economy degenerates into an Arrow-Debreu economy, in which case the first welfare theorem applies. If there is full commitment, then there is no need for a means of payment in the decentralized market, in which case the bankers will get zero payoff at each date.

[5] An alternative tractable framework that also creates a role for a medium of exchange is the large household model in Shi (1997).

4 Efficient Allocations

In this section, we solve the problem of a planner who can enforce all transfers at zero cost. This means that a solution to the planner's problem is an unconstrained efficient allocation. We also assume that the planner treats entrepreneurs of the same generation equally. Thus, he will assign the same consumption level to each member of a given generation. Given these assumptions, an efficient allocation is obtained in the usual way: Given some minimum utility level U_t^e assigned to each entrepreneur of generation t, for all generations, and some minimum utility levels U and U^s assigned to each banker and each seller at date $t = 0$, respectively, an efficient allocation maximizes the lifetime utility of each buyer subject to the participation and resource constraints.

It should be clear that the planner will fund only the entrepreneurs who are sufficiently productive. This means that each entrepreneur whose productivity level γ is greater than or equal to a specific marginal type $\gamma_t^p \in [0, \bar{\gamma}]$ will receive e units of the daytime good to undertake his project at date t, whereas the types $\gamma \in [0, \gamma_t^p)$ will not operate their projects. We refer to the type γ_t^p as the date-t marginal entrepreneur. Thus, the planner's problem consists of choosing an allocation

$$\left\{x_t^b, x_t^s, x_t, x_t^e, q_t, n_t, i_t, k_{t+1}, \gamma_t^p\right\}_{t=0}^{\infty}$$

to maximize the lifetime utility of the buyer

$$\sum_{t=0}^{\infty} \beta^t \left[x_t^b + u\left(q_t\right)\right], \tag{1}$$

subject to the daytime resource constraint

$$x_t^b + x_t^s + x_t^e + x_t + i_t = 0, \tag{2}$$

the nighttime resource constraint

$$q_t = F\left(k_t, n_t\right), \tag{3}$$

the restrictions imposed by the investment technology

$$k_{t+1} = \hat{k} \int_{\gamma_t^p}^{\bar{\gamma}} \gamma g\left(\gamma\right) d\gamma, \tag{4}$$

$$i_t = e\left[1 - G\left(\gamma_t^p\right)\right], \tag{5}$$

the entrepreneurs' participation constraints

$$x_t^e \geq U_{t-1}^e, \tag{6}$$

the bankers' participation constraint

$$\sum_{t=0}^{\infty} \beta^t x_t \geq U, \tag{7}$$

and the sellers' participation constraint

$$\sum_{t=0}^{\infty} \beta^t \left[x_t^s - c\left(n_t\right)\right] \geq U^s,$$

taking the initial capital stock $k_0 = k\left(\gamma_{-1}^p\right) > 0$ and the required utility levels $\left\{U_{t-1}^e\right\}_{t=0}^{\infty}$, U, and U^s as given. Notice that any Pareto optimal allocation solves the problem described above for a particular choice of required utility levels $\left\{U_{t-1}^e\right\}_{t=0}^{\infty}$, U, and U^s, and that any solution to the problem above is a Pareto optimal allocation.

Let $k\left(\gamma_t^p\right) \equiv \hat{k} \int_{\gamma_t^p}^{\bar{\gamma}} \gamma g\left(\gamma\right) d\gamma$ denote the aggregate capital stock available at the beginning of date $t+1$ as a function of the date-t marginal entrepreneur γ_t^p. The first-order conditions are given by

$$\beta u' \left[F\left(k\left(\gamma_t^p\right), n_{t+1}\right)\right] F_k\left(k\left(\gamma_t^p\right), n_{t+1}\right) \hat{k} \gamma_t^p = e, \tag{8}$$

$$u' \left[F\left(k\left(\gamma_{t-1}^p\right), n_t\right)\right] F_n\left(k\left(\gamma_{t-1}^p\right), n_t\right) = c'\left(n_t\right), \tag{9}$$

for all $t \geq 0$. To marginally increase each buyer's consumption within a household at date $t+1$ without changing the effort level that each seller within a household exerts at date $t+1$, the planner needs to give up e units of the daytime good at date t at the margin to increase the amount of the capital good available for production at date $t+1$. The left-hand side in (8) gives the marginal benefit of an extra unit of capital at date $t+1$, whereas the right-hand side gives the marginal resource cost at date t. Similarly, to marginally increase each buyer's consumption within a household at date t given a predetermined stock of capital at the beginning of date t, the planner needs to instruct each seller within a household to exert more effort in the night subperiod. Condition (9) guarantees that the marginal disutility of effort equals the marginal benefit of consuming an extra unit of the nighttime good.

A stationary solution to the planner's problem involves $\gamma_t^p = \gamma^*$ and $n_t = n^*$ for all $t \geq 0$, with γ^* and n^* satisfying

$$\beta u' \left[F \left(k \left(\gamma^* \right), n^* \right) \right] F_k \left(k \left(\gamma^* \right), n^* \right) \hat{k} \gamma^* = e, \tag{10}$$

$$u' \left[F \left(k \left(\gamma^* \right), n^* \right) \right] F_n \left(k \left(\gamma^* \right), n^* \right) = c' \left(n^* \right). \tag{11}$$

We also need the initial capital stock to be equal to $k(\gamma^*)$. In the Appendix, we show the existence and uniqueness of a stationary solution to the planner's problem for at least some specifications of preferences and technologies.

5 Free Banking

In this section, we describe the equilibrium outcome of an economy with free banking: any agents can lend to entrepreneurs. Because buyers and sellers are able to produce the numeraire, they can lend as much as they want to. Bankers can also lend but they first need to borrow the numeraire through the sale of bank notes or by using retained earnings.[6]

To finance his investments at date t, a banker raises numeraire funds by selling perfectly distinguishable bank notes. Each note is costless to issue, and it is a promise to repay one unit of the numeraire at date $t+1$ to the note holder. Throughout the paper, we restrict attention to symmetric equilibria in which all notes trade at the same price ϕ_t, so that the notes issued by bankers are perfect substitutes. Agents in the economy take the sequence of prices $\{\phi_t\}_{t=0}^{\infty}$ as given when making their individual decisions. Agents are willing to hold bank notes for two reasons: They may offer them a return greater than that paid on any other asset, and they can be used as a means of payment in the decentralized market.

After he sold notes, a banker invests the proceeds in the credit market or in the storage technology, or both. At date $t+1$, he uses the revenue from his investments to repay his creditors, and he consumes or reinvests the remaining profits.

The goal of this section is to characterize equilibrium allocations in the absence of intervention in lending practices. Because bankers cannot commit to repay creditors, we only assume the existence of a regulator whose exclusive role is to punish those bankers who default on their liabilities. As in Hellmann, Murdock and Stiglitz (2000), the regulator has access to a bank's balance sheet at the end of each period

[6] In this economy, the money supply is completely endogenous: The bankers issue private debt that can be used as a medium of exchange so that the aggregate supply of bank notes depends entirely on the banking sector's willingness to expand its aggregate balance sheet.

and the regulator can punish any banker who defaults on his liabilities by revoking his "franchise" and garnishing his assets.[7]

5.1 Credit Market

In the day subperiod, there is a perfectly competitive credit market for one-period loans in which the entrepreneurs can borrow units of the numeraire to fund their investment projects. Let R_t denote the gross interest rate that prevails in this market at date t. In the absence of intervention, the equilibrium interest rate must be

$$R_t = \beta^{-1}. \tag{12}$$

If $R_t > \beta^{-1}$, then any buyer or seller will wish to supply an infinite amount of resources. If $R_t < \beta^{-1}$, then the supply of funds will be zero because agents have the option of using the storage technology to transfer resources from one period to the next.

A type-γ entrepreneur has a profitable project if and only if $\rho_{t+1}\hat{k}\gamma - e\beta^{-1} \geq 0$, where ρ_t denotes the price of one unit of capital in terms of the date-t numeraire. The value of a type-γ entrepreneur's project in terms of the date-$(t+1)$ numeraire is $\rho_{t+1}\hat{k}\gamma$ gives t, whereas $eR_t = e\beta^{-1}$ gives the repayment that the entrepreneur needs to make at date $t+1$. Hence, a type-γ entrepreneur has a profitable project if and only if its surplus is positive. Given the relative price of capital ρ_{t+1}, any type-γ entrepreneur for whom

$$\rho_{t+1}\hat{k}\gamma \geq e\beta^{-1} \tag{13}$$

will find it optimal to borrow at date t. Thus, given ρ_{t+1}, we can define the date-t marginal entrepreneur γ_t^m as the type satisfying

$$\gamma_t^m = \frac{e}{\beta\rho_{t+1}\hat{k}}. \tag{14}$$

This means that any entrepreneur indexed by $\gamma \in [0, \gamma_t^m]$ will find it optimal to borrow in the credit market to fund his project, whereas the types $\gamma \in [\gamma_t^m, \bar{\gamma}]$ will choose not to fund their projects. Thus, the aggregate demand for loans at date t is given by

$$\ell_t = e\left[1 - G\left(\gamma_t^m\right)\right],$$

[7] When verifying a bank's balance sheet is costly, Gu, et. al. (forthcoming) provide a rational for why there should be few agents intermediating investment.

and the aggregate amount of capital available at date $t+1$ is given by

$$k_{t+1} = \hat{k} \int_{\gamma_t^m}^{\bar{\gamma}} \gamma g(\gamma)\, d\gamma \equiv k(\gamma_t^m). \tag{15}$$

5.2 Buyer's Problem

Let $w_t^b(a,l)$ denote the value function for a buyer who enters the day subperiod holding $a \in \mathbb{R}_+$ notes and $l \in \mathbb{R}_+$ real loans, and let $v_t^b(k,a,l)$ denote the value function for a buyer who enters the night subperiod holding a portfolio of $k \in \mathbb{R}_+$ units of capital, $a \in \mathbb{R}_+$ notes, and $l \in \mathbb{R}_+$ real loans. The Bellman equation for a buyer in the day subperiod is given by

$$w_t^b(a,l) = \max_{(x,k',a',l') \in \mathbb{R} \times \mathbb{R}_+^3} \left[x + v_t^b(k',a',l') \right],$$

subject to the budget constraint

$$x + \rho_t k' + \phi_t a' + l' = \beta^{-1} l + a.$$

Here k' denotes the amount of capital that the buyer accumulates at the end of the day subperiod, a' denotes his choice of note holdings at the end of the day subperiod, and l' denotes the amount of loans made in the credit market. Because of quasi-linear preferences, the value $w_t^b(a,l)$ is an affine function of the form $w_t^b(a,l) = a + \beta^{-1} l + w_t^b(0,0)$, with the intercept $w_t^b(0,0)$ given by

$$w_t^b(0,0) = \max_{(k',a',l') \in \mathbb{R}_+^3} \left[-\rho_t k' - \phi_t a' - l' + v_t^b(k',a',l') \right]. \tag{16}$$

Let p_{t+1} denote the price of one unit of the date-t consumption good in terms of the date-$(t+1)$ numeraire. The Bellman equation for a buyer holding a portfolio of k' units of capital, a' notes, and l' real loans in the night subperiod is given by

$$v_t^b(k',a',l') = \max_{q \in \mathbb{R}_+} \left[u(q) + \beta w_{t+1}^b(a' - p_{t+1} q, l') \right], \tag{17}$$

subject to the liquidity constraint

$$p_{t+1} q \leq a'. \tag{18}$$

Using the fact that $w_t^b(a,l)$ is an affine function, we can rewrite the Bellman equation

(17) as follows:

$$v_t^b(k', a', l') = \max_{q \in \mathbb{R}_+} [u(q) - \beta p_{t+1} q] + \beta a' + l' + \beta w_{t+1}^b(0,0).$$

First, notice that there is no benefit of accumulating capital (capital cannot be used as a medium of exchange and fully depreciates from one period to the next). Therefore, the buyer optimally chooses $k' = 0$. Second, because $\left(\partial w_t^b / \partial l\right)(a, l) = \beta^{-1}$, the buyer is willing to supply any amount of resources in the credit market.

The liquidity constraint (18) may either bind or not, depending on the buyer's note holdings. In particular, notice that

$$\frac{\partial v_t^b}{\partial a}(k', a', l') = \begin{cases} \frac{1}{p_{t+1}} u'\left(\frac{a'}{p_{t+1}}\right) \text{ if } a' < p_{t+1}\hat{q}(p_{t+1}); \\ \\ \beta \text{ if } a' > p_{t+1}\hat{q}(p_{t+1}); \end{cases}$$

where $\hat{q}(p_{t+1}) = (u')^{-1}(\beta p_{t+1})$. If the liquidity constraint does not bind, then the marginal utility of an extra note equals β, which is simply the discounted value of the payoff of one unit of the daytime good at date $t+1$. If the liquidity constraint binds, then the marginal utility of an extra note is greater than β. In this case, the notes offer a liquidity premium. Since the buyer can always use the storage technology, he will hold notes if and only if he obtains a liquidity premium or the return on notes is greater than the return to storage.

The first-order condition for the optimal choice of note holdings on the right-hand side of (16) is given by

$$-\phi_t + \frac{\partial v_t^b}{\partial a}(k', a', l') \leq 0,$$

with equality if $a' > 0$. If $\phi_t > \beta$, then the optimal choice of note holdings will be given by

$$u'\left(\frac{a'}{p_{t+1}}\right) = \phi_t p_{t+1}, \tag{19}$$

so that notes offer a liquidity premium. Because of quasi-linear preferences, all buyers choose to hold the same quantity of notes at the end of the day market. Thus, condition (19) gives the aggregate demand for notes as a function of the relative price of the nighttime good p_{t+1} and the price of notes ϕ_t. A higher price for the notes reduces the amount of notes demanded. The effect of the relative price p_{t+1} on the demand for notes depends on the curvature of the utility function $u(q)$. If $-[u''(q)q]/u'(q) < 1$, then an increase in p_{t+1} reduces the demand for notes, holding ϕ_t constant. If $-[u''(q)q]/u'(q) > 1$, then an increase in p_{t+1} results in a higher demand for notes.

5.3 Seller's Problem

Let $w_t^s(a,l)$ denote the value function for a seller who enters the day subperiod holding $a \in \mathbb{R}_+$ notes and $l \in \mathbb{R}_+$ real loans, and let $v_t^s(k,a,l)$ denote the value function for a seller who enters the night subperiod holding $k \in \mathbb{R}_+$ units of capital, $a \in \mathbb{R}_+$ notes, and $l \in \mathbb{R}_+$ real loans. The Bellman equation for a seller in the day subperiod is given by

$$w_t^s(a,l) = \max_{(x,k',a',l')\in\mathbb{R}\times\mathbb{R}_+^3} \left[x + v_t^s(k',a',l')\right],$$

subject to the budget constraint

$$x + \rho_t k' + \phi_t a' + l' = \beta^{-1} l + a.$$

Here k' denotes the amount of capital that the seller accumulates at the end of the day subperiod, a' denotes his choice of note holdings at the end of the day subperiod, and l' denotes the amount of real loans made in the current daytime credit market. Similarly, the value $w_t^s(a,l)$ is an affine function, $w_t^s(a,l) = a + \beta^{-1}l + w_t^s(0,0)$, with the intercept $w_t^s(0,0)$ given by

$$w_t^s(0,0) = \max_{(k',a',l')\in\mathbb{R}_+^3} \left[-\rho_t k' - \phi_t a' - l' + v_t^s(k',a',l')\right]. \tag{20}$$

The Bellman equation for a seller with a portfolio of k' units of capital, a' notes, and l' real loans in the night subperiod is given by

$$v_t^s(k',a',l') = \max_{n\in\mathbb{R}_+} \left[-c(n) + \beta w_{t+1}^s\left(p_{t+1}F(k',n) + a', l'\right)\right]. \tag{21}$$

Using the fact that $w_t^s(a,l)$ is an affine function, we can rewrite the right-hand side of (21) as follows:

$$\max_{n\in\mathbb{R}_+} \left[-c(n) + \beta p_{t+1} F(k',n)\right] + \beta a' + l' + \beta w_{t+1}^s(0,0).$$

The first-order condition for the optimal choice of nighttime effort is given by

$$c'(n) = \beta p_{t+1} F_n(k',n). \tag{22}$$

Because $(\partial v_t^s/\partial k)(k',a',l') = \beta p_{t+1} F_k(k',n)$, the first-order condition for the optimal choice of capital on the right-hand side of (20) is given by

$$\rho_t = \beta p_{t+1} F_k(k', n). \tag{23}$$

Thus, conditions (22) and (23) determine the demand for capital and the nighttime effort decision as a function of the relative price of the consumption good p_{t+1} and the relative price of capital ρ_t. Combining (22) with (23), we obtain the following condition:

$$\frac{\rho_t}{c'(n)} = \frac{F_k(k', n)}{F_n(k', n)}. \tag{24}$$

Because $(\partial w_t^s / \partial l)(a, l) = \beta^{-1}$, the seller is willing to supply any amount of resources in the credit market. Finally, the first-order condition for the optimal choice of note holdings is given by

$$-\phi_t + \beta \leq 0,$$

with equality if $a' > 0$. This means that the seller does not hold notes if $\phi_t > \beta$.

5.4 Banker's Problem

Now we describe the decision problem of a banker. Let $w_t(b_{t-1}, i_{t-1})$ denote the value function for a banker with debt b_{t-1} and assets i_{t-1} at the beginning of date t. The banker's assets at the beginning of date t consist of real loans made at date $t-1$ and units invested in the storage technology at date $t-1$, whereas the banker's debt refers to the amount of notes issued at date $t-1$. As we have seen, the marginal return on the banker's assets is given by β^{-1} in the absence of intervention, whether he invests in the credit market or in the storage technology. Thus, the banker's decision problem can be formulated as follows:

$$w_t(b_{t-1}, i_{t-1}) = \max_{(x_t, i_t, b_t) \in \mathbb{R}_+^3} [x_t + \beta w_{t+1}(b_t, i_t)] \tag{25}$$

subject to his budget constraint

$$i_t + x_t + b_{t-1} = \beta^{-1} i_{t-1} + \phi_t b_t$$

and the debt limit

$$b_t \leq \bar{B}_t.$$

Here i_t denotes the amount of resources (units of the daytime good) that the banker decides to invest at date t. In other words, i_t gives the banker's assets at the beginning of date $t+1$. When making his investment decisions at each date, the banker takes as given the sequence of debt limits $\left\{\bar{B}_t\right\}_{t=0}^{\infty}$, the marginal return on his assets β^{-1}, and the sequence of prices $\{\phi_t\}_{t=0}^{\infty}$.

If $\phi_t > \beta$, then the banker finds it optimal to borrow up to his debt limit, i.e., he will choose $b_t = \bar{B}_t$. Because the return paid on his notes (his cost of funds) is lower than the return on his assets, he makes a positive profit by borrowing and investing the proceeds in the storage technology.

It is important to note that a banker is indifferent between immediately consuming and reinvesting the proceeds from his previous profits (his retained earnings), because the return on his assets equals his rate of time preference. Therefore, a solution to the banker's optimization problem is $i_t = \phi_t \bar{B}_t$, which means that the banker invests all funds he has borrowed at date t but does not invest his own funds. Thus, the balance sheet of a typical banker will have no equity, only debt. In this case, the banker's consumption at date t is simply given by

$$x_t = \bar{B}_{t-1}\left(\beta^{-1}\phi_{t-1} - 1\right).$$

We refer to the bank franchise *value* as the lifetime utility associated with a particular choice of the return on the banker's assets, the sequence of debt limits, and the sequence of prices for the banker's liabilities. At each date t, the bank franchise value is given by

$$w_t\left(\bar{B}_{t-1}, \phi_{t-1}\bar{B}_{t-1}\right) = \sum_{\tau=t}^{\infty} \beta^{\tau-t}\bar{B}_{\tau-1}\left(\beta^{-1}\phi_{\tau-1} - 1\right).$$

Perfect competition in the credit market implies that the return on the banker's assets is the smallest possible, β^{-1} at each date, which lowers the bank franchise value. As we will see, the introduction of regulation will play a crucial role in increasing the return on the banker's assets, thus raising the bank's franchise value.

5.5 Aggregate Note Holdings

Let a_t denote the date-t aggregate note holdings. For any price $\phi_t > \beta$, the liquidity constraint (18) is binding, in which case the value of the notes in circulation must equal the value of the aggregate production at night,

$$a_t = p_{t+1} F\left(k\left(\gamma_{t-1}^{m}\right), n_t\right). \tag{26}$$

This is the equation for the quantity theory of money. Note that the aggregate production depends on the current capital stock and the effort level that each seller is willing to exert in order to produce the consumption good. Combining (19) with (26), we obtain

$$u'\left[F\left(k\left(\gamma_{t-1}^{m}\right), n_t\right)\right] = \phi_t p_{t+1}.$$

Using (22) to substitute for p_{t+1}, we get the following equilibrium condition:

$$u'\left[F\left(k\left(\gamma_{t-1}^{m}\right), n_t\right)\right] = \frac{\phi_t}{\beta} \frac{c'\left(n_t\right)}{F_n\left(k\left(\gamma_{t-1}^{m}\right), n_t\right)}. \tag{27}$$

This condition determines the equilibrium effort decision, given the predetermined capital stock. The price of notes ϕ_t influences this decision in the following way: A lower price for the bankers' notes increases the return on these notes and the buyer's expenditure decision, thus raising the relative price p_{t+1} and inducing each seller to exert more effort.

As we have seen, the choice of the date-t marginal entrepreneur is given by (14). Using (24) to substitute for ρ_{t+1}, we obtain the following equilibrium condition:

$$\beta u'\left[F\left(k\left(\gamma_t^m\right), n_{t+1}\right)\right] F_k\left(k\left(\gamma_t^m\right), n_{t+1}\right) \hat{k} \gamma_t^m = e\frac{\phi_{t+1}}{\beta}. \tag{28}$$

This condition determines the equilibrium capital accumulation decision at date t given the nighttime effort decision at date $t+1$. A lower value for ϕ_{t+1} results in a larger capital stock at date $t+1$, holding n_{t+1} constant.

We can use (27) and (28) to implicitly define the functions $\gamma_{t-1}^m = \gamma^m\left(\phi_t\right)$ and $n_t = n\left(\phi_t\right)$. Using these functions, we can define the aggregate production of the nighttime good by $q\left(\phi_t\right) = F\left[k\left(\gamma^m\left(\phi_t\right)\right), n\left(\phi_t\right)\right]$. Then, the aggregate note holdings as a function of the price ϕ_t are given by

$$a\left(\phi_t\right) = \frac{u'\left[q\left(\phi_t\right)\right] q\left(\phi_t\right)}{\phi_t}. \tag{29}$$

5.6 Equilibrium

To define an equilibrium, we need to specify the sequence of debt limits $\left\{\bar{B}_t\right\}_{t=0}^{\infty}$ in such a way that the bankers are willing to supply the amount of notes other agents demand and are willing to fully repay their creditors (note holders). We take two steps to define a sequence of debt limits satisfying these two conditions. First, for any given sequence of prices $\{\phi_t\}_{t=0}^{\infty}$, we set

$$\bar{B}_t = a\left(\phi_t\right) \tag{30}$$

at each date t. This condition guarantees that each banker is willing to supply the amount of notes in (29) at the price ϕ_t. Then, given this choice for the individual debt limits, we need to verify whether a particular choice for the price sequence $\{\phi_t\}_{t=0}^{\infty}$ implies that each banker does not want to renege on his liabilities at any date. As we have seen, a banker who reneges on his liabilities will no longer be able to issue notes. Moreover, he will have his assets seized. Thus, a particular price sequence $\{\phi_t\}_{t=0}^{\infty}$ is consistent with the solvency of each banker if and only if

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} a\left(\phi_{\tau-1}\right)\left(\beta^{-1}\phi_{\tau-1}-1\right) \geq a\left(\phi_{t-1}\right)\left(\beta^{-1}\phi_{t-1}-1\right)+\phi_t a\left(\phi_t\right)$$

holds at each date t. As in Alvarez and Jermann (2000), these solvency constraints allow the banker to borrow as much as possible without inducing him to default on his liabilities. The left-hand side gives the beginning-of-period continuation utility. The right-hand side gives the current payoff the banker gets if he decides *not* to invest the resources he has borrowed at date t. In this case, he can increase his current consumption by the amount $a\left(\phi_t\right)\phi_t$, but he will permanently lose his note-issuing privileges at date $t+1$. We can rewrite the solvency constraints above as follows:

$$-\phi_t a\left(\phi_t\right)+\sum_{\tau=t+1}^{\infty} \beta^{\tau-t} a\left(\phi_{\tau-1}\right)\left(\beta^{-1}\phi_{\tau-1}-1\right) \geq 0. \tag{31}$$

As in Alvarez and Jermann, we want to allow the bankers to borrow as much as they can and, at the same time, make sure that they do not want to default. If we define debt limits that are not too tight, then we can rewrite the solvency constraints as follows:

$$-\phi_t a\left(\phi_t\right)+\beta\hat{w}_{t+1}=0,$$

where $\hat{w}_t = w_t\left(a\left(\phi_{t-1}\right), \phi_{t-1}a\left(\phi_{t-1}\right)\right)$. We can also rewrite (25) as follows:

$$\hat{w}_t = a\left(\phi_{t-1}\right)\left(\beta^{-1}\phi_{t-1}-1\right)+\beta\hat{w}_{t+1}.$$

Note that the term $a\left(\phi_{t-1}\right)\left(\beta^{-1}\phi_{t-1}-1\right)$ gives the banker's current profit. It depends on the amount of notes issued at the previous date as well as the price at which the banker sold his notes at that time. Specifically, at date $t-1$, the banker received the amount $a\left(\phi_{t-1}\right)\phi_{t-1}$ in exchange for his notes. He invested this amount in both real loans and the storage technology, obtaining the revenue $\beta^{-1}a\left(\phi_{t-1}\right)\phi_{t-1}$ at date t. Because each note is a promise to pay one unit of the daytime good, his current liabilities are $a\left(\phi_{t-1}\right)$. Thus, his profit will be given by the difference between the revenue $\beta^{-1}a\left(\phi_{t-1}\right)\phi_{t-1}$ and the repayment $a\left(\phi_{t-1}\right)$. As we have seen, he will immediately consume any profit he makes. Finally, throughout the paper, we will restrict attention to non-autarkic equilibria for which the price sequence $\{\phi_t\}_{t=0}^{\infty}$ is bounded.

Definition 1 *An equilibrium is an array* $\left\{\gamma_t^m, n_t, a_t, \bar{B}_t, \phi_t, R_t\right\}_{t=0}^{\infty}$ *satisfying (12), (27), (28), (29), (30), and (31) with equality at each date* t*, given the initial capital stock.*

5.7 Welfare Properties

Now we want to show any equilibrium allocation in the absence of intervention (even though we have not shown existence yet) is inefficient. If we compare equations (27) and (28) with the solution to the planner's problem, given by equations (8) and (9), setting $\phi_t = \beta$ at each date $t \geq 0$ makes the choices of the marginal entrepreneur and the nighttime effort level exactly the same as those in the planner's solution. Thus, $\phi_t = \beta$ for all $t \geq 0$ is a necessary condition for efficiency: the optimal return on notes at each date should be given by β^{-1}. But condition (31) implies that the banker's solvency constraints are necessarily violated in this case, so we cannot have an equilibrium with $\phi_t = \beta$ for all $t \geq 0$. This means that any allocation that can be implemented in the absence of intervention in the credit market is *not* Pareto optimal. We summarize these findings in the following proposition.

Proposition 2 *Any equilibrium allocation in the absence of intervention in lending practices is inefficient.*

Why are the bankers unwilling to supply the socially efficient amount of money? As we have seen, the return on the banker's assets is the same as the return to storage and the rate of time preference. Perfect competition in the credit market implies that there is no markup over the return to storage. When lenders compete for borrowers in the credit market, the return that each one of them gets is lower, which in turn restricts the bankers' ability to pay a higher return on their notes.

It may be worth stressing that this result is not due to the participation of buyers and sellers in the credit market. One could argue that one role of banks is to pool resources from many agents to channel to fewer entrepreneurs. We fail to capture this aspects because each buyer or seller can generate enough resources to lend to an entrepreneur. However, in a previous version of the model, we assumed that only bankers could lend and we showed that bankers' competition was enough to also drive the lending rate to β^{-1}.

To implement the optimal return on notes, we must drive the value of the bank future profits to zero, which is inconsistent with the solvency constraints. As a result, there exists an upper bound on the return bankers are willing to offer on their liabilities without inducing them to default. Any return above this bound makes the banker prefer to default on his liabilities. Such a bound exists because the return on the banking sector's assets is relatively low when there is free entry in the credit market.

The previous result says that any kind of regulation that seeks to restrict competition on the liability side of banks' balance sheets, such as the interest rate cap proposed by Hellmann, Murdock and Stiglitz (2000), will result in an inefficient amount of private money, regardless of the kind of intervention that is carried out on the asset side. Regulation Q in the U.S. is an example of a regulatory measure aimed at restricting the return that banks are allowed to pay to their depositors. Our analysis predicts that these measures necessarily lead to an inefficient amount of bank liquidity creation.

5.8 Existence

To show existence, we will restrict attention to stationary equilibria for which the aggregate amount of notes issued at each date is constant over time. In the Appendix, we discuss the characterization of non-stationary equilibria. In the case of stationary allocations, we have $\phi_t = \phi$, $\gamma_t^m = \gamma^m$, $n_{t+1} = n$, $\bar{B}_t = \bar{B}$, and $a_t = a$ for all $t \geq 0$. We must also have $R_t = \beta^{-1}$ for all t.

Note that we can use (27) and (28) to define the choices of the marginal entrepreneur γ^m and the nighttime effort level n as a function of the price ϕ and then define the aggregate note holdings in the same way. Finally, any stationary equilibrium must also satisfy the solvency constraints (31). In particular, a stationary solution satisfies these constraints if and only if

$$-\phi a(\phi) + \frac{\beta}{1-\beta} a(\phi) \left(\beta^{-1}\phi - 1\right) \geq 0. \tag{32}$$

Because $a(\phi) > 0$ for any $\phi > \beta$, condition (32) holds if and only if

$$\phi \geq 1.$$

This means that the bankers are willing to supply any amount of notes for which the return on these notes is nonpositive. In other words, in the case of perfect competition in the credit market, the bankers need to charge for their liquidity services in order to be individually rational for them to redeem their notes at par. As we have seen, this result has a crucial implication for the welfare properties of equilibrium allocations. In particular, the nonpositive-return-on-notes property arises in the case of stationary allocations and implies that any stationary equilibrium is necessarily inefficient.

The following proposition establishes existence and uniqueness for some specifications of preferences and technologies.

Proposition 3 *Suppose that* $u(q) = (1-\sigma)^{-1}(q^{1-\sigma}-1)$*, with* $0 < \sigma < 1$*,* $c(n) = n$*, and* $F(k,n) = k^{\alpha}n^{1-\alpha}$*, with* $0 < \alpha < 1$*. Suppose also that* $g(\gamma) = 1$ *for all* $0 \leq \gamma \leq 1$ *and* $g(\gamma) = 0$ *otherwise. Then, there exists a unique non-autarkic stationary equilibrium for which* $\phi_t = 1$ *for all* $t \geq 0$*.*

Under these specifications of preferences and technologies, it is straightforward to show that the aggregate amount of notes $a(\phi)$ is strictly decreasing in ϕ. This means that the lack of intervention in the credit market results in an inefficiently small amount of money, in which case the price of notes will be too high to allow society to achieve a Pareto optimal allocation. This result suggests that we can mitigate the commitment problem only by increasing the return on the banker's assets, which can be accomplished through banking regulation.

6 Rents Through Regulation

In the previous section, we have shown that a free banking system fails to deliver an efficient allocation of resources as bankers are not generating enough rents. Our results suggest that it is efficient to raise the return on the banking sector's assets, as it allows us to "relax" the bankers' solvency constraints without impairing the efficient provision of money.

In this section, we consider a regulatory mechanism that achieves the efficient outcome. While we leave implementation issues aside, we offer different ways to interpret the mechanism in a later section. The mechanism looks as if there is interest rate controls, in the sense that banks will act as if the regulator can set different interest rates for different types of borrowers (which type is observable). We let $r_t(\gamma)$ denote the interest rate offered to a type-γ entrepreneur, which is the interest rate that will prevail in the market for loans to type-γ entrepreneurs. So, the goal of the regulator is to find the minimum interest rates $r_t(\gamma)$ that imply a sufficiently high return on the bankers' assets to allow them to supply the socially efficient amount of notes. We start by describing the regulatory mechanism.

6.1 A Regulatory Mechanism

Here we describe the regulatory mechanism (or just mechanism for short) in the daytime credit market. The mechanism announces a return function $\hat{R}_{t+1}(i_t)$ to each banker. It promises to deliver $i_t \hat{R}_{t+1}(i_t)$ units of the date-$(t+1)$ numeraire if the banker decides to invest i_t units of the date-t numeraire. Then, the mechanism collects all funds raised from the bankers and allocates these resources to fund entrepreneurs in the credit market and invest in the storage technology. Because of the possibility of using the storage technology, the participation constraint for the banker is given by

$$\hat{R}_{t+1}(i_t) \geq \beta^{-1}. \tag{33}$$

We will restrict attention to return functions of the form:

$$\hat{R}_{t+1}(i_t) = \begin{cases} \beta^{-1} + \mu_t \text{ if } i_t < e\left[1 - G\left(\gamma^m\left(\phi_{t+1}\right)\right)\right], \\ \\ \beta^{-1} \text{ if } i_t \geq e\left[1 - G\left(\gamma^m\left(\phi_{t+1}\right)\right)\right], \end{cases} \tag{34}$$

where $\mu_t \geq 0$ denotes the date-t markup over the return to storage. At each date t, the mechanism chooses a portfolio that devotes the amount $e\left[1 - G\left(\gamma^m\left(\phi_{t+1}\right)\right)\right]$ to fund entrepreneurs and invests the remaining resources in the storage technology. In this way, we can guarantee that any entrepreneur whose project has a positive surplus, given the price ϕ_{t+1}, will be able to get funding in the credit market provided that the bankers (the suppliers of those funds) have enough resources (raised from the sale of notes and their own retained earnings).

Naturally, the mechanism faces a resource constraint. It finances the return function by a transfer $r_t(\gamma)$ for each unit borrowed by an entrepreneur $\gamma \in [0, \bar{\gamma}]$, above and beyond the free banking equilibrium rate $R_t = \beta^{-1}$. This transfer has to satisfy the type-γ entrepreneur's participation constraint:

$$\rho_{t+1}\hat{k}\gamma - \left[\beta^{-1} + r_t(\gamma)\right] e \geq 0. \tag{35}$$

Also, we must have that, given the transfer $r_t(\gamma)$, the announced return function $\hat{R}_{t+1}(i_t)$ satisfies

$$\left(\beta^{-1} + \mu_t\right)\left[1 - G\left(\gamma^m\left(\phi_{t+1}\right)\right)\right] \leq \int_{\gamma^m(\phi_{t+1})}^{\bar{\gamma}} \left[\beta^{-1} + r_t(\gamma)\right] g(\gamma)\, d\gamma. \tag{36}$$

The left-hand side gives the amount of resources that the mechanism promised at date t to deliver at date $t+1$, and the right-hand side gives the total repayment received at date $t+1$ from the entrepreneurs who were funded at date t. Thus, condition (36) guarantees that the announced return function $\hat{R}_{t+1}(i_t)$ is feasible. Finally, the participation constraints (33) and (35) imply that the transfer $r_t(\gamma)$ can neither be too large nor too small:

$$\beta^{-1} \leq \beta^{-1} + r_t(\gamma) \leq \beta^{-1} \frac{\gamma}{\gamma^m(\phi_{t+1})} \tag{37}$$

for each type $\gamma \geq \gamma^m(\phi_{t+1})$.

Definition 4 *Given a sequence of prices $\{\phi_t\}_{t=0}^{\infty}$, a mechanism consists of a sequence of markups $\{\mu_t\}_{t=0}^{\infty}$ and a sequence of transfers $\{r_t(\gamma)\}_{t=0}^{\infty}$ satisfying (36) and (37) at each date.*

The mechanism specifies a sequence of markups and transfers as a function of the sequence of prices $\{\phi_t\}_{t=0}^{\infty}$.

Note that setting $\mu_t = 0$ at each date gives us the free banking solution that we have analyzed in the previous section. We now characterize equilibria for which the markup μ_t is positive at each date so that the bankers will be able to extract some of the surplus from the entrepreneurs. As a result, the average return on the bankers' assets will be higher.

The mechanism does not affect the behaviour of buyers and sellers. Therefore, it is straightforward to show that the buyer will choose not to accumulate capital and that his demand for notes will also be given by (19). Also, a seller's decision for capital and nighttime effort are still given by (22) and (23). As in the previous section, the seller will not hold notes if $\phi_t > \beta$.

6.2 Banker's Problem

Given the mechanism, the banker's decision problem can now be formulated as follows:

$$w_t(b_{t-1}, i_{t-1}) = \max_{(x_t, i_t, b_t) \in \mathbb{R}_+^3} [x_t + \beta w_{t+1}(b_t, i_t)]$$

subject to his budget constraint

$$i_t + x_t + b_{t-1} = \hat{R}_t(i_{t-1}) i_{t-1} + \phi_t b_t$$

and the debt limit

$$b_t \leq \bar{B}_t.$$

The return function $\hat{R}_{t+1}(i_t)$ is given by (34) with $\mu_t > 0$. The banker takes the announced return functions as given when making his individual decisions, as well as the sequence of debt limits $\{\bar{B}_t\}_{t=0}^{\infty}$ and prices $\{\phi_t\}_{t=0}^{\infty}$.

As before, if $\phi_t > \beta$, then we have $b_t = \bar{B}_t$ at the optimum, so the banker finds it optimal to borrow up to his debt limit. If the banker has enough funds at date t, then the optimal choice for i_t is such that $i_t \geq e[1 - G(\gamma^m(\phi_{t+1}))]$ because $\mu_t > 0$. If the investment amount i_t is lower than $e[1 - G(\gamma^m(\phi_{t+1}))]$, then the return to each incremental amount invested at date t is greater than the rate of time preference. In this case, the banker would be better off if he increased his investment at date t. If the investment at date t exceeds $e[1 - G(\gamma^m(\phi_{t+1}))]$, then the return to each extra unit invested at date t equals the rate of time preference. In this case, the banker is indifferent between immediately consuming and investing one extra unit. This means that $i_t = \phi_t \bar{B}_t$ is part of a solution to the banker's decision problem provided that $\phi_t \bar{B}_t \geq e[1 - G(\gamma^m(\phi_{t+1}))]$. We will later show that this will be the case in equilibrium.

6.3 Equilibrium

To construct an equilibrium, we follow the same steps as in the previous section. We need to find a sequence of debt limits that guarantees that the bankers are willing to supply the amount of notes other people demand and are willing to fully repay their creditors at each date. The banker's solvency constraints are now given by

$$-\phi_t a(\phi_t) + \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} [\Pi(\phi_{\tau-1}, \phi_\tau, \mu_{\tau-1}) - a(\phi_{\tau-1})] \geq 0 \qquad (38)$$

at each date $t \geq 0$, where the date-t revenue $\Pi(\phi_{t-1}, \phi_t, \mu_{t-1})$ is given by

$$\Pi(\phi_{t-1}, \phi_t, \mu_{t-1}) \equiv \mu_{t-1} e[1 - G(\gamma^m(\phi_t))] + \beta^{-1}\phi_{t-1} a(\phi_{t-1}).$$

The solvency constraints (38) are similar to those that we have obtained in the previous section, except that now the banker's date-t revenue has increased by the amount $\mu_{t-1} e[1 - G(\gamma^m(\phi_t))]$. The definition of an equilibrium is now straightforward.

Definition 5 *An equilibrium is an array* $\{\gamma_t^m, n_t, a_t, \bar{B}_t, \phi_t, R_t, r_t(\gamma), \mu_t\}_{t=0}^{\infty}$ *satisfying (12), (27), (28), (29), (30), (36), (37), and (38) with equality at each date* t*, given the initial capital stock.*

Notice that buyers and sellers can still lend to entrepreneurs, which forces the interest rate in the credit market to $R_t = \beta^{-1}$. However, it will turn out that in equilibrium, neither buyers nor sellers will lend to entrepreneurs. So the equilibrium appears as if buyers and sellers have been excluded from participating in the credit market. We come back to this interpretation later.

6.4 Existence

To show existence, we will restrict attention to stationary equilibria in which the aggregate amount of notes issued at each date is constant over time. In the Appendix, we characterize non-stationary equilibria. First, consider a solution to the banker's decision problem when $\phi_t = \phi$ and $\bar{B}_t = a(\phi)$ at each date $t \geq 0$. In this case, we have $b_t = a(\phi)$ and $i_t = \phi a(\phi)$. Second, the following result guarantees that, at any given price ϕ, the bankers will be able to raise enough resources from the sale of notes in order to finance all entrepreneurs whose projects have a positive surplus.

Lemma 6 *For any given $\phi > \beta$, we have $\phi a(\phi) > e[1 - G(\gamma^m(\phi))]$.*

This results implies that neither buyers nor sellers are lending directly to entrepreneurs in equilibrium as they are indifferent doing so.

Finally, we need to find the set of stationary prices ϕ for which the solvency constraints hold. Given a stationary markup $\mu > 0$, any price ϕ satisfying

$$-\phi a(\phi) + \frac{\beta}{1-\beta}\left[\hat{\Pi}(\phi,\mu) - a(\phi)\right] \geq 0 \tag{39}$$

implies that the repayment of creditors is individually rational for each banker. Here the value $\hat{\Pi}(\phi,\mu)$ is defined by

$$\hat{\Pi}(\phi,\mu) \equiv \mu e[1 - G(\gamma^m(\phi))] + \phi a(\phi)\beta^{-1},$$

which gives the banker's revenue at each date as a function of the price ϕ and the markup μ. The markup μ must satisfy the following condition:

$$\beta^{-1} < \beta^{-1} + \mu \leq \beta^{-1}\frac{\int_{\gamma^m(\phi)}^{\bar{\gamma}} \gamma g(\gamma)\, d\gamma}{\gamma^m(\phi)[1 - G(\gamma^m(\phi))]} \equiv \hat{R}(\phi). \tag{40}$$

Here $\hat{R}(\phi)$ gives the average return on the banking sector's loan portfolio when the type-γ entrepreneur, for each type $\gamma \geq \gamma^m(\phi)$, obtains no surplus from borrowing. The corresponding transfer is given by

$$r(\gamma) = \beta^{-1}\frac{\gamma - \gamma^m(\phi)}{\gamma^m(\phi)}, \tag{41}$$

(40) implies that the average return $\beta^{-1}+\mu$ on the banking sector's loan portfolio can range from the competitive return β^{-1} to the monopolist return $\hat{R}(\phi)$, depending on the regulatory mechanism. Indeed, given a particular choice of the transfer $\{r(\gamma)\}_{\gamma\geq\gamma^m(\phi)}$, the markup μ is given by

$$\mu = \frac{\int_{\gamma^m(\phi)}^{\bar{\gamma}} r(\gamma)\, g(\gamma)\, d\gamma}{[1 - G(\gamma^m(\phi))]}. \tag{42}$$

Thus, given any price ϕ, a stationary mechanism consists of a stationary markup μ and a transfer scheme $\{r(\gamma)\}_{\gamma\geq\gamma^m(\phi)}$ satisfying (37) and (42) for each $\gamma \geq \gamma^m(\phi)$.

One immediate consequence of the existence of a positive markup is that the average return on the banker's assets is higher than the average return he gets in the case of perfect competition by the amount $\mu e[1 - G(\gamma^m(\phi))]$. As a consequence, the set of stationary prices satisfying the solvency constraints must be larger than the one we obtain in the case of unregulated lending because a higher return on assets essentially relaxes the solvency constraints. The following proposition establishes existence and uniqueness of a non-autarkic stationary equilibrium in the presence of regulation.

Proposition 7 *Suppose that* $u(q) = (1-\sigma)^{-1}(q^{1-\sigma} - 1)$*, with* $0 < \sigma < 1$*,* $c(n) = n$*, and* $F(k, n) = k^{\alpha}n^{1-\alpha}$*, with* $0 < \alpha < 1$*. Suppose also that* $g(\gamma) = 1$ *for all* $0 \leq \gamma \leq 1$ *and* $g(\gamma) = 0$ *otherwise. Then, there exists a unique non-autarkic stationary equilibrium for which* $\phi_t = \bar{\phi}$ *for all* $t \geq 0$*, where* $\bar{\phi} < 1$*.*

With a positive markup, it is possible to have an equilibrium in which the return on notes is strictly positive. As should be expected, a positive markup raises the return on the banking sector's assets, mitigating the commitment problem associated with the note-issuing privileges. As a result, there exists an equilibrium in which the bankers are willing to pay a strictly positive return on their notes. In other words, there exists an equilibrium in which the price of bank liabilities is lower, and the aggregate supply of these liabilities is larger than those that we have obtained in the absence of regulation.

6.5 Welfare Properties

Now we turn to the welfare implications of the mechanism. In particular, we want to know whether the mechanism allows us to implement the optimal return on notes.

Proposition 8 *If* β *is sufficiently close to one, then an equilibrium with* $\phi_t = \beta$ *for all* $t \geq 0$ *exists.*

For any μ sufficiently close to the upper bound, given by the monopolist markup ($\mu = \hat{R}(\phi) - \beta^{-1}$), it is possible to have an equilibrium in which the return on notes equals the rate of time preference. In this case, we eliminate the opportunity cost of holding money, maximizing the surplus from trade in the decentralized night market. Because any other allocation that makes at least one entrepreneur better off necessarily makes a banker worse off, we conclude that setting $\phi_t = \beta$ for all $t \geq 0$ is both necessary and sufficient for efficiency.

The mechanism expands the set of equilibrium allocations to include the efficient one. Without such mechanism, bankers compete on the asset side of their balance sheets and can only get a positive franchise value if they offer a low return on their liabilities. The role of the regulator is to increase the return on the banking sector's assets by shifting some of the entrepreneurs' surplus to bankers. This is important because such an intervention will allow bankers to increase the return on their liabilities, thus favouring the provision of liquidity. As we have shown, bankers are willing to supply the optimum quantity of money (i.e., the amount that results in a Pareto optimal allocation) only if the average return on their assets is sufficiently close to the return that a monopolist banker would obtain.

However, a monopolist banker would not choose an efficient allocation because he would certainly not choose the price of his liabilities to be $\phi_t = \beta$ at each date. We have assumed a perfectly competitive market for the bankers' notes so that banks bid up the return on private money. This is crucial for the efficiency of the system. The fact that a monopolist would obtain a high return on his assets does not mean that he would be willing to offer the socially efficient return on his liabilities. To obtain efficiency, a monopolist banker would have to be regulated as well.

An important corollary that follows immediately is that narrow banking (a system in which banks hold 100% in reserves) cannot provide the efficient amount of liquidity. Indeed, narrow banking does not offer any means to increase the return on the banking sector's assets.

6.6 Interpretations of the Regulatory Mechanism

There are several ways to interpret the mechanism.

A rather direct interpretation is one of a regulated mutual fund in which all bankers invest their resources, while the regulator imposes restrictions on lending practices by any other agents.[8] In this way the fund can behave as a monopolist on the credit market, extracting enough rents from entrepreneurs to redistribute to

[8] A quick inspection of the buyers' and sellers' problem reveals that their other decisions will not change following their exclusion from the credit market.

banks. At the same time, banks still compete for deposits so that they can bid the return on banknotes up to β^{-1}. Let us emphasize that we have assumed that buyers and sellers have enough resources to lend to an entrepreneur. In facts, depositors are usually too "small" to lend to an entrepreneur by itself. If this was the case in our model, we would not have to restrict their entry in the credit market. Still, bankers themselves would compete their rents away, and the mutual fund is able to coordinate banks behavior to extract some of the entrepreneurs' surplus.

The rules of the fund determine the amount of resources that will be devoted to finance entrepreneurs in the now uncompetitive credit market and to invest in the storage technology. The regulator prohibits any other agents to make loans on their own. This means that bankers can either invest in the fund or in the storage technology. The lending strategy of the fund then determines the marginal return on each unit of its capital (the banks' deposits in the fund). Different choices for the interest rates $\beta^{-1} + r_t(\gamma)$ by the fund will imply different values for the markup μ_t, determining the profitability of the fund. Notice that the transfer $r_t(\gamma)$ is now the interest rate that the funds charge entrepreneurs above and beyond the competitive market rate. This mutual fund may look like a nationalization of the banking system. However, this is only half correct as all banks need to compete for deposits.

While our interpretation of the mechanism as a mutual fund may leave some of the readers perplexed, we can also interpret it as a regulation that 1) restricts lending from buyers and sellers if necessary, 2) limits banks' entry and 3) requires a floor on the return on banknotes. Such a policy will create some monopoly rent for bankers. In fact, it does look like the prohibition of branch banking in the U.S. free banking era together with the requirement that the banker held enough capital in the form of specie (i.e. gold and silver) or government bonds.

Yet another interpretation of the mechanism is a simple fiscal scheme designed to guarantee that bankers get some of the entrepreneurs surplus. The taxpayers are the entrepreneurs, who need to borrow in the credit market to fund their investment projects. They each pay according to their profit a tax which equals to the transfer $r_t(\gamma)e$. The proceeds is then given back to bankers, and bankers only, as a function of their portfolio. Thus bankers benefit from higher returns. This is independent of whether buyers and sellers are active in the credit market.

Because this transfer artificially raises the return on the banking sector's assets, one could think that bankers are necessarily better off with the mechanism in place. This is not necessarily the case. Recall that the ultimate goal of the regulator is to induce bankers to pay a high return on their liquid liabilities, so it is not necessarily true that the higher return on the banking sector's assets means that bankers make a bigger profit in the presence of intervention. Even though the regulator wants to make sure that some of the surplus from trade in the credit market goes to the bankers, it wants them to eliminate the consumers' opportunity cost of holding bank liabilities, which boils down to paying them for holding bank notes. It is important to keep in mind that the goal of the regulator is not to increase the bankers' profit but to guarantee that they pay a high return on their liquid liabilities.

7 Conclusion

We have shown that an unregulated banking system is unable to supply an efficient amount of liquid liabilities. In the absence of intervention, the return on the bankers' assets will be relatively low because of competition in the credit market. This makes the option of defaulting on their liabilities more attractive. Thus, the bankers will be willing to offer to pay only a low return on their liabilities, creating a cost for their liability holders (that they are willing to bear because these liabilities provide them with a transaction service). For this reason, any equilibrium allocation in the absence of intervention is necessarily inefficient.

In view of this inefficiency, we have considered the possibility of regulating the banking system. One way to induce bankers to supply an efficient amount of private money is to sufficiently raise the return on their assets. The regulator's goal is to ensure that the bankers get some of the surplus from the borrowers in the credit market. We have also offered several alternative interpretations for the kind of intervention necessary to achieve this result. It can either be viewed as the introduction of an incentive-compatible taxation scheme, or as a restriction on competition.

So far, we have left aside the role of banks as risk transformers, whereby banks undertake risky investments but issue relatively safe debt, or alternatively whereby banks' assets are information sensitive while they issue information-insensitive liabilities (an idea that dates back to Gorton and Pennacchi, 1990, but has regained some traction recently; see Gorton, 2010). This is clearly an important issue that will impact the optimal provision of liquidity, and we leave it for future work.

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8 Appendix

8.1 Existence of a Unique Stationary Solution to the Planner's Problem

Here we show the existence of a unique stationary solution to the planner's problem for some specifications of preferences and technologies. In particular, we assume that $u(q) = (1-\sigma)^{-1}(q^{1-\sigma}-1)$, with $0<\sigma<1$, $c(n) = n$, and $F(k,n) = k^{\alpha}n^{1-\alpha}$, with $0<\alpha<1$. We also assume that $g(\gamma) = 1$ for any $0 \leq \gamma \leq 1$ and $g(\gamma) = 0$ otherwise. In this case, conditions (10) and (11) become

$$n = \chi\left[\gamma^{-1}\left(1-\gamma^{2}\right)^{1-\alpha+\alpha\sigma}\right]^{\frac{1}{(1-\alpha)(1-\sigma)}} \equiv Z(\gamma), \tag{43}$$

$$n = \lambda\left(1-\gamma^{2}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \equiv H(\gamma), \tag{44}$$

respectively, where the constants χ and λ are defined as

$$\chi \equiv \left(\frac{1}{2}\right)^{\frac{1-\alpha+\alpha\sigma}{(1-\alpha)(1-\sigma)}}\left[\frac{e}{\alpha\beta\hat{k}^{\alpha(1-\sigma)}}\right]^{\frac{1}{(1-\alpha)(1-\sigma)}}, \tag{45}$$

$$\lambda \equiv \left[(1-\alpha)\left(\frac{\hat{k}}{2}\right)^{\alpha(1-\sigma)}\right]^{\frac{1}{\alpha+\sigma(1-\alpha)}}. \tag{46}$$

Notice that $Z'(\gamma) < 0$ for all $\gamma \in (0,1)$. Also, we have that $\lim_{\gamma\to 0} Z(\gamma) = +\infty$ and $\lim_{\gamma\to 1} Z(\gamma) = 0$. This means that the function $Z(\gamma)$ is strictly decreasing in the open interval $(0,1)$. With respect to the function $H(\gamma)$, we have that $H'(\gamma) < 0$ and $H''(\gamma) < 0$ for all $\gamma \in (0,1)$. Also, we have that $\lim_{\gamma\to 0} H(\gamma) = \lambda$ and $\lim_{\gamma\to 1} H(\gamma) = 0$. This means that the function $H(\gamma)$ is strictly decreasing and concave in the open interval $(0,1)$. This means that a unique interior solution exists.

8.2 Proof of Proposition 3

Suppose that $u(q) = (1-\sigma)^{-1}(q^{1-\sigma}-1)$, with $0<\sigma<1$, $c(n) = n$, and $F(k,n) = k^{\alpha}n^{1-\alpha}$, with $0<\alpha<1$. Suppose also that $g(\gamma) = 1$ for any $0 \leq \gamma \leq 1$ and $g(\gamma) = 0$ otherwise. In this case, conditions (27) and (28) become

$$n = \chi^{e}(\phi)\left[\gamma^{-1}\left(1-\gamma^{2}\right)^{1-\alpha+\alpha\sigma}\right]^{\frac{1}{(1-\alpha)(1-\sigma)}} \equiv Z^{e}(\gamma,\phi), \tag{47}$$

$$n = \lambda^e(\phi)\left(1-\gamma^2\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \equiv H^e(\gamma,\phi), \tag{48}$$

respectively, where the functions $\chi^e(\phi)$ and $\lambda^e(\phi)$ are given by

$$\chi^e(\phi) = \left(\frac{1}{2}\right)^{\frac{1-\alpha+\alpha\sigma}{(1-\alpha)(1-\sigma)}} \left[\frac{e\phi}{\alpha\beta^2\hat{k}^{\alpha(1-\sigma)}}\right]^{\frac{1}{(1-\alpha)(1-\sigma)}},$$

$$\lambda^e(\phi) = \left[(1-\alpha)\frac{\beta}{\phi}\left(\frac{\hat{k}}{2}\right)^{\alpha(1-\sigma)}\right]^{\frac{1}{\alpha+\sigma(1-\alpha)}}.$$

Notice that $d\chi^e/d\phi > 0$, whereas $d\lambda^e/d\phi < 0$. Also, we have that $\chi^e(\beta) = \chi$ and $\lambda^e(\beta) = \lambda$, where χ and λ are given by (45) and (46), respectively. For any fixed $\phi > \beta$, we have that $\partial Z^e/\partial\gamma < 0$ for all $\gamma \in (0,1)$, $\lim_{\gamma\to 0} Z^e(\gamma,\phi) = +\infty$, and $\lim_{\gamma\to 1} Z^e(\gamma,\phi) = 0$. For any fixed $\phi > \beta$, we also have that $\partial H^e/\partial\gamma < 0$ and $\partial^2 H^e/\partial\gamma^2 < 0$ for all $\gamma \in (0,1)$, $\lim_{\gamma\to 0} H^e(\gamma,\phi) = \lambda^e(\phi)$, and $\lim_{\gamma\to 1} H^e(\gamma,\phi) = 0$. Thus, for any fixed $\phi > \beta$, a unique interior solution exists. Moreover, the Implicit Function Theorem implies that $d\gamma^m/d\phi > 0$ and $dn/d\phi < 0$.

As we have seen, condition (32) holds if and only if $\phi \geq 1$. In particular, it holds with equality if and only if $\phi = 1$. Thus, there exists a unique non-autarkic stationary equilibrium for which $\gamma^m = \gamma^m(1)$, $n = n(1)$, $a = a(1)$, where $a(1)$ is given by

$$a(1) = \left(\frac{\hat{k}}{2}\right)^{\alpha(1-\sigma)} \left[1-\gamma^m(1)^2\right]^{\alpha(1-\sigma)} n(1)^{(1-\alpha)(1-\sigma)}. \tag{49}$$

Q.E.D.

8.3 Proof of Lemma 6

Note that we can rewrite the expression for the aggregate note holdings as follows:

$$\phi a(\phi) = \frac{e\phi F\left[k\left(\gamma^m(\phi)\right), n(\phi)\right]}{\beta^2\hat{k}\gamma^m(\phi) F_k\left[k\left(\gamma^m(\phi)\right), n(\phi)\right]}.$$

For any price $\phi > \beta$, we have

$$\begin{aligned}
\frac{e\phi F\left[k\left(\gamma^m\left(\phi\right)\right), n\left(\phi\right)\right]}{\beta^2 \hat{k}\gamma^m\left(\phi\right) F_k\left[k\left(\gamma^m\left(\phi\right)\right), n\left(\phi\right)\right]} &> \frac{e\phi k\left(\gamma^m\left(\phi\right)\right)}{\beta^2 \hat{k}\gamma^m\left(\phi\right)} \\
&> \frac{ek\left(\gamma^m\left(\phi\right)\right)}{\beta \hat{k}\gamma^m\left(\phi\right)} \\
&> \frac{ek\left(\gamma^m\left(\phi\right)\right)}{\hat{k}\gamma^m\left(\phi\right)} \\
&= \frac{e\int_{\gamma^m(\phi)}^{\bar{\gamma}} \gamma g\left(\gamma\right) d\gamma}{\gamma^m\left(\phi\right)} \\
&> e\left[1 - G\left(\gamma^m\left(\phi\right)\right)\right].
\end{aligned}$$

Q.E.D.

8.4 Proof of Proposition 7

Suppose that $u\left(q\right) = \left(1-\sigma\right)^{-1}\left(q^{1-\sigma}-1\right)$, with $0 < \sigma < 1$, $c\left(n\right) = n$, and $F\left(k, n\right) = k^{\alpha} n^{1-\alpha}$, with $0 < \alpha < 1$. Suppose also that $g\left(\gamma\right) = 1$ for any $0 \leq \gamma \leq 1$ and $g\left(\gamma\right) = 0$ otherwise. Note that we can rewrite (39) as follows:

$$-\phi a\left(\phi\right) + \frac{\beta}{1-\beta} a\left(\phi\right)\left(\phi\beta^{-1} - 1\right) + \frac{\beta}{1-\beta}\mu\left[1 - \gamma^m\left(\phi\right)\right] \geq 0, \tag{50}$$

where

$$a\left(\phi\right) = \phi^{-1}\left(\frac{\hat{k}}{2}\right)^{\alpha(1-\sigma)}\left[1 - \gamma^m\left(\phi\right)^2\right]^{\alpha(1-\sigma)} n\left(\phi\right)^{(1-\alpha)(1-\sigma)}.$$

We have already shown that

$$-\phi a\left(\phi\right) + \frac{\beta}{1-\beta} a\left(\phi\right)\left(\phi\beta^{-1} - 1\right) \geq 0$$

if and only if $\phi \geq 1$. This means that there exists $\bar{\phi} < 1$ such that, for $\phi = \bar{\phi}$, there exist $\left\{r\left(\gamma\right)\right\}_{\gamma \geq \gamma^m\left(\bar{\phi}\right)}$ and $\mu > 0$ satisfying (42), (**??**), and (50). **Q.E.D.**

8.5 Proof of Proposition 8

Suppose now that $r\left(\gamma\right)$ is given by (41) for any given ϕ. Then, (39) can be written as

$$\frac{e}{\beta}\left[\frac{1}{2\gamma^m\left(\phi\right)} + \frac{\gamma^m\left(\phi\right)}{2} - 1\right] - \left(\frac{1-\phi}{\phi}\right)\left(\frac{\hat{k}}{2}\right)^{\alpha(1-\sigma)}\left[1 - \gamma^m\left(\phi\right)^2\right]^{\alpha(1-\sigma)} n\left(\phi\right)^{(1-\alpha)(1-\sigma)} \geq 0.$$

Taking the limit as $\phi \to \beta$ from above, the left-hand side of this expression converges to

$$e\left(\frac{1}{2\gamma_\beta^*}+\frac{\gamma_\beta^*}{2}-1\right)-(1-\beta)\left(\frac{\hat{k}}{2}\right)^{\alpha(1-\sigma)}\left[1-\left(\gamma_\beta^*\right)^2\right]^{\alpha(1-\sigma)}\left(n_\beta^*\right)^{(1-\alpha)(1-\sigma)},$$

where $\left(\gamma_\beta^*, n_\beta^*\right)$ denotes the solution to the planner's problem [i.e., the unique interior solution to the system (43)-(44)] for any given discount factor $\beta < 1$. As $\beta \to 1$ from below, we have that $0 < \lim_{\beta\to 1}\gamma_\beta^* < 1$. This means that there exists $\beta < 1$ sufficiently close to one such that the expression above is strictly positive. Therefore, we have constructed an equilibrium in which $\phi_t = \beta$,

$$1 + r_t(\gamma) = \beta^{-1}\frac{\gamma}{\gamma_\beta^*},$$

for each $\gamma \geq \gamma_\beta^*$, and

$$\mu_t = \frac{\beta^{-1}}{2}\left(\frac{1}{\gamma_\beta^*}-1\right).$$

for all $t \geq 0$. **Q.E.D.**

8.6 Full Commitment

To make it clear that lack of commitment is the only friction in the environment, we show in this subsection that if we assume full commitment, the model degenerates into an Arrow-Debreu economy, in which case the first welfare theorem applies. In the case of full commitment, buyers can acquire the nighttime good from sellers on credit because they can fully commit to repay their debts in the following day subperiod. This means that agents will no longer need a medium of exchange to trade in the night market because repayment promises can be perfectly enforced. An immediate implication of this result is that the bankers will no longer be able to sell notes at a price higher than β. In other words, the cost of funds for bankers will be given by β^{-1}, which will render their decision problem trivial. Because they can only reinvest the proceeds in the storage technology or into loans in the credit market yielding β^{-1}, they get zero payoff at each date.

We can formulate the buyer's problem in the following way. Let $w_t^b(d, l)$ denote the value function for a buyer who enters the day subperiod with debt $d \in \mathbb{R}_+$ and holding $l \in \mathbb{R}_+$ real loans made at the previous date, and let $v_t^b(k, l)$ denote the value function for a buyer who holds a portfolio of k units of capital and l real loans at the beginning of the night subperiod. The Bellman equation for a buyer in the day subperiod is given by

$$w_t^b(d,l) = \max_{(x,l',k') \in \mathbb{R} \times \mathbb{R}_+^2} \left[x + v_t^b(k',l') \right],$$

subject to the budget constraint

$$x + \rho_t k' + d + l' = \beta^{-1} l.$$

Here d denotes the amount of the daytime good that the buyer needs to produce in order to repay his outstanding debt from the purchase of the nighttime good in the previous period, and l' denotes the amount of real loans made in the current day subperiod. Note that $w_t^b(d,l) = -d + \beta^{-1} l + w_t^b(0,0)$, with the intercept $w_t^b(0,0)$ given by

$$w_t^b(0,0) = \max_{(k',l') \in \mathbb{R}_+^2} \left[-\rho_t k' - l' + v_t^b(k',l') \right].$$

The Bellman equation for a buyer with a portfolio of k' units of capital and l' real loans in the night subperiod is given by

$$v_t^b(k',l') = \max_{q \in \mathbb{R}_+} \left[u(q) + \beta w_{t+1}^b(p_{t+1} q, l') \right],$$

Because there is no benefit of accumulating capital, the buyer optimally chooses $k' = 0$. The optimal choice of q satisfies the following first-order condition:

$$u'(q) = \beta p_{t+1}. \tag{51}$$

Because $\left(\partial w_t^b / \partial l\right)(d,l) = \beta^{-1}$, the buyer is willing to lend any amount in the daytime credit market.

The seller's decision problem is as follows. Let $w_t^s(c,l)$ denote the value function for a seller who enters the day subperiod with credit $c \in \mathbb{R}_+$ and holding $l \in \mathbb{R}_+$ real loans made at the previous date, and let $v_t^s(k,l)$ denote the value function for a seller who holds a portfolio of k units of capital and l real loans at the beginning of the night subperiod. The Bellman equation for a seller in the day subperiod is given by

$$w_t^s(c,l) = \max_{(x,l',k') \in \mathbb{R} \times \mathbb{R}_+^2} \left[x + v_t^s(k',l') \right],$$

subject to the budget constraint

$$x + \rho_t k' + l' = \beta^{-1} l + c.$$

Here c denotes the units of the daytime good to which the seller is entitled in the current day subperiod due to his production in the previous night market, and l' denotes the amount of real loans made in the current day subperiod. Note that $w_t^s(c,l) = c + \beta^{-1}l + w_t^s(0,0)$, with the intercept $w_t^s(0,0)$ given by

$$w_t^s(0,0) = \max_{(k',l')\in\mathbb{R}_+^2} \left[-\rho_t k' - l' + v_t^s(k',l')\right].$$

The Bellman equation for a seller with a portfolio of k' units of capital and l' real loans in the night subperiod is given by

$$v_t^s(k',l') = \max_{n\in\mathbb{R}_+} \left[-c(n) + \beta w_{t+1}^s\left(p_{t+1}F(k',n), l'\right)\right].$$

Using the fact that $w_t^s(c,l)$ is an affine function, we can rewrite the right-hand side of the previous expression as

$$\max_{n\in\mathbb{R}_+} \left[-c(n) + \beta p_{t+1}F(k',n)\right] + l' + \beta w_{t+1}^s(0,0).$$

This means that the optimal choices of nighttime effort and capital accumulation are also given by (22) and (23). Because $(\partial w_t^s/\partial l)(d,l) = \beta^{-1}$, the seller is willing to lend any amount in the daytime credit market.

Using (22) and (51), we obtain the following equilibrium condition:

$$u'\left[F\left(k\left(\gamma_{t-1}^m\right), n_t\right)\right] = \frac{c'(n_t)}{F_n\left(k\left(\gamma_{t-1}^m\right), n_t\right)}. \tag{52}$$

The choice of the date-t marginal entrepreneur is still given by (14). Thus, combining (14) with (22) and (23), we obtain another equilibrium condition:

$$\beta u'\left[F\left(k\left(\gamma_t^m\right), n_{t+1}\right)\right] F_k\left(k\left(\gamma_t^m\right), n_{t+1}\right)\hat{k}\gamma_t^m = e. \tag{53}$$

Then, an equilibrium can be defined as any sequence $\{\gamma_t^m, n_t\}_{t=0}^{\infty}$ satisfying (52) and (53), given the initial capital stock. Note that the entrepreneurs who find it optimal to borrow in the daytime credit market are able to consume all surplus from their projects. Because the producers of the daytime good (buyers and sellers) are indifferent, they are willing to supply exactly the amount of funds demanded by the entrepreneurs whose projects have a positive surplus, which guarantees that the daytime credit market clears at each date.

Note that (52) and (53) are the same marginal conditions as those that we have obtained for the planner's problem. This means that if an equilibrium exists, it is

Pareto optimal.

8.7 Non-Stationary Equilibria

In this subsection, we consider the existence of non-stationary equilibria in which the solvency constraints hold with equality at each date. Using the same terminology as Alvarez and Jermann (2000), we characterize equilibria in which the solvency constraints are not too tight. Consider first the case of unregulated bank lending. Let $\hat{w}_t$ denote the banker's discounted lifetime utility at the beginning of date t. Then, the equations defining the equilibrium dynamics of $\hat{w}_t$ and ϕ_t are given by

$$\hat{w}_t = a\left(\phi_{t-1}\right)\left(\beta^{-1}\phi_{t-1} - 1\right) + \beta\hat{w}_{t+1} \tag{54}$$

and

$$\phi_t a\left(\phi_t\right) = \beta\hat{w}_{t+1}. \tag{55}$$

Combining these two conditions, we can define an equilibrium as a sequence of prices $\{\phi_t\}_{t=0}^{\infty}$ satisfying

$$\phi_t a\left(\phi_t\right) = a\left(\phi_{t-1}\right), \tag{56}$$

given an initial condition $\phi_0 > 0$. The initial price of notes must be such that it guarantees market clearing at date $t = 0$, given the predetermined capital stock available for the production of the nighttime good at date $t = 0$.

Note that there exists at least one stationary solution: $\phi_{t-1} = \phi_t = 1$. Suppose now that $u\left(q\right) = \left(1-\sigma\right)^{-1}\left(q^{1-\sigma} - 1\right)$, with $0 < \sigma < 1$, $c\left(n\right) = n$, and $F\left(k,n\right) = k^{\alpha}n^{1-\alpha}$, with $0 < \alpha < 1$. Suppose also that $g\left(\gamma\right) = 1$ for any $0 \leq \gamma \leq 1$ and $g\left(\gamma\right) = 0$ otherwise. Using the Implicit Function Theorem, we find that

$$\frac{d\phi_t}{d\phi_{t-1}} = \frac{a'\left(\phi_{t-1}\right)}{\phi_t a'\left(\phi_t\right) + a\left(\phi_t\right)} > 0.$$

In particular, we have

$$\left.\frac{d\phi_t}{d\phi_{t-1}}\right|_{\phi_{t-1}=\phi_t=1} = \frac{a'\left(1\right)}{a'\left(1\right) + a\left(1\right)} > 1.$$

If $\phi_{t-1} = \phi_t = 1$ is the unique non-autarkic stationary solution, then we have that, for any initial value $\phi_0 > 1$, the equilibrium price trajectory is strictly increasing and unbounded, so the equilibrium allocation approaches the autarkic allocation as $t \to \infty$. Along this equilibrium path, the debt limits, given by $\bar{B}_t = a\left(\phi_t\right)$, shrink over time and converge to zero, similar to the analysis in Gu, et. al. (2011). This means that liquidity becomes scarcer and more expensive over time, and consumers are able to trade smaller amounts of goods in the decentralized night market.

Consider now the case of regulated lending. Suppose that $r_t(\gamma)$ is given by (41). In this case, the equations defining the equilibrium dynamics of $\hat{w}_t$ and ϕ_t are given by (55) and

$$\hat{w}_t = a(\phi_{t-1})\left(\beta^{-1}\phi_{t-1} - 1\right) + e\beta^{-1}\frac{\left[1-\gamma^m(\phi_t)\right]^2}{2\gamma^m(\phi_t)} + \beta\hat{w}_{t+1}. \tag{57}$$

Combining (55) with (57), we can define an equilibrium as a sequence of prices $\{\phi_t\}_{t=0}^{\infty}$ satisfying

$$a(\phi_{t-1}) = e\beta^{-1}\frac{\left[1-\gamma^m(\phi_t)\right]^2}{2\gamma^m(\phi_t)} + \phi_t a(\phi_t), \tag{58}$$

given an initial condition $\phi_0 > 0$. Suppose that $u(q) = (1-\sigma)^{-1}(q^{1-\sigma} - 1)$, with $0 < \sigma < 1$, $c(n) = n$, and $F(k,n) = k^{\alpha}n^{1-\alpha}$, with $0 < \alpha < 1$. Suppose also that $g(\gamma) = 1$ for any $0 \leq \gamma \leq 1$ and $g(\gamma) = 0$ otherwise. Notice that, for β sufficiently close to one, $\phi_{t-1} = \phi_t = \beta$ is a stationary solution. Again, if this is the unique non-autarkic stationary solution, for any initial condition $\phi_0 > \beta$, the debt limits shrink over time and the price of liquid assets grows unbounded as the economy approaches autarky.