Competitive Differential Pricing

Yongmin Chen* Jianpei Li† Marius Schwartz‡

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Abstract. This paper analyzes differential versus uniform pricing across oligopoly markets that differ in costs of service. We provide necessary and sufficient conditions on general properties of demand for differential pricing to raise or lower profit, consumer surplus, and total welfare, explain why differential pricing is generally beneficial but there are exceptions, and compare the findings to oligopoly third-degree price discrimination. Our conditions nest those for monopoly differential pricing, and are derived by evaluating when each of the welfare measures is convex in marginal cost. Our results help elucidate the welfare effects of prevalent constraints on cost-based differential pricing.

Keywords: differential pricing, price discrimination, demand curvature, cross-price elasticity, pass-through, oligopoly

*University of Colorado Boulder; yongmin.chen@colorado.edu

†University of International Business and Economics (UIBE); lijianpei2007@gmail.com

‡Georgetown University; mariusschwartz@mac.com

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1. INTRODUCTION

For a given product, distinct consumer groups or 'markets' frequently differ in the costs of serving them or in their price elasticities of demand. A large literature studies the welfare effects, relative to uniform pricing, of differential pricing across markets based on demand elasticities—third-degree price discrimination—under monopoly (see Aguirre, Cowan and Vickers, 2010 and references therein). Significant work has also addressed such price discrimination in oligopoly (e.g., Holmes, 1989; and Stole, 2007). There has been little analysis, however, of uniform pricing (UP) versus differential pricing (DP) when markets vary in *costs* of service. Yet in many situations competing firms are constrained from adopting DP that is motivated (at least partly) by cost differences.[1] Uncovering the welfare effects of cost-based DP under imperfect competition can advance our understanding of an open issue in economics and help inform public policy or consumer attitudes towards constraints on DP. This paper takes a step in that direction.

To isolate the role of cost differences, we assume that markets exhibit equal demand elasticities but different costs of service. (Hereafter, unless stated otherwise, DP refers to 'cost-based'.) We compare two pricing regimes, UP or DP, in an industry with price competition between symmetric firms selling differentiated products with general demands. This environment serves two purposes. It reveals how the welfare properties of DP under monopoly (Chen and Schwartz, 2015) may change when moving to oligopoly absent firm asymmetries, and it permits a natural comparison to symmetric oligopoly *price discrimination* analyzed by Holmes (1989).

[1] Examples include insurance markets, universal-service mandates on utilities, antidumping rules in international trade, and consumer resistance to add-on pricing such as airline bag fees. The constraints on cost-based DP can stem from various sources: government policy, contractual restrictions, consumer perceptions of the likely effects, or transaction costs. For further discussion and examples, see Chen and Schwartz (2015), Edelman and Wright (2015), and Nassauer (2017).

We provide necessary and sufficient conditions for DP to raise or lower consumer surplus (aggregated across markets), profit, or their sum, 'total welfare' (Propositions 1-3), based on general properties of the demand system: curvature and own- and cross-price elasticities, and how these vary as firms change price equally. We illustrate those conditions for various demand systems, and link them to endogenous effects governing how DP affects consumer surplus or profit, such as the change in average price and the extent of output reallocation between markets.

The pass-through rate from firms' common marginal cost to their symmetric equilibrium price plays a major role in our analysis.[2] Under UP, each firm sets price based on the average of its marginal costs across markets, whereas under DP it sets prices based on each market's specific marginal cost. Thus, moving to DP effectively lowers firms' marginal cost in one market and raises cost in the other market, with the equilibrium price adjustments determined by the integral of pass-through over the relevant ranges of demand. Incorporating the level and change of the pass-through rate, we evaluate when consumer surplus, profit, and total welfare are convex or concave in marginal cost, from which our main results are obtained.

When pass-through exceeds one, DP induces excessive output reallocation between markets, as the price difference 'overshoots' the cost difference. Under monopoly, DP nevertheless raises total welfare for any constant pass-through, because pass-through above one requires very convex demand, in which case moving from UP to DP yields a large beneficial output expansion (Chen and Schwartz, 2015). However, pass-through can be greater in oligopoly than under monopoly independent of demand curvature, because the own cost effect on a firm's price is amplified by the feedback from the rival's price. With this added strategic channel, we show that DP in oligopoly may reduce total welfare via output misallocation, even when the pass-through is constant.

Another contrast to monopoly is that competitive DP may reduce profits. We

[2] For a general analysis of pass-through in various applications see Weyl and Fabinger (2013).

show this can occur in two ways. First, DP can induce excessive output reallocation to the lower-cost market. When pass-through rate exceeds one and cross-elasticity of demand is high, the profit margin under DP ends up smaller in the (low-cost) market that gains output *and* total output does not expand enough to outweigh this misallocation effect. Second, DP can reduce average price, and again without inducing a large (if any) increase in output. Interestingly, the same mechanism operates in Holmes (1989), who compares uniform pricing to price discrimination. But the demand properties generating this outcome obviously differ, since his markets diverge only in demand elasticities while here they diverge only in costs. In our setting, DP can reduce profit if the cross-price elasticity of demand relative to the own-price elasticity is greater at lower prices than at higher prices. Price can then fall by more in the low-cost market than it rises in the other market—and not due to greater demand curvature at lower prices, since larger price reductions motivated by demand convexity would have increased output, as occurs with DP under monopoly.

Consumer surplus can fall or rise with DP, as we shall explain, for the same reasons as under monopoly. However, unlike for monopoly, DP can reduce total welfare even when consumer surplus rises, because profit can fall.

While the welfare effects of competitive DP threfore can be subtle, we are able to derive sufficient conditions for DP to raise consumer surplus, profit, or total welfare that have intuitive interpretations and are satisfied for broad classes of demand functions. These conditions reduce to the respective conditions for monopoly DP—and hence neatly nest the latter—when the cross-price effects vanish.

Overall, our analysis suggests that, while there are exceptions, cost-based DP in symmetric oligopoly is broadly beneficial. Profit and total welfare tend to rise due to cost savings from reallocating output to the lower-cost market by varying prices, and consumers gain from the price dispersion. Furthermore, DP is more beneficial than oligopoly price discrimination for consumers and total welfare. In both cases, consumers gain from the price dispersion by adjusting quantities. But price discrimination has a bias to raise average price which is harmful to consumers, whereas cost-based DP does not for a broad class of demand functions. Regarding total welfare, price discrimination misallocates output across markets while UP does not, whereas when markets differ in costs of service, UP misallocates output and DP can improve the allocation.

In an extension of the model, we allow cost asymmetries between firms, in addition to cost differences across markets. Do new forces emerge, beyond demand properties, that alter the welfare effects of DP? With homogeneous products, DP now can reduce profits or consumer surplus even for 'simple' demand functions such as linear (Proposition 4). Since the underlying forces are intuitive, one might expect the results to hold also if products are minimally differentiated. To check this, we consider symmetrically differentiated products with linear demands; for any cost asymmetry and any degree of differentiation, DP benefits firms and consumers (Proposition 5). The 'discontinuity' arises for homogeneous products because the equilibrium price of any firm with positive sales in homogeneous Bertrand competition is determined solely by the rival's marginal cost; whereas with any product differentiation, the firm's own marginal cost also affects equilibrium prices. This contrast further illustrates that the welfare effects of DP, while broadly favorable, can be quite subtle.

To our knowledge, the only other analysis addressing cost-based DP in oligopoly is by Adachi and Fabinger (2017). Our contributions are complementary. Adachi and Fabinger add cost differences between markets to Holmes' (1989) symmetric oligopoly setting. They provide sufficient conditions for DP to lower or raise total welfare, using similar techniques as Aguirre, Cowan and Vickers (2010; 'ACV'), who study monopoly price discrimination with no cost differences. Their conditions resemble ACV's, in comparing weighted markups between markets at the equilibrium price(s), but are more complex due to the role of cross-price effects in the weights. Those weights include terms such as pass-through rates that ultimately are determined by the demand system. Our analysis of symmetric oligopoly has a narrower scope: we assume equally-elastic demands to focus sharply on the role of cost differences. Using simple arguments on when each welfare measure is a convex function of marginal cost, we provide transparent necessary and sufficient conditions on the demand system for DP to raise or lower consumer surplus, profit, or total welfare, explain the underlying forces, and illustrate with examples. In addition, we explore the effect of cost asymmetries between firms.

Section 2 of the paper presents the model and some preliminary results. Section 3 analyses how DP affects consumers, profits, and total welfare. Section 4 considers a variation of the model where firms have asymmetric costs. Section 5 presents conclusions and potential extensions. All proofs are in the appendix.

2. THE MODEL

Two firms, $i = 1, 2$, each produce one product. When firm i's price is p_i, the demand function for firm i, $i = 1, 2 \neq j$, is $D_i(p_1, p_2)$. There are two distinct groups of consumers or markets, L and H. Firm i's constant marginal cost is c_L to serve group L and c_H to serve group H, with $0 \leq c_L < c_H$. Group L's demand for firm i's product is $\lambda D_i(p_1, p_2)$ and group H's demand is $(1-\lambda) D_i(p_1, p_2)$ for $\lambda \in (0, 1)$. Thus, demands of the two groups are proportional to each other, but the (constant) marginal costs of serving the groups differ.[3]

Firms compete by simultaneously choosing prices, possibly in one of two pricing regimes. Under uniform pricing (UP) each firm can only set a single price for all

[3] We could more generally assume that the demands for firm i by the two consumer groups are $D_{iL}(p_1, p_2)$ and $D_{iH}(p_1, p_2)$, respectively. When the demands are proportional, as we assume here, the two consumer groups for each firm will have the same price elastities, which allows us to focus on differential pricing motivated purely by cost differences between consumer groups.

consumers, whereas under differential pricing (DP) each firm can charge two different prices for the two distinct consumer groups.

Let

$$\bar{c} \equiv \lambda c_L + (1-\lambda) c_H. \tag{1}$$

and, as in Holmes (1989), assume that the firms produce symmetrically differentiated products with $D_i(p_1, p_2)$ being a continuous function satisfying

$$D_i(x, y) = D_j(y, x) \quad \text{for firm } i \neq j = 1, 2. \tag{2}$$

This setting allows us to compare, relative to uniform pricing, the effects of competitive differential pricing by symmetric firms that is motivated by cost differences instead of demand differences as in Holmes (1989).[4]

When $p_i = p_j \equiv p$, we further define the industry demand as

$$D(p) \equiv 2D_i(p, p) \quad \text{for } i = 1, 2. \tag{3}$$

Given firms' symmetry in demands and costs, we will analyze the symmetric equilibria in which both firms charge equal prices under UP and under DP.

Denote the equilibrium price by p^u under UP and by p_k for $k = L, H$ under DP. We assume standard demand conditions such that DP raises price in the high-cost market and reduces price in the low-cost market: $p_L < p^u < p_H$. Let $q^u = D(p^u)$, $q_L = D(p_L)$, and $q_H = D(p_H)$. Then $q_H < q^u < q_L$, and $\Delta q_L \equiv q_L - q^u > 0$ while $\Delta q_H \equiv q_H - q^u < 0$. Define

$$p^d \equiv \lambda p_L + (1-\lambda) p_H \tag{4}$$

as the *average* price under DP weighted by the relative sizes of the two groups, which equal their relative consumption quantities under UP. If $p^d = p^u$, and consumers in

[4]We shall also briefly state the results for the case where the two firms produce a homogeneous product so that $D_i(p_i, p_j)$ is not continuous at $p_i = p_j$. The analysis in this case is straightforward.

the two groups were to maintain the same consumption quantities as under UP, their total expenditure and welfare would be unchanged. We will first show, therefore, that $p^d \leq p^u$ is a sufficient condition for DP to raise aggregate consumer surplus because consumers can advantageously adjust quantities to exploit price dispersion—purchasing more where price is lower and less where price is higher (Waugh, 1944).

Define $g_2(y)$ as the consumer surplus from good 2 at price y when good 1 is not available:

$$g_2(y) \equiv \int_y^\infty D_2(\infty.x)\,dx;$$

and $g_1(y)$ as the incremental consumer surplus from good 1 at price y when the other good is also available at the symmetric price y:

$$g_1(y) \equiv \int_y^\infty D_1(x,y)\,dx.$$

The (aggregate) consumer surplus under uniform pricing is then[5]

$$S^u = \int_{p^u}^\infty D_1(x,p^u)\,dx + \int_{p^u}^\infty D_2(\infty,x)\,dx \quad [= g_1(p^u) + g_2(p^u)\ ] \tag{5}$$

and under differential pricing it is

$$\begin{aligned} S^d &= \lambda\left[\int_{p_L}^\infty D_1(x,p_L)\,dx + \int_{p_L}^\infty D_2(\infty,x)\,dx\right] \\ &\quad + (1-\lambda)\left[\int_{p_H}^\infty D_1(x,p_H)\,dx + \int_{p_H}^\infty D_2(\infty,x)\,dx\right]. \end{aligned} \tag{6}$$

Since $g_2(y)$ is decreasing and convex, we have:

$$\lambda g_2(p_L) + (1-\lambda)\,g_2(p_H) > g_2(\lambda p_L + (1-\lambda)\,p_H) = g_2\left(p^d\right).$$

Since the goods are imperfect substitutes ($0 < \frac{\partial D_1}{\partial p_2} < -\frac{\partial D_1}{\partial p_1}$), $g_1(y)$ is decreasing:

$$\begin{aligned} g_1'(y) &= -D_1(y,y) + \int_y^\infty \frac{\partial D_1(x,y)}{\partial y}dx < -D_1(y,y) - \int_y^\infty \frac{\partial D_1(x,y)}{\partial x}dx \\ &= -D_1(y,y) - D_1(\infty,y) + D_1(y,y) = 0. \end{aligned}$$

[5] We assume that consumer surplus is uniquely defined, independent of the integration path. This requires the demand functions to exhibit equal cross-partial derivatives, as occurs when demand is derived from a quasi-linear utility function so there are no income effects.

Moreover,

$$g_1''(y) = -\frac{\partial D_1(y,y)}{\partial p_1} - \frac{\partial D_1(y,y)}{\partial p_2} - \frac{\partial D_1(y,y)}{\partial p_1} + \int_y^\infty \frac{\partial^2 D_1(x,y)}{\partial y^2} dx > 0,$$

provided that

$$-\frac{\partial D_1(y,y)}{\partial p_1} \geq -\int_y^\infty \frac{\partial^2 D_1(x,y)}{\partial y^2} dx, \tag{7}$$

which we shall maintain as a regularity condition.[6] Thus

$$\lambda g_1(p_L) + (1-\lambda) g_1(p_H) > g_1(\lambda p_L + (1-\lambda) p_H) = g_1\left(p^d\right).$$

Therefore,

$$\begin{aligned}
\Delta S &\equiv S^d - S^u \\
&= \lambda [g_1(p_L) + g_2(p_L)] + (1-\lambda)[g_1(p_H) + g_2(pH)] - [g_1(p^u) + g_2(p^u)] \\
&= [\lambda g_1(p_L) + (1-\lambda) g_1(p_H) - g_1(p^u)] + [\lambda g_2(p_L) + (1-\lambda) g_2(pH) - g_2(p^u)] \\
&> 0 \qquad \text{if} \qquad p^d \leq p^u.
\end{aligned}$$

Remark 1 *DP raises consumer surplus if it does not increase average price.*

This generalizes the result from monopoly and also from a homogeneous-product oligopoly with market demand $D(p)$. There, price dispersion that does not raise average price will benefit consumers in aggregate, due to beneficial adjustments of consumption quantities. With competition and differentiated products, cross-price effects complicate the analysis. But under the regularity condition that ensures convexity of $g_1(y)$, price dispersion for both goods that does not raise their average price again benefits consumers.

Next, let the industry output under DP be $q^d \equiv \lambda q_L + (1-\lambda) q_H$, and also let $\Delta q = q^d - q^u$. Furthermore, let $m_L = p_L - c_L$ and $m_H = p_H - c_H$ be the price-cost

[6] We have not found a violation for any of the demand functions discussed in this paper.

margins for the two consumer groups under DP. The effect of DP on equilibrium industry profit can be decomposed as follows:

$$\Pi^d - \Pi^u = \left[\lambda\left(p_L - c_L\right)q_L + (1-\lambda)\left(p_H - c_H\right)q_H\right] - \left[p^u - \lambda c_L - (1-\lambda)c_H\right]q^u$$

$$\begin{aligned} = \; & \lambda\left(p_L - c_L\right)\left(q_L - q^u\right) + \lambda\left(p_L - c_L\right)q^u + (1-\lambda)\left(p_H - c_H\right)\left(q_H - q^u\right) \\ & + (1-\lambda)\left(p_H - c_H\right)q^u - \left(p^u - \lambda c_L - (1-\lambda)c_H\right)q^u \end{aligned}$$

$$= \left(p^d - p^u\right)q^u + \lambda\left(m_L - m_H\right)\left(q_L - q^u\right) + \lambda m_H\left(q_L - q^u\right) + (1-\lambda)m_H\left(q_H - q^u\right).$$

That is:

$$\Delta\Pi \equiv \Pi^d - \Pi^u = \underbrace{\left(p^d - p^u\right)q^u}_{\text{Average-P Effect}} + \underbrace{\lambda\left(m_L - m_H\right)\Delta q_L}_{\text{Reallocation Effect}} + \underbrace{m_H \Delta q}_{\text{Output Effect}}. \quad (8)$$

Since $\Delta q_L > 0$, the reallocation effect will be positive if the margin is higher in market L than in H at the differential prices (as was true under uniform pricing) and negative if the reverse holds.

Observe that under classic third-degree price discrimination, i.e., markets face different prices but have the same cost of service, the reallocation effect is necessarily negative: output shifts to the market where price fell and, hence, where the margin is lower. Thus, profitable price discrimination requires an increase in output or in the average price. In contrast, (cost-based) DP can be profitable even when output and average price fall, because the reallocation effect will be positive as long as the price difference between markets is less than the cost difference. This distinction will prove useful in subsection 3.5 and we record it as follows:

Remark 2 *Profitable price discrimination requires an increase in average price or in total output, but profitable cost-based differential pricing does not.*

3. WELFARE ANALYSIS

If the two firms supply a homogeneous product, then under UP

$$p^u = \bar{c} = \lambda c_L + (1 - \lambda) c_H,$$

whereas under DP we have $p_L = c_L$ and $p_H = c_H$, with average price

$$p^d = \lambda c_L + (1 - \lambda) c_H = p^u.$$

Since average price is the same under UP and DP, Remark 1 straightforwardly implies that when the firms produce a homogeneous product, consumer surplus is higher under DP, and so is total welfare since profit is zero in both regimes.

However, the results are not obvious when products are (symmetrically) differentiated. The remainder of this section addresses that case. Subsections 3.1-3.3 analyze the effects of DP, compared to UP, on consumer surplus, profits, and total welfare, respectively, using general properties of the demand system. Subsection 3.4 provides illustrative examples using specific demand functions. Subsection 3.5 compares the welfare effects of cost-based pricing to price discrimination (Holmes, 1989).

3.1 Equilibrium Prices and Consumer Surplus

It is helpful to start with the benchmark where each firm i has the same marginal cost c to serve *all* its customers. In equilibrium, firm i chooses p_i to maximize

$$\pi_i (p_1, p_2) = (p_i - c) D_i (p_1, p_2),$$

taking as given p_j, for $i = 1, 2 \neq j$. A sufficient condition for the existence of a unique equilibrium, which we shall maintain, is

$$-\frac{\partial^2 \pi_i}{\partial p_i^2} > \frac{\partial^2 \pi_i (p_1, p_2)}{\partial p_i \partial p_j} > 0. \tag{9}$$

The second inequality implies that firms' prices are strategic complements.

The equilibrium price of both firms, denoted as $p^* \equiv p^*(c)$, satisfies the first-order condition

$$\frac{\partial \pi_i (p^*, p^*; c)}{\partial p_i} = D_i (p^*, p^*) + (p^* - c) \frac{\partial D_i (p^*, p^*)}{\partial p_i} = 0. \tag{10}$$

Thus, the equilibrium price satisfies the familiar inverse elasticity rule: $\frac{p^*-c}{p^*} = \frac{1}{\eta_{ii}(p^*)}$, where

$$\eta_{ii} = \frac{1}{-\frac{\partial D_i(p,p)}{\partial p_i} \frac{p}{D_i(p,p)}} > 0 \tag{11}$$

is the own-price elasticity (Holmes' firm elasticity $e_i^F(p)$). By the symmetry of demand, without loss of generality we can conduct our analysis for all variables associated with one firm, say firm 1. Define:

$$\eta_{12} \equiv \frac{\partial D_1 (p,p)}{\partial p_2} \frac{p}{D_1} > 0 \qquad \text{and} \qquad \eta_r \equiv \frac{\eta_{12}}{\eta_{11}}, \tag{12}$$

where η_{12} is the cross-price elasticity of demand ($e_i^C(p)$ in Holmes 1989), and η_r is the ratio of cross-price elasticity to own-price elasticity.[7] As is customary, we assume $-\frac{\partial D_i(p,p)}{\partial p_i} > \frac{\partial D_i(p,p)}{\partial p_j}$, so that $\eta_r < 1$.

From (10), we obtain the equilibrium pass-through rate as

$$\begin{aligned} p^{*\prime}(c) &= -\frac{-\frac{\partial D_1(p^*,p^*)}{\partial p_1}}{2\frac{\partial D_1(p^*,p^*)}{\partial p_1} + \frac{\partial D_1(p^*,p^*)}{\partial p_2} + (p^*(c) - c)\frac{d}{dp^*}\frac{\partial D_1(p^*,p^*)}{\partial p_1}} \\ &= \frac{1}{2 - \frac{\eta_{12}}{\eta_{11}} + \alpha} = \frac{1}{2 - \eta_r(p^*) + \alpha(p^*)}, \end{aligned} \tag{13}$$

where

$$\alpha(p) \equiv \frac{(p-c)}{p} \frac{p}{\frac{\partial D_1(p,p)}{\partial p_1}} \frac{d}{dp} \frac{\partial D_1(p,p)}{\partial p_1} \tag{14}$$

[7] For market i and equal prices p by both firms, Holmes defines the elasticity ratio as $e_i^C(p) / e_i^I(p)$, where $e_i^I(p)$ is the industry demand elasticity when firms change price equally, and $e_i^F(p) = e_i^I(p) + e_i^C(p)$ (e_i^F, e_i^I, e_i^C all are > 0). In his notation, our elasticity ratio, instead, is $e_i^C(p) / e_i^F(p)$.

is the adjusted-concavity of $D_1(p,p)$, or the elasticity of the slope of firm 1's demand $\frac{\partial D_1(p,p)}{\partial p_1}$, adjusted by $\frac{(p-c)}{p}$. Thus, $\alpha(p) > 0$ or < 0 as the slope of each firm's demand, $\frac{\partial D_1(p,p)}{\partial p_1}$, decreases (becomes a larger negative number) or increases in p; we shall refer to these two situations respectively as each firm's demand being concave or convex (in symmetric price p). Notice that $p^{*'}(c) > 0$ under assumption (9).[8]

Equation (13) suggests that a marginal increase in c affects equilibrium price under competition through two channels. One is the adjusted-concavity of each firm's demand, α: a higher c leads to a smaller price increase, or $p^{*'}(c)$ is lower, when demand is concave ($\alpha < 0$) than when it is convex ($\alpha > 0$), as under monopoly (Bulow and Pfleiderer, 1983). The second channel is the elasticity ratio of firm demand: *ceteris paribus,* a higher η_r raises $p^{*'}(c)$. A common increase in marginal cost c will raise also the rival's price, which magnifies the firm's own price increase given that prices are strategic complements. This cross-effect increases with η_r and provides a force in favor of a higher pass through under competition than under monopoly.

Now consider our actual case where each firm has marginal costs c_L and c_H to serve the two consumer groups. Under uniform pricing, given firm 2's price p^u, firm 1 chooses a single p_1 to maximize its profit

$$(p_1 - c_L)\,\lambda D_1(p_1, p^u) + (p_1 - c_H)(1-\lambda)\,D_1(p_1, p^u) = (p_1 - \bar{c})\,D_1(p_1, p^u),$$

where, under the assumption that each firm draws customers symmetrically from L and H (i.e., in proportions λ and $1-\lambda$), the firm's average marginal cost is $\bar{c}$. The equilibrium uniform price is thus simply $p^u \equiv p^*(\bar{c})$, obtained by setting $c = \bar{c}$ in

[8] For a monopolist with demand $q = D(p)$, its monopoly price $p^m \equiv p^m(c)$ has pass-through $p^{m'}(c) = \frac{1}{2-\sigma}$, where $\sigma \equiv \frac{p^m - c}{p^m}\frac{p^m}{-D'(p^m)}D''(p^m)$ is the curvature of the inverse demand function $P(q) = D^{-1}(q)$ (e.g., Chen and Schwartz, 2015). Thus $p^{*'}(c)$ differs from $p^{m'}(c)$ due to the additional term $-\eta_r$ in the denominator and also to the difference in the curvature term for the demand function; whereas with monopoly we would have $\eta_r = 0$ and $\alpha = -\sigma$.

(10). Each firm's equilibrium profit under uniform pricing is

$$\pi^u = (p^u - \bar{c})\, D_1 (p^u, p^u)\,. \tag{15}$$

Under differential pricing, the profit function of firm 1 from group L is

$$\pi_{1L} = (p_{1L} - c_L)\, \lambda D_1 (p_{1L}, p_{2L})\,,$$

and the equilibrium price for group L is $p^* (c_L) \equiv p_L$, obtained by setting $c = c_L$ in (10). Similarly, the equilibrium price for group H is $p^* (c_H) \equiv p_H$. Therefore, each firm's equilibrium profit under differential pricing is

$$\pi^d = \lambda \left[p_L - c_L\right] D_1 (p_L, p_L) + (1 - \lambda) \left[p_H - c_H\right] D_1 (p_H, p_H)\,. \tag{16}$$

We are now ready to compare the uniform price, p^u, with the average price paid under DP weighted by the relative sizes of the two groups, defined in (4): $p^d \equiv \lambda p^* (c_L) + (1 - \lambda)\, p^*(c_H)$. As shown in Section 2, the difference between p^u and p^d affects the change in both consumer surplus and in profits from a move to DP. Since

$$p^u = p^* (\bar{c}) = p^* (\lambda c_L + (1 - \lambda)\, c_H)\,,$$

we have $p^u \gtreqless p^d$ when

$$p^u = p^* (\lambda c_L + (1 - \lambda)\, c_H) \gtreqless \lambda p^* (c_L) + (1 - \lambda)\, p^* (c_H) = p^d.$$

That is, DP lowers average price if $p^* (c)$ is concave, and DP raises average price if $p^* (c)$ is convex.

From (13),

$$p^{*''}(c) = [\eta_r'(p^*) - \alpha'(p^*)] \left[p^{*'} (c)\right]^3. \tag{17}$$

Thus, $p^{*''}(c)$ has the same sign as $\eta_r'(p^*) - \alpha'(p^*)$, implying:

Remark 3 *Cost-based competitive differential pricing does not raise average price if and only if* $\eta_r'(p^*) \leq \alpha'(p^*)$.

As shown in (13), the pass-through rate becomes smaller as η_r falls or α rises. The condition $\eta_r'(p^*) \le \alpha'(p^*)$ says that the cross-elasticity relative to own-elasticity, η_r, increases more slowly with firms' common price p^* than does the curvature term α. Then pass-through will be smaller in market H, where a move to DP raises the virtual marginal cost (from $\bar{c}$ to c_H) and price, than in market L (where the virtual marginal cost falls), explaining why average price does not rise. Both η_r and α are constant for many familiar demand functions—including linear, constant elasticity, log, and logit demands (see subsection 3.4 below). For these demands, cost-based DP does not raise average price.

From Remark 1 and Remark 3, it follows that if $\eta_r'(p^*) \le \alpha'(p^*)$ or, equivalently, if $p^{*''}(c) \le 0$, DP will raise aggregate consumer surplus.

But even if $p^{*''}(c) > 0$, so that $p^d > p^u$, consumer surplus can still be higher under DP than under UP since consumers gain from price dispersion. We can thus derive a tighter necessary and sufficient condition for DP to raise consumer surplus. Define consumer surplus as a function of c as:

$$\begin{aligned} s(c) &\equiv S(p^*, p^*) = \int_{p^*(c)}^{\infty} D_1(x, p^*)\, dx + \int_{p^*(c)}^{\infty} D_2(\infty, y)\, dy \\ &= \int_{p^*(c)}^{\infty} D_1(x, p^*)\, dx + \int_{p^*(c)}^{\infty} D_1(x, \infty)\, dx, \end{aligned}$$

where the second equality is due to the symmetry of D_i. Also, when the two firms charge a symmetric price p, define the price elasticity and the adjusted price elasticity of market (or industry) demand respectively by

$$\eta(p) = -\frac{D'(p)}{D(p)}p; \qquad \hat{\eta}(p) = -\frac{[D'(p) + \delta'(p)]}{D(p) + \delta(p)}p, \tag{18}$$

where

$$\delta(p) = \int_p^{\infty} \left[\frac{\partial D_1(p, x)}{\partial x} - \frac{\partial D_1(x, p)}{\partial p} \right] dx.$$

The adjustment term $\delta(p)$, which is zero for linear demand, reflects the cross-price effect between the two products and the demand curvature.

Proposition 1 *Consumer surplus is higher under differential pricing than under uniform pricing if (19) holds, and is lower if the inequality in (19) is reversed.*

$$\frac{\alpha'(p) - \eta_r'(p)}{2 - \eta_r(p) + \alpha(p)} p + \hat{\eta}(p) > 0. \tag{19}$$

Condition (19) can be understood as follows. Differential pricing affects consumer surplus both through an average price effect and a price dispersion effect. The term $\frac{\alpha' - \eta_r'}{2 - \eta_r + \alpha} p$ has the same sign as $p^{*''}(c)$, and indicates the direction of change in average price. When $\alpha_r' \geq \eta'$, DP does not raise the average price, in which case consumer surplus increases due to the change in quantities purchased (Remark 1). The term $\hat{\eta}$ is the adjusted price elasticity of market demand. When $\hat{\eta}$ is higher, consumers are more capable of making quantity adjustments and thus benefit more from the price dispersion. Therefore, even when DP raises average price ($\alpha' < \eta_r'$), if $\hat{\eta}$ is sufficiently high, consumer surplus will still be higher under DP than under UP; whereas if DP does not raise average price, it increases consumer surplus for any values of $\hat{\eta} > 0$. Thus, DP raises consumer surplus if $[\alpha(p) - \eta_r(p)]$ does not decrease too fast.

Under monopoly DP (Chen and Schwartz, 2015), the corresponding condition for DP to raise S is

$$\frac{\sigma'(q)}{2 - \sigma(q)} + \frac{1}{q} > 0 \Longleftrightarrow \frac{-\sigma'(q) q'(p)}{2 - \sigma(q)} p + \eta > 0.$$

We can consider $\sigma(q)$ as corresponding to $-\alpha(p)$ and $-\sigma'(p) q'(p)$ to $\alpha'(p)$. Thus, the condition for DP to increase consumer surplus under competition, (19), reduces to that under monopoly for which $\eta_r' = \eta_r = 0$ and $\hat{\eta} = \eta$. It is not clear whether DP is more favorable to consumers (relative to UP) under competition than under monopoly; but in both cases DP has no tendency to raise average price for a broad class of demand functions and, hence, tends to raise consumer surplus.

3.2 Profit

Equilibrium profit for firm 1 under marginal cost c is

$$\pi^*(c) = [p^*(c) - c] \, D_1(p^*(c), p^*(c)).$$

Thus, using the envelope theorem:

$$\pi^{*'}(c) = -\underbrace{D_1(p^*(c), p^*(c))}_{\text{direct effect}} + \underbrace{[p^*(c) - c] \frac{\partial D_1(p^*, p^*)}{\partial p_2} p^{*'}(c)}_{\text{rival's effect}}.$$

Since

$$\begin{aligned} [p^*(c) - c] \frac{\partial D_1(p^*, p^*)}{\partial p_2} p^{*'}(c) &= D_1(p^*, p^*) \frac{p^*(c) - c}{p^*(c)} \frac{p^*(c)}{D_1(p^*, p^*)} \frac{\partial D_1(p^*, p^*)}{\partial p_2} p^{*'}(c) \\ &= D_1(p^*, p^*) \frac{\eta_{12}}{\eta_{11}} p^{*'}(c) = D_1(p^*, p^*) \eta_r p^{*'}(c), \end{aligned}$$

we can rewrite $\pi^{*'}(c)$ as

$$\pi^{*'}(c) = -D_1(p^*(c), p^*(c)) \left[1 - p^{*'}(c)\eta_r\right]. \tag{20}$$

For a monopolist, an increase in c lowers profit because its profit margin for each unit of sales is reduced by the increase in c. Under competition, this margin reduction is alleviated by the increase in the rival's price associated with the higher c, as reflected in the additional term $-p^{*'}(c)\eta_r$ in (20). Thus, $\pi^{*'}(c) < 0$ if and only if $p^{*'}(c)\eta_r < 1$. Condition (9) for a unique equilibrium does not ensure $p^{*'}(c)\eta_r < 1$, and we will consider also $p^{*'}(c)\eta_r \geq 1$.

The result below is derived by analyzing when $\pi^*(c)$ is convex or concave. It provides the condition for DP to raise or lower profit, in terms of the demand system parameters $\eta_r = \eta_r(p)$, $\alpha = \alpha(p)$, and $\eta = \eta(p)$:

Proposition 2 *Profit is higher under differential pricing than under uniform pricing if (21) holds, and is lower if the inequality in (21) is reversed.*

$$\frac{\eta_r(\eta_r' - \alpha')}{[2 - \eta_r + \alpha]^2} p + \eta_r' p + \eta \left[1 - \frac{\eta_r}{2 - \eta_r + \alpha}\right] > 0. \tag{21}$$

Notice first that for a monopolist, in (21) $\eta_r = \eta_r' = 0$, yielding simply $\eta > 0$, which always holds. It represents a monopolist's gain from adjusting outputs across markets in response to mean-preserving cost dispersion and is proportional to the elasticity of demand η, similar to the flexibility gain for consumer surplus.[9] Thus, as with the condition for DP to raise consumer surplus, the condition for DP to raise profit under competition embeds and generalizes the condition under monopoly. Next, consider the three terms in (21) for our oligopoly case.

The first term in (21), $\frac{\eta_r(\eta_r'-\alpha')}{[2-\eta_r+\alpha]^2}p$, can be written as $p^{*\prime}(c)\eta_r\left[\frac{(\eta_r'-\alpha')}{[2-\eta_r+\alpha]}p\right]$. The bracketed term reflects the change in average price when moving to DP (hence takes the opposite sign to the first term in condition (19) for consumer surplus). It affects the firm's profit via the *rival's* price response to the common cost shocks, and in proportion to the cross-elasticity term, η_r. An increase in average price due to DP boosts industry profit because competition under uniform pricing forces price too low from the standpoint of the industry. Average price tends to rise if $\alpha' < 0$, i.e., if firm demand becomes less concave at higher prices. The pass-through rate then is higher in market H where price rises than in market L where price falls, so the net effect of the rival's price adjustments is to raise average price.

The middle term, $\eta_r'p$, reflects an output externality. Each firm sets its price based on marginal cost and firm elasticity of demand, η_{11}, incorporating the effect of its price only on its own output, not on the rival. But when both firms adjust prices equally, output is determined by market elasticity, η, where $\eta = \eta_{11} - \eta_{12}$. Since $\eta_r \equiv \eta_{12}/\eta_{11} = 1 - \eta/\eta_{11}$, if $\eta_r'(p) > 0$, market elasticity relative to firm elasticity is lower at higher prices than at lower prices. Moving to DP then induces a positive output externality on the rival firm: each firm ignores (a) that its price increase in market H expands the rival's output and (b) that its price decrease in market L

[9] Recall that moving from uniform pricing to differential pricing can be analyzed as introducing mean-preserving cost dispersion.

reduces the rival's output—but the former exceeds the latter when $\eta_r' > 0$. Oligopoly DP then yields a larger output than predicted based on each firm's own-elasticity, boosting profit. In addition, when $\eta_r' > 0$ DP tends to raise average price (see first term in (21)), which also boosts profit. Of course, both effects are reversed if $\eta_r' < 0$, and DP then can reduce profit, as we will show.

The last term, $\eta\left[1-\frac{\eta_r}{2-\eta_r+\alpha}\right]$, can be written as $\eta\left[1-p^{*'}(c)\eta_r\right]$ in equilibrium: a monopolist's gain (η) from adjusting prices and outputs across markets in response to cost dispersion, modified in oligopoly by the impact of the rival's symmetric price responses ($\eta p^{*'}(c)\eta_r > 0$). Suppose $0 < p^{*'}(c)\eta_r < 1$. In market H, where moving to DP effectively raises marginal cost, the firm loses from the cost increase, but less than if it were a monopolist. The reverse occus in market L, where moving to DP effectively lowers marginal cost. Since the rival's response in each market does not outweigh the own-cost effect, cost dispersion still benefits the firm, in proportion to the elasticity of market demand. However, when $p^{*'}(c)\eta_r > 1$, the output reallocation effect will be *negative* and DP can *reduce* profit.

Why Differential Pricing Can Reduce Profits. The inequality in (21) can be reversed: unlike for monopoly, competitive DP can reduce profit relative to uniform pricing. This may happen when $\eta_r' < 0$, in which case it is possible that DP reduces both average price and total output, and profit falls due to the first two terms in (21), as we will illustrate in Example 4 of subsection 3.4

Another channel for DP to lower profit is $p^{*'}(c)\eta_r > 1$. In Example 3 of subsection 3.4 (constant-elasticity demand function), η_r and α are constant, so the first two terms in (21) are zero. And for certain parameter values, $p^{*'}(c)\eta_r > 1$, hence the third term in (21) is negative. Recall from (20) that $p^{*'}(c)\eta_r > 1$ yields the 'unusual' case where a cost increase raises industry profits. This requires $p^{*'}(c) > 1$ (since $\eta_r < 1$), so the profit margin rises; and, furthermore, η_r large enough (though still < 1), so the price rise is driven sufficiently by the strategic interaction due to cross-elasticity, hence industry output does not fall too much.[10] The reverse pattern occurs in market L: DP then reduces profit since the price drop exceeds the cost reduction, and without inducing a sufficient output expansion. But why does overall profit fall?

Observe that

$$m_L - m_H = c_H - c_L - \int_{c_L}^{c_H} p^{*'}(c)\, dc \; > (=) < 0 \;\text{ if }\; p^{*'}(c) < (=) > 1. \tag{22}$$

Thus, when $p^{*'}(c) > 1$ the profit margin in market L under DP is lower than in H, hence the output reallocation to L harms profit, by (8). For a monopolist, DP nevertheless raises profit for any constant $p^{*'}(c)$, because $p^{*'}(c) > 1$ requires demand to be sufficiently convex that the monopolist's opposite price changes across markets expand total output enough to outweigh the harmful misallocation effect (Chen and Schwartz, 2015). In oligopoly, however, $p^{*'}(c) = \frac{1}{2-\eta_r+\alpha} > 1$ can arise not only from demand curvature, but also from the cross-elasticity term, η_r. Thus, an equilibrium pass-through above 1 need not imply a large output expansion from DP. Indeed, $p^{*'}(c)\eta_r > 1$ requires not only $p^{*'}(c) > 1$, but also η_r sufficiently above zero, highlighting the difference from monopoly.

Although it is possible for symmetric DP to lower profit, arguably the required demand conditions are rather special. Assuming the 'normal' case where a common cost shock moves industry profit in the opposite direction ($p^{*'}(c)\eta_r < 1$),[11] there is a systematic force pushing towards greater profit: the output reallocation effect captured by the last term in (21).

Comparing (19) and (21) shows that both profit and consumer surplus benefit from

[10] For a given own-price elasticity η_{11}, a larger cross-elasticity η_{12} implies a lower market-demand elasticity η. Thus, for $\eta_r \equiv \eta_{12}/\eta_{11}$ large (though still < 1), η_{11} can be sufficiently high to render a unilateral price increase unprofitable, yet η can be sufficiently low that a joint price increase by all firms, induced by a common cost increase, can be profitable.

[11] Weyl and Fabinger (2013, fn. 16) consider the alternative case "unlikely to be empirically relevant in many symmetric industries."

greater scope for output reallocation under DP, a larger η in (21) or its analogue $\hat{\eta}(p)$ in (19); whereas a higher $\alpha' - \eta'_r$, which reduces the increase (if any) of average price, benefits consumers but harms profit. As we shall illustrate in subsection 3.4, while it is possible that DP lowers consumer surplus or profit under competition, both (19) and (21) are satisfied for many familiar demand functions.

3.3 Total Welfare

Given marginal cost c and the associated equilibrium price $p^*(c)$, the equilibrium total welfare in a market can be written as

$$\begin{aligned} W(p^*(c)) &\equiv w^*(c) = s^*(c) + 2[p^*(c) - c] D_1(p^*(c), p^*(c)) \\ &= s^*(c) + [p^*(c) - c] D(p^*(c)). \end{aligned}$$

Thus,

$$\begin{aligned} w^{*\prime}(c) &= -[D(p^*) + \delta(p^*)] p^{*'}(c) \\ &\quad + \left[p^{*'}(c) - 1\right] D(p^*(c)) + [p^*(c) - c] D'(p^*(c)) p^{*'}(c) \\ &= p^{*\prime}(c) [D'(p^*(c))(p^*(c) - c) + \delta(p^*)] - D(p^*(c)). \end{aligned}$$

But because $p^* \frac{D'(p^*(c))}{D(p^*(c))} = -\eta_{11} + \eta_{12}$ and $\frac{(p^*(c) - c)}{p^*} = \frac{1}{\eta_{11}}$, we have

$$\begin{aligned} w^{*\prime}(c) &= -D(p^*(c)) \left[p^{*\prime}(c) \left(1 - \eta_r - \frac{\delta(p^*)}{D(p^*(c))}\right) + 1\right] \\ &\equiv -D(p^*(c)) [1 + p^{*\prime}(c)(1 - \hat{\eta}_r)]. \end{aligned} \quad (23)$$

Notice that for a single product, $\hat{\eta}_r \equiv \left(\eta_r + \frac{\delta(p^*)}{D(p^*(c))}\right) = 0$, so that a marginal increase in c affects w through the profit effect $-D(p^*(c)$ and the consumer surplus effect $-D(p^*(c)p^{*\prime}(c)$. Under competition between two products, the output reduction for the industry due to a higher c is adjusted because of the cross-price effects on output.

By analyzing when $w^*(c)$ is convex we obtain the following result, which provides the condition for DP to raise or lower total welfare in terms of the demand parameters $\eta_r = \eta_r(p)$, $\alpha = \alpha(p)$, $\eta = \eta(p)$, and $\hat{\eta}_r = \hat{\eta}_r(p)$:

Proposition 3 *DP increases total welfare if (24) holds, and DP reduces total welfare if (24) is reversed.*

$$\frac{(\alpha' - \eta_r')(1 - \hat{\eta}_r)}{[2 - \eta_r + \alpha]^2} p + \hat{\eta}_r' p + \eta \left[1 + \frac{1 - \hat{\eta}_r}{2 - \eta_r + \alpha}\right] > 0. \tag{24}$$

The first term corresponds to the first terms in (19) and (21), reflecting the net effect of the average price change on consumer surplus and profit: when $(\alpha' - \eta_r') \geq 0$, or the average price does not increase, its net effect on consumer surplus and profit is non-negative, provided that $\hat{\eta}_r \leq 1$.[12] The second term, $\hat{\eta}_r' p$, is similar to the the middle term in (21) and captures the output externality under competition in each of the two markets, H and L. Together, the first and second terms in (21) reflect how DP affects total welfare through the change in total output. The last term on the LHS of (24), $\eta\left[1 + \frac{1-\hat{\eta}_r}{2-\eta_r+\alpha}\right]$, corresponds to the output adjustment effect due to differential pricing: moving from UP to DP creates price dispersion for consumers and cost dispersion for firms, leading to beneficial adjustments in outputs for both sides, and a higher η magnifies this advantageous adjustment. DP can raise total welfare due to both the (beneficial) output reallocation and output expansion, but neither of them is necessary for DP to raise welfare.[13]

The condition for monopoly DP to raise total welfare (Chen and Schwartz, 2015, condition (A1B)) can be written as

$$\eta \left[1 + \frac{1}{2 - \sigma(q^*)}\right] + p \frac{-\sigma'(q^*) D'(p)}{[2 - \sigma(q^*)]^2} > 0,$$

[12] Since the demand curve is downward sloping, a change in average price can affect total output, and hence its effects on consumer surplus and profit do not entirely offset each other.

[13] Importantly, output reallocation under $p_H - c_H < p_L - c_L$ can improve total welfare even when total output decreases, a key difference between cost-based DP and price discrimination.

and monopoly DP lowers total welfare if the above inequality is reversed. As with consumer surplus and profit, the condition for DP to raise total welfare under oligpoly, (24), embeds and generalizes that under monopoly: $\eta_r = \eta_r' = \hat{\eta}_r = 0$ for the single-product monopolist, while α in oligopoly corresponds to $-\sigma$ under monopoly and α' corresponds to $-\sigma'(q^*) D'(p)$.

Like its counterparts for consumer surplus and profit, (19) and (21), condition (24) is met for a broad class of demands, such as those with constant α and η, but not always; see Examples 3 and 4 below. Finally, observe that under monopoly, DP always increases profit, hence total welfare rises under broader conditions than does consumer surplus (compare Propositions 1 and 2 of Chen and Schwartz, 2015). In oligopoly, DP can reduce profit, hence consumer surplus may rise yet total welfare fall, as in Examples 3 and 4. The reverse occurs in Example 5: consumer surplus falls but total welfare rises. Thus, the conditions for DP to raise consumer surplus or welfare, (19) and (24), are no longer nested.

3.4 Examples

Below we provide several examples showing that the conditions in Propositions 1-3 for differential pricing to benefit consumers, profits, and overall welfare, respectively, are met by many familiar demand functions, though not always. The examples also illustrate the underlying economic effects that generate these gains or losses. To that end, recall from (8) that the output reallocation due to DP affects profit positively when $m_L > m_H$ and negatively when $m_L < m_H$; and that $m_L > m_H$ if $p^{*'}(c) < 1$ while $m_L < m_H$ if $p^{*'}(c) > 1$, from (22).

In Examples 1-3, average price under DP equals the uniform price, hence consumer surplus rises due to consumers' gain from adjusting outputs. In Examples 1-2, profit also rises (hence, so does total welfare), though for different reasons, identified in (8): total output is unchanged but the output reallocation is beneficial (Example

1); or output increases while the output reallocation effect is neutral (Example 2). In Example 3, total output expands but the output reallocation is harmful and, depending on the parameters, profit can rise, fall, or remain unchanged, and similarly for total welfare.

In Example 4, average price under DP is lower than the uniform price, hence consumer surplus again rises; but total output can fall despite the reduction in average price, which can reduce profit and total welfare, but through a different channel than in Example 3.

In Example 5, DP raises the average price and, for certain parameter values, it reduces consumer surplus, but the output reallocation is sufficiently beneficial that profit and total welfare rise even when total output falls.

Example 1 *Linear demand (DP increases consumer surplus and profit):*

$$q_i = I - b_1 p_i + b_2 p_j, \qquad I > 0 \text{ and } b_1 > b_2.$$

Then, $\eta_r = \frac{b_2}{b_1}$, $\eta = p\frac{b_1 - b_2}{I-(b_1-b_2)p}$, $\alpha = 0$.

It follows that both (19) and (21) hold, hence DP increases consumer surplus and profit. Average price and total output are the same under UP and DP, so the gains in consumer surplus and profit come solely from reallocating output between markets.

Example 2 *Log demand function (DP increases consumer surplus and profit):*

$$p_i(q_i, q_j) = A - \log[Bq_i + q_j], \qquad B > 1.$$

Then, $\eta_r = \frac{1}{B}$, $\alpha(p) = -\frac{B-1}{B} < 0$, $p^*(c) = c + \frac{B-1}{B}$, *and* $p^{*\prime}(c) = 1$.

Again, both (19) and (21) hold. As in Example 1, DP does not change average price, but now it increases total output because $D(p)$ is convex:

$$q^d = \lambda D(p_L) + (1-\lambda) D(p_H) > D(\lambda p_L + (1-\lambda) p_H) = D(p^u) = q^u.$$

This output expansion is the sole reason why profit rises, since the output reallocation here is neutral for profit: $p^{*'}(c) = 1$ implies $m_L = m_H$.

Example 3 *Constant Elasticity Demand (DP increases consumer surplus, but profit and total welfare can rise or fall):*

$$q_i = kp_i^{-a}p_j^b, \qquad a > b > 0,\ a > 1,$$

Then, $p^*(c) = \frac{a}{a-1}c$, $p^{*'}(c) = \frac{a}{a-1} > 1$, $\eta_r = \frac{b}{a}$, $\alpha = -\frac{a-b+1}{a}$, *and* $\eta = a - b$.

Average price is unchanged, as $\eta_r' = \alpha' = 0$, hence consumer surplus rises, and total output expands since $D(p) = D_i(p,p) = kp^{-a}p^b = kp^{-(a-b)}$ is convex. Profits benefit from this output expansion. But $p^{*'}(c) > 1$ implies that the margin under DP is lower in market L than in H, $m_L < m_H$, so the output reallocation to L harms profit, from (8). Using (21), and given that η_r, α, and η (> 0) are constant, $\Delta\Pi$ takes the sign of $(1 - \frac{\eta_r}{2-\eta_r+\alpha}) = (1 - p^{*'}(c)\,\eta_r) = (1 - \frac{b}{a-1})$. Three cases are possible: (i) $p^{*'}(c)\,\eta_r < 1 \iff a > b+1 \implies$ profit rises; (ii) $p^{*'}(c)\,\eta_r = 1 \iff a = b+1 \implies$ profit is unchanged; and (iii) $p^{*'}(c)\,\eta_r > 1 \iff a < b+1 \implies$ profit falls. The beneficial output expansion outweighs the harmful output reallocation in case (i) and the reverse occurs in case (iii). In case (ii) the effects just offset, leaving profit unchanged. Total welfare W likewise can move in any direction. For instance, using (24), it can be shown that for $a = 2$, DP raises W if $b < 1.85$, leaves W unchanged if $b = 1.85$, and lowers W if $b \in (1.85, 2)$

CES demand yields the same patterns for consumer surplus (increase) and profit (no change) as in case (ii) above:

$$q_i = A\frac{p_i^{-1/(1-\rho)}}{p_1^{-\rho/(1-\rho)} + p_2^{-\rho/(1-\rho)}}, \qquad 0 \le \rho \le 1 \quad \text{and} \quad A > 0.$$

Then: $p^*(c) = c\frac{2-\rho}{\rho}$, $p^{*'}(c) = \frac{2-\rho}{\rho} > 1$, $\eta_r = \frac{\rho}{2-\rho}$, $\alpha = -4\frac{\rho-1}{\rho-2}$, and $\eta = 1$. Since

η_r and α are constant, average price is unchanged, hence output expands because demand $D(p) = D_i(p, p) = \frac{1}{2}Ap^{-1}$ is convex. The output reallocation is excessive for profit, since $p^{*'}(c) > 1$, and on balance profit is unchanged (as $p^{*'}(c)\,\eta_r = 1$).

Example 4 *Binomial Logit demand with outside option (DP increases consumer surplus but can reduce total output, profit, and welfare):*

$$q_i = \frac{e^{-p_i}}{e^{-p_i} + e^{-p_j} + A}, \qquad 0 \leq A < 1.$$

Then, $p^* = c + \frac{A+2e^{-p^*}}{A+e^{-p^*}}$, $p^{*'}(c) = \frac{\left(A+e^{-p^*}\right)^2}{e^{-2p^*}+3Ae^{-p^*}+A^2}$; $\eta_r = \frac{e^{-2p}}{e^{-p}(A+e^{-p})}$, $\eta_r' = -A\frac{e^{-p}}{(A+e^{-p})^2} < 0$; $\alpha = -\frac{A^2}{(A+e^{-p})^2} < 0$, $\alpha' = -2A^2\frac{e^{-p}}{(A+e^{-p})^3} < 0$; $\eta = \hat{\eta} = A\frac{p}{A+2e^{-p}}$.

In Example 4, condition (19) always holds and, hence, DP raises consumer surplus. Profit and welfare also rise under DP if $A = 0$, for the standard logit demand. However, conditions (21) and (24) can be reversed, so that DP can lower profit and welfare when $A > 0$. For instance, suppose $A = 0.01$, $c_L = 0$, $c_H = 2$, and $\lambda = 0.5$. Then: $m_L^d = 1.94 > m_H^d = 1.71$, so the output reallocation benefits profit and total welfare. But DP lowers average price, from $p^u = 2.85$ to $p^d = 2.82$, and also lowers total output, from $q^u = 0.92$ to $q^d = 0.90$, causing profit to fall: $\Pi^d = 1.65 < \pi^u = 1.71$; and the output reduction also causes welfare to fall, albeit only slightly.

The price and output changes can be understood as follows. Demand is more convex at higher prices ($\alpha' < 0$), which pushes the pass-through $p^{*'}(c)$ to be increasing; however, the cross-elasticity relative to own-elasticity of demand is smaller at higher prices ($\eta_r' < 0$), which pushes $p^{*'}(c)$ to be decreasing, and this effect dominates since $\eta_r' < \alpha'$. Hence DP lowers average price. Importantly, this reduction in average price is driven solely by firms' profit-maximizing response to the cross-elasticity effect ($\eta_r' < 0$), and runs counter to the demand curvature effect ($\alpha' < 0$), which explains why total output fell: $\alpha' < 0$ implies that demand became more convex in the market where price rose than in the market where price fell.

Example 5 *Modified Logit demand (DP raises average price, and can reduce output and consumer surplus,but total welfare rises):*

$$q_i = \frac{1}{1+e^{2p_i - p_j - k}}, \qquad\qquad k \geq 0.$$

Then, $p^* = c + \frac{1}{2}\frac{e^{-k+p*}+1}{e^{-k+p*}}$, $p^{*\prime}(c) = \frac{e^{-k+p}}{e^{-k+p}+\frac{1}{2}} < 1$, $\eta_r = \frac{1}{2}$, $\alpha = -\frac{1}{2}\frac{e^{-k+p}-1}{e^{-k+p}}\frac{1}{2}$, $\alpha' = -\frac{1}{2}e^{k-p} < 0$, $\eta = p\frac{e^{-k+p}}{e^{-k+p}+1}$.

In this example, DP raises average price because $\alpha' - \eta' < 0$. Since $\eta_r' = 0$, average price rises solely because $\alpha'(p) < 0$: demand becomes more convex at higher prices, which yields an increasing pass-through rate. The increase in average price can outweigh consumers' gain from adjusting outputs, causing consumer surplus to be lower under DP. For instance, suppose $c_L = 0$, $c_H = 2$, $\lambda = 0.5$, $\bar{c} = 1$, $k = 4$. Then: $S^d - S^u = -0.06 < 0$; $\Pi^d - \Pi^u = 0.1 > 0$; $W^d - W^u = 0.04 > 0$. Differential pricing here reduces total output, $\Delta q = -6.86 \times 10^{-3}$. Profit and total welfare nevertheless increase due to the output reallocation to market L, where the margin remains higher than in market H under DP ($m_L = 2.58 > m_H = 1.41$).

Summarizing our findings, DP raises both consumer surplus and profit when the demand curvature (α) and the elasticity ratio (η_r) do not change too fast relative to the elasticity of market demand (η). In particular, DP raises both consumer surplus and profit when α and η_r are constant, such as for linear, constant elasticity, log, CES (no effect on profit), and standard logit demands.

3.5 Comparison to Oligopoly Price Discrimination

For symmetric oligopoly, Holmes (1989) analyzed price discrimination rather than our cost-based differential pricing. Our results exhibit similarities to his findings as well as differences. In both settings, differential pricing—cost-based or demand-based—may reduce profit relative to uniform pricing, unlike for monopoly. Holmes shows this can occur if the market with the smaller elasticity of demand has the larger cross-price elasticity between firms. Price discrimination then lowers price in the 'wrong market' and can reduce total output. In our setting, markets differ only in costs, but DP still can reduce output and profit if cross-price elasticity relative to the own-price elasticity for the common demand function is greater at lower prices than at higher prices (Example 4). Then price can fall by more in the low-cost market than it rises in the other market—and not due to greater demand curvature at lower prices, since a reduction in average price motivated by demand convexity would increase output, as occurs with DP under monopoly. In addition, we showed that cost-based DP can reduce profit through a second force: excessive output reallocation between markets when $p^{*'}(c)\eta_r > 1$ (Example 3).

Turning to differences, cost-based DP is more likely to benefit consumers than is price discrimination. From Remark 2, profitable price discrimination requires an increase in total output or in average price; indeed, price discrimination has a tendency to raise average price, which of itself harms consumers.[14] By contrast, DP can raise profit even if average price does not rise (Examples 1-3), which ensures that consumers also benefit. The cost savings achieved by reallocating output to the lower-cost market provide firms an incentive to adopt DP also under demand conditions that do not yield an increase in average price (or in output), and consumers benefit from the price dispersion by adjusting their consumption patterns. In our environment, we saw that cost-based DP does not raise average price for several familiar demand functions.

Total welfare also is more likely to rise with cost-based DP than with price discrimination. Discrimination misallocates output between markets, hence an increase

[14] "There is a sense in which discrimination increases "average" price; the increase in price in the strong market above the uniform price is "large" relative to the decrease in the weak-market price." (Holmes 1989, p. 248.) This price bias of price discrimination under monopoly is further discussed by Chen and Schwartz (2015, pp. 449-451).

in total output is necessary but not sufficient for total welfare to rise.[15] By contrast, cost-based DP can increase welfare when output remains constant, as with linear demand (Example 1),[16] or even when output decreases (Example 5), due to the favorable output reallocation to the lower-cost market.

4. FIRMS WITH ASYMMETRIC COSTS

We now extend our model to allow cost asymmetries between firms, in addition to cost differences across markets. Suppose that firms have different costs for serving a given consumer group (or market): firm i has costs (c_{iL}, c_{iH}), where $c_{iH} > c_{iL}$, for $i = 1, 2$. The firms are still assumed symmetric in demand: they produce either homogeneous products or symmetrically differentiated products. Our interest here is to explore whether cost asymmetries introduce new forces, beyond demand-side factors such as pass-through, that alter the welfare properties of DP relative to UP.

4.1 Homogeneous Products

We consider two scenarios of cost asymmetries:

(1) *Global Cost Advantage*: the same firm, say firm 1, has a cost advantage in serving both consumer groups: $c_{1L} < c_{2L}$ and $c_{1H} < c_{2H}$;

(2) *Local Cost Advantage*: each firm has a cost advantage in serving a different group. Without loss of generality, let $c_{1L} < c_{2L}$ and $c_{1H} > c_{2H}$, with $\bar{c}_1 \leq \bar{c}_2$.

[15] Holmes (1989, fn.2) notes that this well-known result under monopoly also "holds for this oligopoly analysis."

[16] In Holmes' setting, where demand differs between the two consumer groups, price discrimination can increase or decrease output even with linear demand, depending on his elasticity-ratio condition, which compares relative market-demand elasticities to relative cross-price elasticities. When output falls, total welfare also must fall, because price discrimination misallocates output. Moreover, in a subset of his cases where output falls, profit also falls.

Gobal Cost Advantage

We adopt the standard assumption for Bertrand competition with asymmetric costs: the lower-cost firm can capture the market by pricing at the rival's marginal cost. Assume also that firms' costs are not too far apart, so the lower-cost firm sets price below its monopoly level, at the rival's cost. Under DP, competition occurs market-by-market and the equilibrium prices in the two markets are therefore:

$$p_L = \max\{c_{1L}, c_{2L}\} = c_{2L}; \qquad p_H = \max\{c_{1H}, c_{2H}\} = c_{2H}. \tag{25}$$

Under uniform pricing, we assume that the firm with the lower average cost can capture both markets by pricing at the other firm's *average cost.* Therefore, the equilibrium uniform price is given by

$$p^u = \max\{\bar{c}_1, \bar{c}_2\} = \bar{c}_2. \tag{26}$$

The next result shows that, while DP benefits consumers, profits can readily fall. The profit comparison depends on the difference in marginal costs of serving the two markets for firm 1 ($\Delta c_1 \equiv c_{1H} - c_{1L} > 0$) relative to firm 2 ($\Delta c_2 \equiv c_{2H} - c_{2L} > 0$).

Lemma 1 *For any given pair of costs* $\{(c_{1L}, c_{1H}), (c_{2L}, c_{2H})\}$ *with* $c_{1L} < c_{2L}$*,* $c_{1H} < c_{2H}$*, and* $\Delta c_i \equiv c_{iH} - c_{iL} > 0$*,* $i = 1, 2$*:*
(i) $p^d = p^u$*, and hence* $S^d > S^u$*; (ii) with linear demand,* $\pi^d > \pi^u$ *if* $\Delta c_1 > \Delta c_2$ *and* $\pi^d < \pi^u$ *if* $\Delta c_1 < \Delta c_2$*; (iii) relative to linear demand,* $\pi^d - \pi^u$ *and* $W^d - W^u$ *are higher if demand is strictly convex and lower if demand is strictly concave.*

Part (i) is straightforward. Next, consider profit. With linear demand, total output as well as average price are the same under DP and UP, hence the change in firm 1's profit is determined entirely by the reallocation effect in (8). Firm 1's prices are set equal to firm 2's costs: $p_L = c_{2L}$ and $p_H = c_{2H}$. Thus, the difference in firm 1's profit margins under DP between markets L and H is $(c_{2L} - c_{1L}) - (c_{2H} - c_{1H}) = \Delta c_1 - \Delta c_2$.

The output reallocation under DP raises profit if the margin is higher in market L, which occurs if $\Delta c_1 > \Delta c_2$, and lowers profit if $\Delta c_1 < \Delta c_2$.[17] Intuitively, firm 1 is harmed by being constrained to adopt a price differential larger than the difference in its costs. Third, in market H where price rises under DP, output decreases less if demand is strictly convex instead of linear, while in market L where price falls under DP, output increases by more if demand is strictly convex instead of linear. Hence relative to linear demand, $\pi^d - \pi^u$ and $W^d - W^u$ are both higher if demand is strictly convex, and the conclusion is reversed for strictly concave demand.

Local Cost Advantage

Now suppose firm 1 has the cost advantage for market L and firm 2 has the advantage for H: $c_{1L} < c_{2L} < c_{2H} < c_{1H}$, with $\bar{c}_1 \leq \bar{c}_2$. Under DP, firm 1 serves market L at price $p_L = c_{2L} < \bar{c}_2$ and firm 2 serves market H at price $p_H = c_{1H} > c_{2H} > \bar{c}_2$. Under UP, we assume that each firm cannot refuse to serve its higher-cost market; it must be willing to sell in both markets or none. Suppose also that at equal prices ($p_1 = p_2$), if $\bar{c}_1 < \bar{c}_2$, firm 1 can capture both markets at price $\bar{c}_2$, while if $\bar{c}_1 = \bar{c}_2$, the firms split both markets equally. In both cases, the equilibrium uniform price is $p^u = \bar{c}_2$, and moving to DP lowers price in market L and raises price in market H, but raises average price.[18] The next result shows that while profit necessarily rises, consumer surplus can fall without requiring unusual demand conditions.

Lemma 2 *For any given pair of costs $\{(c_{1L}, c_{1H}), (c_{2L}, c_{2H})\}$ with $c_{1L} < c_{2L} < c_{2H} < c_{1H}$, and $\bar{c}_1 \leq \bar{c}_2$: (i) price is higher under DP than under UP: $p^d \equiv \lambda p_L + (1-\lambda) p_H > p^u$; (ii) consumer surplus is higher under DP ($S^d > S^u$) if cost differences within*

[17] With linear demand, total welfare rises if $\Delta c_1 \geq \Delta c_2$, but can fall if $\Delta c_1 < \Delta c_2$. As $\Delta c_1 \to 0$, the output allocation under uniform pricing converges to the first-best, but is inefficient under differential pricing, since $p_{1H} - p_{1L} = \Delta c_2 > 0$, hence $W^d < W^u$.

[18] We have considered a variant of this scenario, where firm 1 has lower cost to serve market A than market B and the reverse holds for firm 2. Differential pricing then raises price in both markets.

markets, $\Delta_L \equiv c_{2L} - c_{1L}$ *and* $\Delta_H \equiv c_{1H} - c_{2H}$*, are sufficiently small, but* $S^d < S^u$ *if* $c_{2H} - c_{2L}$ *is sufficiently small. (iii) Profits always rise:* $\pi^d > \pi^u$*.*

The results in Lemma 2 can be understood as follows. The uniform price is determined by the firm with the higher *average* of the marginal costs across markets. Under DP, each market's price is set by the higher of the two firms' marginal costs for that market. Since cost heterogeneity is greater market-by-market than on average, the average price is higher under DP. In other words, under UP there is less cost dispersion between the two firms, intensifying their competition and resulting in a lower equilibrium (average) price.

Consumer surplus is subject to opposing effects: it increases due to the price dispersion, but decreases due to the rise in average price. When the cost difference between firms within each market ($c_{2k} - c_{1k}$, $k = L, H$) is sufficiently small, the average price under DP converges to the uniform price, hence the price dispersion effect dominates and DP raises consumer surplus. On the other hand, if the cost difference between markets for firm 2 ($c_{2H} - c_{2L}$) is small, then $p^u = \bar{c}_2$ is close to c_{2L}, so that moving to DP lowers price in market L only slightly but raises price in market H substantially. As Example 6 below shows, however, DP can lower consumer surplus even when $c_{2H} - c_{2L}$ is not 'small.'

Industry profits always rise with DP, for three reasons: as with monopoly, output is reallocated to the lower-cost market; in addition, DP now leads to each market being served by the efficient firm in that market (firm 2 replaces 1 in market H) and, furthermore, DP raises average price by relaxing the competitive constraint.

Example 6 *(DP can raise or lower consumer surplus.) Let* $\lambda = 1/2$*,* $D(p) = 10 - p$*.*
(i) $S^d > S^u$*. Suppose costs are* $c_{1L} = 3, c_{2L} = 3.1$*,* $c_{2H} = 4.9, c_{1H} = 5$*. Then* $\bar{c}_1 = \bar{c}_2 = 4 = p^u$*,* $p_L = 3.1$*,* $p_H = 5$*,* $p^d = 4.05 > p^u$*, and* $S^d = 18.15 > S^u = 18$*.*
(ii) $S^d < S^u$*. Suppose costs are* $c_{1L} = 3, c_{2L} = 4$*,* $c_{2H} = 6, c_{1H} = 7$*. Then* $\bar{c}_1 = \bar{c}_2 =$

$5 = p^u$, $p_L = 4$, $p_H = 7$, $p^d = 5.5 > p^u$, *and* $S^d = 11.25 < S^u = 12.5$.

Summarizing, when the cost advantage in each market is held by a different firm, DP has opposing effects on aggregate consumer surplus. The price dispersion is beneficial, but the increase in average price is harmful. The first effect dominates if the cost heterogeneity across markets is large relative to that between firms in a given market, while the second effect dominates in the reverse scenario.

Proposition 4 below consolidates results from Lemmas 1 and 2 to summarize that DP can reduce profit or consumer surplus under "familiar" demands—such as linear demand—when firms produce homogeneous products and have asymmetric costs.

Proposition 4 *Suppose firms offer homogeneous products and have asymmetric costs.*
(i) Under a Global Cost Advantage ($c_{1L} < c_{2L}$ and $c_{1H} < c_{2H}$), with linear demand, differential pricing reduces profit if $c_{1H} - c_{1L} < c_{2H} - c_{2L}$.
(ii) Under a Local Cost Advantage ($c_{1L} < c_{2L}$, $c_{1H} > c_{2H}$ and $\bar{c}_1 \leq \bar{c}_2$), differential pricing reduces consumer surplus if $c_{2H} - c_{2L}$ is sufficiently small.

4.2 Differentiated Products with Linear Demands

The above results, however, appear to hinge on products being homogeneous. As we now show, for differentiated products with linear demand, DP *always* raises consumer surplus and profit under asymmetric firm costs, as in our main model where firms have symmetric costs.

We consider the following linear demand system:

$$q_i \;=\; a - p_i + \gamma\, p_j \quad \text{for } \; i,\, j \in \{1, 2\} \; (j \neq i)\,, \tag{27}$$

where $a > 0$ and $\gamma \in (0, 1)$ are parameters. The respective inverse demand system is

$$P_i = \frac{a\,(1+\gamma) - q_i - \gamma\, q_j}{1 - \gamma^2}.$$

This system reflects the demands of a representative consumer with utility function:

$$U(q_1, q_2) \;=\; \frac{1}{1-\gamma^2}\left[a(1+\gamma)(q_1+q_2) - \frac{1}{2}\left(q_1^2 + 2\gamma\, q_1 q_2 + q_2^2\right)\right]. \tag{28}$$

The two firms may have different costs of serving the same consumer group; i.e., c_{1k} may differ from c_{2k}, for $k = L, H$, in arbitrary ways—including the cases of global and local advantages—with

$$\bar{c}_i = \lambda c_{iL} + (1-\lambda)\, c_{iH}.$$

Under uniform pricing (with an equal split of consumers between the firms), firm $i's$ profit function is:

$$\pi_i = (p_1 - \bar{c}_i)\,(a - p_i + \gamma\, p_j).$$

The equilibrium uniform prices and outputs by the two firms are:

$$p_1^u = \frac{2(a+\bar{c}_1) + \gamma(a+\bar{c}_2)}{4-\gamma^2}, \qquad p_2^u = \frac{2(a+\bar{c}_2) + \gamma(a+\bar{c}_1)}{4-\gamma^2}. \tag{29}$$

Similarly, under differential pricing, for $k = L, H$, the equilibrium differential prices are:

$$p_{1k} = \frac{2(a+c_{1k}) + \gamma(a+c_{2k})}{4-\gamma^2}, \qquad p_{2k} = \frac{2(a+c_{2k}) + \gamma(a+c_{1k})}{4-\gamma^2}. \tag{30}$$

It follows that the average prices under DP and under UP are equal:

$$p_1^d = \lambda p_{1L} + (1-\lambda)\, p_{1H} = p_1^u; \qquad p_2^d = \lambda p_{2L} + (1-\lambda)\, p_{2H} = p_2^u.$$

Furthermore, it is straightforward to verify that each firm's output is the same under DP and under UP.

In the appendix, we show that (aggregate) consumer surplus as a function of the two firms' outputs q_1, q_2) and industry profit as a function of the two firms' marginal costs (c_1, c_2) are both strictly convex functions, which leads to the following result:

Proposition 5 *If firms produce differentiated products with a linear demand system given in (27), then consumer surplus, industry profit, and total welfare are all higher under differential pricing than under uniform pricing regardless of the cost asymmetry between firms or the extent of product differentiation.*

5. CONCLUSION

The welfare properties in oligopoly of uniform versus differential pricing when markets differ in costs of service have gone largely unexplored, despite the prevalence of industries where firms are constrained from adopting cost-based differential pricing. In a standard oligopoly setting where firms face symmetric demands, we showed that the effects of DP on consumers and firms are subtle, depending on whether products are homogeneous or differentiated and whether firms are symmetric in costs or not.

With symmetric firm costs and homogeneous products, (cost-based) differential pricing obviously maximizes consumer welfare whereas uniform pricing does not, while profits are zero in both regimes. Under differentiated products, differential pricing increases consumer surplus and profits under conditions met by many standard demand functions. The systematic force driving higher profit is cost savings from reallocating output between markets by adjusting prices; consumers benefit from this price dispersion provided average price does not rise too much. Profit *can fall* with differential pricing, unlike for monopoly, as can consumer surplus, but such outcomes require demand conditions that seem rather stringent.

When firms have asymmetric costs, however, differential pricing can reduce profit or, under a different cost configuration, reduce consumer surplus even with standard demand functions such as linear. However, the beneficial role of differential pricing appears to be restored when firms produce differentiated products, at least with linear demands. A natural extension would be to explore the implications of asymmetries between firms also on the demand side.

APPENDIX

Proof of Proposition 1. First,

$$\begin{aligned} S'(p,p) &= -D_1(p,p) + \int_p^\infty \frac{\partial D_1(x,p)}{\partial p} dx - D_1(p,\infty) \\ &= -2D_1(p,p) - \int_p^\infty \left[\frac{\partial D_1(p,x)}{\partial x} - \frac{\partial D_1(x,p)}{\partial p}\right] dx = -D(p) - \delta(p). \end{aligned}$$

Hence,

$$s'(c) = S'(p^*,p^*)\, p^{*'}(c) = -\left[D(p^*) + \delta(p^*)\right] p^{*'}(c),$$

$$s''(c) = -\left[D'(p^*) + \delta'(p^*)\right]\left[p^{*'}(c)\right]^2 - \left[D(p^*) + \delta(p^*)\right] p^{*''}(c).$$

Next, from (17):

$$s''(c) = -\left[D'(p^*) + \delta'(p^*)\right]\left[p^{*'}(c)\right]^2 - \left[D(p^*) + \delta(p^*)\right]\left[\eta_r'(p^*) - \alpha'(p^*)\right]\left[p^{*'}(c)\right]^3 > 0$$

$$\begin{aligned} &\Longleftrightarrow -\left[D'(p^*) + \delta'(p^*)\right] - \left[D(p^*) + \delta(p^*)\right]\left[\eta_r'(p^*) - \alpha'(p^*)\right] p^{*'}(c) > 0 \\ &\Longleftrightarrow -\frac{\left[D'(p^*) + \delta'(p^*)\right] p^*(c)}{\left[D(p^*) + \delta(p^*)\right]} + \left[\alpha'(p^*) - \eta_r'(p^*)\right] p^*(c)\, p^{*'}(c) > 0 \\ &\Longleftrightarrow \left[\alpha'(p^*(c)) - \eta_r'(p^*(c))\right] \frac{p^*(c)}{2 - \eta_r(p^*(c)) + \alpha(p^*(c))} + \hat{\eta}(p^*(c)) > 0. \end{aligned}$$

Therefore, if (19) holds, $s(c)$ is convex and

$$\begin{aligned} S^u &= S(p^u(\bar{c}), p^u(\bar{c})) = s(\bar{c}) = s(\lambda c_L + (1-\lambda) c_H) \\ &< \lambda s(c_L) + (1-\lambda) s(c_H) = \lambda S(p^*(c_L)) + (1-\lambda) S(p^*(c_H)) = S^d. \end{aligned}$$

Similarly, if (19) is reversed, then $s(c)$ is concave and DP lowers consumer surplus. ■

Proof of Proposition 2. We derive the condition for equilibrium profit $\pi^*(c)$ to be convex as follows:

$$\pi^{*''}(c) = \frac{dD_1(p^*,p^*)}{dp^*} p^{*'}(c)\left[p^{*'}(c)\eta_r - 1\right] + D_1(p^*,p^*))\left[p^{*''}(c)\eta_r + p^{*'}(c)\eta_r'\right] > 0$$

$$\underset{\text{since } D_1=\frac{1}{2}D}{\Longleftrightarrow} \frac{D'(p^*(c))}{D(p^*(c))} p^{*'}(c) \left[p^{*'}(c)\eta_r - 1\right] + \left[(\eta_r' - \alpha') \left[p^{*'}(c)\right]^3 \eta_r + p^{*'}(c)\eta_r'\right] > 0$$

$$\Longleftrightarrow \frac{-D'(p^*(c))}{D(p^*(c))} \left[1 - p^{*'}(c)\eta_r\right] + \left[(\eta_r' - \alpha') \left[p^{*'}(c)\right]^2 \eta_r + \eta_r'\right] > 0$$

$$\Longleftrightarrow \eta(p^*) \left[1 - \frac{\eta_r(p^*)}{2 - \eta_r(p^*) + \alpha(p^*)}\right] + p^* \left[\frac{\eta_r(p^*)\left[\eta_r'(p^*) - \alpha'(p^*)\right]}{\left[2 - \eta_r(p^*) + \alpha(p^*)\right]^2} + \eta_r'(p^*)\right] > 0.$$

Denote the equilibrium profits of each firm under uniform pricing by $\pi^u = \pi^*(\bar{c})$, and under differential pricing by $\pi^d = \lambda\pi^*(c_L) + (1-\lambda)\pi^*(c_H)$. Then, when $\pi^{*''}(c) \gtreqless 0$,

$$\pi^u = \pi^*(\bar{c}) \lesseqgtr \lambda\pi^*(c_L) + (1-\lambda)\pi^*(c_H) = \pi^d.$$

Therefore, for $\eta_r = \eta_r(p)$, $\alpha = \alpha(p)$, and $\eta = \eta(p)$: $\pi^{*''}(c) > 0$ or $\pi^d > \pi^u$ if (21) holds, and $\pi^d < \pi^u$ if (21) is reversed. ■

Proof of Proposition 3. From (23):

$$\begin{aligned} w^{*''}(c) &= -D'(p^*(c))p^{*'}(c)\left[1 + p^{*\prime}(c)(1-\hat{\eta}_r)\right] - D(p^*(c))\left[p^{*''}(c)(1-\hat{\eta}_r) - p^{*\prime}(c)\hat{\eta}_r'\right] \\ &= -D'(p^*(c))p^{*'}(c)\left[1 + p^{*\prime}(c)(1-\hat{\eta}_r)\right] \\ &\quad -D(p^*(c))\left[(\eta_r' - \alpha')\left[p^{*'}(c)\right]^3(1-\hat{\eta}_r) - p^{*\prime}(c)\hat{\eta}_r'\right] \\ &\gtreqless 0 \end{aligned}$$

$$\Longleftrightarrow \frac{-D'(p^*(c))}{D(p^*(c))}\left[1 + p^{*\prime}(c)(1-\hat{\eta}_r)\right] - \left[\left(\eta_r' - \alpha'\right)\left[p^{*'}(c)\right]^2(1-\hat{\eta}_r) - \hat{\eta}_r'\right] \gtreqless 0 \Longleftrightarrow$$

$$\eta(p^*)\left[1 + \frac{1 - \hat{\eta}_r(p^*)}{2 - \eta_r(p^*) + \alpha(p^*)}\right] + p^*\left[\frac{\left[\alpha'(p^*) - \eta_r'(p^*)\right](1 - \hat{\eta}_r(p^*))}{\left[2 - \eta_r(p^*) + \alpha(p^*)\right]^2} + \hat{\eta}_r'(p^*)\right] \gtreqless 0.$$

When $w^{*''}(c) \gtreqless 0$, we have

$$\begin{aligned} W(p^*(\bar{c})) &= w^*(\bar{c}) = w^*(\lambda c_L + (1-\lambda)c_H) \\ &\lesseqgtr \lambda w^*(c_L) + (1-\lambda)w^*(c_H) \\ &= \lambda W(p^*(c_L)) + (1-\lambda)W(p^*(c_H)). \end{aligned} \tag{31}$$

■

Proof of Lemma 1. (i) From (25) and (26):

$$p^d \equiv \lambda p_L + (1-\lambda)p_H = \lambda c_{2L} + (1-\lambda)c_{2H} \equiv p^u.$$

Then $S^d > S^u$ holds, from Remark 1.

(ii) For profit we only need consider firm 1, since the higher-cost rival earns no profit under either pricing regime. Given $p^d = p^u$, linear demand implies that total output also remains unchanged:

$$q^d = \lambda D(p_L) + (1-\lambda)D(p_H) = D(\lambda p_L + (1-\lambda)p_H) = D(p^u) = q^u.$$

With $p^d = p^u$ and $q^d = q^u$, (8) implies that $sign(\Pi^d - \Pi^u) = sign(m_L - m_H)$. Since $p_L = c_{2L}$ and $p_H = c_{2H}$, we have

$$m_L - m_H = (c_{2L} - c_{1L}) - (c_{2H} - c_{1H}) = c_{1H} - c_{1L} - (c_{2H} - c_{2L}) \equiv \Delta c_1 - \Delta c_2.$$

(iii) From (25) and (26), the prices p_L, p_H and p^u are determined by firm 2's marginal costs independent of the curvature of $D(p)$. Suppose $D(p)$ is strictly convex. Consider the linear demand $L(p)$ that is tangent to $D(p)$ at p^u.[19] Uniform pricing yields the same price and output with $L(p)$ or $D(p)$, hence the same profit and welfare. But under differential pricing, since $p^d = p^u$, outputs in both markets will be greater with $D(p)$ than with $L(p)$. Since firm 1's margins in both markets are positive ($p_L = c_{2L} > c_{1L},\ p_H = c_{2H} > c_{1H.}$), profit and total welfare will be higher with $D(p)$ than with $L(p)$. The reverse holds if $D(p)$ is strictly concave. ■

Proof of Lemma 2. (i) Price. $p^d \equiv \lambda c_{2L} + (1-\lambda)\, c_{1H} > \lambda c_{2L} + (1-\lambda)\, c_{2H} \equiv \bar{c}_2 = p^u$.

(ii) Consumer surplus.

$$S^u = s\left(\bar{c}_2\right) = \int_{\bar{c}_2}^{\infty} D\left(p\right) dp$$

[19] The ensuing argument is inspired by Malueg (1994).

$$S^d = \lambda s(c_{2L}) + (1-\lambda) s(c_{1H}) = \lambda \int_{c_{2L}}^{\infty} D(p)\,dp + (1-\lambda) \int_{c_{1H}}^{\infty} D(p)\,dp$$

When $\Delta_L \equiv c_{2L} - c_{1L} \to 0$ and $\Delta_H \equiv c_{1H} - c_{2H} \to 0$, $\lambda c_{2L} + (1-\lambda) c_{1H} \to \bar{c}_2$, and hence, because $s(p)$ is strictly convex,

$$S^u = s(\bar{c}_2) \to s(\lambda c_{2L} + (1-\lambda) c_{1H}) < \lambda s(c_{2L}) + (1-\lambda) s(c_{1H}) = S^d.$$

On the other hand,

$$S^d - S^u = \lambda \int_{c_{2L}}^{\bar{c}_2} D(p)\,dp - (1-\lambda) \int_{\bar{c}_2}^{c_{1H}} D(p)\,dp < 0$$

when $c_{2L} \to c_{2H}$ so that $c_{2L} \to \bar{c}_2$.

(iii). Profits**.** Under uniform pricing, firm 2's profit is zero, but under differential pricing, each firm earns positive profit. Total profits under the two regimes are

$$\Pi^u = (\bar{c}_2 - \bar{c}_1)D(\bar{c}_2)$$
$$\Pi^d = \pi_1^d + \pi_2^d = \lambda(c_{2L} - c_{1L})D(c_{2L}) + (1-\lambda)(c_{1H} - c_{2H})D(c_{1H})$$

Thus,

$$\begin{aligned}
\Pi^d - \Pi^u &= \lambda(c_{2L} - c_{1L})D(c_{2L}) - (\bar{c}_2 - \bar{c}_1)D(\bar{c}_2) + (1-\lambda)(c_{1H} - c_{2H})D(c_{1H}) \\
&> \lambda(c_{2L} - c_{1L})D(c_{2L}) - \lambda(c_{2L} - c_{1L})D(\bar{c}_2) - (1-\lambda)(c_{2H} - c_{1H})D(\bar{c}_2) \\
&= \lambda(c_{2L} - c_{1L})\left[D(c_{2L}) - D(\bar{c}_2)\right] + (1-\lambda)(c_{1H} - c_{2H})D(\bar{c}_2) > 0.
\end{aligned}$$

■

Proof of Proposition 5. From (29) and (27), the equilibrium outputs by the two firms under UP are:

$$q_1^u = \frac{2a - (2-\gamma^2)\bar{c}_1 + \gamma(a + \bar{c}_2)}{4-\gamma^2}, \qquad q_2^u = \frac{2a - (2-\gamma^2)\bar{c}_2 + \gamma(a + \bar{c}_1)}{4-\gamma^2}.$$

Similarly, From (30) and (27), the equilibrium outputs by the two firms under DP, for $k = L, H$, are:

$$q_{1k} = \frac{2a - (2-\gamma^2)c_{1k} + \gamma(a + c_{2k})}{4-\gamma^2}, \qquad q_{2k} = \frac{2a - (2-\gamma^2)c_{2k} + \gamma(a + c_{1k})}{4-\gamma^2}.$$

It follows that the output for each firm under DP are

$$q_1^d = \lambda q_{1L} + (1-\lambda) q_{1H} = q_1^u; \qquad q_2^d = \lambda q_{2L} + (1-\lambda) q_{2H} = q_2^u.$$

Consumer surplus as a function of (q_1, q_2) is

$$\begin{aligned} S &\equiv \tilde{s}(q_1, q_2) = U(q_1, q_2) - p_1 q_1 - p_2 q_2 \\ &= \frac{a(1+\gamma)(q_1+q_2) - \frac{1}{2}\left(q_1^2 + 2\gamma q_1 q_2 + q_2^2\right)}{1-\gamma^2} \\ &\quad - \frac{a(1+\gamma) - q_1 - \gamma q_2}{1-\gamma^2} q_1 - \frac{a(1+\gamma) - q_2 - \gamma q_1}{1-\gamma^2} q_2 \\ &= \frac{1}{2}\frac{q_1^2 + q_2^2 + 2\gamma q_1 q_2}{1-\gamma^2}. \end{aligned}$$

We have

$$\frac{\partial^2 \tilde{s}(q_1, q_2)}{\partial q_1^2} = \frac{1}{1-\gamma^2} = \frac{\partial^2 s(q_1, q_2)}{\partial q_2^2} > 0,$$

$$\frac{\partial^2 \tilde{s}(q_1, q_2)}{\partial q_1 \partial q_2} = \frac{\gamma}{1-\gamma^2} = \frac{\partial^2 s(q_1, q_2)}{\partial q_2 \partial q_1}, \qquad \left(\frac{1}{1-\gamma^2}\right)^2 - \left(\frac{\gamma}{1-\gamma^2}\right)^2 > 0.$$

Thus $\tilde{s}(q_1, q_2)$ is strictly convex. Therefore DP raises consumer surplus because

$$\begin{aligned} S^u &= \tilde{s}(q_1^u, q_2^u) = s\left(q_1^d, q_2^d\right) = \tilde{s}(\lambda q_{1L} + (1-\lambda) q_{1H},\ \lambda q_{2L} + (1-\lambda) q_{2H}) \\ &< \lambda \tilde{s}(q_{1L}, q_{2L}) + (1-\lambda)\tilde{s}(q_{1H}, q_{2H}) = S^d. \end{aligned}$$

Equilibrium industry profit, when firm $i's$ marginal cost is c_i for $i = 1, 2$, is

$$\Pi \equiv \tilde{\pi}(c_1, c_2) = \frac{\left(2a - 2c_1 + a\gamma + \gamma c_2 + \gamma^2 c_1\right)^2}{(\gamma-2)^2(\gamma+2)^2} + \frac{\left(2a - 2c_2 + a\gamma + \gamma c_1 + \gamma^2 c_2\right)^2}{(\gamma-2)^2(\gamma+2)^2}.$$

Then

$$\frac{\partial^2 \tilde{\pi}(c_1, c_2)}{\partial c_1^2} = 2\frac{-3\gamma^2 + \gamma^4 + 4}{(\gamma-2)^2(\gamma+2)^2} = \frac{\partial^2 \tilde{\pi}(c_1, c_2)}{\partial c_2^2} > 0, \qquad \frac{\partial^2 \tilde{\pi}}{\partial c_2 \partial c_1} = \frac{4\gamma\left(\gamma^2 - 2\right)}{(\gamma-2)^2(\gamma+2)^2},$$

$$\left(2\frac{-3\gamma^2 + \gamma^4 + 4}{(\gamma-2)^2(\gamma+2)^2}\right)^2 - \left(4\gamma\frac{\gamma^2 - 2}{(\gamma-2)^2(\gamma+2)^2}\right)^2 = 4(\gamma-1)^2\frac{(\gamma+1)^2}{(\gamma-2)^2(\gamma+2)^2} > 0.$$

Thus $\tilde{\pi}(c_1, c_2)$ is strictly convex, and hence DP raises industry profit because

$$\begin{aligned}\Pi^u &= \pi(\bar{c}_1, \bar{c}_2) = \pi(\lambda c_{1L} + (1-\lambda) c_{1H},\ \lambda c_{2L} + (1-\lambda) c_{2H}) \\ &< \lambda \pi(c_{1L}, c_{2L}) + (1-\lambda) \pi(c_{1H}, c_{2H}) = \Pi^d.\end{aligned}$$

It follows that total welfare is higher under DP than under UP: $W^d > W^u$. ■

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