Preferences, Prices, and Performance in Monopoly and Duopoly*

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Abstract: This paper develops a new approach to discrete choice demand in multiproduct industries, using copulas to separate the marginal distribution of consumer values for each product from their dependence relationship. Application of the copula approach to monopoly and duopoly markets reveals how price, profit and consumer welfare vary with the strength, diversity, and dependence of preferences. The comparative statics of demand strength and preference diversity, both properties of the marginal distribution, are remarkably similar across market structures. Preference dependence, disintangled from preference diversity as a distinct indicator of product differentiation, is a key determinant of how prices differ between multiproduct industries and single-product monopoly.

Keywords: Multiproduct industry, discrete choice demand, copula.

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1. INTRODUCTION

This paper develops a new framework for studying how consumer preferences determine equilibrium firm conduct and market performance in multiproduct industries. At the heart of this new framework is the copula approach to modelling the distribution of consumer preferences in a discrete choice model of product differentiation. This approach disintangles the effects of the marginal distributions of consumer values for a product variety from the dependence relations between varieties captured by a copula. Our analysis uncovers several unifying principles in the economics of monopoly and duopoly, extending and developing new insights for economic literatures on product differentiation, pricing, and business strategy.

Our main model of product differentiation considers two symmetric varieties of a good. We focus on three dimensions of consumer preferences: demand strength, preference diversity, and preference dependence. Demand strength and preference diversity are measured respectively by the mean (μ) and variance (σ) of the marginal distribution of consumer values for each variety. Preference dependence is measured by a parameter (θ) ordering a copula family according to conditional stochastic dominance. Loosely speaking, greater preference dependence means that more consumers regard the two varieties as close substitutes. Thus preference dependence is an intuitive measure of the degree of product differentiation.

The copula framework advances the standard approach to product differentiation in discrete choice demand models by relaxing in a neat way the typical assumption of independent consumer values. Under more general conditions of preference dependence, we re-examine firm conduct and market performance in a standard discrete choice model of consumer demand. The market can be either a monopoly or a duopoly, and each firm produces either one or two goods. This gives rise to three possible market structures of interest: single-product monopoly, horizontally-differentiated multiproduct monopoly, and horizontally-differentiated duopoly. The strategic variables are prices, which firms choose simultaneously under duopoly competition.

We show how demand strength and preference diversity affect equilibrium prices, profits, and consumer welfare for monopoly and duopoly market structures. First, prices, profits and consumer surplus are all higher with stronger demand. Second, firm
profits increase in preference diversity for "low-demand" products ($\mu \leq 0$); while for “high-demand” products ($\mu > 0$) profits exhibit a U-shaped relationship with $\sigma$, first decreasing and then increasing. Third, prices and consumer surplus both increase in preference diversity if $\mu \leq 0$. These comparative-static results are similar across all three market structures and under various preference dependence conditions, providing unifying principles in the economics of monopoly and duopoly. Moreover, the effects of preference diversity on profits clarify a key result in Johnson and Myatt (2006) for "variance-ordered distributions" for a single-product monopoly, and, under general preference dependence, extend their insights to horizontally-differentiated multiproduct monopoly and to horizontally-differentiated price-setting duopoly.

We also consider how preference dependence affects prices and profits in multiproduct industries. The standard approach to discrete-choice models of differentiated oligopoly, pioneered by Perloff and Salop (1985), typically assumes independence between consumer values for different varieties. Our analysis advances the literature on product differentiation by showing that, under certain sufficient conditions, both prices and profits in multiproduct industries decrease as preferences become more positively dependent or less negatively dependent, suggesting that preference dependence is a useful dimension along which to measure product differentiation.

Furthermore, our analysis leads to two new results concerning how prices differ across market structures: (i) Extending Chen and Riordan (2008), we find that single-monopoly price is higher than symmetric duopoly price if the hazard rate of the marginal distribution is non-decreasing and preferences are positively dependent, but lower if the hazard rate is non-increasing and preferences are negatively dependent. (ii) The symmetric multiproduct monopoly price is higher than the single-product

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1 The results require appropriate regularity conditions for each market structure. The comparative statics of prices and duopoly profits require somewhat stronger sufficient conditions because of the strategic interdependence of price decisions.

2 Johnson and Myatt (2006) show insightfully that firm profit is maximized with either minimum or maximum preference diversity for a single-product monopolist, and, under the assumption of preference independence between varieties, for a multiproduct monopoly with vertically differentiated products and for a quantity-setting oligopoly. They find interesting implications of this result for business strategy in areas such as advertising, marketing, and product design.

3 Anderson, dePalma, and Thisse (1992) provides an excellent overview of discrete-choice models of product differentiation.

4 As we noted above, preference diversity, the usual measure of product differentiation used in the literature under preference independence, has non-monotonic relations with profits if $\mu > 0$. 
monopoly price, provided that preferences possess a uniform dependence property (i.e., if preferences are uniformly either positively dependent, or independent, or negatively dependent).

We formulate our main model in Section 2, establish the comparative statics of preferences under various market structures in Section 3, and compare prices across markets in Section 4. Section 5 illustrates our findings and additional results by numerically analyzing a particular class of preferences described by exponential marginal distributions and the Fairlie-Gumbel-Morgenstern (FGM) family of copulas. Section 6 concludes. Proofs are in the Appendix.

2. PREFERENCES AND DEMAND

Consumers are assumed to purchase at most one of two possible varieties of a good, referred to as X and Y. A consumer’s utility for X is \( w(x) \) and for Y is \( w(y) \), with \( x \) and \( y \) each uniformly distributed on \([0, 1]\). For convenience, \( w(\cdot) \) is assumed to be a strictly-increasing and twice-differentiable function, with a subinterval on which \( w(x) > 0 \). The utility of the "outside good" is normalized to zero. If \( p \) is the price of X and \( r \) is the price of Y, then a type \((x, y)\) consumer purchases X if \( w(x) - p \geq \max\{w(y) - r, 0\} \) and Y if \( w(y) - r > \max\{w(x) - p, 0\} \). If only X is available, then the consumer purchases it if \( w(x) - p \geq 0 \).

The population of consumers, whose size is normalized to 1, is described by a symmetric copula \( C(x, y) \), which is a bivariate uniform distribution satisfying \( C(x, 1) = x = C(1, x) \), and \( C(x, 0) = 0 = C(0, x) \). For convenience, \( C(x, y) \) is assumed to be twice differentiable on \([0, 1]^2\) with a joint density given by \( C_{12}(x, y) \equiv \frac{\partial^2 C(x, y)}{\partial x \partial y} \). The copula determines the statistical dependence of consumer values for the two varieties. In particular, \( C_1(x, y) \equiv \frac{\partial C(x, y)}{\partial x} \) is the conditional distribution of \( y \) given \( x \), and \( C_{11}(x, y) \equiv \frac{\partial^2 C(x, y)}{\partial x^2} < 0 \) (\( > 0 \)) indicates positive (negative) stochastic dependence. The independence copula is \( C(x, y) = xy \). Positive (negative) stochastic dependence implies positive (negative) quadrant dependence, i.e. \( C(x, y) > xy \) (\( C(x, y) < xy \)).

We parameterize the preference distribution along three dimensions: demand strength,
preference diversity, and preference dependence. Let

\[ \mu = \int_0^1 w(x)dx \quad \text{and} \quad \sigma^2 = \int_0^1 [w(x) - \mu]^2 dx \]

denote the mean and variance of consumer values, and define the normalized utility \( u(x) = \frac{w(x) - \mu}{\sigma} \). Assume the marginal distribution of consumer values for X is \( F\left(\frac{w_x - \mu}{\sigma}\right) \equiv u^{-1}\left(\frac{w_x - \mu}{\sigma}\right) \), where \( w_x \equiv w(x) \); and similarly for Y.\(^6\) The joint distribution of values can then be written as \( C\left(F\left(\frac{w_x - \mu}{\sigma}\right), F\left(\frac{w_y - \mu}{\sigma}\right)\right) \).\(^7\) A family of copulas, \( C(x, y; \theta) \), indexed by parameter \( \theta \), satisfies the monotonic dependence ranking property (MDR) if \( \partial C_{11}(x, y; \theta) / \partial \theta \equiv C_{11\theta}(x, y; \theta) < 0 \) for interior \((x, y)\). MDR implies \( C_\theta(x, y; \theta) \equiv \partial C(x, y; \theta) / \partial \theta > 0 \) (Nelsen, 2006).

Summarizing, the distribution of consumer preferences for the two goods is completely characterized by the marginal distribution function \( F(u) \), the copula \( C(x, y; \theta) \),\(^8\) and parameters \( \mu \) and \( \sigma \) which respectively measure demand strength and preference diversity. The copula indicates the nature of preference dependence, and, for copula families satisfying MDR, parameter \( \theta \) further measures the degree of preference dependence. Introduced by Chen and Riordan (2008), the copula approach to product variety has the advantage of disintangling the effects of the marginal distribution of consumer values for each variety from their dependence relationship.

**Bivariate exponential case.** We shall use the following parametric functions to illustrate results:

\[ u(x) = -\left[1 + \ln(1 - x)\right], \quad (1) \]

\[ C(x, y) = xy + \theta xy(1 - x)(1 - y). \quad (2) \]

\(^6\)The support for \( F(\cdot) \) is extended in the usual way, i.e., \( F(u) = 0 \) for \( u < u(0) \) and \( F(u) = 1 \) for \( u > u(1) \), unless \( u(0) = -\infty \) and/or \( u(1) = \infty \).

\(^7\)By Sklar’s Theorem (Nelsen, 2006), it is without loss of generality to represent joint distribution of consumers’ values for two products by a copula and marginal distributions. However, our symplifying assumptions do confine our analysis to all joint distributions that are symmetric and have the same form of marginal distributions for X and Y.

\(^8\)The parameter \( \theta \) is suppressed notationally for results that hold \( \theta \) fixed or that do not require the MDR property.
In this case, the normalized utility $u = u(x)$ has an exponential distribution

$$F(u) = 1 - e^{-u-1}$$  \hspace{1cm} (3)$$

with $E\{u\} = 0$, and $Var(u) = 1$. The copula belongs to the Fairlie-Gumbel-Morgenstern (FGM) family for which $\theta \in [-1, 1]$. Members of the FGM family exhibit positive stochastic dependence if $\theta > 0$, negative dependence if $\theta < 0$, and independence if $\theta = 0$. The FGM copula density is

$$C_{12}(x, y) = 1 + \theta(2x - 1)(2y - 1).$$ \hspace{1cm} (4)$$

The resulting bivariate exponential distribution $C(F(u), F(u))$ was introduced by Gumbel (1960).\(^9\) Our numerical analyses focuses on the extreme cases of positive and negative dependence ($\theta = 1$ and $\theta = -1$) and independence ($\theta = 0$).\(^{10}\)

The demand for X is calculated by integrating over the acceptance set;

$$Q(\bar{p}, \bar{r}; \theta) = 1 - C(F(\bar{p}), F(\bar{r}); \theta) - \int_{F(\bar{r})}^{1} C_{2}(F(u(x) + \bar{p} - \bar{r}), y; \theta) \, dy$$ \hspace{1cm} (5)$$

$$= \int_{F(\bar{p})}^{1} C_{1}(x, F(u(x) + \bar{r} - \bar{p}); \theta) \, dx$$

where $\bar{p} \equiv \frac{p - \mu}{\sigma}$ and $\bar{r} \equiv \frac{r - \mu}{\sigma}$. The demand for Y is calculated similarly. Thus, the two good are always substitutes because

$$\frac{\partial Q(\bar{p}, \bar{r}; \theta)}{\partial F} = \int_{F(\bar{p})}^{1} C_{12}(x, F(u(x) + \bar{r} - \bar{p}); \theta) \, f(u(x) + \bar{r} - \bar{p}) \, dx > 0. \hspace{1cm} (6)$$

If only good X is available, then its demand is obtained by setting $F(\bar{r}) = 1$ in (5), or

$$Q^{g}(\bar{p}) = 1 - F(\bar{p}).$$ \hspace{1cm} (7)$$

\(^{9}\)Gumbel (1960) introduced several bivariate exponential distributions. This is the second one.

\(^{10}\)The FGM family of copulas has a limited range of positive and negative dependence; so the extreme cases, $\theta = 1$ and $\theta = -1$, do not indicate perfectly positive and negative dependence.
As discussed in Chen and Riordan (2008), the introduction of product Y at the same price has two effects on demand for product X. The "market share effect" is a downward shift in the demand for product X, as some consumers switch to a more attractive alternative. The "price sensitivity effect" is change in the slope of the demand curve for product X.

While complementary to classical demand theory, the copula approach has the advantage of linking endogenous demand elasticities to more fundamental properties of preference distributions. First, the price elasticity of demand for a single available product is a function on demand strength and preference diversity:

$$
\eta^X \equiv \left| \frac{p}{Q^o(\bar{p})} \frac{\partial Q^o(\bar{p})}{\partial p} \right| = \lambda(\bar{p}) \frac{p}{\sigma} \quad (8)
$$

where $\lambda(\bar{p}) \equiv \frac{f(\bar{p})}{1-F(\bar{p})}$ is the hazard rate and $p > 0$. $\eta^X$ decreases with $\mu$ if and only if $\lambda(\bar{p})$ is increasing. $\eta^X$ decreases with $\sigma$ if $p \geq \mu$ and $\lambda(\bar{p})$ is non-decreasing, or if $p \leq \mu$ and $\lambda(\bar{p})$ is non-increasing function; the relationship is ambiguous otherwise. Second, with both varieties available, the cross- and own-price elasticities are:

$$
\eta^{XY} \equiv \left| \frac{\partial Q(\bar{p}, \bar{r}; \theta)}{\partial r} \frac{r}{Q(\bar{p}, \bar{r}; \theta)} \right| = \left[ \int_{F(\bar{p})}^{1} C_{12} (x, F(u(x) + \bar{r} - \bar{p}), x; \theta) f(u(x) + \bar{r} - \bar{p}) dx \right] \frac{r}{\sigma Q(\bar{p}, \bar{r}; \theta)}
$$

$$
\eta^{XX} \equiv \left| \frac{\partial Q(\bar{p}, \bar{r}; \theta)}{\partial p} \frac{p}{Q(\bar{p}, \bar{r}; \theta)} \right| = \eta^{XY} \frac{p}{r} + \frac{C_1(F(\bar{p}), F(\bar{r}); \theta) f(\bar{p}) p}{\sigma Q(\bar{p}, \bar{r}; \theta)}
$$

If the two variants are priced identically, i.e. $\bar{r} = \bar{p}$, these simplify to:

$$
\eta^{XY*} = \frac{2p}{\sigma [1 - C(F(\bar{p}), F(\bar{p}); \theta)]}.
$$
\[
\eta^{XX^*} = \eta^{XY^*} + \frac{2pC_1(F(p), F(p); \theta) f(p)}{\sigma [1 - C(F(p), F(p); \theta)]}
\]

Note that \(\eta^{XX^*}\) depends on price sensitivity at both the intensive and extensive margins, i.e. both the density of consumers indifferent between X and Y and those indifferent between X and the outside good, whereas \(\eta^{XY^*}\) only depends on price sensitivity at the intensive margin. A sufficient condition for \(\eta^{XY^*}\) to decrease as \(\theta\) increases is

\[
\frac{1}{t} \int C_{12\theta}(x, x; \theta) f(u(x)) dx \geq 0 \tag{9}
\]

for any \(t \in [0, 1]\), where \(C_{12\theta}(x, y; \theta) \equiv \partial C_{12}(x, y; \theta)/\partial \theta\). Condition (9) holds if greater preference dependence is associated with greater density of consumer preferences along the diagonal of the type space, meaning that more purchasing consumers value the two goods similarly and therefore are sensitive to a unilateral price cut starting from a symmetric situation. While \(C_{12\theta}(x, x; \theta) \geq 0\) is an intuitive property, it does not appear to be a general implication of MDR.\(^{11}\) The condition does hold for an FGM copula, for which

\[
C_{12\theta}(x, x; \theta) = (2x - 1)^2 \geq 0. \tag{10}
\]

### 3. COMPARATIVE STATICS

The copula approach enables us to derive new results on how prices, profits, and consumer welfare vary with the strength (\(\mu\)), diversity (\(\sigma\)) and dependence (\(\theta\)) of the distribution of consumer preferences. As we shall discuss later, these comparative-static results have interesting implications and broad applications. To proceed, we make a couple of additional simplifying assumptions for both this and the next section. First, the average cost of production for each variety is constant, and without loss of generality normalized to zero. An appropriate interpretation of the normalization is that consumers reimburse the firm for the cost of producing the product in addition to paying a markup \(p\). Consequently, \(\mu\) can be interpreted as stand-

\(^{11}\)To understand the ambiguity, observe that \(C_{12\theta}(x, x; \theta) = dC_{1\theta}(x, x; \theta)/dx - C_{11\theta}(x, x; \theta)\). While \(-C_{11\theta}(x, x; \theta) > 0\) by MDR, \(dC_{1\theta}(x, x; \theta)/dx\) may be either positive or negative. In the FGM case, \(dC_{1\theta}(x, x; \theta)/dx < 0\) in an intermediate range of \(x\), although \(-C_{11\theta}(x, x; \theta) > 0\) still dominates in this range.
ing for mean demand minus the constant average variable cost, and naturally can be either positive or negative.\(^{12}\) Second, equilibrium prices exist uniquely and are interior under all market structures.\(^{13}\) Together with the symmetry of \(C(\cdot, \cdot)\), this simplifying assumption implies that equilibrium is symmetric. These maintained assumptions facilitate comparative statics and comparisons of outcomes for different market structures.

**Monopoly**

We start with the familiar case of a single-product monopolist who produces \(X\). While this does not require a consideration of preference dependence, our analyses of various cases nevertheless have a common structure in key respects, and the single-product monopoly case will also serve as a point of comparison for firm conduct. For this reason, it is worthwhile to elucidate carefully the single-product monopoly problem. Furthermore, our analysis of this base case clarifies and expands aspects of Johnson and Myatt (2006)'s seminal analysis of the effect of preference dispersion on monopoly profit.

The structure of our analysis hinges on the price normalization introduced in our discussion of demand. The single-product monopolist's (gross) profit function is

\[
\pi^m (\bar{p}) = \sigma (\bar{p} + \bar{\mu}) [1 - F(\bar{p})] 
\]

where \(\bar{p}\) is the normalized price and \(\bar{\mu} \equiv \frac{\mu}{\sigma}\) is the strength-diversity ratio. The profit-maximizing normalized price \((\bar{p}^m)\) satisfies

\[
(\bar{p}^m + \bar{\mu}) \lambda (\bar{p}^m) = 1
\]

at an interior solution, where \(\lambda (\bar{p})\) is the hazard rate determining the elasticity of

\(^{12}\)This interpretation requires an appropriate adjustment of the elasticity formulas. Suppose consumers pay \(p + c\) where \(c \geq 0\) is the constant marginal cost of production. Then the single-product price elasticity of demand becomes \(\lambda (\bar{p}) \frac{p + c}{\sigma} \equiv \frac{p + c}{\bar{p}} \eta^X\) with \(\eta^X\) given by (8). The other elasticity formulas require similar adjustment.

\(^{13}\)For convenience, we refer to optimal prices under monopoly as equilibrium prices. An interior price satisfies \(p \in (w(0), w(1))\), so the market is neither shut down nor fully covered. Consequently, profit functions are differentiable at equilibrium prices.
demand. The following regularity condition guarantees a unique local maximum.

**A1:** \( d [(\bar{p} + \bar{\mu}) \lambda (\bar{p})] / d\bar{p} > 0. \)

Thus the familiar assumption of a monotonically increasing hazard rate \( \lambda'(u) \geq 0 \) is sufficient but not necessary for **A1**. The maximum profit is \( \pi^m \equiv \pi^m (\bar{p}^m) \) and the consumer welfare is

\[
w^m = \sigma \int_{\bar{p}^m}^{u(1)} [1 - F(\bar{p})] d\bar{p}.
\]

(13)

Defining normalized profits and consumer surplus as \( \pi^n = \pi^m / \sigma \) and \( w^n = w^m / \sigma \), we first establish how normalized price, profit, and consumer welfare vary with the strength-diversity ratio.

**Lemma 1** Given **A1**: (i) \( \frac{d \pi^n}{d \bar{\mu}} < 0 \), and there exists some \( \bar{\mu}^m > 0 \) such that \( \bar{p}^m \gg 0 \) if \( \bar{\mu} \leq \bar{\mu}^m \); (ii) \( \frac{d \pi^n}{d \mu} > 0 \); and (iii) \( \frac{d \pi^n}{d \bar{\mu}} > 0 \).

The lemma characterize the normalized price, profit, and consumer welfare effects of a shift in the strength-diversity ratio. Part (i) follows easily from **A1**: the function \( (\bar{p} + \bar{\mu}) \lambda (\bar{p}) \) is increasing in \( \bar{p} \) and crosses 1 once from below; and increase in \( \bar{\mu} \) shifts up the entire function and moves the intersection point to the left. Part (ii) follows quickly from applying the envelope theorem to (11), and part (iii) follows from simple differentiation and part (i).

Lemma 1 leads immediately to the following proposition establishing how single-product monopoly conduct and performance depend on demand strength and preference diversity.

**Proposition 1** Given **A1**: (i) \( \frac{d \pi^m}{d \mu} \gg 0 \) if \( \mu' (\cdot) \gg 0 \); \( \frac{d \pi^m}{d \sigma} > 0 \); \( \frac{d w^m}{d \mu} > 0 \); (ii) \( \frac{d \pi^m}{d \sigma} > 0 \) if \( \bar{\mu} \leq \bar{\mu}^m \) and \( \mu' (\cdot) \geq 0 \); \( \frac{d \pi^m}{d \sigma} \ll 0 \) if \( \bar{\mu} \gg \bar{\mu}^m \); \( \frac{d w^m}{d \sigma} > 0 \) if \( \bar{\mu} \leq 0 \).

As one might expect, profit and consumer welfare under single-product monopoly are both increasing in demand strength, and so is monopoly price if the hazard rate is increasing. If we interpreting \( \mu \) as mean consumer value net of constant marginal cost, then \( \frac{d \pi^m}{d \bar{\mu}} \) is the pass-through rate, i.e. the rate at which an increase in translates to a higher markup. The amount of pass-through depends on the slope of the hazard.
rate, i.e. \( \lambda' > (\leq) 0 \) corresponds to less (more) than full pass through as discussed by Weyl and Fabinger (2009).

Johnson and Myatt (2006) studied families of "variance-ordered" distributions for which demand strength is a differentiable function of preference dispersion, i.e. \( \mu = \mu(\sigma) \) with \( \mu'(\sigma) \geq 0 \). An important result of their analysis under this assumption is that \( \pi^m \) is a quasi-convex function of \( \sigma \), which implies that a single-product monopolist seeks either to maximize or minimize preference diversity.\(^{14}\) Part (ii) of Proposition 1 details this result for the special case of constant demand strength, i.e. \( \mu'(\sigma) = 0 \), and clarifies for this case that the monopolist seeks to increase preference diversity when demand is relatively weak (\( \mu < \sigma \bar{\mu}^m \)) but seeks to decrease diversity when demand is relatively strong (\( \mu > \sigma \bar{\mu}^m \)). Thus, since \( \bar{\mu}^m > 0 \) from Lemma 1, \( \pi^m \) is increasing in \( \sigma \) for \( \mu \leq 0 \), but is a U-shaped function of \( \sigma \) when \( \mu > 0 \), first decreasing and then increasing.

Our analysis goes further than Johnson and Myatt (2006) by explicitly considering the consumer welfare effects of demand strength and preference dispersion. Part (ii) of Proposition 1 shows that the firm’s incentive to increase \( \sigma \) for weak-demand products (\( \mu \leq 0 \)) coincides with the consumer interests. In those cases, even though higher \( \sigma \) leads to higher prices, it also leads to higher output and to higher average values for consumers who actually purchase. As a result, consumer welfare goes up. If \( \mu > 0 \), higher \( \sigma \) will still increase the values of consumers who purchase the product, but it now also reduces output, so the effect on consumer welfare is no longer clear cut. Nevertheless, if \( \mu > 0 \), \( \sigma \) is sufficiently large, and \( \lambda'(\cdot) \geq 0 \), then it can be shown that a further increase in \( \sigma \) increases both profit and consumer welfare.

We next turn to a price-setting multiproduct monopoly producing both varieties of the good,\(^{15}\) who optimally charges the same price for the two symmetric variants.

\(^{14}\)Johnson and Myatt (2006) establishes a preference for extremes for an even broader family of distributions ordered by a decreasing sequence of "rotation" points. Johnson and Myatt (2006) also showed that \( q^m \) is a convex function of \( \sigma \) if \( \mu'(\sigma) \geq 0 \). By equation (7), however, \( q^m \) is a decreasing function of \( \bar{\mu}^m \). Therefore, because \( dq^m/d\sigma = f(\bar{\mu}^m)[d\bar{\mu}^m/d\bar{\mu}][\mu/\sigma^2] \), part (i) of Lemma 1 clarifies for the constant mean case that \( q^m \) increases with \( \sigma \) if \( \mu < 0 \) and, conversely, \( q^m \) decreases with \( \sigma \) if \( \mu > 0 \).

\(^{15}\)Johnson and Myatt (2006) studied a quantity-setting monopoly selling a line of vertically-differentiated products to a one-dimensional population of consumers. In contrast, we study a price-setting monopoly selling horizontally-differentiated products to a two-dimensional population.
Defining \(\bar{p}\) and \(\bar{x}\) as before, the multiproduct monopolist’s profit function is

\[
\pi^{mm}(\bar{\rho}) = \sigma(\bar{\rho} + \bar{x}) [1 - C(F(\bar{\rho}), F(\bar{\rho}))].
\]  

(14)

The profit-maximizing normalized price \(\bar{\rho}^{mm}\) satisfies

\[
(\bar{\rho}^{mm} + \bar{\mu}) \lambda^C(\bar{\rho}^{mm}) = 1
\]

(15)

where

\[
\lambda^C(\bar{\rho}) \equiv \frac{2C_1(F(\bar{\rho}), F(\bar{\rho}))}{1 - C(F(\bar{\rho}), F(\bar{\rho}))} f(\bar{\rho}).
\]

(16)

A key observation is that \(\lambda^C(\bar{\rho})\) is in fact the hazard rate corresponding to cumulative distribution function \(F^C(\bar{\rho}) \equiv C(F(\bar{\rho}), F(\bar{\rho}))\) on support \([u(0), u(1)]\).

The appropriate regularity condition, which is satisfied in our FGM-exponential case, serves the same role as for the single-product monopoly case:

\textbf{A2}: \(d[(\bar{\rho} + \bar{\mu}) \lambda^C(\bar{\rho})] / d\bar{\rho} > 0\).

Note that the first-order condition (15) is the same as (12) for single-product monopoly, except for the difference in the relevant hazard rates. Therefore, the comparative statistics of \(\bar{\rho}^{mm}, \pi^{mm}, \) and \(\bar{\omega}^{mm}\) with respect to \(\bar{\mu}\) follow from Lemma 1, with the maximum profit being \(\pi^{mm} \equiv \sigma\bar{\pi}^{mm} \equiv \pi^{mm}(\bar{\rho}^{mm})\), and consumer surplus

\[
\bar{\omega}^{mm} \equiv \sigma\bar{\omega}^{mm} = \int_{\bar{\rho}^{mm}}^{u(1)} [1 - C(F(\bar{\rho}), F(\bar{\rho}))] d\bar{\rho}.
\]

The comparative statics with respect to \(\mu\) and \(\sigma\) are therefore essentially the same as under single-product monopoly and have the similar intuition:

\textbf{Proposition 2} Given A2: there exists \(\mu^{mm} > 0\) such that (i) \(\frac{d\pi^{mm}}{d\mu} \geq 0\) if \(\lambda^C(\mu) \geq 0\); \(\frac{d\pi^{mm}}{d\mu} > 0\); (ii) \(\frac{d\pi^{mm}}{d\sigma} > 0\) if \(\bar{\mu} \leq \mu^{mm}\) and \(\lambda^C(\cdot) \geq 0\); \(\frac{d\pi^{mm}}{d\sigma} \geq 0\) if \(\bar{\mu} \leq \mu^{mm}\); \(\frac{d\omega^{mm}}{d\sigma} > 0\) if \(\mu \leq 0\).

The important difference is that consumer demand for the multiproduct problem depends on preference dependence. To investigate how preference dependence affects
outcomes under multiproduct monopoly, we consider copula families satisfying MDR, and define \( \lambda^C (\tilde{p}; \theta) \) accordingly. A useful property of an MDR copula family is that the conditional copula \( C_1 (x, x; \theta) \) increases (decreases) in \( \theta \) when \( x \) is small (large). This implies that greater positive dependence shifts up the hazard rate for the multiproduct monopolist when market coverage is high enough.

**Lemma 2** Given MDR, there exists some \( u^* \in (u(0), u(1)) \) such that \( \frac{\partial \lambda^C (\tilde{p}; \theta)}{\partial \theta} > 0 \) if \( \tilde{p} \leq u^* \).

Furthermore, it is straightforward that the market is fully covered, or nearly so, if demand is sufficiently strong.\(^{16}\) This consideration leads to the conclusion that prices decrease with preference dependence if demand is sufficiently strong. The profit of the multiproduct monopolist, however, always decreases with greater dependence, whether or not price increases because of the resulting downward shift in demand.

**Proposition 3** Given \( A2 \) and MDR: (i) for any \( \theta \), there exists some \( \bar{\mu}^* \) such that \( p^{mm} > u(0) \) when \( \bar{\mu} = \bar{\mu}^* \) and \( \frac{dP^{mm}}{d\theta} < 0 \) if \( \bar{\mu} \geq \bar{\mu}^* \); and (ii) \( \frac{d\pi^{mm}}{d\theta} < 0 \).

Therefore, a multiproduct monopolist prefers that consumer values for its two products are less positively (more negatively) dependent. This is intuitive, since the more similar are product varieties the less valuable is a choice. Thus a higher \( \theta \) reduces output at any given price and hence reduces equilibrium profit, while the effect of \( \theta \) on equilibrium price is more subtle. The lower output under a higher \( \theta \) motivates the firm to lower price, but the slope of the demand curve also changes with \( \theta \), possibly having an opposing effect on price. Both effects work in the same direction if demand is sufficiently strong. It is possible, however, that \( p^{mm} \) increases with \( \theta \) if demand is sufficiently weak. For example, in the bivariate exponential case introduced above, numerical analysis shows that \( p^{mm} \) increases in \( \theta \) if \( \bar{\mu} \) is below a critical value.

The effects of greater preference dependence on consumer welfare \( (w^{mm}) \) is also ambiguous in general. On the one hand, a higher \( \theta \) shifts down the demand curve, thus reducing consumer surplus at any price. On the other hand, a higher \( \theta \) might

\(^{16}\)Let \( \bar{\mu}^o = \frac{1}{u(0)} - u(0) \). Then the market is fully covered for \( \bar{\mu} \geq \bar{\mu}^o \) and almost fully covered for \( \bar{\mu} = \bar{\mu}^o - \epsilon \) and \( \epsilon \) a small positive number. Our maintained interiority assumption implicitly assumes \( \bar{\mu} < \bar{\mu}^o \).
result in a lower price, as when demand is sufficiently strong, which increases consumer surplus given the demand curve. However, if more dependence leads to higher prices, as it is sometimes the case when $\bar{\mu}$ is low, then greater dependence reduces consumer welfare. This general ambiguity persists even in the neighborhood of independence and for strong demand. For the bivariate exponential special case, however, numerical analysis shows that consumer welfare increases with preference dependence when demand is sufficiently strong.

**Duopoly**

We now suppose the two products are sold by symmetric single-product firms. The profit function of Firm X is $\pi^d(\bar{p}, \bar{r}) = \sigma(\bar{p} + \bar{r})Q(\bar{p}, \bar{r})$. In equilibrium, $\bar{p} = \bar{r} = \bar{p}^d$, satisfying

$$ (\bar{p}^d + \bar{\mu}) h(\bar{p}^d) = 1, \quad (17) $$

where we define the adjusted hazard rate under duopoly competition as

$$ h(\bar{p}) = \lambda^C(\bar{p}) + \frac{2}{1 - C(F(\bar{p}), F(\bar{p}))} \int_{F(\bar{p})}^1 c(x, x) f(u(x)) \, dx, \quad (18) $$

which is the hazard rate under multiproduct monopoly adjusted by an extra term. The extra term measures the business-stealing effect when both firms charge the same price, i.e. the percentage demand increase from a price cut resulting from customers who change allegiance. Notice that if $\bar{p} \leq u(0)$, then $\lambda^C(\bar{p}) = 0$ and $h(\bar{p}) = 2 \int_0^1 c(x, x) f(u(x)) \, dx$. Thus $u(0)$ is the critical price separating the equilibrium regimes of fully covered versus non-fully covered markets.

We modify the regularity condition for unique comparative statics:

**A3:** $d[(\bar{p} + \bar{\mu}) h(\bar{p})] / dp > 0$.

**A3** is implied by $h'(\bar{p}) \geq 0$ and is also satisfied in our FGM-exponential case. With the appropriate regularity condition in hand, the comparative static for symmetric duopoly are similar to those for multiproduct monopoly, thus further extending insights of Johnson and Myatt (2006) to horizontally-differentiated price-setting.
duopoly. Each firm’s equilibrium profit and consumer welfare are, respectively:

\[
\pi^d \equiv \sigma \bar{\pi}^d = \frac{1}{2} \sigma (\bar{p}^d + \bar{\mu}) \left[ 1 - C \left( F \left( \bar{p}^d \right) , F \left( \bar{p}^d \right) \right) \right],
\]

(19)

\[
w^d \equiv \sigma \bar{w}^d = \sigma \int_{\bar{p}^d}^{u(1)} [1 - C \left( F \left( \bar{p} \right) , F \left( \bar{p} \right) \right)] d\bar{p}.
\]

(20)

The equilibrium condition (17) has the same form as (12). Thus, the effects of \( \bar{\mu} \) on \( \bar{\pi} \) and \( \bar{\omega} \) under duopoly is qualitatively similar to those for monopoly, whereas the effect on profits is similar under the sufficient condition of a non-decreasing adjusted hazard rate \( h(\cdot) \):

**Lemma 3** Given **A3**: (i) \( \frac{d\bar{\pi}^d}{d\bar{\mu}} < 0 \), and there exists some \( \bar{\mu}^d > 0 \) such that \( \bar{p}^d \gtrless 0 \) if \( \bar{\mu} \lesssim \bar{\mu}^d \). (ii) \( \frac{d\bar{\omega}^d}{d\bar{\mu}} > 0 \) if \( h'(\cdot) \geq 0 \). (iii) \( \frac{d\bar{\omega}^d}{d\bar{\mu}} > 0 \).

The reason for the additional condition on \( h(\cdot) \) to ensure \( \frac{d\bar{w}^d}{d\bar{\mu}} > 0 \) is the following. An increase in \( \bar{\mu} \) directly increases demand, with a positive effect on profits; but it also has an indirect effect through adjustment in the rival’s price that can potentially lower equilibrium price \( p^d \).\(^{17}\) If \( h'(\cdot) \geq 0 \), then \( \frac{d\bar{p}^d}{d\bar{\mu}} = \frac{d(p^d + \bar{\mu})}{d\mu} \geq 0 \); hence higher \( \bar{\mu} \) leads to higher \( p^d \) and higher \( \pi^d \). The comparative statics of \( \bar{\mu} \) and \( \sigma \) follow straightforwardly with a similar form as for the monopoly cases.

**Proposition 4** Given **A3**: (i) \( \frac{d\bar{\pi}^d}{d\bar{\mu}} \gtrless 0 \) if \( h'(p) \gtrless 0 \); \( \frac{d\bar{\omega}^d}{d\bar{\mu}} > 0 \) if \( h'(\cdot) \geq 0 \); \( \frac{d\bar{\omega}^d}{d\bar{\mu}} > 0 \).

(ii) \( \frac{d\bar{\pi}^d}{d\sigma} > 0 \) if \( \bar{\mu} \leq \bar{\mu}^d \) and \( h'(\cdot) \geq 0 \); \( \frac{d\bar{\omega}^d}{d\sigma} \gtrless 0 \) if \( \bar{\mu} \lesssim \bar{\mu}^d \) and \( h'(\cdot) \geq 0 \); \( \frac{d\bar{\omega}^d}{d\sigma} > 0 \) if \( \mu \leq 0 \).

We further consider how equilibrium outcomes are affected by preference dependence under duopoly. It is intuitive to expect that duopoly competition intensifies with more preference dependence, as more consumers regard the two varieties to be close substitutes. In general, however, the effect of preference dependence on prices and profits is ambiguous. As under multiproduct monopoly, the regularity condition is not enough to ensure that prices monotonically decrease with \( \theta \). For while a higher

\(^{17}\)For a multiproduct monopolist, this second effect is zero due to the envelope theorem because the business-stealing effect is internalized.
\( \theta \) results in a lower output, motivating a lower price (market share effect), it also affects the slope of the residual demand curve, potentially providing an incentive to raise price (price sensitivity effect). Under duopoly, a unilateral marginal reduction in price affects the slope of a duopolist’s residual demand on both an extensive margin (market expansion) and the intensive margin (business stealing). The ambiguity of the price sensitivity effect on the extensive margin explains why more substitutability between the two goods (e.g. \( C_{12\theta}(x, x) \geq 0 \)) may not be sufficient to conclude that \( p^d \) decreases with \( \theta \). We next identify sufficient conditions under which \( p^d \) and \( \pi^d \) decrease with \( \theta \).

We first identify a lemma providing technical conditions that are sufficient for \( \frac{\partial h(p)}{\partial \theta} > 0 \), which, together with \( A3 \), immediately implies \( \frac{dp^d}{d\theta} < 0 \).

Lemma 4 Given \( A2, A3 \) and \( MDR \): \( p^d \) decreases in \( \theta \) if

\[
h(u) + \frac{f'(u)}{f(u)} \geq 0 \tag{21}
\]

and

\[
\frac{d^2 \ln f(u)}{du^2} \geq \frac{2f^2(u(x))}{C_{\theta}(x, x)} - C_{11\theta}(x, x).
\]

Using the technical lemma, the next proposition identifies simple plausible sufficient conditions for \( \frac{dp^d}{d\theta} < 0 \) and \( \frac{d\pi^d}{d\theta} < 0 \). Part (i) extends Proposition 3 to the duopoly case when the market is fully or almost fully covered. A sufficiently strong demand ensures that higher \( \theta \) increases price sensitivity on the extensive margin (as for multiproduct monopoly), and condition (9), under which the two varieties are closer substitutes for higher \( \theta \), ensures that the same on the intensive margin. The other parts of the propositions use Lemma 4 to verify more basic sufficient conditions. Part (ii) invokes positive dependence and limited log-curvature of the marginal density (e.g. when \( f \) is approximately uniform or exponential). Part (iii) invokes stronger log-curvature restrictions on the marginal density (e.g. when \( f \) is approximately uniform) without imposing restrictions on the copula.

Proposition 5 Given \( A2, A3 \) and \( MDR \): \( p^d \) and \( \pi^d \) decrease in \( \theta \) if one of the following conditions is satisfied: (i) \( \bar{\mu} \) is sufficiently large and (9) holds at \( t = 0 \); (ii)
$C_{11} < 0$ and $\left| \frac{d^2 \ln f(x)}{dx^2} \right|$ is sufficiently small; or (iii) $\frac{d \ln f(x)}{dx}$ and $\frac{d^2 \ln f(x)}{dx^2}$ both are not too negative.

The standard discrete choice oligopoly theory of product differentiation, pioneered by Perloff and Salop (1985), typically assumes independence of values for different varieties. We contribute to this literature by showing that preference dependence is a useful indicator of product differentiation. In fact, while profits increase in $\sigma$ when $\mu$ is relatively small ($\mu < \sigma \mu^d$), profits always monotonically decrease in $\theta$ under multiproduct monopoly, and profits also monotonically decrease in $\theta$ under duopoly for all $\mu$ when $f$ is approximately uniform or when $f$ is approximately exponential and $C$ is positively dependent. The effect of more preference dependence on consumer welfare appears to be ambiguous generally, with a higher $\theta$ lowering both consumer demand and equilibrium prices. In the bivariate exponential special case, however, $w^d$ increases in $\theta$.

There are several reasons why we are interested in the comparative-static results of this section, under both monopoly and duopoly and in a setting of general preference dependence. First, the effects of preference strength ($\mu$) and diversity ($\sigma$) on prices, profits and consumer welfare are remarkably similar across all three market structures, suggesting unifying principles in the economics of monopoly and duopoly. Additionally, the effects of $\sigma$ on profits extends the important work of Johnson and Myatt (2006). Second, the effects of preference dependence ($\theta$) on prices and profits suggest a new way to think about product differentiation. Both $\sigma$ and $\theta$ can be interpreted as indicators of the degree of product differentiation: higher $\sigma$ indicates more heterogeneity of consumer values for each product, while higher $\theta$ indicates greater similarity of these values between products for a randomly chosen consumer. They have rather different economic meanings and it is important to disentangle their effects in a general theory of product differentiation. Third, the key preference parameters have intuitive interpretations. As argued by Johnson and Myatt (2006), firms can use advertising, marketing, and product design strategies to influence preference diversity ($\sigma$), and by extension demand strength ($\mu$) and preference dependence ($\theta$).

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18 Preference dispersion sometimes is interpreted as an indicator of horizontal differentiation under the assumption of independent values, because with greater dispersion more consumers regard the two goods to be substantially different.
The comparative statics of $\mu$, $\sigma$, and $\theta$ can then have implications for business strategies in these and possibly other areas. For example, Proposition 5 loosely suggests that two competing single-product firms might have a mutual incentive to design or promote their products so that consumer values are less positively dependent or more negatively dependent. Furthermore, since government regulation and other interventions can influence firm strategies, the comparative statics may have implications for public policies. Finally, as discussed earlier, these fundamental preference parameters link to the price elasticities of classical demand theory, thus providing richer interpretations of the comparative static of consumer demand.

4. PRICE AND MARKET STRUCTURE

The copula approach also enables us to derive new results on how prices differ across market structures, disentangling the roles of the marginal distributions and the dependence relationship. This will provide important insights on how market structure affects firm conduct. Given that we have three market structures under consideration, there are three relevant prices to be compared with: $p^m$, $p^d$, and $p^{mm}$.

We start with comparing equilibrium prices under single-product monopoly and under duopoly. While Chen and Riordan (2008) finds a sufficient condition for $p^m \geq p^d$ when the marginal distribution is exponential (i.e. $\lambda'(\cdot) = 0$), it has been an open question how the prices compare for arbitrary marginal distributions, which we can now answer with the following result:

**Proposition 6** Given $A1$ and $A3$: if $C_{11} < 0$ (positive dependence) and $\lambda'(p) \geq 0$, then $p^m > p^d$; and if $C_{11} > 0$ (negative dependence) and $\lambda'(p) \leq 0$, then $p^m < p^d$.

Therefore, positive dependence and a non-decreasing hazard rate for the marginal distribution ensures that duopoly competition lowers prices; conversely, negative dependence and a non-increasing hazard rate ensures that competition raises prices.\(^{19}\)

This result can be understood as follows. A duopolist sells less output at the monopoly price, $p^m$, and thus a slight price reduction at $p^m$ is less costly to the duopolist since it applies to a smaller output. This "market share effect" is a stan-

\(^{19}\)Chen and Riordan (2007) and Perloff, Suslow, and Sequin (1995) present more specific models of product differentiation in which entry can result in higher prices.
dard reason why one expects more competition to lower price. However, as Chen and Riordan (2008) discuss, there is a potentially offsetting "price sensitivity effect" when products are differentiated. Since a duopolist sells on a different margin from a monopolist, the slope of a duopolist’s (residual) demand curve differs from the slope of the single-product monopolist’s demand curve. Furthermore, greater negative dependence makes it more difficult for the duopolist to win over marginal consumers who value its own product less but its rival’s product more. Similarly, a non-increasing hazard rate tends to put less consumer density on the duopolist’s intensive margin, further reducing price sensitivity. Together, negative dependence and a non-increasing hazard are sufficient for the price sensitivity effect to dominate the market share effect, resulting in a higher price under duopoly competition.

Although preference dependence and the number of firms are different economic concepts, our analysis suggests a common theme between their effects on equilibrium prices. Both greater preference dependence and more firms represent increased competition. Each has a market share effect—lower output—that favors lower prices, but each may also have a price sensitivity effect—potentially steepening the residual demand curve—that favors higher prices. Propositions 6 and 5 give the respective sufficient conditions for the net effect to lower prices.

Next, we compare the prices for the multiproduct monopoly with those under single-product monopoly and symmetric duopoly.

**Proposition 7** Given A1-A3: (i) \( p_{mm} > p^d \); and (ii) \( p_{mm} > p^m \) if \( C_{11}(x, x) \) has a uniform sign for \( x \in (0, 1) \) (i.e., for \( x \in (0, 1) \), either \( C_{11}(x, x) \geq 0 \) or \( C_{11}(x, x) = 0 \), or \( C_{11}(x, x) \leq 0 \)).

As one might expect, \( p_{mm} > p^d \), or prices for two substitutes are higher under monopoly than under competition, as shown in Chen and Riordan (2008). The familiar intuition is that a multiproduct monopolist internalizes the negative effects of reducing one product’s price on profits from the other product. The comparison of prices under multiproduct monopoly (\( p_{mm} \)) and single-product monopoly (\( p^m \)) is

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20 The argument in the proof of Proposition 6 can be adapted to show more formally that, with \( \lambda' (\cdot) \leq 0 \), the (residual) demand curve of a duopolist, given by (5), is indeed steeper than that of the monopolist if \( C (\cdot, \cdot) \) is negatively dependent, independent, or has sufficiently limited positive dependence.
more subtle. The multiproduct monopolist has higher total output at \( p^m \) than the single-product monopolist, which motivates it to raise its symmetric price above \( p^m \). But, as with the duopoly comparison, the marginal consumers of the multiproduct monopolist differ from those of the single-product monopolist, which can potentially make the slope of the multiproduct monopolist’s demand curve steeper than that of the single-product monopolist. Interestingly, the market share effect unambiguously dominates, provided that \( C_{11} (x, x) \) has a uniform sign for \( x \in (0, 1) \), which of course is ensured if \( C (\cdot, \cdot) \) is negatively dependent, independent, or positively dependent. The uniform dependence condition is a sufficient for the hazard rate corresponding to the pricing problem of the multiproduct monopolist is below that of the single-product monopolist.

Under general preference distributions, Propositions 6 and 7 largely settle the question of how prices differ across market structures under monopoly and duopoly.

5. NUMERICAL ANALYSIS

We now use the FGM-exponential case, for which \( f (\cdot) \) is loglinear and \( \lambda (p) \) is a constant, and for which \( C(x, y; \theta) \) exhibits a limited range of dependence as \( \theta \) varies between \(-1\) and \( 1\), to illustrate graphically comparative static properties and the comparisons of conduct and performance across markets. The numerical analysis also considers endogenous market structure in the spirit of Shaked and Sutton (1990).

[Insert Figure 1 about here]

Figures 1 shows how normalized price (\( \bar{p} \)) varies with preferences and across market structures for the FGM-exponential case. There are three panels, corresponding to negative dependence (\( \theta = -1 \)), independence (\( \theta = 0 \)), and positive dependence (\( \theta = 1 \)). The horizontal axis in each graph is measured with respect to \( q^m = 1 - F (\bar{p}^m) \), the profit-maximizing single-product monopoly market share. From Lemma (??) there is a monotonic negative relationship between \( \bar{p}^m \) and \( \bar{p} \), and, therefore, a monotonic positive relationship between \( q^m \) and \( \bar{p} \). Consequently, the graphs effectively describe how industry outcomes vary with the strength-diversity ratio (\( \bar{\mu} \)) over the relevant
Confirming our analytical results, we observe: (1) $\bar{p}$ decrease with $\bar{\mu}$ under all three market structures. (2) $\bar{p}^d$ decrease in $\theta$, and $\bar{p}^{mm}$ decreases in $\theta$ only for $\bar{\mu}$ sufficiently high;\(^{22}\) (3) $\bar{p}^{mm}$ is always the highest, whereas $\bar{p}^m > \bar{p}^d$ for $\theta = 1$ but $\bar{p}^m < \bar{p}^d$ for $\theta = -1$ (negative dependence), $\bar{p}^m = \bar{p}^d$ for $\theta = 0$ (independence), and $\bar{p}^m > \bar{p}^d$ for $\theta = 1$ (positive dependence).

[Insert Figures 2 and 3 about here]

Figure 2 and Figure 3 conduct the same exercises for normalized profit ($\bar{\pi}$) and consumer welfare ($\bar{w}$). Again confirming our analytical results, we observe: (1) $\bar{\pi}$ and $\bar{w}$ increase with $\bar{\mu}$ under all three market structures;\(^{23}\) (2) $\bar{\pi}^{mm}$ and $\bar{\pi}^d$ decrease as $\theta$ increases; (3) $\bar{\pi}^{mm} > \bar{\pi}^m > \bar{\pi}^d$. A numerical analysis (not shown) also confirms that the effects of $\theta$ on consumer welfare is ambiguous: $\bar{w}^{mm}$ is decreasing in $\theta$ except for large values of $\bar{\mu}$, in which case greater positive dependence can deliver more consumer welfare than independence because price is lower. Duopoly competition creates the most consumer welfare, while a multiproduct monopoly creates more consumer welfare than single-product monopoly except when demand is sufficiently strong. This case of very strong demand is interesting. The higher price of a multiproduct monopoly inefficiently reduces the quantity demanded, reducing welfare to more than offset the benefits of greater product variety for consumers. Even then, however, total welfare clearly is higher under multiproduct monopoly because the widening profit gap offsets the consumer surplus loss. Not surprisingly, the adverse consumer welfare effect of greater variety is more pronounced, and occurs in a wider range of circumstances, when preference dependence is more negative.

We further employ the FGM-exponential case to study endogenous market structure, thus extending Shaked and Sutton (1990) to a discrete choice demand setting with horizontally-differentiated products and price-setting firms. Consider a two-stage game in which, in stage one, two firms simultaneously decide whether or not to

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\(^{21}\)In this case, $p^m = 1 - \bar{\mu}$, $q^m = e^{-(2 - \bar{\mu})}$, and $\bar{\pi}$ ranges between $-\infty$ and 2 as $q^m$ ranges between 0 and 1.

\(^{22}\)While the effect of $\theta$ on $\bar{p}^{mm}$ is quantitatively small, a close comparison of the panels shows that that $\bar{p}^{mm}$ decreases in $\theta$ for $\bar{\mu}$ above a critical value but increases in $\theta$ for $\bar{\mu}$ below. Thus the numerical analysis shows that if is difficult to go beyond Proposition 3.

\(^{23}\)Since $p^d < p^m$ when $\theta = -1$, the market is fully covered in duopoly when $q^m < 1$ but sufficiently high. In this range, $\pi^d$ is flat.
incur a fixed cost $K$ to offer each of the two products, and, in stage two, simultaneously set prices for active products. Obviously, there is no pure strategy subgame perfect equilibrium in which both firms produce the same variety. Thus, depending on stage one choices, the possible market structures are single-product monopoly, multiproduct monopoly, and duopoly. Figure 5 graphs that indicate certain critical normalized fixed costs ($\tilde{K} = K/\sigma$) for product selection. As before, graphs for negative and positive dependence correspond to the extremes of the FGM family, $\theta = -1$ and $\theta = 1$. If $\tilde{K} \leq \tilde{\pi}^d$, then duopoly is an equilibrium. If $\tilde{K} \leq \tilde{\pi}^{mm} - \tilde{\pi}^m$, then multiproduct monopoly is also an equilibrium. If $\tilde{\pi}^m \geq \tilde{K} > \tilde{\pi}^d$, then single-product monopoly is a unique equilibrium; and if $\tilde{\pi}^d \geq \tilde{K} > \tilde{\pi}^{mm} - \tilde{\pi}^m$, then duopoly is a unique equilibrium. Finally, for $\tilde{\pi}^{mm} - \tilde{\pi}^m \geq \tilde{K}$, duopoly and multiproduct monopoly coexist as equilibria, and prices are lower and consumers are better off with duopoly (as shown in Figure 1).

[Insert Figure 4 about here]

Reading the diagrams in Figure 4 from left to right indicates how increasing demand strength changes the structure of equilibrium. When $\tilde{K}$ is low, equilibrium market structure shifts from no production, to single-product monopoly, to duopoly, to multiple equilibria as demand strength increases. Thus duopoly equilibrium always emerges before the multiproduct monopoly equilibrium as $\tilde{\mu}$ increases. For intermediate $\tilde{K}$, there is a non-monotonicity; when demand strength becomes sufficiently great, multiple equilibria give way to a unique duopoly equilibrium. For higher $\tilde{K}$, the multiproduct monopoly becomes impossible for any relevant $\tilde{\mu}$, and for even higher $\tilde{K}$ so does the duopoly equilibrium. These conclusions hold over the entire range of preference dependence for the bivariate exponential case.\footnote{Shaked and Sutton (1990) showed in a linear-demand representative consumer model similarly that a multiproduct-monopoly equilibrium is never a unique equilibrium.} Furthermore, comparing the three panels, positive (negative) dependence decreases (increases) the range of $\tilde{K}$ supporting multiproduct market structures.
6. CONCLUSION

Using copulas to describe the distribution of consumer preferences is a convenient and intuitive approach to discrete choice demand in multiproduct industries. The approach leads to several sets of conclusions about how preferences matter for the industrial organization of monopoly and duopoly. First, with certain qualifications, prices, profits, and consumer welfare all increase in demand strength, and they also all increase in preference diversity when demand is low ($\mu \leq 0$); but profit first decreases and then increases in preference diversity when demand is high ($\mu > 0$). These comparative statics are robust to varying degrees of preference dependence across monopoly and duopoly market structures. Second, preference dependence can be disintangled from preference diversity as a distinct indicator of product differentiation in multiproduct industries, in the sense that greater dependence leads to lower prices and profits under certain conditions. Third, for an initially monopolized market, the entry of a competitor with a horizontally differentiated product lowers (raises) market prices if preferences are positively (negatively) dependent and the hazard rate of the marginal distribution is non-decreasing (non-increasing); and, under a uniform dependence condition, a single-product monopolist will raise price when it adds a horizontally differentiated product.

There are many possible directions for further research using the copula approach. One is to extend our analysis to oligopoly markets with arbitrary numbers of firms and product varieties. While it is not entirely clear in general how to characterize preference dependence with more than three varieties, there are special cases, including a generalization of the FGM copula, that appear promising. Another direction for future research is to use the copula approach to model the consumer preferences over product characteristics, and to endogenize product design and market structure in a two-stage game in which rival firms in the first stage select product characteristics, and in the second stage set prices, thus further extending Shaked and Sutton (1990) to endogenize the nature of product differentiation. Chen and Riordan (2009) contains some preliminary analyses along these lines, including an example demonstrating how horizontal competition between low-quality duopolists can foreclose a high-quality

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monopoly to the detriment of industry profit, consumer welfare, and social welfare. A third related direction is to study situations in which some consumers may want to purchase both products as in the product bundling literature. While McAfee, McMillian, and Whinston (1989) allow for arbitrary preference dependence in a standard product bundling model, the copula approach suggests an analytically more convenient and perhaps also more useful characterization of preference dependence. The copula approach can also be applied to other areas of applied microeconomics, such as the economics of search (e.g., Anderson and Renault 1999; Schultz and Stahl 1996; Bar Isaac, Caruana, and Cunat 2010) and the economic analysis of horizontal mergers. Finally, the copula approach to discrete choice demand models, and the rich set of predictions that our analysis has already generated and that can be further developed concerning how market characteristics affect prices, profits, consumer surplus, and market structures, might open up interesting new directions for empirical industrial organization research.

REFERENCES


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26 Product bundling when consumer values for two products are correlated have also been studied, for example, in Schmalensee (1984), Nalebuff (2004), and Armstrong and Vickers (2008).

27 See, for example, Chen and Savage (forthcoming) for an empirical analysis of how preference dispersion affects prices and price differences between monopoly and duopoly markets for Internet services.


APPENDIX: PROOFS

Proof of Proposition 1. (i) First, from (12) and part (i) of Lemma 1,
\[ \frac{dp^m}{d\mu} = \frac{1}{\sigma} \frac{dp^m}{d\bar{\mu}} = \frac{d(\bar{p}^m + \bar{\mu})}{d\bar{\mu}} \geq 0 \text{ if } \lambda'(p) \geq 0. \] (22)

Next, \( \frac{ds^m}{d\mu} = \frac{d\bar{s}^m}{d\bar{\mu}} > 0 \) and \( \frac{dw^m}{d\mu} = \frac{d\bar{w}^m}{d\bar{\mu}} > 0 \), from Lemma 1.

(ii) First, from part (i) of Lemma 1:
\[ \frac{dp^m}{d\sigma} = \bar{p}^m + \sigma \frac{dp^m}{d\bar{\sigma}} = \bar{p}^m - \bar{\mu} \frac{dp^m}{d\bar{\mu}} = \frac{1}{\sigma} \left[ p^m - \mu \left( 1 + \frac{d(p^m)}{d\bar{\mu}} \right) \right] = \frac{1}{\sigma} \left[ p^m - \mu \left( \frac{d(\bar{p}^m + \bar{\mu})}{d\bar{\mu}} \right) \right]. \]

Therefore, from (22), since \( p^m > 0 \) at an interior optimum, \( \frac{dp^m}{d\sigma} > 0 \) if \( \mu \leq 0 \) and \( \lambda'(p) \geq 0 \). If \( \mu > 0 \), then Lemma 1 implies \( 1 + \frac{dp^m}{d\bar{\mu}} < 1 \) and \( \frac{dp^m}{d\sigma} > \frac{1}{\sigma} (p^m - \mu) \equiv \bar{p}^m \geq 0 \) if \( \bar{\mu} \leq \bar{\mu}^m \).

Next, \( \bar{p}^m \geq 0 \) if \( \bar{\mu} \leq \bar{\mu}^m \) from Lemma 1 implies
\[ \frac{d\pi^m}{d\sigma} = \bar{\pi}^m + \sigma \frac{d\pi^m}{d\bar{\sigma}} = \bar{\pi}^m + \sigma \frac{\partial \pi^m}{\partial \bar{\mu}} \frac{d\bar{\mu}}{d\sigma} = \bar{\pi}^m - \bar{\mu} \left[ 1 - F(\bar{p}^m) \right] = \bar{p}^m \left[ 1 - F(\bar{p}^m) \right] \geq 0 \text{ if } \bar{\mu} \leq \bar{\mu}^m. \]

Finally, since \( \frac{dw^m}{d\sigma} > 0 \) from part (iii) of Lemma 1, if \( \mu \leq 0 \):
\[ \frac{dw^m}{d\sigma} = \bar{w}^m + \sigma \frac{d\bar{w}^m}{d\sigma} = \int_{\bar{p}^m}^{u(1)} [1 - F(\bar{p})] d\bar{p} - \bar{\mu} \frac{d\bar{w}^m}{d\bar{\mu}} > 0. \]

Proof of Lemma 2. Given MDR, for any \( \bar{p} > F^{-1}(0) \) and for all \( \theta \),
\[ \int_{0}^{F(\bar{p})} C_1(x,x;\theta) \, dx = \frac{1}{2} \int_{0}^{F(\bar{p})} \frac{dC(x,x;\theta)}{dx} \, dx = \frac{1}{2} C(F(\bar{p}), F(\bar{p}); \theta) \]
increases in \( \theta \), which is possible only if \( C_{1\theta}(x,x;\theta) \equiv \frac{\partial C_1(x,x;\theta)}{\partial \theta} > 0 \) for all \( \theta \) if \( x \) is sufficiently close to zero. Similarly, \( C_{1\theta}(x,x;\theta) < 0 \) for all \( \theta \) if \( x \) is sufficiently close to 1. Thus there must exist \( x_1 > 0 \) such that, for all \( \theta \), \( C_{1\theta}(x_1,x_1;\theta) = 0 \) and
Proof of Proposition 3. (i) From (15), for any \( \theta \), let \( \bar{\mu}^* \) be such that \([u^* + \bar{\mu}^*] \lambda^C (u^*; \theta) = 1\), where \( u^* \geq F^{-1}(x_1) > u(0) \) is defined in Lemma 2. Then, \( \bar{\mu}^{mm} = u^* > u(0) \) if \( \bar{\mu} = \bar{\mu}^* \). If \( \bar{\mu} \geq \bar{\mu}^* \), \( \bar{\mu}^{mm} \leq u^* \) and Lemma 2 implies \( \frac{\partial \lambda^C (\bar{\mu}^{mm}; \theta)}{\partial \theta} > 0 \). It follows from (15) that \( \frac{d\bar{\mu}^{mm}}{d\theta} < 0 \) and hence \( \frac{d\bar{\mu}^{mm}}{d\theta} < 0 \). (ii) holds from application of the envelope theorem to (14) and \( C_\theta > 0 \). ■

Proof of Lemma 4. Integrating by parts, we can rewrite

\[
C_1 (F (\bar{\mu}), F (\bar{\mu})) f (\bar{\mu}) + \frac{1}{F (\bar{\mu})} \frac{dC_1 (x, x)}{dx} f (u (x)) dx - \frac{1}{F (\bar{\mu})} C_{11} (x, x) f (u (x)) dx
\]

\[
h (\bar{\mu}) = 2 \frac{f (u (1)) - \int_{F (\bar{\mu})}^{1} C_1 (x, x) \frac{f^2 (u (x))}{f (u (x))} dx - \int_{F (\bar{\mu})}^{1} C_{11} (x, x) f (u (x)) dx}{1 - C (F (\bar{\mu}), F (\bar{\mu}))}
\]

Then, since

\[
- \int_{F (\bar{\mu})}^{1} C_1 (x, x) \frac{f^2 (u (x))}{f (u (x))} dx = \int_{F (\bar{\mu})}^{1} \frac{f^2 (u (x))}{f (u (x))} \frac{1}{2} [1 - C (x, x)]
\]

\[
= - \frac{f^2 (\bar{\mu})}{f (\bar{\mu})} \frac{1}{2} [1 - C (F (\bar{\mu}), F (\bar{\mu}))] - \int_{F (\bar{\mu})}^{1} \frac{1}{2} [1 - C (x, x)] d \left( \frac{f^2 (u (x))}{f (u (x))} \right),
\]

\[27\]}
Thus, if (21) holds,

\[ \partial h(\bar{p}) = \frac{\int_0^1 \left[ \frac{C_1(x,x)}{f(x)} - 2C_1(x,x)f(u(x)) \right] dx + \left[ h(\bar{p}) + \frac{f(u(x))}{f(x)} \right] C_\theta(F(\bar{p}), F(\bar{p}))}{1 - C(F(\bar{p}), F(\bar{p}))} \]

because \( C_\theta > 0 \) and \( C_{11\theta} < 0 \) by MDR. From (17) and A3 we have \( \frac{d\theta}{d\bar{p}} < 0 \).

**Proof of Proposition 5.** First, observe that \( \frac{d\pi^d}{d\theta} \) has the same sign as \( \frac{d\pi^d}{d\bar{p}} \). Second, observe that \( \pi^d \) decreases in \( \theta \) when \( \frac{d\pi^d}{d\bar{p}} < 0 \), because

\[ \frac{d\pi^d}{d\theta} = \frac{\partial \pi^d}{\partial \theta} + \frac{\partial \pi^d}{\partial \bar{p}} \frac{d\bar{p}}{d\theta} \]

\[ = -\frac{1}{2} (\bar{p} \sigma + \mu) C_\theta(F(\bar{p}^d), F(\bar{p}^d); \theta) + \frac{\partial \pi^d}{\partial \bar{p}} \frac{d\bar{p}}{d\theta} < 0, \]

where \( C_\theta(F(\bar{p}^d), F(\bar{p}^d); \theta) > 0 \) from (??), and \( \frac{\partial \pi^d}{\partial \bar{p}} > 0 \) by the envelope theorem and by the fact that a firm’s demand increases in the other firm’s price. Given these observations we focus on sufficient conditions for \( \frac{d\pi^d}{d\theta} < 0 \) for each part of the proposition.

(i) From the proof of Proposition 3, \( \lambda^C(\bar{p}; \theta) \) increases with \( \theta \) if \( \bar{p} \) is sufficiently large so that \( \bar{p} \) is sufficiently close to \( u(0) \). Therefore, from MDR and (18), if (9) holds for \( t = 0 \), then \( h(\bar{p}; \theta) \) increases with \( \theta \); and from (17), \( \bar{p}^d \) and hence \( p^d \) decrease with \( \theta \).

(ii) If \( \frac{d^2 \ln f(x)}{dx^2} \to 0 \) and \( C_{11} < 0 \), then

\[ h(\bar{p}) + \frac{f(u(1)) - \int_0^1 C_{11}(x,x)f(u(x)) \, dx}{f(\bar{p})} \to 2 \frac{f(u(1)) - \int_0^1 C_{11}(x,x)f(u(x)) \, dx}{1 - C(F(\bar{p}), F(\bar{p}))} > 0 \]
and \( \frac{d^2 \ln f(u)}{du^2} > \frac{2f^2(u(x))}{C_{11\theta}(x,x)} \) since \( C_{11\theta}(x,x) < 0 \), thus satisfying Lemma (4).

(iii) Finally, if \( \frac{d\ln f(x)}{dx} \) and \( \frac{d^2 \ln f(x)}{dx^2} \) both are not too negative, then \( h(u) + \frac{f(u)}{f'(u)} \geq 0 \) and \( \frac{d^2 \ln f(u)}{du^2} > \frac{2f^2(u(x))}{C_{11\theta}(x,x)} \) by MDR, thus satisfying Lemma (4).

**Proof of Proposition 6.** From (16), (18) and \( u(x) = F^{-1}(x) \), for any \( \bar{p} \):

\[
h(\bar{p}) = \frac{2C_1(F(\bar{p}), F(\bar{p})) f(\bar{p})}{1 - C(F(\bar{p}), F(\bar{p}))} + \frac{2 \int_0^1 (1-x) c(x, x) \frac{f(u(x))}{1-f(u(x))} dx}{1 - C(F(\bar{p}), F(\bar{p}))}.
\]

Notice that

\[
\int_0^1 (1-x) c(x, x) dx = \int_0^1 (1-x) \left[ \frac{dC_1(x, x)}{dx} - C_{11}(x, x) \right] dx.
\]

Thus, if \( \lambda'(p) \geq 0 \), then

\[
h(\bar{p}) \geq \frac{2C_1(F(\bar{p}), F(\bar{p})) f(\bar{p})}{1 - C(F(\bar{p}), F(\bar{p}))}
\]

\[
+ \frac{2f(\bar{p})}{1-f(\bar{p})} \left\{ - [1 - F(\bar{p})] C_1(F(\bar{p}), F(\bar{p})) + \frac{1-C(F(\bar{p}), F(\bar{p}))}{2} - \int_0^1 (1-x) C_{11}(x, x) dx \right\}
\]

\[
= \lambda'(\bar{p}) \left[ 1 - \frac{2 \int_{F(\bar{p})}^1 (1-x) C_{11}(x, x) dx}{1 - C(F(\bar{p}), F(\bar{p}))} \right] > \lambda(\bar{p}) \text{ if } C_{11}(x, x) < 0. \quad (24)
\]

Therefore, from (12) and (17), using A1 and A3, we have \( \bar{p}^m > \bar{p}^d \) and hence \( p^m > p^d \) if \( \lambda'(\cdot) \geq 0 \) and \( C_{11} < 0 \).

Similarly, if \( \lambda'(\cdot) \leq 0 \) and \( C_{11} > 0 \), the inequality in (24) will be reversed, which
proves $h(\bar{p}) < \lambda(\bar{p})$, implying that $\bar{p}^m < \bar{p}^d$ and hence $\bar{p}^m < p^d$.

Proof of Proposition 7. (i) Since $h(\bar{p}) > \lambda_C(\bar{p})$ from (18), comparing (15) and (17) leads to $\bar{p}^d < \bar{p}^{mm}$.

(ii) It suffices to show that $\lambda_C(\bar{p}) < \lambda(\bar{p})$ for all $\bar{p} \in (u(0), u(1))$.

First, if, for all $x \in (0, 1)$, $C_{11}(x, x) \geq 0$ or $C_{11}(x, x) = 0$, then

$$\frac{\lambda_C(\bar{p})}{\lambda(\bar{p})} = \frac{2C_1(F(\bar{p}), F(\bar{p}))f(\bar{p})}{1-C(F(\bar{p}), F(\bar{p}))} = \frac{2[1-F(\bar{p})]C_1(F(\bar{p}), F(\bar{p}))}{1-C(F(\bar{p}), F(\bar{p}))}$$

$$= \frac{[1 - C(F(\bar{p}), F(\bar{p}))] - 2 \int_{F(\bar{p})}^{1} (1-x)C_{11}(x, x) dx - 2 \int_{F(\bar{p})}^{1} (1-x)C_{12}(x, x) dx}{1-C(F(\bar{p}), F(\bar{p}))}$$

$$= \frac{2 \int_{F(\bar{p})}^{1} (1-x)[C_{11}(x, x) + C_{12}(x, x)] dx}{1-C(F(\bar{p}), F(\bar{p}))} < 1.$$  

Next, suppose $C_{11}(x, x) \leq 0$ for all $x \in (0, 1)$. Then, $C_1(x, x) \leq \frac{C(x,x)}{x}$ for all $x \in (0, 1)$ since

$$C(x, x) = \int_{0}^{x} C_1(t, x) dt \geq \int_{0}^{x} C_1(x, x) dt = C_1(x, x)x.$$  

Hence

$$\frac{\lambda_C(u(x))}{\lambda(u(x))} = \frac{2(1-x)C_1(x,x)}{1-C(x,x)} \leq \frac{2(1-x)C(x,x)}{x[1-C(x,x)]}.$$  

Now, suppose to the contrary $\frac{\lambda_C(u(x))}{\lambda(u(x))} = \frac{2(1-x)C_1(x,x)}{1-C(x,x)} \geq 1$. Then $\frac{2(1-x)C(x,x)}{x[1-C(x,x)]} \geq 1$. But since

$$\lim_{x \to 0} \frac{2(1-x)C(x,x)}{x[1-C(x,x)]} = 2 \lim_{x \to 0} \frac{-C(x,x) + (1-x)2C_1(x,x)}{1-C(x,x) - 2xC_1(x,x)} = 0,$$

$$\lim_{x \to 1} \frac{2(1-x)C(x,x)}{x[1-C(x,x)]} = 2 \lim_{x \to 1} \frac{-C(x,x) + (1-x)2C_1(x,x)}{1-C(x,x) - 2xC_1(x,x)} = 1,$$
It follows that \( \pi \geq 0 \) because the curve \( u(x) + u(y) = F(\bar{p}) \) in the copula unit square is inside the square formed by \( x = F(\bar{p}) = y \). Thus, if \( \bar{\mu} \geq 0 \), since \( \bar{p} = \bar{p} - \bar{\mu} \), we have

\[
\pi^B = \pi^B(\bar{p}^B) \geq \pi^B(\bar{p}^{mm}) > \pi^mm(\bar{p}^{mm}) = \pi^{mm}.
\]

Next, if \( C(\cdot, \cdot; \theta) \rightarrow \min \{x, y\} \), we would have \( \Pr (X = Y) \rightarrow 1 \). Hence, for any given \( p > 0 \) such that \( q^{mm}(\bar{p}) \geq 0 \),

\[
q^B(\bar{p}) = q^B \left( \frac{\bar{p}}{\sigma} - 2\bar{\mu} \right) > q^B(2\bar{p}) \rightarrow q^{mm}(\bar{p}) \text{ as } C(x, y) \rightarrow \min \{x, y\}.
\]

It follows that \( \pi^B > \pi^{mm} \) if \( C \) is sufficiently positively dependent.

Finally, if \( C(\cdot, \cdot; \theta) \rightarrow \max \{0, x + y - 1\} \), we would have \( \Pr (X + Y = 1) \rightarrow 1 \). Hence, if \( C \) is sufficiently negatively dependent, we would have \( x + y \rightarrow 1 \) in probability. Hence, if in addition \( \mu < 0 \) is sufficiently small, for almost all \( x \) and \( y \) that satisfy the dependence relationship, we would have \( w(x) + w(y) \leq 0 \), or \( u(x) + u(y) \leq \bar{p} \) even if \( p = 0 \), implying that \( \pi^B \rightarrow 0 < \pi^d \), where \( \pi^d \) is bounded.
above 0 when $C$ is sufficiently negatively dependent. ■
Figure 1
Prices in the Bivariate Exponential Case

(a) Negative dependence

(b) Independence

(c) Positive dependence

\[ \bar{p}^{mm} > \bar{p}^{d} > \bar{p}^{m} \]

\[ \bar{p}^{mm} > \bar{p}^{m} = \bar{p}^{d} \]

\[ \bar{p}^{mm} > \bar{p}^{m} > \bar{p}^{d} \]

Normalized Prices

Monopoly Market Share \((q^m)\)
Figure 2
Profits in the Bivariate Exponential Case

(a) Negative dependence
\[ \bar{\pi}^{mm} > \bar{\pi}^{m} > \bar{\pi}^{d} \]

(b) Independence
\[ \bar{\pi}^{mm} > \bar{\pi}^{m} > \bar{\pi}^{d} \]

(c) Positive dependence
\[ \bar{\pi}^{mm} > \bar{\pi}^{m} > \bar{\pi}^{d} \]

Normalized Profits

Monopoly Market Share \((q^{m})\)
Figure 3
Consumer Welfare in the Bivariate Exponential Case

(a) Negative dependence
\[ \bar{w}^d > \max\{\bar{w}^m, \bar{w}^{mm}\} \]

(b) Independence
\[ \bar{w}^d > \max\{\bar{w}^m, \bar{w}^{mm}\} \]

(c) Positive dependence
\[ \bar{w}^d > \max\{\bar{w}^m, \bar{w}^{mm}\} \]
Figure 4
Critical Fixed Costs in the Bivariate Exponential Case

(a) Negative dependence

(b) Independence

(c) Positive dependence