TI 2021-065/VII
Tinbergen Institute Discussion Paper

How Do Start-up Acquisitions Affect the Direction of Innovation?

Esmée Dijk[1]
José Luis Moraga-González[1]
Evgenia Motchenkova[1]

[1] Vrije Universiteit Amsterdam

Tinbergen Institute is the graduate school and research institute in economics of Erasmus University Rotterdam, the University of Amsterdam and Vrije Universiteit Amsterdam.

Contact: discussionpapers@tinbergen.nl

More TI discussion papers can be downloaded at https://www.tinbergen.nl

Tinbergen Institute has two locations:

Tinbergen Institute Amsterdam
Gustav Mahlerplein 117
1082 MS Amsterdam
The Netherlands
Tel.: +31(0)20 598 4580

Tinbergen Institute Rotterdam
Burg. Oudlaan 50
3062 PA Rotterdam
The Netherlands
Tel.: +31(0)10 408 8900

How Do Start-up Acquisitions Affect the Direction of Innovation?*

Esmée S.R. Dijk †
José L. Moraga-González ‡
Evgenia Motchenkova §

June 25, 2021

Abstract

A start-up engages in an investment portfolio problem by choosing how much to invest in a "rival" project, which threatens the position of an existing incumbent, and a "non-rival" project. Anticipating its acquisition by the incumbent, the start-up strategically distorts its portfolio of projects to increase the (expected) acquisition rents. Depending on parameters, such a strategic distortion may result in an alignment or a misalignment of the direction in which innovation goes relative to what is socially optimal. Moreover, prohibiting acquisitions may increase or decrease consumer surplus. The more (less) the rival project threatens the incumbent and the less (more) the non-rival project appropriates the social surplus, the more likely is that consumers benefit (suffer) following an acquisition. These results are robust to acquisitions where the acquirer takes over the research facilities of the start-up.

Keywords: start-ups, acquisitions, mergers, innovation portfolios, competition policy, antitrust

JEL Classification: O31, L13, L41

*We thank Marie-Laure Allain, Eric Bartelsman, Richard Gilbert, Yassine Lefouili, Igor Letina, Georgios Stamatopoulos and Markus Reisinger for their useful comments. The paper has also benefited from presentations at the MaCCI Annual Conference 2021 and the 12th Paris Conference on Digital Economics.

†Vrije Universiteit Amsterdam and Tinbergen Institute. E-mail: e.s.r.dijk@vu.nl

‡Vrije Universiteit Amsterdam and University of Groningen. E-mail: j.l.moragagonzalez@vu.nl. Moraga is also affiliated with the Tinbergen Institute, the CEPR, and the Public-Private Sector Research Center (IESE, Barcelona).

§Vrije Universiteit Amsterdam. E-mail: e.i.motchenkova@vu.nl. Motchenkova is also affiliated with the Tinbergen Institute and the Tilburg Law and Economics Center.

1 Introduction

The potential (anti-)competitive effects of start-up acquisitions have recently raised much scholarly and practitioner attention. Though consolidation processes between firms are certainly not new, big companies in sectors as varied as digital, pharmaceutical and healthcare have acquired a disproportional number of start-ups in the last few years. For example, according to McLaughlin (2020), Amazon, Facebook, Google, Apple and Microsoft bought 21 firms in 2019 and only in the first half of 2020, these big five corporations had already acquired 27 smaller companies. Admittedly, many of the start-up acquisitions we have recently witnessed may have been motivated by the creation of value via quality upgrading and the filling of gaps in the acquirer's product portfolio. However, the acquiring companies have grown so large that, absent strong competition within the market, the fear is that start-up acquisitions are suppressing nascent competition that would otherwise benefit consumers. In fact, Cunningham et al. (2019) provide empirical support for the idea that a significant share of the acquisitions observed in the pharmaceutical industry are aimed at discontinuing the innovative products of the target firms and so forestall future competition. As a result, there has recently emerged a general debate among academicians and policy makers (see e.g. Bryan and Hovenkamp (2020b); Cabral (2020); Katz (2020); Scott-Morton et al. (2019); Crémer et al. (2019); Furman et al. (2019); OECD (2020)) about whether a more active antitrust intervention is sufficient or merger policy needs reform to address start-up acquisitions.[1] This paper adds to this debate by studying the impact of prohibiting start-up acquisitions on their portfolios of innovation projects. In doing so, the paper's focus is not just on the harm to consumers caused by the elimination of a future competitor, but also on how a start-up acquisition affects the direction of innovation.

There are at least two important aspects that make start-up acquisitions different from standard mergers. The first is based on the notion of "entry for buyout" in the spirit of Rasmusen (1988), which refers to the idea that the mere anticipation of being bought by giant companies may heavily influence start-up's business strategy (see also Cabral (2020), Hollenbeck (2020), Katz (2020), Letina et al. (2020), and Motta and Peitz (2020)). Thus, while building their portfolio of research projects, and anticipating an acquisition, start-ups may pay close attention to the direction large corporations go and give more or less weight to projects that might fit the interests of potential acquirers compared to other, non-rival, projects. Interestingly, the way in which this "innovation for buyout" effect may affect the direction of innovation is *a priori* indeterminate. On the one hand, projects that create much added value for the incumbent firms may be given priority because these

[1] Though the literature on mergers is extensive, most of the merger writings focus on the acquisition of *existing* firms with mature technologies and products. The earlier literature studied the impact of mergers on prices, insider and outsider profits and consumer surplus (Salant et al., 1983; Deneckere and Davidson, 1985). Subsequent work examined the trade-off between the increase in market power implied by mergers and the potential efficiency gains arising from either the supply-side (Williamson, 1968; Farrell and Shapiro, 1990) or the demand-side (Klemperer and Padilla, 1997; Moraga-González and Petrikaitė, 2013). Only very recently has the literature incorporated innovation incentives into the analysis of mergers (Federico et al., 2018; Motta and Tarantino, 2017). In doing this, a couple of papers have pointed out that it is important to look at how mergers affect the portfolio of research projects firms choose to engage in (Gilbert, 2019; Letina, 2016; Moraga-González et al., 2019). It is this latest angle that this paper intends to develop within the context of start-up acquisitions.

projects generate in turn high negotiation rents for the start-ups. On the other hand, projects that highly disrupt the dominant position of potential acquirers and thus generate little added acquisition value may fall out of the start-ups' priority agenda because they create low negotiation rents. Whether such project portfolio adjustment is socially desirable may depend on whether rival projects create more value for consumers than alternative non-rival ones.

The second important reason is that many start-up acquisitions occur at a time in which the target firm is still hardly, or not at all, active in the (relevant) market. Instead, many start-ups are bought during the early stages of their research and development program. A canonical example is that of pharmaceutical firms, which often buy start-ups at their incipient phases of their maturation (Krieger et al., 2017). By taking over the research facilities of the target firms, decisions over the project portfolio change hands from the start-ups to the acquirers. Because the acquirers also anticipate an increase in market power in the product market, their choice of project portfolio is modulated by a different "replacement effect" (Arrow, 1962) compared to that of the start-ups. *A priori*, it is not clear whether this replacement effect is stronger for incumbents than for start-ups (Greenstein and Ramey, 1998; Chen and Schwartz, 2013; Motta and Peitz, 2020).

These reflections lead us to ask how these strategic project portfolio decisions affect the direction of innovation and social welfare. To address this question we formulate a novel model of an industry with an incumbent operating in a single market and an entrant start-up.[2] The start-up engages in an investment portfolio problem. Specifically, the start-up chooses how to allocate its funding across two projects. One of the projects is a "rival" project, in the sense that it is meant to challenge the incumbent's dominant position in its traditional market. If successful, this rival project results in a product of higher quality than that of the incumbent. In case of failure, the start-up enters the incumbent's market with the same product as the incumbent. The alternative project is a "non-rival" project, that is, a project for which the start-up does not face competition. The two projects also differ from one another in their difficulty, that is, in the probability with which a given effort results in a successful innovation, and in their social returns.

Because firms are motivated by the private returns of the projects in which they engage and they thus neglect part of the social return, start-ups tend to hold biased portfolios of projects. We then ask whether start-up acquisitions aggravate or ameliorate such market distortion. We address this question in two settings. In the first setting, a start-up, anticipating its acquisition, strategically invests to maximize the rents it gets from the integration process. This modelling, which is well suited to identify the "innovation for buyout" effect on project portfolio choice, fits the case of acquisitions in the digital industry where, often, acquirers buy start-ups after the outcome of their research projects is (to a large extent) known. A recent example is the acquisition of *Vilnyx* by *Apple* to incorporate the former's technology of analysis of video's visual, text and audio content into Apple's apps (Gurman, 2020). In the second setting, the acquirer takes over the research

[2]We also study the case in which there are multiple incumbents. With n incumbents, not all acquisitions are incentive-compatible in our baseline model. More details are provided later.

labs of the start-up, and thereby its investment portfolio choice. This setting, which serves well to examine how the strength of the "replacement effect" shapes portfolio choice, is better tailored to markets such as those for pharmaceuticals, in which many start-up acquisitions occur during the early phases of drug development (Krieger et al., 2017; Cunningham et al., 2019).

To identify the "strategic" portfolio effect of start-up acquisitions, we compare the outcome of a three-stage acquisition game with the outcome of a benchmark two-stage no-acquisition game. Specifically, in the three-stage acquisition game, the start-up first chooses its portfolio of investments; in the second stage, after observing the outcome of its research efforts, the start-up and the incumbent bargain over the surplus generated by the acquisition; finally, in the last stage, firms compete in the market. In the benchmark no-acquisition two-stage game, the start-up first chooses its portfolio of investments and then, upon observing the results of the research projects, the start-up and the incumbent engage in competition.

We first show that, anticipating an acquisition, the start-up strategically distorts its investment to maximize its acquisition rents. The direction in which the portfolio of projects is adjusted depends on whether the acquisition rents when the rival project turns successful are greater or smaller than the acquisition rents when the rival project fails. The acquisition rents stem from the monopolization of the product market and it turns out that which acquisition rents are greater depends on parameters. Specifically, the acquisition rents when the rival project turns successful are larger than when the project fails provided that the difference in the quality of the existing product and that of the innovative product is not very large. In such a case, anticipating an acquisition, the start-up strategically increases investment in the rival project and, correspondingly, decreases investment in the non-rival one. By contrast, when the rival project threatens the position of the incumbent much because the difference in the quality of the existing product and that of the innovative product is very large, the acquisition rents when the rival project turns successful are very small (because the start-up's profits are already very high before an acquisition) and the start-up strategically reduces investment in the rival project and, correspondingly, raises investment in the non-rival one.

We then examine the social impact of the start-up's strategic adjustment of the investment portfolio and provide competition policy recommendations. We show that an acquisition may result in an alignment or a misalignment of the private and the social incentives to invest. This implies that an acquisition may improve the direction of innovation or worsen it. The former occurs when the project that generates larger social gains receives more investment in anticipation of an acquisition. Specifically, when the quality gap is small the start-up reallocates funding by reducing investment in the non-rival project and increasing it in the rival project, the acquisition improves the direction of innovation provided that the consumer gains from the non-rival project are relatively small compared to the private gains. Likewise, when the quality gap is large and the start-up invests less in the rival project and more in the non-rival project, this reduces the portfolio distortion when consumers benefit significantly from the non-rival project compared to the private gains.

Although the acquisition of the start-up may reduce the project portfolio distortion, this is not necessarily welfare improving because the acquisition also increases the quantity distortion. Therefore, prohibiting acquisitions of potential competitors involves a trade-off. We show conditions under which a reduction in the innovation distortion is sufficiently large so as to make an acquisition welfare improving, despite its associated quantity distortion. When the quality difference between the innovative and the existing products is relatively small, prohibiting the acquisition of the start-up is always consumer welfare increasing. Despite the fact that the start-up's portfolio of investments may be more in line with what consumers prefer, the consumer welfare gains associated to a smaller innovation distortion are too little to offset the consumer welfare losses originating from the negative price effects associated to a larger quantity distortion. When the quality difference is high, things are different. The negative price effects need not dominate the positive portfolio adjustment effect. In fact, when consumer benefits associated with the non-rival project are large enough, then prohibiting the acquisition of potential entrants is consumer welfare reducing. Based on this result, blanket prohibitions of start-up acquisitions are not warranted and competition policy should address them case by case.

In Section 7 of the paper we turn our attention to the alternative setting in which the acquirer takes over the research facilities of the start-up. We find that when the quality gap between the incumbent's product and the successful start-up's product is small, the acquirer benefits relatively more from obtaining a high-quality product than the entrant does, while both benefit equally from the non-rival project. Hence, the acquirer invests more in the rival project and less in the non-rival one than the start-up. When the quality difference is sufficiently large, it is the opposite and investment in the rival project decreases after an acquisition.

This result that the acquirer may invest more in the rival project compared to the start-up is in contrast with the theoretical results in Cunningham et al. (2019) and Motta and Peitz (2020). The difference in results stems from the facts that in our model both the start-up and the acquirer face replacement effects and that these are of different magnitude. In equilibrium, both the start-up and the acquirer choose an investment portfolio so that the marginal returns from the rival and non-rival projects are equalised. The start-up's marginal returns from the rival project are proportional to the profits difference between a Cournot seller of high quality and a Cournot seller of low quality. That is, a successful start-up that sells high quality replaces an unsuccessful start-up that sells low quality. By contrast, the acquirer's marginal returns are proportional to the difference in the profits of a monopolist of high quality and a monopolist of low quality. As it turns out, the relative magnitude of these two replacement effects depend on parameters. When the quality difference is small, the start-up's marginal gains from investing in the rival project are lower than the acquirer's marginal gains. By contrast, when the quality difference is large, it is the opposite.[3]

[3]Motta and Peitz (2020) present a result similar to Cunningham *et al.* (2018) but conjecture that it is theoretically possible that an entrant's incentives to invest fall short of those of an incumbent. In this regard, our model of vertical product differentiation with Cournot competition provides an instance in which the Arrow replacement effect of an

From a social welfare point of view, also in this setting the acquisition of the start-up by the incumbent may result in an alignment between the private incentives to invest and the social incentives, thereby putting the direction in which the market innovates more in line with the socially optimal one. Moreover, we find that prohibiting the acquisition of potential entrants is consumer welfare reducing under conditions similar to those when the acquisition takes place after the outcomes of the research projects become known.

We examine the robustness of these results in a number of additional extensions. In one of the extensions we consider the case in which there are $n \geq 2$ incumbent firms. Interestingly, we find that the "innovation for buyout" effect always results in an increase in investment in the rival project, which, under conditions, may improve the direction of innovation. The result is related to the *merger paradox* that arises under Cournot competition. In fact, acquisitions are no longer incentive-compatible when the start-up's investment in the rival project turns unsuccessful. Because acquisition rents only accrue to the start-up when the rival project turns successful, the start-up's incentives to invest in the rival project go up compared to the case in which acquisitions are prohibited. Consumer surplus turns out not to improve, due to the price effects of start-up acquisitions. We also study the case in which there are $n \geq 2$ incumbent firms in the alternative setting in which the acquirer takes over the research facilities of the start-up. In this second setting, there are no qualitative differences in our results.

In another extension we consider the case in which, in addition to vertical product differentiation, there is also significant horizontal differentiation. In such a case, when the rival project is successful, the acquirer maximizes profits by commercializing the two existing rival products. The bargaining rents when the rival project is successful turn out to be always larger than those when the project is unsuccessful. As a result, anticipating its acquisition, the start-up always distorts its investment in the rival project upwards. In this setting, prohibiting acquisitions increases consumer surplus.

In yet another extension we consider the case of drastic innovations. When the innovation is drastic, a successful start-up can monopolize the market. In this case the "innovation for buyout" effect always results in a decrease in investment in the rival project. This is because acquisition rents only accrue to the start-up when the rival project turns unsuccessful. As in the main model, therefore, the direction of innovation may improve and consumer surplus may increase. In the alternative setting in which the acquirer takes over the research facilities of the start-up, we find the same result. This is because the start-up always faces a larger Arrow replacement effect compared to the acquirer.

Finally, we examine the case in which the start-up cannot enter the market in case of project failure. In this case, our results on the way acquisitions affect the direction of innovation remain the same. However, the impact of prohibitions of acquisitions on consumer surplus depends on the

entrant can be larger or smaller than that of an incumbent (see also Greenstein and Ramey (1998) and Chen and Schwartz (2013)).

timing of acquisitions.

Related literature

Our paper contributes to the recent surge in the academic interest about the effects of mergers on innovation. One branch of this literature focuses on the impact of mergers between existing firms on innovation. Some papers look at the case of single-project firms and centre around the question how mergers affect expenditures in R&D (e.g. Motta and Tarantino (2017); Federico et al. (2018); Bourreau et al. (2018). Other papers, more related to ours, have examined how mergers impact the variety and diversity of R&D projects firms engage in (e.g. Letina (2016); Gilbert (2019); Moraga-González et al. (2019)).[4]

A second branch of the literature focuses on the acquisition of potential competitors. Cunningham et al. (2019) present a model where an entrant with a single multi-stage project may be acquired by an existing firm. The focus of their theoretical analysis is on whether the entrant or the acquirer has greater incentives to continue to develop the project further once an initial stage is complete. Because of the Arrow replacement effect, the acquirer has weaker incentives to develop the project further so under some parameters the project is discontinued upon acquisition. This is more likely the greater the overlap between the interests of the acquirer and the entrant's project, and the fewer competitors in the market. Cunningham et al. (2019) also present an empirical analysis of the pharmaceutical industry corroborating these insights. They estimate that around 5-7% of the acquisitions are killer acquisitions. Motta and Peitz (2020) feature a single-project entrant that may be acquired by an existing incumbent. Like Cunningham et al. (2019), their focus is on the likelihood of project killing after the acquisition of the potential entrant. They also study the probable impact of acquisitions on consumer surplus. They find that whenever the start-up company has the ability to continue to develop its project, an acquisition (weakly) reduces consumer surplus. Acquisitions may only be beneficial for consumers when the entrant does not have the resources to develop the project while the incumbent does (see also Fumagalli et al. (2020)). Our model differs from these two papers in two important regards. First, we examine how an acquisition impacts project portfolio choice. In our paper, the decision of a firm is not whether to continue or discontinue a project, but how much effort to allocate across a portfolio of projects. This implies that lowering investment in a project is not *per se* consumer welfare reducing because such a decision frees up resources that can be allocated to other projects. Second, our model is one of vertical product differentiation and, as it turns out, whether the Arrow replacement effect is larger for the incumbent or for the start-up depends on parameters.

[4] In fact, in terms of its model, our paper is related to Moraga-González et al. (2019) where mergers between existing firms are examined in a symmetric setting in which firms invest in a portfolio of research projects of different profitability and social value. The main difference between our paper and theirs in that our model is tailored to the phenomenon of start-up acquisitions, which allows us to identify the "innovation for buyout" effect on project portfolio choice. Moreover, we explicitly model the trade-off between the portfolio effects and the price effects of acquisitions.

The strategic innovation effect of start-up acquisitions is also studied in Cabral (2018), Bryan and Hovenkamp (2020a), Gilbert and Katz (2021), Hollenbeck (2020), Katz (2020) and Letina et al. (2020). In Cabral (2018), Hollenbeck (2020) and Katz (2020) the "innovation for buyout" effect of start-up acquisitions is generally beneficial (see also the discussion in Cabral (2020)), although Cabral (2018) and Katz (2020) also put forward situations in which the "innovation for buyout" effect of start-up acquisitions is harmful. The latter occurs when innovators have a choice between different types of innovations. In Cabral (2018) this choice is between incremental and radical innovation, while in Katz (2020) the innovator chooses product quality. They show conditions under which the "innovation for buyout" effect results in less radical innovation and lower quality. In a model where a start-up does not have production capabilities, Bryan and Hovenkamp (2020b) study the start-up's incentives to transfer its technology either to a dominant, more efficient, firm or to its less efficient competitor. They show that anticipating its acquisition, the start-up gears its innovative effort towards the interests of the dominant firm. Our paper also shows that, depending on parameters, the "innovation for buyout" effect may be beneficial or harmful to consumers. The mechanism is however different because what is important in our model is whether acquisitions result in an alignment or a misalignment of the investment portfolio of the start-up. Gilbert and Katz (2021) also study how the prospect of a merger affects the direction of the entrant's investment. In their model, direction is defined as the extent to which the entrant's product (horizontally) differs from that of the incumbent. Like in our paper, they find that competition policy solely based on price effects may be socially suboptimal. Specifically, competition policy ought to account for how the possibility of mergers affects entrant's incentives to imitate the incumbent's product or alternatively introduce an innovative product.

Another paper on the acquisition of potential competitors is by Letina et al. (2020). In contrast to the work of Cunningham et al. (2019) and Motta and Peitz (2020), they focus on the impact of acquisitions on the variety of projects undertaken by an entrant firm and an incumbent firm. They show that prohibiting start-up acquisitions lowers the variety of projects the entrant and the incumbent activate and hence increases project duplication and reduces the likelihood of successful innovation, which reduces welfare. The most important difference between our paper and theirs in that the portfolio of projects firms can invest in are intended for different markets, with different profitability and social value. In addition, we explicitly model the trade-off between the portfolio effects and the price effects of acquisitions.

Finally, in a model with network externalities Kamepalli et al. (2020) show how acquisitions may result in too little entry. This occurs because the consumers' propensity to adopt a new entrant's technology decreases when they anticipate an acquisition to take place. Katz (2020) and Motta and Shelegia (2021) also study "defensive" strategies by incumbent firms. Katz (2020) mentions the "incumbency for buyout effect," the idea that permissive policy towards acquisitions may trigger defensive investments by the incumbents to deter entry. Motta and Shelegia (2021) focus on how an incumbent firm may deploy a "defensive" (product-copying) strategy to prevent that a start-up develops a rival product, rather than a complementary one, that challenges its dominant position. They show that start-up acquisitions may increase the incentives of the start-up to develop the rival product.

The rest of the paper is organized as follows. Section 2 describes the model. In Section 3 we solve for the socially optimal investment portfolio. In Sections 4 and 5 we derive the profit-maximizing investment portfolios and the corresponding market outcomes in the case of no-acquisition and acquisition. Section 6 examines how, anticipating its acquisition by the incumbent, the start-up distorts its investment portfolio to maximize the acquisition rents. This section also presents the impact of prohibiting acquisitions on the direction of innovation and consumer welfare. Section 7 models acquisitions in which the acquirer takes over the research facilities of the start-up, compares the project portfolio of the start-up with that of the acquirer and derives the implications for the direction of innovation and consumer welfare. Section 8 shows that our results are robust to the relaxation of some assumptions. Finally, Section 9 provides some concluding remarks. Proofs not provided in the main text are relegated to the Appendix.

2 Model

We consider an industry with an incumbent (I) and a start-up (E).[5] The incumbent operates in a single market, referred to as market A, where it sells a product of quality $\underline{s} > 0$. The start-up faces an investment portfolio problem, namely, it may make an investment to challenge the incumbents' position in market A, or it may put effort to introduce an alternative, independent from the incumbent's business, product B. Specifically, the start-up has a fixed budget (or a fixed number of scientists) and its decision is how to allocate the funding (or the researchers) across two projects, denoted A and B. Project A is a *rival* project in the sense that it is meant to enter the incumbent's market A. Project B is a *non-rival* (from the incumbent's point of view) project.

We normalise the entrant's fixed investment budget to 1. Let x denote the start-up's investment in project A and, correspondingly, let $1 - x$ be the start-up's investment in project B. We assume that investment in a project does not guarantee success, but increases the probability of success. Specifically, let $\tau(x, \epsilon_A)$ denote the probability with which project A is successful, in which case the start-up enters the incumbent's market A offering a product of higher quality $\overline{s}$ than that of the incumbent, with $\underline{s} < \overline{s} < 2\underline{s}$.[6] The probability of success $\tau(x, \epsilon_A)$ is increasing in x and decreasing in ϵ_A, which is a shift parameter that proxies for the difficulty of the project. If investment in project A results in failure, which occurs with probability $1 - \tau(x, \epsilon_A)$, then the start-up competes

[5] Later in Sections 6.2 and 7.6 we discuss the case in which there are n incumbents. The case of n incumbents presents some important differences. For example, when there are n incumbents not all start-up acquisitions are incentive-compatible.

[6] The assumption that $\overline{s} < 2\underline{s}$ rules out drastic innovations, that is, situations in which the incumbent firm would be forced to exit the market after a successful innovation of the entrepreneur. This assumption does not alter the main insights of our paper. For a more detailed discussion, see Section 8.

face-to-face with the incumbent in market A offering the same quality $\underline{s}$.[7] Demand in market A stems from a unit mass of consumers with the well-known quality-augmented quadratic utility function introduced in Sutton (1997, 2001):

$$U^A = \sum_{i=1}^{2} \left[\alpha q_i - \left(\frac{\beta q_i}{s_i} \right)^2 \right] - \sigma \sum_{i=1}^{2} \sum_{j<i} \frac{\beta q_i}{s_i} \frac{\beta q_j}{s_j} - \sum_{i=1}^{2} p_i q_i.$$

For tractability reasons, we assume away horizontal product differentiation by setting the parameter $\sigma = 2$.[8] Utility maximization yields the system of demands for the (possibly) vertically differentiated products of the start-up and the incumbent:

$$p_i = \alpha - \frac{2\beta^2 q_i}{s_i^2} - \frac{2\beta^2}{s_i} \sum_{j \neq i} \frac{q_j}{s_j}, \ i, j = E, I, \ i \neq j.$$

The start-up and the incumbent engage in quantity competition in market A. We normalise the marginal cost of production to zero.

Similarly, the probability with which product B is successful is denoted by $\eta(1-x, \epsilon_B)$, where $1-x$ is the investment in project B and ϵ_B is its intrinsic difficulty. This probability is increasing in $1-x$ and decreasing in ϵ_B. As mentioned before, project B is non-rival. We assume that project B delivers a profit π_B to the start-up in case of success and a surplus U_B to consumers. Otherwise, in case of failure, the project does not deliver anything.

We further specify the success probabilities as in Tullock contests:

$$\tau(x, \epsilon_A) = \frac{x}{x + \epsilon_A}, \text{ and } \eta(1-x, \epsilon_B) = \frac{1-x}{1-x+\epsilon_B},$$

where $\epsilon_A, \epsilon_B > 0$. With this formulation, a project becomes a sure success if its difficulty goes to zero, and a sure failure if its difficulty goes to infinity. This well-known functional form ensures that all our investment portfolio problems are strictly concave in own investment effort and therefore the first order conditions (FOCs) for expected profit maximization are necessary and sufficient for maxima. Moreover, when $\epsilon_A, \epsilon_B \to 0$ all our decision problems have interior solutions.

The choice of investment portfolio affects the likelihood with which projects A and B are realized and thus which products the market probably delivers to consumers. In what follows we shall informally speak about the *direction of innovation* implied by the investment portfolio choice of the agents. We start our analysis with a comparison of the outcome of a three-stage *acquisition* game with the outcome of a benchmark two-stage *no-acquisition* game. In the first stage of the

[7] We have also examined the case in which the start-up cannot enter the market in case of failure to innovate in project A. Details of such an analysis are placed in the extended working paper version of this paper (**?**). It turns out that in this case the portfolio adjustment after an acquisition depends on whether acquisitions take place before investment or after investment.

[8] We refer the reader to Section 8.3 for a discussion of the effects of substantial horizontal product differentiation. An important difference with horizontal product differentiation is that the acquirer becomes a multi-product monopolist.

three-stage *acquisition* game, *the investment portfolio stage*, the start-up chooses its portfolio of investments. In the second stage, *the acquisition stage*, after observing the outcome of its research efforts in projects A and B, the start-up and the incumbent bargain over the surplus generated by the acquisition. In this stage, we implement the Nash bargaining solution and assume that δ is the bargaining power of the start-up and, correspondingly, $1 - \delta$ that of the incumbent. In the third stage, *the market competition stage*, firms set their optimal quantities in market A to maximize their profits. In the benchmark *no-acquisition* two-stage game, the start-up first chooses its portfolio of investments and then, upon observing the results of the research projects, the start-up and the incumbent engage in Cournot competition. We solve for the subgame perfect Nash equilibria of these games. By comparing them we identify the impact of allowing for the acquisition of the start-up.

The above timing of moves in the acquisition game is adequate to model acquisitions in environments in which start-ups have developed their products before they are bought. This timing of moves, which is in line with Letina et al. (2020) seems a good modelling choice for the acquisitions of big-tech firms such as Facebook. An alternative modelling choice is one in which the incumbent buys the start-up during the research phase, thus taking over its research facilities and the investment portfolio decision. This modelling, which is in line with the paper of Cunningham et al. (2019), seems more suited to pharmaceutical markets where the acquirer intervenes in the last stages of drug development. We examine the implications of this alternative modelling in Section 7.

3 Social optimum

Before solving the games outlined above, we examine the social optimum. Specifically, we consider a social planner who chooses investment levels in projects A and B, as well as the production level in market A to maximize consumer surplus.[9] Our results do not qualitatively change if we consider a second-best approach in which the social planner chooses investment levels in projects A and B but has no control over the quantities the start-up and the incumbent offer in the market. For details, see the footnotes to Propositions 1 and 2 as well as Sections 6.1 and 7.5.

In the market competition stage, conditional on the outcome of the investment portfolio stage, the social planner chooses the quantity that maximizes consumer surplus. Suppose that project A is successful. In that case, the social planner only offers the high-quality product and chooses a quantity such that the price equals the marginal cost. Otherwise, when project A fails, the planner offers the existing, previously available, low-quality product at marginal cost. It is straightforward to derive the planner's optimal quantities. The corresponding levels of the surplus consumers obtain

[9] During the last decade, using consumer welfare as the standard for competition enforcement has become the European norm.

at the optimal quantities is given by:

$$\overline{U}_A^o = \frac{\alpha^2 \overline{s}^2}{4\beta^2}, \quad \underline{U}_A^o = \frac{\alpha^2 \underline{s}^2}{4\beta^2},$$

where $\overline{U}_A^o$ denotes the level of surplus in case of success and $\underline{U}_A^o$ in case of failure.

In the portfolio investment stage, the social planner chooses a portfolio of investments to maximize consumer surplus. Let x_o denote the socially optimal investment in project A, and correspondingly, $1 - x_o$ be the optimal investment in project B. The socially optimal investment x_o must maximize the expression:

$$\mathbb{E}U^o(x_o) = \frac{x_o}{x_o + \epsilon_A}\overline{U}_A^o + \frac{\epsilon_A}{x_o + \epsilon_A}\underline{U}_A^o + \frac{1 - x_o}{1 - x_o + \epsilon_B}U_B.$$

The first two terms of this expression represent the expected consumer surplus from investing in project A. With probability $\frac{x_o}{x_o+\epsilon_A}$, the social planner's investment in project A is successful and the surplus $\overline{U}_A^o$ corresponding to the high-quality product is realized. With probability $\frac{\epsilon_A}{x_o+\epsilon_A}$ project A fails, in which case the surplus $\underline{U}_A^o$ corresponding to the low-quality project is obtained. The third term is the expected surplus from investing in project B. With probability $\frac{1-x_o}{1-x_o+\epsilon_B}$, project B is successful, which yields the consumer surplus U_B. With the remaining probability, project B fails and no surplus is obtained.

The FOC for consumer surplus maximization is given by:

$$\frac{\epsilon_A}{(x_o + \epsilon_A)^2}\left(\overline{U}_A^o - \underline{U}_A^o\right) - \frac{\epsilon_B}{(1 - x_o + \epsilon_B)^2}U_B = 0$$

This FOC, which is necessary and sufficient for an interior social optimum, implies that the social planner should continue to increase its investment in project A until the marginal surplus from project A equals the marginal surplus from project B. Note that the marginal surplus from a project is proportional to the increase in the innovation surplus that results from success in such a project relative to failure. This innovation surplus is $\overline{U}_A^o - \underline{U}_A^o$ for project A and U_B for project B. Solving for x_o gives the socially optimal investment portfolio.

Lemma 1. *Assume that* $\frac{\epsilon_A\epsilon_B}{(1+\epsilon_A)^2} < \frac{4\beta^2}{\alpha^2(\overline{s}^2-\underline{s}^2)}U_B < \frac{(1+\epsilon_B)^2}{\epsilon_A\epsilon_B}$. *Then, the socially optimal investment in project A is equal to:*

$$\hat{x}_o = \frac{1 + \epsilon_B - \epsilon_A\sqrt{\frac{\epsilon_B U_B}{\epsilon_A(\overline{U}_A^o - \underline{U}_A^o)}}}{1 + \sqrt{\frac{\epsilon_B U_B}{\epsilon_A(\overline{U}_A^o - \underline{U}_A^o)}}}. \quad (1)$$

The corresponding socially optimal investment in project B is $1 - \hat{x}_o$.

The parameter condition given in the Lemma ensures that the socially optimal investment level $\hat{x}_o$ is interior, that is, $0 < \hat{x}_o < 1$. In such a case, the social planner chooses to activate both projects A and B. Notice that this is always the case when $\epsilon_A, \epsilon_B \to 0$. Obviously, parameters can be chosen for which the planner chooses to "kill" one of the projects. In what follows we shall ignore those cases.[10]

The socially optimal investment level $\hat{x}_o$ depends on the parameters of the model in a natural way. First, it is increasing in ϵ_B and decreasing in ϵ_A. That is, the optimal investment shifts towards a particular project when such a project gets easier to realize relative to the alternative project. Further, $\hat{x}_o$ is increasing in $\overline{U}^o_A - \underline{U}^o_A$ and decreasing in U_B. That is, investment shifts towards a given project when the innovation surplus increase due to success in such a project gets higher relative to the innovation surplus increase from success in the alternative project. The surplus increase from innovation in project A is increasing in the demand intercept α, decreasing in the demand slope β, increasing in $\overline{s}$ and decreasing in $\underline{s}$.

In the two next sections, we solve for the subgame perfect equilibria of the benchmark no-acquisition game and the acquisition game. We shall compare the outcomes of these two games in Section 6 with the socially optimal investment portfolio given in Lemma 1.

4 The no-acquisition game

In the benchmark no-acquisition two-stage game, the start-up first chooses its portfolio of investments and then, upon observing the results of the research projects, the start-up and the incumbent engage in Cournot competition. To solve the game, we proceed by backward induction. We start the analysis of the game by the market competition stage. After this, we fold the game backwards and examine the investment portfolio stage.

4.1 Market competition stage

In this stage the start-up and the incumbent engage in Cournot competition. When setting their quantities, the start-up and the incumbent take into account whether or not the start-up has succeeded in project A. Hence, there are two types of subgames to examine.

In the first subgame, the start-up's effort in project A is successful and it enters market A offering quality $\overline{s}$, while the incumbent offers a product of quality $\underline{s}$. It is straightforward to solve for the quantities that constitute a Nash equilibrium. The corresponding profits of the start-up and the incumbent, denoted $\overline{\pi}^E_A$ and $\underline{\pi}^I_A$ respectively, and consumer surplus are equal to:

$$\overline{\pi}^E_A = \frac{\alpha^2(2\overline{s}-\underline{s})^2}{18\beta^2}, \quad \underline{\pi}^I_A = \frac{\alpha^2(2\underline{s}-\overline{s})^2}{18\beta^2}, \quad \overline{U}_A = \frac{\alpha^2}{36\beta^2}(\overline{s}+\underline{s})^2.$$

In the second subgame, the start-up fails to innovate and both firms, the start-up and the incumbent, offer quality $\underline{s}$. Standard derivations yield the start-up and the incumbent equilibrium

[10] The recent literature on the acquisitions of potential competitors (see e.g. Cunningham et al. (2019) and Motta and Peitz (2020)) has emphasized this "killing" by modelling the decisions of the acquirer as a discrete choice between "continue" and "discontinue" a project. In our model, the investment decision is a continuous variable and draining the resources allocated to a project may also be interpreted as "killing" it.

profits and consumer surplus:

$$\pi_A^E = \pi_A^I = \pi_A^* = \frac{\alpha^2 \underline{s}^2}{18\beta^2}, \quad \underline{U}_A = \frac{\alpha^2 \underline{s}^2}{9\beta^2}.$$

4.2 Investment portfolio stage

In this stage, the start-up chooses its portfolio of investments to maximize its (expected) profits. In doing so, the start-up anticipates the outcomes of the quantity-setting games that ensue after the result of its research effort in project A is realized.

The expected payoff to a start-up that chooses to invest an amount x_E in project A and correspondingly an amount $1 - x_E$ in project B equals:

$$\mathbb{E}\pi^E(x_E) = \frac{x_E}{x_E + \epsilon_A}\overline{\pi}_A^E + \frac{\epsilon_A}{x_E + \epsilon_A}\pi_A^* + \frac{1 - x_E}{1 - x_E + \epsilon_B}\pi_B. \tag{2}$$

The first two terms of this expression represent the expected payoff from investing in project A. With a probability equal to $\frac{x_E}{x_E+\epsilon_A}$, the start-up's investment in project A is successful, in which case it obtains the Cournot profit corresponding to a firm selling a high-quality product and competing with an incumbent selling a low-quality product, i.e. $\overline{\pi}_A^E$. With probability $\frac{\epsilon_A}{x_E+\epsilon_A}$, the start-up's effort in project A is unsuccessful. In this case, both the incumbent and the start-up earn the symmetric Cournot profit corresponding to the situation in which they both offer a low-quality product, i.e. π_A^*. The third term is the expected profit from investing in project B. With probability $\frac{1-x_E}{1-x_E+\epsilon_B}$, the B innovation is successful, which yields a profit of π_B to the start-up. With the remaining probability, the project is unsuccessful and the start-up gets zero profits in market B.

The necessary and sufficient condition for profit maximization is:

$$\frac{\epsilon_A}{(x_E + \epsilon_A)^2}\left(\overline{\pi}_A^E - \pi_A^*\right) - \frac{\epsilon_B}{(1 - x_E + \epsilon_B)^2}\pi_B = 0.$$

This equation says that the start-up will continue to invest in project A until the marginal profit from project A equals the marginal profit from project B. Note that the marginal profit from project A is proportional to the increase in the profit that results from success in such a project relative to failure. This profit increase is $\overline{\pi}_A^E - \pi_A^*$ for project A, and π_B for project B. Solving for x_E gives the start-up's profit-maximizing investment portfolio.

Lemma 2. *Assume that* $\frac{\epsilon_A\epsilon_B}{(1+\epsilon_A)^2} < \frac{9\beta^2}{2\alpha^2\overline{s}(\overline{s}-\underline{s})}\pi_B < \frac{(1+\epsilon_B)^2}{\epsilon_A\epsilon_B}$. *Then, the start-up's profit-maximizing investment in project A is:*

$$\hat{x}_E = \frac{1 + \epsilon_B - \epsilon_A\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A(\overline{\pi}_A^E - \pi_A^*)}}}{1 + \sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A(\overline{\pi}_A^E - \pi_A^*)}}}. \tag{3}$$

The corresponding investment in project B is equal to $1 - \hat{x}_E$.

The parameter constellation given in the Lemma ensures that the start-up's profit-maximizing investment level $\hat{x}_E$ is interior. Notice that when $\epsilon_A, \epsilon_B \to 0$, $\hat{x}_E$ is always interior. As it is the case for the socially optimal investment portfolio, the investment level $\hat{x}_E$ is increasing in ϵ_B and decreasing in ϵ_A, reflecting the fact that the more difficult project B is relative to A, the more attractive project A becomes compared to project B. Furthermore, $\hat{x}_E$ is increasing in $\overline{\pi}_A^E - \pi_A^*$ and decreasing in π_B. Thus, the start-up's investment incentives depend on the relative profitability of the projects. The higher the extra profits a successful innovation in project A delivers relative to a successful innovation in B, the higher the investment level $\hat{x}_E$. The relative gains from successful innovation in project A are increasing in α, decreasing in β, increasing in $\overline{s}$ and decreasing in $\underline{s}$. Obviously, the previously mentioned parameters have the opposite impact on $1 - \hat{x}_E$.

4.3 Portfolio efficiency

In this section, we compare the equilibrium of the benchmark no-acquisition game with the social optimum.

Proposition 1. *In the no-acquisition game, the start-up's investment effort in project A is excessive (and therefore its investment in project B insufficient) from the point of view of consumer surplus maximization if and only if* $\pi_B < \frac{8\overline{s}}{9(\overline{s}+\underline{s})} U_B$.[11]

Proof. See the Appendix. □

Proposition 1 demonstrates that the portfolio of investments of the start-up is generally misaligned with the social incentives. The proposition also shows that the bias may be in favor of project A or in favor of project B, depending on parameters. To understand the condition in the proposition, note that both the social planner and the start-up make their choice of portfolio to equalize the expected gains from a successful project A relative to failure, and the expected gains from a successful project B relative to failure. However, while the start-up cares about profit gains, the planner cares about consumer gains. The condition thus stems from a comparison of the relative consumer surplus gains across projects, i.e. the ratio $\frac{\overline{U}_A^o - \underline{U}_A^o}{U_B}$, with the relative profit gains across projects, i.e. the ratio $\frac{\overline{\pi}_A^E - \pi_A^*}{\pi_B}$. Everything else constant, a bias in favor of project A is more likely to occur when project B is not very profitable (low π_B) but however has a large social value attached to it (large U_B). Likewise, this is more probable the higher $\overline{s}$ and the lower $\underline{s}$, which increases the private gains from project A relative to those of project B. It is obvious that this portfolio bias causes the innovation to go in a socially suboptimal direction.

[11] If we consider the second-best approach, the condition for the start-up's investment effort in project A to be excessive is modified to $\pi_B < \frac{8\overline{s}}{\overline{s}+3\underline{s}} U_B$. This follows from a comparison between the second-best socially optimal investment portfolio with $\hat{x}_E$. To obtain the second-best socially optimal investment portfolio we just replace $\overline{U}_A^o - \underline{U}_A^o$ with $\overline{U}_A - \underline{U}_A$ in expression (1).

5 The acquisition game

We now solve the three-stage acquisition game. In the first stage of this game the start-up chooses its portfolio of investments. In the second stage, after observing the outcome of the research effort, the start-up and the incumbent bargain over the surplus generated by the acquisition. In the third stage of the game firms choose their quantities to maximize their profits. To solve the game we proceed backwards.

5.1 Market competition stage

We start by considering subgames in which the acquirer and the start-up have not reached an agreement in the acquisition stage and therefore the acquisition does not materialize. In such a case, the start-up and the incumbent produce as in the no-acquisition game of Section 5. The profits the start-up and the incumbent make when the start-up's investment in project A is successful are $\overline{\pi}_A^E$ and $\underline{\pi}_A^I$, otherwise they make a profit equal to π_A^*.

Consider now subgames in which the acquirer and the start-up agree to the acquisition. In that case, the acquirer becomes a monopolist in market A. Again, there are two types of subgames to consider. In the first subgame, the start-up's effort in project A is successful, in which case the acquirer offers a high-quality product in market A. In the alternative subgame, the start-up's effort in project A is unsuccessful and the acquirer offers a low-quality product.

Standard derivations yield the acquirer's profits from market A conditional on the project being successful or unsuccessful, denoted $\overline{\pi}_A^m$ and $\underline{\pi}_A^m$ respectively, as well as the corresponding consumer surplus levels, denoted $\overline{U}_A^m$ and $\underline{U}_A^m$:

$$\overline{\pi}_A^m = \frac{\alpha^2 \overline{s}^2}{8\beta^2}, \quad \underline{\pi}_A^m = \frac{\alpha^2 \underline{s}^2}{8\beta^2}, \quad \overline{U}_A^m = \frac{\alpha^2 \overline{s}^2}{16\beta^2}, \quad \underline{U}_A^m = \frac{\alpha^2 \underline{s}^2}{16\beta^2}.$$

5.2 Acquisition stage

In this stage, the incumbent and the start-up negotiate over the surplus that the acquisition generates. We implement the Nash bargaining solution.

We start by noting that the profits from project B do not affect the bargaining process because there are no bargaining rents. Suppose the start-up's investment effort in project A is successful. In this case, the surplus generated by the acquisition is $\overline{\pi}_A^m - (\overline{\pi}_A^E + \underline{\pi}_A^I)$, where $\overline{\pi}_A^E$ and $\underline{\pi}_A^I$ play the role of disagreement payoffs. The bargaining outcome is the result of the problem:

$$\max_{s_E, s_I} (s_E - \overline{\pi}_A^E)^\delta (s_I - \underline{\pi}_A^I)^{1-\delta}$$
$$\text{subject to } s_E + s_I = \overline{\pi}_A^m,$$

where the parameter δ captures the bargaining power of the start-up and $1-\delta$ that of the incumbent. Substituting the constraint into the objective function, taking the FOC and solving we obtain the

Nash bargaining solution:

$$\overline{s}_E = \overline{\pi}^E_A + \delta(\overline{\pi}^m_A - (\overline{\pi}^E_A + \underline{\pi}^I_A))$$
$$\overline{s}_I = \underline{\pi}^I_A + (1-\delta)(\overline{\pi}^m_A - (\overline{\pi}^E_A + \underline{\pi}^I_A))$$

The Nash bargaining outcome prescribes that each agent receives its disagreement payoff plus a share of the bargaining surplus that is proportional to its bargaining power.

When the start-up's investment effort in project A is not successful, the surplus generated by the acquisition is $\underline{\pi}^m_A - 2\pi^*_A$, where π^*_A plays the role of disagreement payoff for each of the firms. The bargaining outcome is then the result of the problem:

$$\max_{s_E, s_I} (s_E - \pi^*_A)^\delta (s_I - \pi^*_A)^{1-\delta}$$
$$\text{subject to } s_E + s_I = \underline{\pi}^m_A.$$

Proceeding as before, we obtain the Nash bargaining solution:

$$\underline{s}_E = \pi^*_A + \delta(\underline{\pi}^m_A - 2\pi^*_A)$$
$$\underline{s}_I = \pi^*_A + (1-\delta)(\underline{\pi}^m_A - 2\pi^*_A).$$

Because the surplus created by an acquisition is strictly positive no matter the outcome of the research project, the acquisition always occurs in equilibrium.

5.3 Investment portfolio stage

In this stage, the start-up chooses its portfolio of investments to maximize its (expected) profits. In doing so, the start-up anticipates the outcomes of the bargaining games that ensue after the result of its research effort in project A is realized.

The expected profit of the start-up that invests x_E into project A and correspondingly $1 - x_E$ into project B is:

$$\mathbb{E}\pi^E(x_E) = \frac{x_E}{x_E + \epsilon_A} \left(\overline{\pi}^E_A + \delta(\overline{\pi}^m_A - \overline{\pi}^E_A - \underline{\pi}^I_A)\right) + \frac{\epsilon_A}{x_E + \epsilon_A} \left(\pi^*_A + \delta(\underline{\pi}^m_A - 2\pi^*_A)\right) + \frac{1 - x_E}{1 - x_E + \epsilon_B} \pi_B \quad (4)$$

As before the first two terms show the expected payoff from investing in project A. With probability $\frac{x_E}{x_E+\epsilon_A}$, the start-up's project is successful and obtains the high quality product, in which case the start-up gets its disagreement payoff $\overline{\pi}^E_A$ plus a share δ of the surplus created by the acquisition, $\overline{\pi}^m_A - \overline{\pi}^E_A - \underline{\pi}^I_A$. With probability $\frac{\epsilon_A}{x_E+\epsilon_A}$, the entrant fails to obtain the high quality product and obtains the competitive profit π^*_A plus a share δ of the surplus created by the acquisition, $\underline{\pi}^m_A - 2\pi^*_A$. The third term shows the expected profit from investing in project B, which yields a payoff of π_B

with probability $\frac{1-x_E}{1-x_E+\epsilon_B}$.

The necessary and sufficient condition for profit maximization is:

$$\frac{\epsilon_A}{(x_E+\epsilon_A)^2}\left[\overline{\pi}_A^E+\delta(\overline{\pi}_A^m-\overline{\pi}_A^E-\underline{\pi}_A^I)-(\pi_A^*+\delta(\underline{\pi}_A^m-2\pi_A^*))\right]+\frac{\epsilon_B}{(1-x_E+\epsilon_B)^2}\pi_B=0$$

This FOC can be interpreted similarly as those above. The start-up will continue to invest in project A until the marginal profit from project A equals the marginal profit from project B. What is different here is that the marginal profit from investing in project A now incorporates the negotiation outcome of the bargaining game. As a result, provided that the start-up's bargaining power is non-negligible, the start-up will distort its portfolio of investments in anticipation of the rents it can obtain in the bargaining process. Solving for x_E gives the equilibrium investment of the start-up when it anticipates its acquisition:

Lemma 3. *Assume that* $\frac{\epsilon_A\epsilon_B}{(1+\epsilon_A)^2}<\frac{72\beta^2}{\alpha^2((16\overline{s}-\delta(11\overline{s}-21\underline{s}))(\overline{s}-\underline{s}))}\pi_B<\frac{(1+\epsilon_B)^2}{\epsilon_A\epsilon_B}$ *Then, anticipating that the start-up will be acquired after the outcome of its research projects are realized, the start-up's profit-maximizing investment in project A is:*

$$\tilde{x}_E=\frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A\left(\overline{\pi}_A^E+\delta(\overline{\pi}_A^m-\overline{\pi}_A^E-\underline{\pi}_A^I)-\pi_A^*-\delta(\underline{\pi}_A^m-2\pi_A^*)\right)}}}{1+\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A\left(\overline{\pi}_A^E+\delta(\overline{\pi}_A^m-\overline{\pi}_A^E-\underline{\pi}_A^I)-\pi_A^*-\delta(\underline{\pi}_A^m-2\pi_A^*)\right)}}} \tag{5}$$

The corresponding investment in project B is $1-\tilde{x}_E$.

The condition in the Lemma ensures that the investment level $\tilde{x}_E$ is interior. Moreover, the parameters of the model affect investment in the same way as in the no-acquisition case.

5.4 Portfolio efficiency

As we did for the benchmark no-acquisition game, we now compare the equilibrium investment of the acquisition game with the social optimum.

Proposition 2. *In the acquisition game, the start-up's investment effort in project A is excessive (and therefore its investment in project B insufficient) from the point of view of consumer surplus maximization if and only if* $\pi_B<\frac{16\overline{s}-\delta(11\overline{s}-21\underline{s})}{18(\overline{s}+\underline{s})}U_B$.[12]

Proof. See the Appendix. □

Also in the case of an acquisition, depending on parameters, the portfolio of investments of the start-up may be biased towards project A or project B compared to the socially optimal portfolio of investments. As explained above for the no-acquisition game, the condition in the proposition stems from a comparison of the relative consumer surplus gains across projects, i.e.

[12] If we consider the second-best approach, the condition for the start-up's investment effort in project A to be excessive is modified to $\pi_B<\frac{16\overline{s}-\delta(11\overline{s}-21\underline{s})}{2(\overline{s}+3\underline{s})}U_B$.

the ratio $\frac{\overline{U}_A^o - \underline{U}_A^o}{U_B}$, with the relative profit gains across projects after an acquisition, i.e. the ratio $\frac{\overline{\pi}_A^E + \delta(\overline{\pi}_A^m - \overline{\pi}_A^E - \underline{\pi}_A^I) - (\pi_A^* + \delta(\underline{\pi}_A^m - 2\pi_A^*))}{\pi_B}$. It is obvious that this bias causes the innovation to go in a socially suboptimal direction. The question that arises is whether an acquisition improves or worsens the direction of innovation. We address this question in the next section.

6 The "innovation for buyout" effect of acquisitions

In this section we compare the outcome of the benchmark no-acquisition game with that of the acquisition game. We start with the impact of an acquisition on the portfolio of investments of the start-up. Later in this section, we evaluate the impact of an acquisition from a consumer welfare perspective.

Proposition 3. *For any* $\delta \in (0, 1]$*:*

(i) Relative to the no-acquisition case, the start-up puts less effort into project A (and therefore more effort into project B), i.e. $\tilde{x}_E < \hat{x}_E$*, if and only if* $\frac{21}{11}\underline{s} < \overline{s} < 2\underline{s}$*.*

(ii) Relative to the no-acquisition case, the start-up puts more effort into project A (and therefore less effort into project B), i.e. $\hat{x}_E < \tilde{x}_E$*, if and only if* $\underline{s} < \overline{s} < \frac{21}{11}\underline{s}$*.*

Moreover, the difference $\hat{x}_E - \tilde{x}_E$ *is greater the larger the start-up's bargaining power* δ*.*

Proof. See the Appendix. □

Proposition 3 describes how the start-up adjusts its investment portfolio in anticipation of an acquisition. As explained after Lemmas 2 and 3, the start-up invests so as to equalize the marginal gains from investing in project A to the marginal gains from investing in project B. Each of these marginal gains are proportional to the additional rents a successful innovation generates compared to failure. In the acquisition game the start-up shares in the bargaining surplus, which causes the entrant's incentives to invest in the no-acquisition game to be different from those in the acquisition game. They differ precisely in the additional rents a successful project A generates. Specifically, in the no-acquisition game, the additional rents a successful innovation generates in market A are equal to $\overline{\pi}_A^E - \pi_A^*$ while in the acquisition game they are equal to $\overline{\pi}_A^E + \delta(\overline{\pi}_A^m - \overline{\pi}_A^E - \underline{\pi}_A^I) - (\pi_A^* + \delta(\underline{\pi}_A^m - 2\pi_A^*))$. Thus, anticipating an acquisition, the start-up distorts its investment in a direction that depends on whether the bargaining surplus generated by the acquisition in case of a successful project is greater or smaller than the bargaining surplus in case of an unsuccessful project. That is, when

$$\underbrace{\overline{\pi}_A^m - \overline{\pi}_A^E - \underline{\pi}_A^I}_{\text{bargaining surplus if } A \text{ succeeds}} < \underbrace{\underline{\pi}_A^m - 2\pi_A^*}_{\text{bargaining surplus if } A \text{ fails}} \tag{6}$$

the start-up will invest less in project A (and more in B) in anticipation of the acquisition as compared to when an acquisition is not possible. In terms of the primitive parameters of the model, this inequality holds whenever $\overline{s}$ is very large compared to $\underline{s}$. The reason for this is intuitive. When $\overline{s}$ is very large compared to $\underline{s}$, the rents from monopolization of the product market are very limited because $\overline{\pi}_A^m$ and $\overline{\pi}_A^E$ are very close to one another. In fact, in the limit when $\underline{s} \to \overline{s}$, $\overline{\pi}_A^E \to \overline{\pi}_A^m$ and $\underline{\pi}_A^I \to 0$ and therefore the LHS of (6) converges to zero. Meanwhile, the RHS of (6) is bounded above zero. When the inequality is reversed, the start-up will invest more in project A (and less in B), which occurs when the difference between $\overline{s}$ and $\underline{s}$ is relatively small. We finally remark that when the start-up does not have bargaining power whatsoever ($\delta = 0$), the start-up's investment portfolio remains the same as in the no-acquisition game. This is natural because the start-up does not share in the extra rents the acquisition generates.

Our next result combines Propositions 1, 2 and 3 to derive a result on how an acquisition affects the direction of innovation activity from the point of view of surplus maximization.

Proposition 4. *(i) Assume that $\frac{21}{11}\underline{s} < \overline{s} < 2\underline{s}$ so that the start-up, anticipating its acquisition, reduces investment in project A and increases it in project B. Then:*

- *if $\pi_B < \frac{16\overline{s}-\delta(11\overline{s}-21\underline{s})}{18(\overline{s}+\underline{s})}U_B$, then $\hat{x}_o < \tilde{x}_E < \hat{x}_E$ and thus an acquisition improves the direction of innovation;*
- *if $\pi_B > \frac{8\overline{s}}{9(\overline{s}+\underline{s})}U_B$, then $\tilde{x}_E < \hat{x}_E < \hat{x}_o$ and thus an acquisition worsens the direction of innovation;*
- *finally, if $\frac{16\overline{s}-\delta(11\overline{s}-21\underline{s})}{18(\overline{s}+\underline{s})}U_B < \pi_B < \frac{8\overline{s}}{9(\overline{s}+\underline{s})}U_B$, then $\tilde{x}_E < \hat{x}_o < \hat{x}_E$ and thus an acquisition causes a move from over- to underinvestment in project A.*

(ii) Assume that $\overline{s} < \frac{21}{11}\underline{s}$ so that the start-up, anticipating its acquisition, increases investment in project A and decreases it in project B. Then:

- *if $\pi_B > \frac{16\overline{s}-\delta(11\overline{s}-21\underline{s})}{18(\overline{s}+\underline{s})}U_B$, then $\hat{x}_E < \tilde{x}_E < \hat{x}_o$ and thus an acquisition improves the direction of innovation;*
- *if $\pi_B < \frac{8\overline{s}}{9(\overline{s}+\underline{s})}U_B$, then $\hat{x}_o < \hat{x}_E < \tilde{x}_E$ and thus an acquisition worsens the direction of innovation;*
- *finally, if $\frac{8\overline{s}}{9(\overline{s}+\underline{s})}U_B < \pi_B < \frac{16\overline{s}-\delta(11\overline{s}-21\underline{s})}{18(\overline{s}+\underline{s})}U_B$, then $\hat{x}_E < \hat{x}_o < \tilde{x}_E$ and thus an acquisition causes a move from under- to overinvestment in project A.*

Proposition 4 shows that an acquisition may result either in an alignment or in a misalignment of the private incentives to invest with the social incentives. In particular, Proposition 4 points out two regions of parameters in which the direction of innovation improves when the start-up anticipates its acquisition, and two regions of parameters in which the direction of innovation worsens. These four regions of parameters are depicted in Figure 1, where we have set $U_B = 1$ and $\delta = 0.85$, along with two parameter regions for which it is not clear whether an acquisition results in a move closer or further away from the socially optimal investment portfolio. In this figure, the blue curve depicts the condition in Proposition 1 which divides the parameter space into an upper region where the investment in project A in the no-acquisition game is insufficient compared to the socially optimal portfolio and a lower region where the opposite occurs. Likewise, the red curve describes the condition in Proposition 2 which divides the parameter space into an upper region where the investment in project A in the acquisition game is insufficient compared to the socially optimal portfolio and a lower region where the opposite happens.

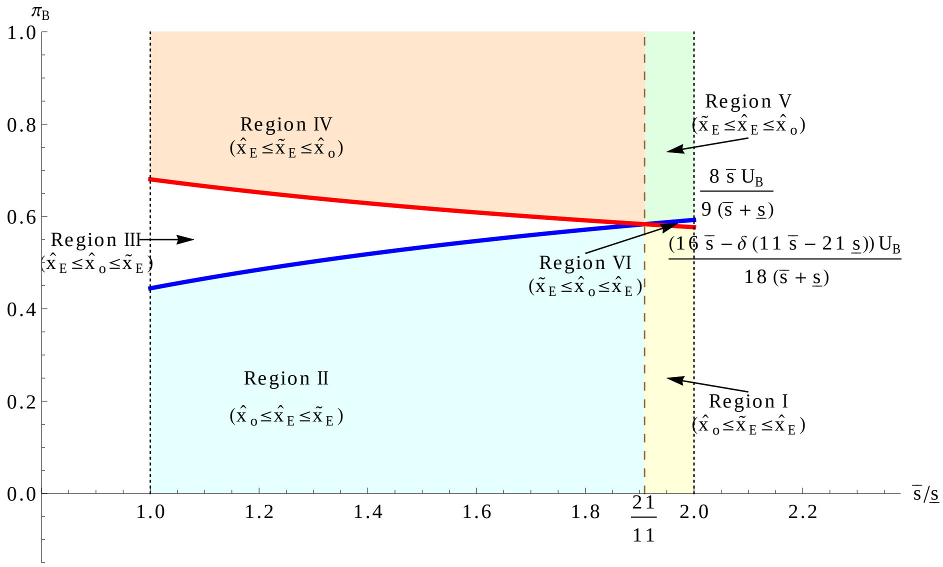

Figure 1: Private and socially optimal innovation portfolios

Specifically, when parameters fall in region I, the start-up, anticipating its acquisition, will decrease its investment in project A and increase it in project B. Because in this region of parameters, investment in A is excessive and in B insufficient from the point of view of social welfare maximization, it follows that the direction of innovation shall improve when the start-up anticipates its acquisition. A similar result holds in Region IV. When the parameters of the model fall in this region, the start-up, anticipating its acquisition, will increase its investment in project A and decrease it in project B. Because in this region of parameters, investment in A is insufficient while investment in B is excessive from the point of view of social welfare maximization, it follows that the direction of innovation will also improve when acquisitions are allowed.

By contrast, when parameters fall in regions II and V, allowing for acquisitions will worsen the direction of innovation. In Region II, an acquisition results in an increase in investment in A and a decrease in investment in B. Because in this region of parameters, investment in A is excessive while investment in B is insufficient from the point of view of social welfare maximization, the direction of innovation worsens when the start-up anticipates its acquisition. In Region V we have a similar observation because this is a region of parameters in which investment in A is insufficient and in B excessive and an acquisition results in even less investment in A and more in B.

Finally, Regions III and VI represent parameter spaces where the impact of an acquisition on the direction of innovation is ambiguous. The reason for this is that in these two regions of parameters the market moves from a portfolio where investment in project A is excessive and in B insufficient to a portfolio where investment in A is insufficient and in B excessive, or viceversa. Whether the direction of innovation improves or worsens depends on the other parameters of the model.

Proposition 4 shows that, depending on parameters, the acquisition of the start-up may result in an alignment or a misalignment of its portfolio of investments and that of the social planner. On this metric only, however, we cannot derive definitive conclusions about the impact of start-up acquisitions on consumer welfare. The reason for this is that an acquisition also results in negative price effects. We now explore the impact of start-up acquisitions on the welfare of consumers. To do this, we compare the expression for consumer surplus corresponding to the no-acquisition case:

$$\mathbb{E}U(\hat{x}_E) = \frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\overline{U}_A + \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\underline{U}_A + \frac{1 - \hat{x}_E}{1 - \hat{x}_E + \epsilon_B}U_B, \tag{7}$$

to the one corresponding to the acquisition case:

$$\mathbb{E}U(\tilde{x}_E) = \frac{\tilde{x}_E}{\tilde{x}_E + \epsilon_A}\overline{U}_A^m + \frac{\epsilon_A}{\tilde{x}_E + \epsilon_A}\underline{U}_A^m + \frac{1 - \tilde{x}_E}{1 - \tilde{x}_E + \epsilon_B}U_B. \tag{8}$$

The following result provides the consumer surplus implications of a prohibition of start-up acquisitions.

Proposition 5. *(i) Assume that* $\frac{21}{11}\underline{s} < \overline{s} < 2\underline{s}$ *so that, by Proposition 3,* $\tilde{x}_E < \hat{x}_E$*. Then, there exists* $\tilde{U}_B > 0$ *such that for all* $U_B < \tilde{U}_B$*, a prohibition of acquisitions results in an increase in consumer surplus. For* $U_B > \tilde{U}_B$*, a prohibition of acquisitions results in a decrease in consumer surplus.*

(ii) Alternatively, assume that $\overline{s} < \frac{21}{11}\underline{s}$ *so that, by Proposition 3,* $\tilde{x}_E > \hat{x}_E$*. Then, a prohibition of acquisitions results in an increase in consumer surplus.*

Proof. See the Appendix. □

Proposition $5(i)$ implies that when the quality difference is relatively large, whether the investment portfolio effects or the price effects of an acquisition have a dominating influence is ambiguous. The investment portfolio effect decreases expected consumer surplus in market A but increases expected consumer surplus in market B. However, when $\overline{s}$ is large compared to $\underline{s}$, the decrease in expected consumer surplus in market A is relatively small because the increase in the quantity distortion is rather limited. As a result, when consumer surplus in market B is sufficiently large, i.e. $U_B > \tilde{U}_B$, the decrease in the innovation distortion has a dominating influence over the increase in the quantity distortion in market A. Consequently, a prohibition of acquisitions reduces the overall expected consumer surplus. Otherwise, when $U_B < \tilde{U}_B$, expected consumer surplus increases if acquisitions are prohibited.

Proposition 5(ii) states that when the quality difference is relatively small, prohibiting acquisitions results in an increase in consumer surplus. When $\overline{s}$ is not very large compared to $\underline{s}$, the increase in the quantity distortion is sizeable and, even though the innovation distortion may become smaller after an acquisition, this effect is not sufficiently strong to offset the negative effects that arise from the increase in the quantity distortion. In different words, when the quality difference is not very large, the associated negative price effects of an acquisition dominate the investment portfolio effects.

We illustrate Proposition 5 in Figure 2. In this graph we illustrate the parameter region where consumer surplus is higher when acquisitions are allowed relative to the case in which acquisitions are forbidden.[13]

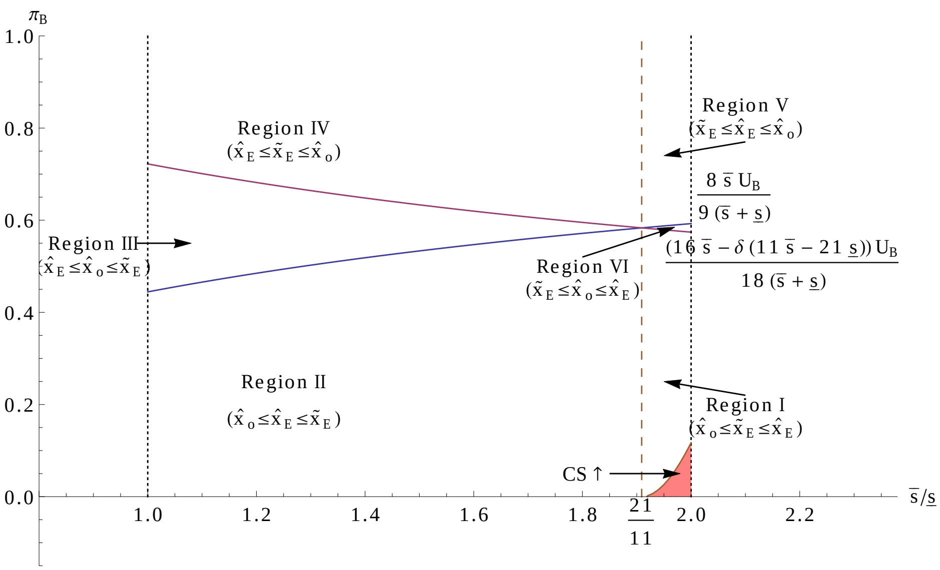

Figure 2: Consumer surplus improving start-up acquisitions

6.1 The second-best

Proposition 4 on the impact of start-up acquisitions on the direction of innovation can easily be extended to the case in which the social planner chooses the portfolio of investments to maximize consumer surplus but has no control over the quantities firms put in the market. For this extension we again make use of Propositions 1 and 2 but in this case we use the conditions corresponding to the second-best approach, which are given in the footnotes to the Propositions.

Following the same reasoning as that around Proposition 4, we can build Figure 3, whose

[13]This figure adds the region of parameters for which CS increases after an acquisition to the previous Figure 1. To construct this region, we set the rest of the parameters of the model to $\alpha = 3, \beta = 6, \epsilon_A = 0.3$ and $\epsilon_B = 10$. Keeping the rest of the parameters fixed, the shaded region covers a larger space of Region I as U_B increases.

economic content is essentially the same as that in Figure 1. (Here, again, $U_B = 1$ and $\delta = 0.85$.) The graph reveals that we can split the range of parameters into six regions that have exactly the same interpretation as those depicted in Figure 1.

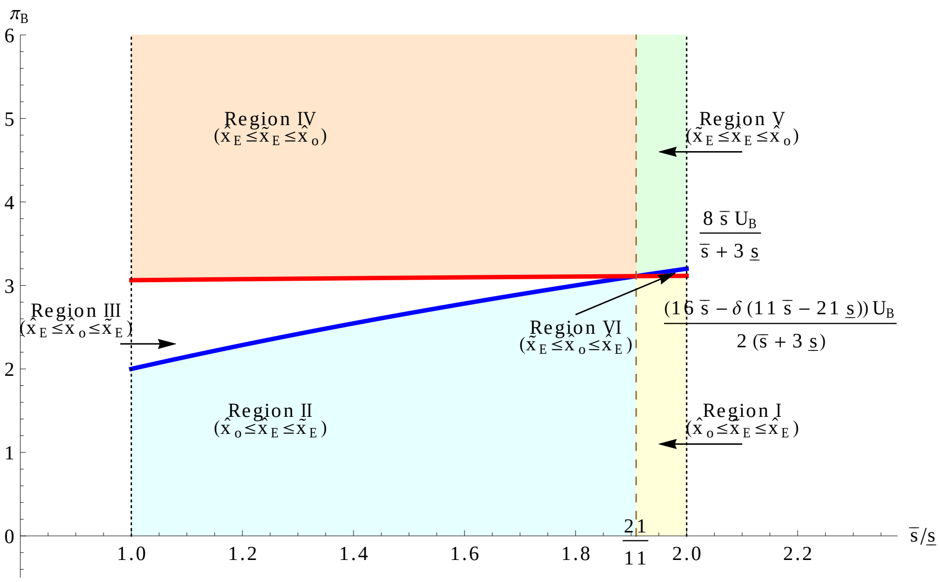

Figure 3: Private and socially second-best optimal innovation portfolios

6.2 The n-incumbents case

In the above analysis, we have made the assumption that there is only one incumbent operating in the "rival" market. In this section we report the results we obtain when there are $n \geq 2$ incumbents. The first observation to make is that with $n \geq 2$ incumbents the *merger paradox* would hold in our model of Cournot competition under some parameter conditions. Specifically, suppose that the start-up's innovation effort in project A turns unsuccessful. Then, the start-up would produce the same quality as that of the the rest of the incumbents. Because the products are otherwise homogeneous, the acquisition would not be incentive compatible. By contrast, suppose that the start-up's innovation effort in project A is successful. In that case, whether the acquisition of the start-up is incentive-compatible or not would depend on the relative magnitude of $\bar{s}$ compared to $\underline{s}$. For $\bar{s}$ sufficiently close to $\underline{s}$, naturally, the merger paradox would still hold and the acquisition would not take place. For $\bar{s}$ sufficiently large compared to $\underline{s}$, the acquisition would be incentive-compatible. As before, we solve the game by backward induction.

6.2.1 No-acquisition game

Starting with the market competition stage in the case of no-acquisition, the profits of the start-up and the incumbents when the start-up's rival project is successful, denoted $\overline{\pi}_A^E$ and $\underline{\pi}_A^I$ respectively,and consumer surplus are equal to:

$$\overline{\pi}_A^E = \frac{\alpha^2}{\beta^2}\left[\frac{(n(\overline{s}-\underline{s})+\overline{s})^2}{2(n+2)^2}\right], \quad \underline{\pi}_A^I = \frac{\alpha^2}{\beta^2}\left[\frac{(2\underline{s}-\overline{s})^2}{2(n+2)^2}\right], \quad \overline{U}_A = \frac{\alpha^2}{\beta^2}\left[\frac{(\overline{s}+n\underline{s})^2}{4(n+2)^2}\right].$$

In the second subgame, the start-up fails to innovate and all firms, the start-up and the incumbents, offer quality $\underline{s}$. This results in firms' profits and consumer surplus equal to:

$$\pi_A^E = \pi_A^I = \pi_A^* = \frac{\alpha^2}{\beta^2}\left[\frac{\underline{s}^2}{2(n+2)^2}\right], \quad \underline{U}_A = \frac{\alpha^2}{\beta^2}\left[\frac{(n+1)^2\underline{s}^2}{4(n+2)^2}\right].$$

Next we consider the start-up's investment portfolio decision. Notice that, apart from the expressions for the start-up's profits, the expected payoff of the start-up is exactly the same as in (2). Therefore, the start-up's profits-maximizing investment level is given by (3) after plugging in the new profit levels. In this case of $n \geq 2$ incumbents, for the equilibrium investment level to be interior we need that the parameters of the model satisfy the inequality $\frac{\epsilon_A \epsilon_B}{(1+\epsilon_A)^2} < \frac{2\beta^2(n+2)^2}{\alpha^2\left[(n(\overline{s}-\underline{s})+\overline{s})^2-\underline{s}^2\right]}\pi_B < \frac{(1+\epsilon_B)^2}{\epsilon_A\epsilon_B}$. As before, notice that this inequality always holds when ϵ_A goes to zero. The same remarks apply for the socially optimal investment portfolio, which is given by (1) after plugging in the new consumer surplus levels. A comparison of the equilibrium investment level to the socially optimal investment level yields the following result.

Proposition 1′. *In the no-acquisition game with $n \geq 2$ incumbents, the start-up's investment effort in project A is excessive (and therefore its investment in project B insufficient) from the point of view of consumer surplus maximization if and only if* $\pi_B < \frac{2((n+1)^2\overline{s}-(n^2-1)\underline{s})}{(n+2)^2(\overline{s}+\underline{s})}U_B$.

6.2.2 The acquisition game

In the case of successful innovation, the profits of the acquirer and the non-merging incumbents, denoted $\overline{\pi}_A^m$ and $\underline{\pi}_A^{nm}$ respectively, and consumer surplus, are equal to:

$$\overline{\pi}_A^m = \frac{\alpha^2}{\beta^2}\left[\frac{(n(\overline{s}-\underline{s})+\underline{s})^2}{2(n+1)^2}\right], \quad \underline{\pi}_A^{nm} = \frac{\alpha^2}{\beta^2}\left[\frac{(2\underline{s}-\overline{s})^2}{2(n+1)^2}\right], \quad \overline{U}_A^m = \frac{\alpha^2}{\beta^2}\left[\frac{(\overline{s}+(n-1)\underline{s})^2}{4(n+1)^2}\right].$$

When the acquirer fails to innovate, all firms, the acquirer and the non-merging incumbents, offer quality $\underline{s}$. Competition results in firms' profits and consumer surplus equal to:

$$\pi_A^m = \pi_A^{nm} = \pi_A^{*m} = \frac{\alpha^2}{\beta^2}\left[\frac{\underline{s}^2}{2(n+1)^2}\right], \quad \underline{U}_A^m = \frac{\alpha^2}{\beta^2}\left[\frac{n^2\underline{s}^2}{4(n+1)^2}\right].$$

Consider now the acquisition stage. In contrast to the single incumbent case, an acquisition does not always generate a surplus for the acquirer and the target firms. When the start-up's rival project fails, an acquisition is not incentive-compatible because $\pi_A^{*m} < 2\pi_A^*$. When the start-up's rival project is successful, an acquisition is incentive-compatible when $0 < \overline{\pi}_A^m - \overline{\pi}_A^E - \underline{\pi}_A^I$, in which case there exists a positive surplus to be shared between the participants in the transaction. This condition holds for $\overline{s}$ sufficiently high; specifically when $\frac{4n^2+6n}{3n^2+6n+2}\underline{s} < \overline{s} < 2\underline{s}$.[14] In the remaining of this section we shall therefore assume this condition holds for otherwise an acquisition would never occur, and competition policy would not have any influence on the resulting equilibrium. The bargaining outcome when the start-up innovates successfully and is acquired by an incumbent is the same as in Section 5 after plugging in the new profit levels.

Next, we consider the investment portfolio stage. With $n \geq 2$ incumbents, the expected profit of the start-up is somewhat different from the one in (4) because the acquisition does not take place in case of project failure. Then, anticipating that the start-up will be acquired when the rival project turns out to be successful, the start-up's profit-maximizing investment in project A is equal to:

$$\tilde{x}_E = \frac{1 + \epsilon_B - \epsilon_A \sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A \left(\overline{\pi}_A^E + \delta(\overline{\pi}_A^m - \overline{\pi}_A^E - \underline{\pi}_A^I) - \pi_A^*\right)}}}{1 + \sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A \left(\overline{\pi}_A^E + \delta(\overline{\pi}_A^m - \overline{\pi}_A^E - \underline{\pi}_A^I) - \pi_A^*\right)}}}. \tag{9}$$

This investment is strictly in between 0 and 1 provided that

$$\frac{\epsilon_A \epsilon_B}{(1+\epsilon_A)^2} < \frac{\beta^2}{\alpha^2} \left[\frac{2(n+2)^2}{(1-\delta)\left((n(\overline{s}-\underline{s})+\overline{s})^2 - \underline{s}^2\right) - \delta(2\underline{s}-\overline{s})^2} + \frac{2(n+1)^2}{(n(\overline{s}-\underline{s})+\underline{s})^2} \right] \pi_B < \frac{(1+\epsilon_B)^2}{\epsilon_A \epsilon_B}.$$

The rest of the funding, $1 - \tilde{x}_E$, will be allocated to project B. A comparison of this investment level to the socially optimal investment level in (1) gives the following result.

Proposition 2′. *In the acquisition game with $n \geq 2$ incumbents, the start-up's investment effort in project A is excessive (and therefore its investment in project B insufficient) from the point of view of consumer surplus maximization if and only if*

$$\pi_B < \frac{2((n+1)^2\overline{s} - (n^2-1)\underline{s})}{(n+2)^2(\overline{s}+\underline{s})} U_B + \frac{2\delta(2\underline{s}-\overline{s})\left[(3n^2+6n+2)\overline{s} - (4n^2+6n)\underline{s}\right]}{(n+1)^2(n+2)^2(\overline{s}^2-\underline{s}^2)} U_B.$$

6.2.3 The impact of an acquisition

Next we compare the outcomes of the no-acquisition and acquisition games.

Proposition 3′. *In the model with $n \geq 2$ incumbents, for any $\delta \in (0,1]$ and $\frac{4n^2+6n}{3n^2+6n+2}\underline{s} < \overline{s} < 2\underline{s}$:*

(i) An acquisition occurs only if the rival project is successful, and

(ii) relative to the no-acquisition case, the start-up puts more effort into project A (and therefore less effort into project B), i.e. $\hat{x}_E < \tilde{x}_E$.

Moreover, the difference $\tilde{x}_E - \hat{x}_E$ increases in the start-up's bargaining power δ.

Proof. See the Appendix. □

[14] Note that this condition may hold for any number of incumbents. This is because the first term is increasing in n and converges to $\frac{4}{3}\underline{s}$ as $n \to \infty$.

Proposition 3′ establishes that the start-up will invest more in project A (and less in B as a result) in anticipation of the acquisition. This is different compared to the case of $n = 1$ and the reason is that acquisitions are not incentive-compatible when the rival project fails. In the no-acquisition game, the additional rents that a successful innovation generates in market A are equal to $\overline{\pi}_A^E - \pi_A^*$ while in the acquisition game they are equal to $\overline{\pi}_A^E - \pi_A^* + \delta(\overline{\pi}_A^m - \overline{\pi}_A^E - \underline{\pi}_A^I)$. The latter is always larger than the former when acquisition generates surplus, which explains the result.

Comparing the thresholds in Propositions 1′, 2′ and 3′ we derive the following result, which extends Proposition 4 to this setting with $n \geq 2$.

Proposition 4′. *In the model with $n \geq 2$ incumbents:*

(i) Assume that $\frac{4n^2+6n}{3n^2+6n+2}\underline{s} < \overline{s} < 2\underline{s}$ so that the start-up, anticipating its acquisition, increases investment in project A and decreases it in project B. Then:

- *if $\pi_B > \frac{2((n+1)^2\overline{s}-(n^2-1)\underline{s})}{(n+2)^2(\overline{s}+\underline{s})}U_B + \frac{2\delta(2\underline{s}-\overline{s})\left[(3n^2+6n+2)\overline{s}-(4n^2+6n)\underline{s}\right]}{(n+1)^2(n+2)^2(\overline{s}^2-\underline{s}^2)}U_B$, then $\hat{x}_E < \tilde{x}_E < \hat{x}_o$ and thus an acquisition improves the direction of innovation;*
- *if $\pi_B < \frac{2((n+1)^2\overline{s}-(n^2-1)\underline{s})}{(n+2)^2(\overline{s}+\underline{s})}U_B$, then $\hat{x}_o < \hat{x}_E < \tilde{x}_E$ and thus an acquisition worsens the direction of innovation;*
- *finally, if $\frac{2((n+1)^2\overline{s}-(n^2-1)\underline{s})}{(n+2)^2(\overline{s}+\underline{s})}U_B < \pi_B < \frac{2((n+1)^2\overline{s}-(n^2-1)\underline{s})}{(n+2)^2(\overline{s}+\underline{s})}U_B + \frac{2\delta(2\underline{s}-\overline{s})\left[(3n^2+6n+2)\overline{s}-(4n^2+6n)\underline{s}\right]}{(n+1)^2(n+2)^2(\overline{s}^2-\underline{s}^2)}U_B$, then $\hat{x}_E < \hat{x}_o < \tilde{x}_E$ and thus an acquisition causes a move from under- to overinvestment in project A.*

(ii) Assume that $\overline{s} < \frac{4n^2+6n}{3n^2+6n+2}\underline{s}$. Then no acquisition occurs and thus there is no change in the innovation portfolio.

In terms of Figure 1, because with $n \geq 2$ incumbents we always have $\hat{x} < \tilde{x}$, we lose regions I, V and VI. We still have region II where the direction of innovation worsens, region III in which the direction of innovation moves from under- to overinvestment in project A and region IV in which the direction of innovation improves. Like in the main model, in region IV, the price effects dominate the innovation effect and therefore an acquisition would decrease consumer surplus. This is confirmed in the following proposition.

Proposition 5′. *In the model with $n \geq 2$ incumbents, assume that $\frac{4n^2+6n}{3n^2+6n+2}\underline{s} < \overline{s} < 2\underline{s}$ so that, by Proposition 3′, $\hat{x}_E < \tilde{x}_E$. Then, a prohibition of acquisitions results in an increase in consumer surplus.*

Proof. See the Appendix. □

7 On the timing of acquisitions: the Arrow replacement effect

In our main model, we have assumed that the incumbent acquires the start-up once the outcome of its research effort is known. As we have argued in the Introduction, this timing seems a sensible modelling choice to model start-up acquisitions in the digital industry. However, in the pharmaceutical industry, many acquisitions take place much earlier in the process. In this section, we assume that the incumbent may acquire the start-up before the outcome of its research effort is known, which means that the acquirer takes over the start-up's research facilities and the innovation portfolio decisions. We then solve an alternative acquisition game where, first, the incumbent and the start-up negotiate over the expected acquisition surplus, second, the acquirer chooses its portfolio of investments and finally the acquirer produces after knowing the outcome of its research efforts in projects A and B.

As before, we solve the game backwards. Because the market competition stage is exactly the same as that in Section 5.1, we move directly to the investment portfolio stage.

7.1 Investment portfolio stage

Let x_m denote the investment effort the acquirer puts in project A and, correspondingly, $1 - x_m$ be its investment in project B. The expected payoff to the acquirer from investing x_m in project A and $1 - x_m$ in project B is:

$$\mathbb{E}\pi^m(x_m) = \frac{x_m}{x_m + \epsilon_A}\overline{\pi}_A^m + \frac{\epsilon_A}{x_m + \epsilon_A}\underline{\pi}_A^m + \frac{1 - x_m}{1 - x_m + \epsilon_B}\pi_B \tag{10}$$

The interpretation of this expected payoff is similar to the interpretation of the payoff in (2).

The FOC is given by:

$$\frac{\epsilon_A}{(x_m + \epsilon_A)^2}\left(\overline{\pi}_A^m - \underline{\pi}_A^m\right) - \frac{\epsilon_B}{(1 - x_m + \epsilon_B)^2}\pi_B = 0$$

As in the no-acquisition case, this equation says that the acquirer will continue to invest in project A until the marginal profit from project A equals the marginal profit from project B. While the marginal profit from project B is exactly identical to that in the no-acquisition case, the marginal profit from project A is different. In particular, the marginal profit from project A is proportional to $\overline{\pi}_A^m - \underline{\pi}_A^m$, which represents the difference between a successful and an unsuccessful project A. This difference is thus the reason for the acquirer to hold a distinct investment portfolio compared to that of the start-up. Solving for x_m we obtain the acquirer's optimal investment portfolio.

Lemma 4. *Assume that* $\frac{\epsilon_A\epsilon_B}{(1+\epsilon_A)^2} < \frac{8\beta^2}{\alpha^2(\overline{s}^2-\underline{s}^2)}\pi_B < \frac{(1+\epsilon_B)^2}{\epsilon_A\epsilon_B}$. *When the acquirer takes over the research facilities of the start-up, the acquirer's profit-maximizing investment in project A is:*

$$\hat{x}_m = \frac{1 + \epsilon_B - \epsilon_A\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A(\overline{\pi}_A^m - \underline{\pi}_A^m)}}}{1 + \sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A(\overline{\pi}_A^m - \underline{\pi}_A^m)}}} \tag{11}$$

The corresponding investment in project B is $1 - \hat{x}_m$.

The parameter restriction in the Lemma ensures that $\hat{x}_m$ is interior. Moreover, the parameters of the model have the same effects on $\hat{x}_m$ as they do on $\hat{x}_E$ in (3)

As we did before, we can examine the nature of the portfolio bias in the case of an acquisition. A comparison of the acquirer's investment portfolio $\hat{x}_m$ with that chosen by the social planner $\hat{x}_o$ leads to the following result:

Proposition 6. *When the acquirer takes over the research facilities of the start-up, the acquirer's investment effort in project A is excessive (and therefore its investment in project B insufficient) from the social point of view if and only if* $\pi_B < \frac{U_B}{2}$.[15]

Proof. See the Appendix. □

As discussed in Section 3, the incentives of the planner are governed by the difference between the social gains from a successful project A and the social gains from a successful project B. Meanwhile, the acquirer cares about the private gains from a successful project A and the private gains from a successful project B. The result follows from a comparison of the relative consumer surplus gains across projects, i.e. the ratio $\frac{\overline{U}_A^o - \underline{U}_A^o}{U_B}$, with the relative profit gains across projects, i.e. the ratio $\frac{\overline{\pi}_A^m - \underline{\pi}_A^m}{\pi_B}$.

7.2 Acquisition stage

Finally, we study the acquisition stage. In this stage, the incumbent and the start-up negotiate over the surplus that the acquisition generates. We implement the Nash bargaining solution. A rejection of the offer of the incumbent yields the profits corresponding to the no-acquisition continuation game, in which case the start-up invests an amount $\hat{x}_E$, which yields a profit $\overline{\pi}_A^E$ in case of project success and a profit π_A^* in case of project failure. Therefore, the start-up's expected profits from rejection are equal to:

$$\mathbb{E}\pi^E(\hat{x}_E) = \frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\overline{\pi}_A^E + \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\pi_A^* + \frac{1 - \hat{x}_E}{1 - \hat{x}_E + \epsilon_B}\pi_B \tag{12}$$

Meanwhile, the expected profits of the incumbent in case of rejection are given by:

$$\mathbb{E}\pi^I(\hat{x}_E) = \frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\underline{\pi}_A^I + \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\pi_A^*$$

The disagreement payoffs for the incumbent and the start-up are thus $\mathbb{E}\pi^I(\hat{x}_E)$ and $\mathbb{E}\pi^E(\hat{x}_E)$, respectively, and the surplus from an acquisition is given by:

$$\mathbb{E}\pi^m(\hat{x}_m) - [\mathbb{E}\pi^E(\hat{x}_E) + \mathbb{E}\pi^I(\hat{x}_E)],$$

[15] If we consider the second-best approach, the condition for the acquirer's investment effort in project A to be excessive is modified to $\pi_B < \frac{9(\overline{s}+\underline{s})}{2(\overline{s}+3\underline{s})}U_B$.

which is strictly positive.[16]

The bargaining outcome is therefore the result of the problem:

$$\max_{s_E, s_I} (s_E - \mathbb{E}\pi^E(\hat{x}_E))^\delta (s_I - \mathbb{E}\pi^I(\hat{x}_E))^{1-\delta}$$
$$\text{subject to } s_E + s_I = \mathbb{E}\pi^m(\hat{x}_m)$$

Substituting the constraint into the objective function, taking the FOC and solving we obtain the Nash bargaining solution:

$$s_E^* = \mathbb{E}\pi^E(\hat{x}_E) + \delta[\mathbb{E}\pi^m(\hat{x}_m) - (\mathbb{E}\pi^I(\hat{x}_E) + \mathbb{E}\pi^E(\hat{x}_E))]$$
$$s_I^* = \mathbb{E}\pi^I(\hat{x}_E) + (1-\delta)[\mathbb{E}\pi^m(\hat{x}_m) - (\mathbb{E}\pi^I(\hat{x}_E) + \mathbb{E}\pi^E(\hat{x}_E))].$$

When the bargaining power of the start-up is zero, it is obvious that the subgame perfect equilibrium offer of the incumbent is equal $\mathbb{E}\pi^E(\hat{x}_E)$.

7.3 The impact of an acquisition

A comparison of the equilibrium investment portfolio under no-acquisition with that under acquisition leads to the following result:

Proposition 7. *Suppose the acquirer takes over the research facilities of the start-up. Then:*

(i) The investment effort put into project A by the acquirer is lower than (and therefore that in project B higher than) that of the start-up, i.e. $\hat{x}_m < \hat{x}_E$, if and only if $\frac{9}{7}\underline{s} < \overline{s} < 2\underline{s}$.

(ii) The investment effort put into project A by the acquirer is higher than (and therefore that in project B is lower than) that of the start-up, i.e. $\hat{x}_E < \hat{x}_m$, if and only if $\overline{s} < \frac{9}{7}\underline{s}$.

Proof. See the Appendix. □

The start-up and the acquirer hold distinct investment portfolios. The reason for this is that the marginal gains from investing in project A differ across the two cases. In the no-acquisition case, the marginal gains from investing in project A are governed by the profit difference $\overline{\pi}_A^E - \pi_A^*$. By contrast, in the acquisition case, the marginal gains from investing in project A are related to the profits difference $\overline{\pi}_A^m - \underline{\pi}_A^m$. When $\overline{\pi}_A^E - \pi_A^* > \overline{\pi}_A^m - \underline{\pi}_A^m$, the start-up's incentive to invest in project A is greater than the acquirer's incentive.

[16] This is because

$$\frac{\hat{x}_m}{\hat{x}_m + \epsilon_A}\overline{\pi}_A^m + \frac{\epsilon_A}{\hat{x}_m + \epsilon_A}\underline{\pi}_A^m + \frac{1-\hat{x}_m}{1-\hat{x}_m+\epsilon_B}\pi_B \geq \frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\left(\overline{\pi}_A^E + \underline{\pi}_A^I\right) + \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}2\pi_A^* + \frac{1-\hat{x}_E}{1-\hat{x}_E+\epsilon_B}\pi_B.$$

This inequality holds true because of the following remarks. First, notice that both the LHS and the RHS of this inequality are exactly the same function of x. Second, observe that $\underline{\pi}_A^m \geq 2\pi_A^*$ and $\overline{\pi}_A^m \geq \overline{\pi}_A^E + \underline{\pi}_A^I$. Finally, given these two observations, an application of the envelope theorem implies the result.

These profit differences represent the incremental gains of the two actors from selling a high-quality product in market A relative to selling a low-quality product. For both actors, we observe the so-called Arrow replacement effect. In fact, when project A turns out to be successful, the start-up replaces its Cournot competitor-self with a low-quality product by a Cournot competitor-self with a high-quality product. By contrast, the acquirer replaces a monopoly-self with a low-quality product by a monopoly-self with a high-quality product. Depending on parameters, the replacement effect of the entrant can be more severe than that of the acquirer. As Proposition 7 states, this is the case when the difference in the quality of the products is small ($\overline{s} < 9\underline{s}/7$). In such a case, the monopolist benefits relatively more from obtaining the high quality product than the entrant does. Hence, the acquirer invests more in project A in the acquisition case than the entrant does in the no-acquisition case. When the difference in the quality of the products is large ($9\underline{s}/7 < \overline{s} < 2\underline{s}$), it is the opposite and investment in project A decreases after an acquisition.

7.4 Welfare effects of acquisitions

Proposition 7 has compared the investment portfolios of the start-up and the acquirer. Specifically, it has provided conditions under which the investment effort put into project A by the start-up is lower (higher) than that of the acquirer, and correspondingly investment effort put into project B is higher (lower). We now ask whether the acquirer's portfolio of investments is more or less aligned with the socially optimal investment portfolio than that of the start-up.

Combining Propositions 1, 6 and 7, we obtain a result on the impact of an acquisition on the direction of innovation.

Proposition 8. *Suppose that the incumbent takes over the research facilities of the start-up. Then:*

(i) Assume that $\frac{9}{7}\underline{s} < \overline{s} < 2\underline{s}$ so that the acquirer reduces investment in project A and increases it in project B. Then:

- *if $\pi_B < \frac{U_B}{2}$, and thus $\hat{x}_o < \hat{x}_m < \hat{x}_E$, the acquisition improves the direction of innovation;*
- *if $\frac{8\overline{s}}{9(\overline{s}+\underline{s})}U_B < \pi_B$, and thus $\hat{x}_m < \hat{x}_E < \hat{x}_o$, the acquisition worsens the direction of innovation;*
- *finally, if $\frac{U_B}{2} < \pi_B < \frac{8\overline{s}}{9(\overline{s}+\underline{s})}U_B$, and thus $\hat{x}_m < \hat{x}_o < \hat{x}_E$, an acquisition causes a move from overinvestment to underinvestment in A;*

(ii) Assume that $\overline{s} < \frac{9}{7}\underline{s}$ so that the acquirer increases investment in project A and decreases it in project B. Then:

- *if $\frac{U_B}{2} < \pi_B$, and thus $\hat{x}_E < \hat{x}_m < \hat{x}_o$, an acquisition improves the direction of innovation;*
- *if $\pi_B < \frac{8\overline{s}}{9(\overline{s}+\underline{s})}U_B$, and thus $\hat{x}_o < \hat{x}_E < \hat{x}_m$, an acquisition worsens the direction of innovation;*

- *finally, if* $\frac{8\bar{s}}{9(\bar{s}+\underline{s})}U_B < \pi_B < \frac{U_B}{2}$*, and thus* $\hat{x}_E < \hat{x}_o < \hat{x}_m$*, an acquisition causes a move from underinvestment to overinvestment in* A*;*

The result in Proposition 8 is similar to that in Proposition 4 in that it provides conditions under which an acquisition may result in an alignment or in a misalignment of the private incentives to invest with the social incentives. Building on this proposition, we construct Figure 4 which is quite similar in nature to Figure 1 and uses the same parameter values. Specifically, we observe a similar split of the parameter space into 6 regions; for two of these regions, the direction of innovation improves after an acquisition takes place.

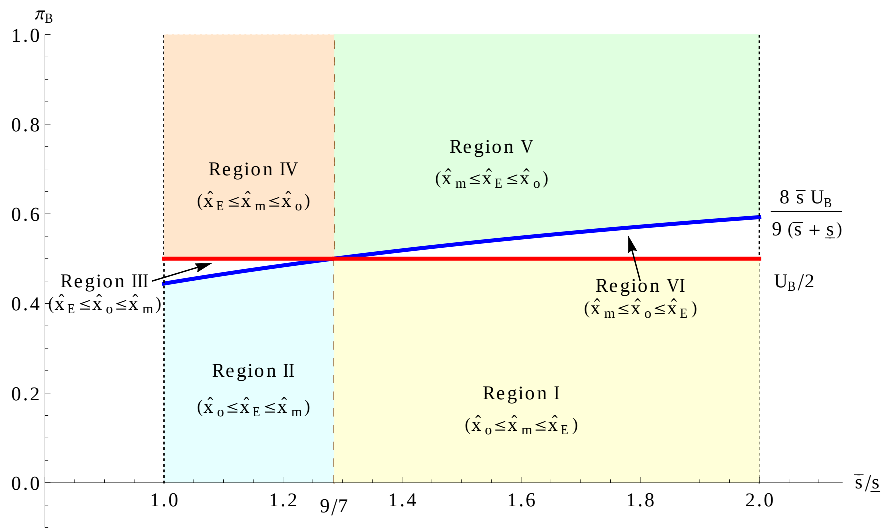

Figure 4: Private and socially optimal innovation portfolios

Our final result in this section takes into account the quantity and the innovation distortions of an acquisition to assess whether its prohibition is consumer welfare improving or not. For this, we compare the expression for consumer surplus corresponding to the case in which the start-up chooses the innovation portfolio:

$$\mathbb{E}U(\hat{x}_E) = \frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\overline{U}_A + \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\underline{U}_A + \frac{1-\hat{x}_E}{1-\hat{x}_E+\epsilon_B}U_B, \tag{13}$$

to the one corresponding to the case in which the acquirer picks the investment levels:

$$\mathbb{E}U(\hat{x}_m) = \frac{\hat{x}_m}{\hat{x}_m + \epsilon_A}\overline{U}_A^m + \frac{\epsilon_A}{\hat{x}_m + \epsilon_A}\underline{U}_A^m + \frac{1-\hat{x}_m}{1-\hat{x}_m+\epsilon_B}U_B. \tag{14}$$

Proposition 9. *Suppose that the incumbent takes over the research facilities of the start-up. Then:*

(i) Assume that $\frac{9}{7}\underline{s} < \bar{s} < 2\underline{s}$ *so that, by Proposition 7,* $\hat{x}_m < \hat{x}_E$*. Then, there exists* $\tilde{U}_B > 0$ *such that for all* $U_B < \tilde{U}_B$*, a prohibition of acquisitions results in an increase in consumer surplus. For* $\tilde{U}_B < U_B$*, a prohibition of acquisitions results in a decrease in consumer surplus.*

(ii) *Alternatively, assume that* $\overline{s} < \frac{9}{7}\underline{s}$ *so that, by Proposition 7,* $\hat{x}_E < \hat{x}_m$*. Then, a prohibition of acquisitions results in an increase in consumer surplus.*

Proof. See the Appendix. □

Proposition 9 is quite similar to Proposition 5 in that it illustrates that prohibiting acquisitions may increase or decrease consumer surplus. We illustrate Proposition 9 in Figure 5, where we set the parameters to the same values as in Figure 2.

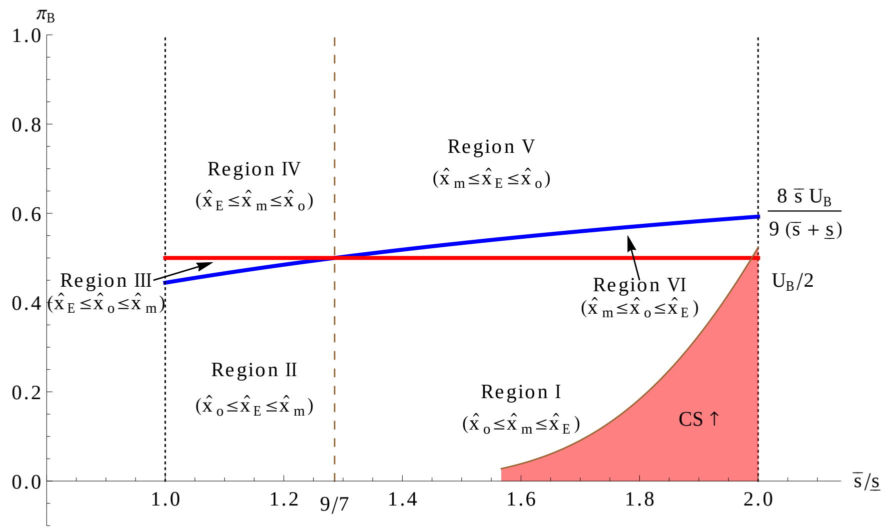

Figure 5: Consumer surplus improving start-up acquisitions

We conclude that also in this case in which the acquirer takes over the research facilities of the start-up, there exists a region of parameters for which an acquisition causes an increase in consumer surplus.

7.5 The second-best

Proposition 8 on the impact of start-up acquisitions on the direction of innovation can easily be extended to the case in which the social planner pursues a second-best allocation. For this extension we make use of the footnotes to Propositions 1 and 6.

Following the same reasoning as that around Proposition 8, we can build Figure 6, which is essentially the same as Figure 4.

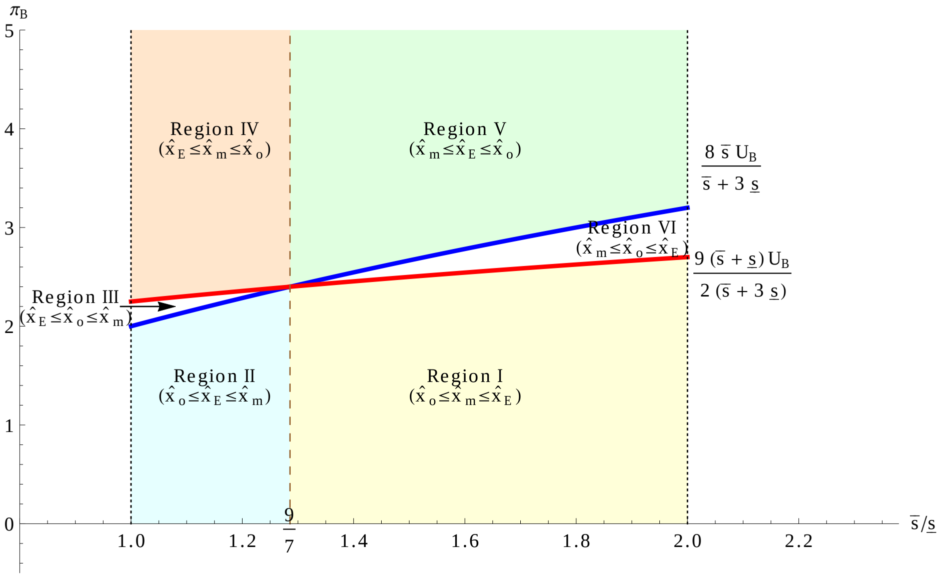

Figure 6: Private and socially second-best optimal innovation portfolios

7.6 The n-incumbents case

In this section, we examine how our result change if there are $n \geq 2$ incumbents, conditional on the acquisition of the start-up being incentive-compatible. We first note that profit and consumer surplus levels in the no-acquisition and acquisition games are identical to those in Section 6.2. Moreover, the no-acquisition investment portfolio decision is identical. We therefore turn to the investment portfolio stage of the acquisition game.

Although the expressions for profits are different, the expected payoff of the start-up is as in equation (10).[17] Therefore, when the acquirer takes over the research facilities of the start-up, the acquirer's profit-maximizing investment in project A is:

$$\hat{x}_m = \frac{1 + \epsilon_B - \epsilon_A \sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A(\overline{\pi}_A^m - \pi_A^{*m})}}}{1 + \sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A(\overline{\pi}_A^m - \pi_A^{*m})}}} \tag{15}$$

This investment level is interior provided that $\frac{\epsilon_A \epsilon_B}{(1+\epsilon_A)^2} < \frac{2\beta^2(n+1)^2}{\alpha^2((n(\bar{s}-\underline{s})+\bar{s})^2-\underline{s}^2)}\pi_B < \frac{(1+\epsilon_B)^2}{\epsilon_A \epsilon_B}$. As before, this condition always holds when ϵ_A and/or ϵ_B are sufficiently small.

Comparing $\hat{x}_m$ in (15) to the socially optimal investment in (1) we get the following result.

Proposition 6′. *In the model with $n \geq 2$ incumbents, when the acquirer takes over the research facilities of the start-up, the acquirer's investment effort in project A is excessive (and therefore its investment in project B insufficient) from the social point of view if and only if* $\pi_B < \frac{2n(n(\bar{s}-\underline{s})+2\underline{s})}{(n+1)^2(\bar{s}+\underline{s})} U_B$.

[17] Note that we denote the symmetric profit of the acquirer and the non-merging firms when they all sell low quality by π_A^{*m} rather than $\underline{\pi}_A^m$.

Comparing the no-acquisition and acquisition portfolio investment decisions we find:

Proposition 7′. *In the model with $n \geq 2$ incumbents, when the acquirer takes over the research facilities of the start-up:*

(i) The investment effort put into project A by the acquirer is lower than (and therefore that in project B higher than) that of the start-up, i.e. $\hat{x}_m < \hat{x}_E$, when $\frac{4n^2+6n-1}{2n^2+4n+1}\underline{s} < \overline{s} < 2\underline{s}$.

(ii) The investment effort put into project A by the acquirer is higher than (and therefore that in project B is lower than) that of the start-up, i.e. $\hat{x}_E < \hat{x}_m$, when $\overline{s} < \frac{4n^2+6n-1}{2n^2+4+1}\underline{s}$.

The intuition behind this result is similar to that developed above for the $n = 1$ case. In the no-acquisition case, the marginal gains from investing in project A are governed by the profit difference $\overline{\pi}_A^E - \pi_A^*$. By contrast, in the acquisition case, the marginal gains from investing in project A are related to the profit difference $\overline{\pi}_A^m - \pi_A^{*m}$. When $\overline{\pi}_A^E - \pi_A^* > \overline{\pi}_A^m - \pi_A^{*m}$, the start-up's incentive to invest in project A is greater than the acquirer's incentive.

Combining the thresholds in Propositions $1'$, $6'$ and $7'$ we observe that six regions similar to those identified in Figure 5 emerge. And the results of Propositions 8 and 9 do not qualitatively change. In particular, Proposition 9 can be reformulated as follows.

Proposition 9′. *In the model with $n \geq 2$ incumbents, suppose the acquirer takes over the research facilities of the start-up. Then:*

(i) Assume that $\frac{4n^2+6n-1}{2n^2+4n+1}\underline{s} < \overline{s} < 2\underline{s}$ so that, by Proposition 7′, $\hat{x}_m < \hat{x}_E$. Then, there exists $\tilde{U}_B > 0$ such that for all $U_B < \tilde{U}_B$, a prohibition of acquisitions results in an increase in consumer surplus. For $\tilde{U}_B < U_B$, a prohibition of acquisitions results in a decrease in consumer surplus.

(ii) Alternatively, assume that $\overline{s} < \frac{4n^2+6n-1}{2n^2+4n+1}\underline{s}$ so that, by Proposition 7′, $\hat{x}_E < \hat{x}_m$. Then, a prohibition of acquisitions results in an increase in consumer surplus.

8 Other extensions

8.1 Drastic innovations

Up to now, we have assumed that $\underline{s} < \overline{s} < 2\underline{s}$. This assumption has signified that both the high- and the low-quality products obtain demand. Suppose now that $\overline{s} > 2\underline{s}$. In this case, if the innovator obtains a high-quality product out of its research effort in project A, then the innovator monopolizes market A. How are our results modified? It turns out that our results are not affected much if we consider this case of disruptive innovations, neither for the timing in which the acquisition takes place after the research results are realized nor for that in which the acquisition occurs before the outcomes of the research projects are known.

To see this, note first that the benchmark no-acquisition investment portfolio is exactly the same as in Lemma 2 but replacing $\overline{\pi}_A^E$ by the monopoly profits $\overline{\pi}_A^m$. Second, anticipating an acquisition, the start-up will always decrease investment in project A, and increase it in project B by implication. The reason for this should be obvious by now because the acquisition rents are equal to zero when project A is successful, whereas they are positive when project A fails. Third, by the same arguments as in Proposition 4, this innovation portfolio adjustment may improve the direction of innovation. Moreover, provided that project B is sufficiently attractive for consumers, start-up acquisitions may be consumer welfare improving. Similar observations apply to acquisitions that occur before the outcomes of the research projects are known. We now provide some details about these observations.

8.1.1 Innovation for buyout

The no-acquisition game

In the case of no-acquisition and successful innovation in project A, the entrant will monopolize market A and obtain the high quality monopoly profit $\overline{\pi}_A^m$ given in Section 5.1 rather than the high quality competitive profit $\overline{\pi}_A^E$. The profit obtained in case of failure remains as in the main model. Therefore, the start-up's profit-maximizing investment is the same as that in Lemma 2 , but replacing $\overline{\pi}_A^E$ by $\overline{\pi}_A^m$. Relative to the main model, the start-up has a higher incentive to invest into project A, since the gains in case of project success are higher. A comparison of the investment level in the no-acquisition game to the socially optimal investment level implies the following result.

Proposition 1$''$. *Suppose the rival project yields a drastic innovation in case of success. In the no-acquisition game, the investment effort put into project A is excessive (and therefore investment put into project B is insufficient) from the point of view of consumer surplus maximization if and only if* $\pi_B < \frac{9\overline{s}^2-4\underline{s}^2}{18(\overline{s}^2-\underline{s}^2)}U_B$.

Proof. See the Appendix. □

Relative to the main model the value π_B for which the investment effort put into project A is exactly right according to the social planner is higher. The range of π_B for which the investment effort in A is excessive (insufficient) is larger (smaller) than in the main model.

The acquisition game

Since a successful start-up already earns the monopoly profit $\overline{\pi}_A^m$ corresponding to a high-quality seller, there are no extra gains from an acquisition. Thus, an acquisition will only be consequential when the start-up rival project turns unsuccessful. In the case of failure the Nash bargaining outcome is as described in Section 5. Therefore, the start-up's profit-maximizing investment can be obtained by replicating the steps in Section 5 but replacing $\overline{\pi}_A^E + \delta(\overline{\pi}_A^m - \overline{\pi}_A^E - \underline{\pi}_A^I)$ by $\overline{\pi}_A^m$.

Proposition 2''. *Suppose the rival project yields a drastic innovation in case of success. In the acquisition game, the investment effort put into project A is excessive (and therefore investment put into project B is insufficient) from the point of view of consumer surplus maximization if and only if* $\pi_B < \frac{9\bar{s}^2-(4+\delta)\underline{s}^2}{18(\bar{s}^2-\underline{s}^2)}U_B$.

Proof. See the Appendix. □

Relative to the main model the value π_B for which the investment effort put into project A is exactly right according to the social planner is higher. The range of π_B for which the investment effort in A is excessive (insufficient) is larger (smaller) than in the main model.

The impact of an acquisition

The impact of an acquisition on the direction of innovation is described in the following Proposition.

Proposition 3''. *Suppose the rival project yields a drastic innovation in case of success. For any* $\delta \in (0,1]$, *relative to the no-acquisition case, the start-up puts less effort into project A (and therefore more effort into project B), i.e.* $\tilde{x}_E < \hat{x}_E$. *Moreover, the difference* $\hat{x}_E - \tilde{x}_E$ *is greater the larger the start-up's bargaining power* δ.

Proof. See the Appendix. □

When project A is successful, the start-up obtains the profit $\overline{\pi}_A^m$ regardless of the acquisition regime. There is no bargaining surplus whatsoever. In contrast, in case of failure there is a bargaining surplus to be shared among the start-up and the acquirer. Anticipating an acquisition and the extra rents generated in case of failure, the entrant distorts its portfolio by investing less (more) into project A (B).

Finally, Proposition 4'' extends Proposition 4 to the case in which project A yields a drastic innovation when successful.

Proposition 4''. *Suppose the rival project yields a drastic innovation in case of success, in which case the start-up, anticipating its acquisition, reduces investment in project A and increases it in project B. Then:*

- *if* $\pi_B < \frac{9\bar{s}^2-(4+\delta)\underline{s}^2}{18(\bar{s}^2-\underline{s}^2)}U_B$, *then* $\hat{x}_o < \tilde{x}_E < \hat{x}_E$ *and thus an acquisition improves the direction of innovation;*
- *if* $\frac{9\bar{s}^2-4\underline{s}^2}{18(\bar{s}^2-\underline{s}^2)}U_B < \pi_B$, *then* $\tilde{x}_E < \hat{x}_E < \hat{x}_o$ *and thus an acquisition worsens the direction of innovation;*
- *finally, if* $\frac{9\bar{s}^2-(4+\delta)\underline{s}^2}{18(\bar{s}^2-\underline{s}^2)}U_B < \pi_B < \frac{9\bar{s}^2-4\underline{s}^2}{18(\bar{s}^2-\underline{s}^2)}U_B$, *then* $\tilde{x}_E < \hat{x}_o < \hat{x}_E$ *and thus an acquisition causes a move from overinvestment to underinvestment in A.*

Proposition 4$''$ shows that an acquisition may result in either an alignment or a misalignment of the private incentives to invest compared with the social incentives. In terms of Figure 1, we are left with three regions. There is one region in which the direction of innovation improves, one in which it worsens and one in which the effect is unclear. These regions are analogous to regions I,V and VI as described in the main model. Furthermore, the prohibition of start-ups would certainly boost consumer welfare if we are in region V, but it may reduce consumer welfare if we are in region I. Proposition 5$''$ below confirms this.

Proposition 5$''$. *Suppose the rival project yields a drastic innovation in case of success so that, by Proposition 3$''$, $\tilde{x}_E < \hat{x}_E$. Then, there exists $\tilde{U}_B > 0$ such that for all $U_B < \tilde{U}_B$, a prohibition of acquisitions results in an increase in consumer surplus. For $U_B > \tilde{U}_B$, a prohibition of acquisitions results in a decrease in consumer surplus.*

Proof. See the Appendix. □

8.1.2 Arrow replacement effect

Suppose now that the rival project yields a drastic innovation in the model in which acquisitions take place before the investment decision is made. Observe that the social optimum and the outcome of the no-acquisition game remain as described in Subsection 8.1.1. Further, the expected payoff and the corresponding optimal investment level of the acquirer are as in the main model of Section 7. Moreover, if acquisitions are allowed they will always occur. This leads to the following result.

Proposition 7$''$. *Suppose the rival project yields a drastic innovation in case of success. Then, relative to the no-acquisition case, the investment effort put into project A decreases (and therefore that in project B increases).*

Proof. See the Appendix. □

Regardless of the parameter values, the Arrow replacement effect of the entrant, which is given by $\overline{\pi}_A^m - \pi_A^*$, is always higher than that of the acquirer, which is given by $\overline{\pi}_A^m - \underline{\pi}_A^m$. Hence, the acquirer's investment effort in project A is always lower than that of the entrant. Therefore, the impact of an acquisition on the direction of innovation and on consumer surplus is qualitatively the same as when the acquisition takes place after the investment decision.

Proposition 8$''$. *Suppose the rival project yields a drastic innovation in case of success. Moreover, suppose that the incumbent takes over the research facilities of the start-up. Then: Assume that $2\underline{s} < \overline{s}$ so that the acquirer reduces investment in project A and increases it in project B. Then:*

- *if $\pi_B < \frac{U_B}{2}$, then $\hat{x}_o < \hat{x}_m < \hat{x}_E$ and thus an acquisition improves the direction of innovation;*
- *if $\frac{9\overline{s}^2 - 4\underline{s}^2}{18(\overline{s}^2 - \underline{s}^2)} U_B < \pi_B$, then $\hat{x}_m < \hat{x}_E < \hat{x}_o$ and thus an acquisition worsens the direction of innovation;*

- *finally, if* $\frac{U_B}{2} < \pi_B < \frac{9\bar{s}^2 - 4\underline{s}^2}{18(\bar{s}^2 - \underline{s}^2)} U_B$*, then* $\hat{x}_m < \hat{x}_o < \hat{x}_E$ *and thus an acquisition causes a move from overinvestment to underinvestment in A.*

The effect on consumer surplus, which is similar to that in Proposition 9, is described below.

Proposition 9″. *Suppose the rival project yields a drastic innovation in case of success so that, by Proposition 7″,* $\hat{x}_m < \hat{x}_E$*. Then, there exists* $\tilde{U}_B > 0$ *such that for all* $U_B < \tilde{U}_B$*, a prohibition of acquisitions results in an increase in consumer surplus. For* $U_B > \tilde{U}_B$*, a prohibition of acquisitions results in a decrease in consumer surplus.*

Proof. See the Appendix. □

8.2 Worthless unsuccessful rival project

In our main model we have assumed that an entrant whose rival project fails is able to enter the market with a product exactly identical to that of the incumbent. In some cases when IP rights are strongly protected, this may not be possible. In this extension we investigate what happens if the entrant can only enter market A if its rival project is successful.

8.2.1 Innovation for buyout

The no-acquisition game

As in the main model, if project A is successful the entrant will enter the market with the high quality product, earning a profit π_A^E. If project A fails, the entrant will not enter the market and obtain zero profits (rather than π_A^*). Therefore, the start-up's profit-maximizing investment is equal to that in Lemma 2 but replacing π_A^* by zero. As a result, relative to the main model, the start-up has a higher incentive to invest into project A.

Proposition 1‴. *Suppose that an unsuccessful rival project is worthless. In the no-acquisition game, the investment effort put into project A is excessive (and therefore investment put into project B is insufficient) from the point of view of consumer surplus maximization if and only if* $\pi_B < \frac{2(2\bar{s} - \underline{s})^2}{9(\bar{s}^2 - \underline{s}^2)} U_B$.

Proof. See the Appendix. □

Relative to the main model, the value π_B for which the investment effort put into project A is exactly right according to the social planner is higher. The range of π_B for which the investment effort in A is excessive (insufficient) is larger (smaller) than in the main model.

The acquisition game

An acquisition generates no additional rents when the start-up's rival project turns unsuccessful. This is because the incumbent already earns the monopoly profit $\underline{\pi}_A^m$. Therefore, the start-up's profit-maximizing investment anticipating its acquisition is equal to that in Lemma 3 but replacing the difference $\pi_A^* - \delta(\underline{\pi}_A^m - 2\pi_A^*)$ by zero. Comparing the profit-maximizing investment level to the socially optimal one gives:

Proposition 2‴. *Suppose that an unsuccessful rival project is worthless. In the acquisition game, the investment effort put into project A is excessive (and therefore investment put into project B is insufficient) from the point of view of consumer surplus maximization if and only if* $\pi_B < \frac{4(2\overline{s}-\underline{s})^2 - \delta(11\overline{s}^2+20\underline{s}^2-32\overline{s}\underline{s})}{18(\overline{s}^2-\underline{s}^2)} U_B$.

Proof. See the Appendix. □

The threshold for the investment effort put into project A to be excessive is again higher than the threshold in the main model.

The impact of an acquisition

The impact of an acquisition on the direction of innovation is described in the following Proposition.

Proposition 3‴. *Suppose that an unsuccessful rival project is worthless. For any* $\delta \in (0, 1]$, *relative to the no-acquisition case, the start-up puts more effort into project A (and therefore less effort into project B), i.e.* $\hat{x}_E < \tilde{x}_E$. *Moreover, the difference* $\tilde{x}_E - \hat{x}_E$ *is greater the larger the start-up's bargaining power* δ.

Proof. See the Appendix. □

When project A is successful and acquisitions are prohibited, the start-up obtains a profit equal to $\overline{\pi}_A^E$. If acquisitions are allowed, the start-up receives the same profit plus some additional bargaining rents. In case of failure, the entrant's payoff is zero regardless of the acquisition regime. Allowing acquisitions then causes the entrant to anticipate the additional rents in the case of success, which makes success relatively more attractive than in the case where acquisitions are prohibited. Hence, the entrant distorts its portfolio by investing more (less) in project A (B).

Finally, Proposition 4‴ extends Proposition 4 to this case in which an unsuccessful rival project does not generate any profit whatsoever.

Proposition 4‴. *Suppose that an unsuccessful rival project is worthless so that the start-up, anticipating its acquisition, increases investment in project A and decreases it in project B. Then:*

- *if* $\frac{4(2\overline{s}-\underline{s})^2 - \delta(11\overline{s}^2+20\underline{s}^2-32\overline{s}\underline{s})}{18(\overline{s}^2-\underline{s}^2)} U_B < \pi_B$, *then* $\hat{x}_E < \tilde{x}_E < \hat{x}_o$ *and thus an acquisition improves the direction of innovation;*

- *if* $\pi_B < \frac{2(2\overline{s}-\underline{s})^2}{9(\overline{s}^2-\underline{s}^2)} U_B$, *then* $\hat{x}_o < \hat{x}_E < \tilde{x}_E$ *and thus an acquisition worsens the direction of innovation;*

- *finally, if* $\frac{2(2\overline{s}-\underline{s})^2}{9(\overline{s}^2-\underline{s}^2)} U_B < \pi_B < \frac{4(2\overline{s}-\underline{s})^2 - \delta(11\overline{s}^2+20\underline{s}^2-32\overline{s}\underline{s})}{18(\overline{s}^2-\underline{s}^2)} U_B$, *then* $\hat{x}_E < \hat{x}_o < \tilde{x}_E$ *and thus an acquisition causes a move from overinvestment to underinvestment in A.*

In terms of Figure 1, only regions II, III and IV remain and the direction of innovation in these regions is affected in the same way as in the main model. The next proposition confirms that consumer surplus may increase or a decrease when acquisitions are prohibited.

Proposition 5‴. *Assume that an unsuccessful rival project is worthless and* $\overline{s} < 2\underline{s}$ *so that, by Proposition 3‴,* $\hat{x}_E < \tilde{x}_E$*. Then, a prohibition of acquisitions can result in an increase or decrease in consumer surplus.*

Proof. See the Appendix. □

8.2.2 Arrow replacement effect

Suppose now that acquisitions take place before the investment decision is made. The social optimum and the no-acquisition game remain as described above in Subsection 8.2. The expected payoff and the corresponding optimal investment level of the acquirer are as in the main model of Section 7. If acquisitions are allowed they will always occur.

Proposition 7‴. *Suppose that the incumbent takes over the research facilities of the start-up. Moreover, suppose that an unsuccessful rival project is worthless. Then, relative to the no-acquisition case, the investment effort put into project A decreases (and therefore that in project B increases).*

Proof. See the Appendix. □

For the entrant, the Arrow replacement effect in market A is $\overline{\pi}_A^m$, whereas, for the acquirer what matters is $\overline{\pi}_A^m - \underline{\pi}_A^m$. The incentive for the entrant to invest into project A is always higher than that of the acquirer. The situation is then qualitatively the same as when the acquisition takes place after the investment decision.

Comparing the investment levels in the no-acquisition and acquisition cases to the socially optimal investment level we obtain the following result:

Proposition 8‴. *Suppose that the incumbent takes over the research facilities of the start-up. Moreover, suppose that an unsuccessful rival project is worthless. Then: Assume that* $\underline{s} < \overline{s} < 2\underline{s}$ *so that the acquirer reduces investment in project A and increases it in project B. Then:*

- *if* $\pi_B < \frac{U_B}{2}$*, then* $\hat{x}_o < \hat{x}_m < \hat{x}_E$ *and thus an acquisition improves the direction of innovation;*
- *if* $\frac{2(2\overline{s}-\underline{s})^2}{9(\overline{s}^2-\underline{s}^2)}U_B < \pi_B$*, then* $\hat{x}_m < \hat{x}_E < \hat{x}_o$ *and thus an acquisition worsens the direction of innovation;*
- *finally, if* $\frac{U_B}{2} < \pi_B < \frac{2(2\overline{s}-\underline{s})^2}{9(\overline{s}^2-\underline{s}^2)}U_B$*, then* $\hat{x}_m < \hat{x}_o < \hat{x}_E$ *and thus an acquisition causes a move from overinvestment to underinvestment in A.*

In terms of Figure 1, what happens in this setting is that only regions I, V and VI remain and the direction of innovation in these regions is affected in the same way as in the main model. The following Proposition extends Proposition 9 and shows that our results remain valid.

Proposition 9'''. *Suppose that the incumbent takes over the research facilities of the start-up. Moreover, suppose that an unsuccessful rival project is worthless so that, by Proposition 7''', $\hat{x}_m < \hat{x}_E$. Then, there exists $\tilde{U}_B > 0$ such that for all $U_B < \tilde{U}_B$, a prohibition of acquisitions results in an increase in consumer surplus. For $U_B > \tilde{U}_B$, a prohibition of acquisitions results in a decrease in consumer surplus.*

Proof. See the Appendix. □

8.3 Horizontal product differentiation

Throughout the paper we have assumed that $\sigma = 2$ which implies that products only differ in quality. In this subsection we report what happens when products are also horizontally differentiated. For this purpose we let $0 < \sigma < 2$ and proceed by solving the model as in the main body of the paper.

Social optimum

The social planner sets price equal to marginal cost. The corresponding levels of the surplus consumers obtain at the optimal quantities is given by:

$$\overline{U}_A^o = \frac{\alpha^2}{\beta^2}\left[\frac{\overline{s}^2 + \underline{s}^2 - \sigma\overline{s}\underline{s}}{(4-\sigma^2)}\right], \quad \underline{U}_A^o = \frac{\alpha^2}{\beta^2}\left[\frac{\underline{s}^2}{(2+\sigma)}\right],$$

where, as in the main body of the paper, $\overline{U}_A^o$ denotes the level of surplus in case the rival project is successful and $\underline{U}_A^o$ otherwise. The new social optimum is then given by (1) after plugging these new expressions for consumer surplus.

The no-acquisition game

Let $\overline{s} < \frac{4}{\sigma}\underline{s}$, which is the new condition that ensures that the rival project is non-drastic. With horizontal product differentiation, provided that the rival project is successful, it is sometimes the case that the acquirer becomes a multi-product monopolist and offers the two qualities in the market. This occurs when $\overline{s} < \frac{2}{\sigma}\underline{s}$. Alternatively, when $\overline{s} > \frac{2}{\sigma}\underline{s}$, as it was the case in the main body of the paper, the acquirer does not find it profitable to offer both qualities and instead offers only the high-quality product. In this section we focus on the former case in which the acquirer puts both products in the market.

In the market competition stage, the profits of the start-up and the incumbent in case of successful innovation, denoted $\overline{\pi}_A^E$ and $\underline{\pi}_A^I$ respectively, and consumer surplus are:

$$\overline{\pi}_A^E = \frac{2\alpha^2}{\beta^2}\left[\frac{(4\overline{s} - \sigma\underline{s})^2}{(16-\sigma^2)^2}\right], \quad \underline{\pi}_A^I = \frac{2\alpha^2}{\beta^2}\left[\frac{(4\underline{s} - \sigma\overline{s})^2}{(16-\sigma^2)^2}\right], \quad \overline{U}_A = \frac{\alpha^2}{\beta^2}\left[\frac{(\overline{s}^2 + \underline{s}^2)(16-3\sigma^2) + \sigma^3\overline{s}\underline{s}}{(16-\sigma^2)^2}\right].$$

In case of innovation failure, the profits of the start-up and the incumbent and consumer surplus are:

$$\pi_A^E = \pi_A^I = \pi_A^* = \frac{2\alpha^2}{\beta^2}\left[\frac{\underline{s}^2}{(4+\sigma)^2}\right], \quad \underline{U}_A = \frac{\alpha^2}{\beta^2}\left[\frac{(2+\sigma)\underline{s}^2}{(4+\sigma)^2}\right]$$

The equilibrium investment portfolio in the no-acquisition game is given by (3), after plugging the new expressions for profits. Comparing the socially optimal investment to the entrant's investment level gives the following.

Proposition 1''''. *Assume that products are also horizontally differentiated, specifically $\overline{s} < \frac{2}{\sigma}\underline{s}$. In the no-acquisition game, the start-up's investment effort in project A is excessive (and therefore its investment in project B insufficient) from the point of view of consumer surplus maximization if and only if $\pi_B < \frac{16(2(\overline{s}+\underline{s})-\sigma\underline{s})}{(\overline{s}+\underline{s}(1-\sigma))(4+\sigma)^2}U_B$.*

Proof. See the Appendix. □

The acquisition game

With substantial horizontal product differentiation, as implied by the parameter restriction $\overline{s} < \frac{2}{\sigma}\underline{s}$, an acquisition is always incentive compatible and, thus, always takes place when it is allowed. After an acquisition, the profits of the acquirer and consumer surplus in case the rival project is successful or unsuccessful are given by the following expressions.

$$\overline{\pi}_A^m = \frac{\alpha^2}{\beta^2}\left[\frac{\overline{s}^2+\underline{s}^2-\sigma\overline{s}\underline{s}}{2(4-\sigma^2)}\right], \quad \overline{U}_A^m = \frac{\alpha^2}{\beta^2}\left[\frac{\overline{s}^2+\underline{s}^2-\sigma\overline{s}\underline{s}}{4(4-\sigma^2)}\right]; \quad \underline{\pi}_A^m = \frac{\alpha^2}{\beta^2}\left[\frac{\underline{s}^2}{2(2+\sigma)}\right], \quad \underline{U}_A^m = \frac{\alpha^2}{\beta^2}\left[\frac{\underline{s}^2}{4(2+\sigma)}\right].$$

In the investment portfolio stage, the expressions for the expected profit and optimal investment level are the same as in the main model, (4) and (5), after plugging in the new profit and surplus expressions. A comparison of the equilibrium investment level of the start-up when it anticipates its acquisition with the socially optimal one gives the following result.

Proposition 2''''. *Assume that products are also horizontally differentiated, specifically $\overline{s} < \frac{2}{\sigma}\underline{s}$. In the acquisition game, the investment effort put into project A is excessive (and therefore investment put into project B is insufficient) from the point of view of consumer surplus maximization if and only if $\pi_B < \frac{16(2(\overline{s}+\underline{s})-\sigma\underline{s})}{(\overline{s}+\underline{s}(1-\sigma))(4+\sigma)^2}U_B + \frac{\delta\left[\overline{s}(16+5\sigma^2)-\underline{s}(\sigma(32+\sigma^2)-(16+5\sigma^2))\right]}{2(\overline{s}+\underline{s}(1-\sigma))(16-\sigma^2)^2}U_B$.*

Proof. See the Appendix. □

The impact of an acquisition

The impact that an acquisition has on the portfolio choice made by the start-up is given by the following result.

Proposition 3''''. *Assume that products are also horizontally differentiated, specifically $\overline{s} < \frac{2}{\sigma}\underline{s}$. For any $\delta \in (0,1]$, relative to the no-acquisition case, the start-up puts more effort into project A (and therefore less effort into project B), i.e. $\hat{x}_E < \tilde{x}_E$. Moreover, the difference $\tilde{x}_E - \hat{x}_E$ is greater the larger the start-up's bargaining power δ.*

Proof. See the Appendix. □

In the no-acquisition game, the additional rents a successful innovation generates in market A are equal to $\overline{\pi}_A^E - \pi_A^*$ while in the acquisition game they are equal to $\overline{\pi}_A^E - \pi_A^* + \delta(\overline{\pi}_A^m - \overline{\pi}_A^E - \underline{\pi}_A^I) - \delta(\underline{\pi}_A^m - 2\pi_A^*)$. With substantial horizontal product differentiation, it turns out that the latter is always larger than the former. Consequently, the start-up will invest more in project A (and less in B) in anticipation of the acquisition as compared to when an acquisition is not allowed.

Comparing the thresholds in Propositions $1''''$, $2''''$ and $3''''$ we derive the following result, which extends Proposition 4 to this setting with substantial horizontal product differentiation.

Proposition 4''''. *Assume that products are also horizontally differentiated, specifically $\overline{s} < \frac{2}{\sigma}\underline{s}$, so that the start-up, anticipating its acquisition, increases investment in project A and decreases it in project B. Then:*

- *if $\frac{16(2(\overline{s}+\underline{s})-\sigma\underline{s})}{(\overline{s}+\underline{s}(1-\sigma))(4+\sigma)^2}U_B + \frac{\delta\left[\overline{s}(16+5\sigma^2)-\underline{s}(\sigma(32+\sigma^2)-(16+5\sigma^2))\right]}{2(\overline{s}+\underline{s}(1-\sigma))(16-\sigma^2)^2}U_B < \pi_B$, then $\hat{x}_E < \tilde{x}_E < \hat{x}_o$ and thus an acquisition improves the direction of innovation;*

- *if $\pi_B < \frac{16(2(\overline{s}+\underline{s})-\sigma\underline{s})}{(\overline{s}+\underline{s}(1-\sigma))(4+\sigma)^2}U_B$, then $\hat{x}_o < \hat{x}_E < \tilde{x}_E$ and thus an acquisition worsens the direction of innovation;*

- *finally, if $\frac{16(2(\overline{s}+\underline{s})-\sigma\underline{s})}{(\overline{s}+\underline{s}(1-\sigma))(4+\sigma)^2}U_B < \pi_B < \frac{16(2(\overline{s}+\underline{s})-\sigma\underline{s})}{(\overline{s}+\underline{s}(1-\sigma))(4+\sigma)^2}U_B + \frac{\delta\left[\overline{s}(16+5\sigma^2)-\underline{s}(\sigma(32+\sigma^2)-(16+5\sigma^2))\right]}{2(\overline{s}+\underline{s}(1-\sigma))(16-\sigma^2)^2}U_B$, then $\hat{x}_E < \hat{x}_o < \tilde{x}_E$ and thus an acquisition causes a move from underinvestment to overinvestment in A.*

The start-up always increases its investment into project A in anticipation of an acquisition. Therefore, in terms of Figure 1, we lose regions I, V and VI. In the remaining regions the effects are the same as in the main model. In region II the direction of innovation worsens, in region III we move from under- to overinvestment in project A and in region IV the direction of innovation improves. Again, our next result confirms that acquisitions always reduce consumer surplus in this setting with horizontal product differentiation.

Proposition 5''''. *Assume that products are also horizontally differentiated, specifically $\overline{s} < \frac{2}{\sigma}\underline{s}$ so that, by Proposition 3'''', $\hat{x}_E < \tilde{x}_E$. Then, a prohibition of acquisitions results in an increase in consumer surplus.*

Proof. See the Appendix. □

9 Conclusions

Start-up acquisitions have recently spurred much interest among politicians, policy makers and academicians. Many have argued that merger policy has been extremely lenient when it comes to start-up acquisitions and have called for reform. Others have warned that blanket prohibitions are not desirable because they may reduce the incentive for innovation. This paper has contributed to this debate by examining start-up acquisitions from a new angle. In particular, we have asked how the palette of innovation projects of a start-up is affected by acquisitions.

To this end, we have formulated a novel model of an industry with an incumbent operating in a single market and an entrant start-up. The start-up engages in an investment portfolio problem by choosing how to allocate funds across a rival project, intended to challenge the incumbent's dominant position, and a non-rival project. We have shown how, motivated by the private returns of the projects, a start-up picks a socially suboptimal portfolio of projects. We have then examined how an acquisition impacts the optimality of the equilibrium portfolio of projects and consumer surplus.

We have shown that, anticipating an acquisition, the start-up, purely motivated by rent-seeking, strategically distorts its investment portfolio in a way that may improve of worsen the direction in which innovation goes. Moreover, when the direction of innovation improves, its improvement may be so large so as to dominate the usual quantity distortion. This result has added to the literature by pointing out a new way in which the "innovation for buyout" argument may increase consumer surplus. Later in the paper we have turned to settings in which the acquirer takes over the research facilities of the start-up. In those settings, we have seen how both the start-up and the acquirer face the so-called "replacement effect" and that it is not necessarily the case that the replacement effect is stronger for acquirers than for start-ups. Also in such settings we have demonstrated that acquisitions may improve or worsen the direction of innovation.

Our results have some antitrust recommendations packed. When acquisitions are allowed, rent-seeking start-ups tend to displace investment from rival to non-rival projects when the former are highly disruptive for existing firms. Because in those situations quantity distortions are expected to be relatively small, provided that non-rival projects benefit consumers much, start-up acquisitions may enhance consumer surplus. By contrast, when rival projects are moderately of little disruptive, quantity distortions are expected to be so large that a prohibition of start-up acquisitions increases consumer welfare. The same result arises when there are many firms in the market, or when products are substantially differentiated horizontally.

Appendix

Proof of Proposition 1

We restrict attention to investment levels that are interior as specified in Lemmas 1 and 2. $\hat{x}_o < \hat{x}_E$ if and only if

$$\frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B U_B}{\epsilon_A(\overline{U}_A^o-\underline{U}_A^o)}}}{1+\sqrt{\frac{\epsilon_B U_B}{\epsilon_A(\overline{U}_A^o-\underline{U}_A^o)}}} < \frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A(\overline{\pi}_A^E-\pi_A^*)}}}{1+\sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A(\overline{\pi}_A^E-\pi_A^*)}}}.$$

Since the LHS and RHS of this expression are decreasing in the term under the square root we have

$$(\overline{U}_A^o-\underline{U}_A^o)\pi_B < (\overline{\pi}_A^E-\pi_A^*)U_B.$$

Using the expression for surplus given in Section 3, and for profits given in Section 4.1 we obtain

$$\frac{\alpha^2(\overline{s}^2-\underline{s}^2)}{4\beta^2}\pi_B < \frac{2\alpha^2\overline{s}(\overline{s}-\underline{s})}{9\beta^2}U_B.$$

Solving we find

$$\pi_B < \frac{8\overline{s}(\overline{s}-\underline{s})}{9(\overline{s}^2-\underline{s}^2)}U_B.$$

Which can be rewritten as

$$\pi_B < \frac{8\overline{s}}{9(\overline{s}+\underline{s})}U_B.$$

□

Proof of Proposition 2

We restrict attention to investment levels that are interior as specified in Lemmas 1 and 3. $\hat{x}_o < \tilde{x}_E$ if and only if

$$(\overline{U}_A^o-\underline{U}_A^o)\pi_B < \left(\overline{\pi}_A^E+\delta(\overline{\pi}_A^m-\overline{\pi}_A^E-\underline{\pi}_A^I)-\pi_A^*-\delta(\underline{\pi}_A^m-2\pi_A^*)\right)U_B.$$

Using the expression for surplus given in Section 3, and for profits given in Section 4.1 and Section 5.1 we obtain

$$\frac{\alpha^2(\overline{s}^2-\underline{s}^2)}{4\beta^2}\pi_B < \frac{\alpha^2\left(16\overline{s}(\overline{s}-\underline{s})+\delta(-11\overline{s}^2-21\underline{s}^2+32\overline{s}\underline{s})\right)}{72\beta^2}U_B.$$

Solving we find

$$\pi_B < \frac{16\overline{s}(\overline{s}-\underline{s})+\delta(-11\overline{s}^2-21\underline{s}^2+32\overline{s}\underline{s})}{18(\overline{s}^2-\underline{s}^2)}U_B.$$

Which can be rewritten as

$$\pi_B < \frac{16\overline{s}-\delta(11\overline{s}-21\underline{s})}{18(\overline{s}+\underline{s})}U_B.$$

□

Proof of Proposition 3

(i) $\tilde{x}_E < \hat{x}_E$ if and only if

$$\overline{\pi}_A^E + \delta(\overline{\pi}_A^m - \overline{\pi}_A^E - \underline{\pi}_A^I) - \pi_A^* - \delta(\underline{\pi}_A^m - 2\pi_A^*) < \overline{\pi}_A^E - \pi_A^*.$$

Dropping the common terms gives

$$\delta\left(\overline{\pi}_A^m - \underline{\pi}_A^m - \overline{\pi}_A^E - \underline{\pi}_A^I + 2\pi_A^*\right) < 0.$$

Using the expression for profits given in Section 4.1 and Section 5.1 we obtain

$$\frac{\alpha^2(-11\overline{s}^2 - 21\underline{s}^2 + 32\overline{s}\underline{s})}{72\beta^2} < 0.$$

Simplifying gives

$$0 < (11\overline{s} - 21\underline{s})(\overline{s} - \underline{s}).$$

Solving we find

$$\frac{21}{11}\underline{s} < \overline{s} < 2\underline{s}.$$

Therefore, $\hat{x}_E < \tilde{x}_E$ when $\underline{s} < \overline{s} < \frac{21}{11}\underline{s}$.

(ii) When $\tilde{x} < \hat{x}$, $\tilde{x}$ is decreasing in δ. When $\hat{x} < \tilde{x}$, $\tilde{x}$ is increasing in δ. Hence, in both cases $\tilde{x} - \hat{x}$ increases in δ. □

Proof of Proposition 5

(i) Suppose that $\frac{21}{11}\underline{s} < \overline{s} < 2\underline{s}$, in which case, $\tilde{x}_E < \hat{x}_E$ and therefore the expected consumer surplus in market B is lower if acquisitions are prohibited.

$$\frac{1 - \hat{x}_E}{1 - \hat{x}_E + \epsilon_B} U_B < \frac{1 - \tilde{x}_E}{1 - \tilde{x}_E + \epsilon_B} U_B. \tag{16}$$

To show the result, we start by noticing that, for $\overline{s} < 2\underline{s}$, we have $\overline{U}_A^m < \overline{U}_A$ so

$$\frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\overline{U}_A^m = \left(\frac{\tilde{x}_E}{\tilde{x}_E + \epsilon_A} + \frac{\epsilon_A}{\tilde{x}_E + \epsilon_A} - \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\right)\overline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\overline{U}_A.$$

Further, because $\underline{U}_A^m < \overline{U}_A^m$,

$$\frac{\tilde{x}_E}{\tilde{x}_E + \epsilon_A}\overline{U}_A^m + \left(\frac{\epsilon_A}{\tilde{x}_E + \epsilon_A} - \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\right)\underline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\overline{U}_A.$$

Furthermore, because $\underline{U}_A^m < \underline{U}_A$, we can write

$$\frac{\tilde{x}_E}{\tilde{x}_E + \epsilon_A}\overline{U}_A^m + \left(\frac{\epsilon_A}{\tilde{x}_E + \epsilon_A} - \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\right)\underline{U}_A^m + \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\underline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\overline{U}_A + \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\underline{U}_A.$$

Simplifying gives:

$$\frac{\tilde{x}_E}{\tilde{x}_E+\epsilon_A}\overline{U}_A^m+\frac{\epsilon_A}{\tilde{x}_E+\epsilon_A}\underline{U}_A^m<\frac{\hat{x}_E}{\hat{x}_E+\epsilon_A}\overline{U}_A+\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\underline{U}_A, \tag{17}$$

which means that the expected consumer surplus in market A is higher if acquisitions are prohibited.

Because the expected consumer surplus is a continuous function of U_B, putting together (16) and (17) implies the result.

(*ii*) Suppose that $\overline{s}<\frac{21}{11}\underline{s}$, in which case, $\hat{x}_E<\tilde{x}_E$ and therefore the expected consumer surplus in market B is higher if acquisitions are prohibited.

$$\frac{1-\tilde{x}_E}{1-\tilde{x}_E+\epsilon_B}U_B<\frac{1-\hat{x}_E}{1-\hat{x}_E+\epsilon_B}U_B. \tag{18}$$

We now show that the expected consumer surplus in market A is also higher if acquisitions are prohibited. For this, we need to show that

$$\frac{\tilde{x}_E}{\tilde{x}_E+\epsilon_A}\overline{U}_A^m+\frac{\epsilon_A}{\tilde{x}_E+\epsilon_A}\underline{U}_A^m<\frac{\hat{x}_E}{\hat{x}_E+\epsilon_A}\overline{U}_A+\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\underline{U}_A. \tag{19}$$

We can rewrite this as:

$$\left(\frac{\hat{x}_E}{\hat{x}_E+\epsilon_A}+\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}-\frac{\epsilon_A}{\tilde{x}_E+\epsilon_A}\right)\overline{U}_A^m+\frac{\epsilon_A}{\tilde{x}_E+\epsilon_A}\underline{U}_A^m<\frac{\hat{x}_E}{\hat{x}_E+\epsilon_A}\overline{U}_A+\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\underline{U}_A.$$

We have $\overline{U}_A^m<\overline{U}_A$ so:

$$\left(\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}-\frac{\epsilon_A}{\tilde{x}_E+\epsilon_A}\right)\overline{U}_A^m+\frac{\epsilon_A}{\tilde{x}_E+\epsilon_A}\underline{U}_A^m<\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\underline{U}_A.$$

Rewriting gives:

$$\frac{\epsilon_A}{\tilde{x}_E+\epsilon_A}\left(\underline{U}_A^m-\overline{U}_A^m\right)+\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\left(\overline{U}_A^m-\underline{U}_A\right)<0. \tag{20}$$

In market A we can rank consumer surplus in two different ways depending on the quality differences.

1. For all $\frac{\overline{s}}{\underline{s}}\in[1,\frac{4}{3})$ we have $\underline{U}_A^m<\overline{U}_A^m<\underline{U}_A<\overline{U}_A$.
2. For all $\frac{\overline{s}}{\underline{s}}\in(\frac{4}{3},\frac{21}{11})$ we have $\underline{U}_A^m<\underline{U}_A<\overline{U}_A^m<\overline{U}_A$.

Therefore, the first term is always negative since $\underline{U}_A^m<\overline{U}_A^m$. The second term is also negative when $\frac{\overline{s}}{\underline{s}}\in[1,\frac{4}{3})$, in which case (20) holds.

When $\frac{\overline{s}}{\underline{s}}\in(\frac{4}{3},\frac{21}{11})$, things are a bit more complicated because $\underline{U}_A<\overline{U}_A^m$. We rewrite (20) as follows:

$$\frac{\hat{x}_E+\epsilon_A}{\tilde{x}_E+\epsilon_A}>\frac{\overline{U}_A^m-\underline{U}_A}{\overline{U}_A^m-\underline{U}_A^m} \tag{21}$$

and notice that the RHS of (21) is increasing in $\overline{s}$. Setting $\overline{s}=\frac{21}{11}\underline{s}$, the RHS of (21) takes on value

0.706 approximately. Therefore, if we show that the LHS of (21) is greater than 0.706, the proof is complete.

To do this, we first note that the LHS of (21) is decreasing in δ. This is because $\tilde{x}$ goes up as δ rises. Therefore, setting $\delta = 1$ for the LHS of (21) it holds that:

$$\frac{\hat{x}_E + \epsilon_A}{\tilde{x}_E + \epsilon_A} > 4\sqrt{\frac{\overline{s}}{5\overline{s} + 21\underline{s}}} \frac{\frac{\alpha}{6\sqrt{2}\beta}\sqrt{(\overline{s} - \underline{s})(5\overline{s} + 21\underline{s})} + \sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A}}}{\frac{\alpha\sqrt{2}}{3\beta}\sqrt{\overline{s}(\overline{s} - \underline{s})} + \sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A}}}$$

$$= 4\sqrt{\frac{\overline{s}}{5\overline{s} + 21\underline{s}}} \frac{\frac{\sqrt{(\overline{s}-\underline{s})(5\overline{s}+21\underline{s})}}{6\sqrt{2}} + \sqrt{z}}{\frac{\sqrt{2\overline{s}(\overline{s}-\underline{s})}}{3} + \sqrt{z}}$$

where we have defined $z \equiv \frac{\epsilon_B \pi_B \beta^2}{\epsilon_A \alpha^2}$. We now show that this last expression is increasing in ϵ_A (and so decreasing in z). In fact, its derivative with respect to z is equal to

$$4\sqrt{\frac{\overline{s}}{5\overline{s} + 21\underline{s}}} \frac{1}{2\sqrt{z}} \frac{-\frac{\sqrt{(\overline{s}-\underline{s})(5\overline{s}+21\underline{s})}}{6\sqrt{2}} + \frac{\sqrt{2\overline{s}(\overline{s}-\underline{s})}}{3}}{\left(\frac{\sqrt{2\overline{s}(\overline{s}-\underline{s})}}{3} + \sqrt{z}\right)^2}.$$

The sign of this expression depends on the sign of

$$-\frac{\sqrt{5\overline{s} + 21\underline{s}}}{6\sqrt{2}} + \frac{\sqrt{2\overline{s}}}{3} = \frac{-\sqrt{5\overline{s} + 21\underline{s}} + 4\sqrt{\overline{s}}}{6\sqrt{2}} < 0 \text{ for all } \frac{\overline{s}}{\underline{s}} \in \left[\frac{4}{3}, \frac{21}{11}\right].$$

Therefore, taking the limit when $z \to \infty$ ($\epsilon_A \to 0$) we can write that

$$\frac{\hat{x}_E + \epsilon_A}{\tilde{x}_E + \epsilon_A} > \lim_{z\to\infty} \left[4\sqrt{\frac{\overline{s}}{5\overline{s} + 21\underline{s}}} \frac{\frac{\sqrt{(\overline{s}-\underline{s})(5\overline{s}+21\underline{s})}}{6\sqrt{2}} + \sqrt{z}}{\frac{\sqrt{2\overline{s}(\overline{s}-\underline{s})}}{3} + \sqrt{z}}\right] = 4\sqrt{\frac{\overline{s}}{5\overline{s} + 21\underline{s}}}.$$

where we have used the L'Hopital rule.

To complete the proof, we note that this last expression is increasing in $\overline{s}/\underline{s}$. Therefore,

$$4\sqrt{\frac{\overline{s}}{5\overline{s} + 21\underline{s}}} > \lim_{\overline{s}/\underline{s}\to 4/3} \left[4\sqrt{\frac{\overline{s}}{5\overline{s} + 21\underline{s}}}\right] = \frac{8}{\sqrt{83}} > 0.706.$$

The proof is now complete. □

Proof of Proposition 1′

$\hat{x}_o < \hat{x}_E$ if and only if[18]

$$\frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B U_B}{\epsilon_A(\overline{U}_A^o-\underline{U}_A^o)}}}{1+\sqrt{\frac{\epsilon_B U_B}{\epsilon_A(\overline{U}_A^o-\underline{U}_A^o)}}} < \frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A(\overline{\pi}_A^E-\pi_A^*)}}}{1+\sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A(\overline{\pi}_A^E-\pi_A^*)}}}.$$

This is equivalent to

$$(\overline{U}_A^o-\underline{U}_A^o)\pi_B < (\overline{\pi}_A^E-\pi_A^*)U_B.$$

Using the expression for surplus given in Section 3, and for profits given in section 6.2.1 we obtain

$$\frac{\alpha^2(\overline{s}^2-\underline{s}^2)}{4\beta^2}\pi_B < \frac{\alpha^2((n(\overline{s}-\underline{s})+\overline{s})^2-\underline{s}^2)}{2(n+2)^2\beta^2}U_B.$$

Solving we find

$$\pi_B < \frac{2\left[(n+1)^2\overline{s}-(n^2-1)\underline{s}\right]}{(n+2)^2(\overline{s}+\underline{s})}U_B.$$

□

Proof of Proposition 2′

$\hat{x}_o < \tilde{x}_E$ if and only if[19]

$$\frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B U_B}{\epsilon_A(\overline{U}_A^o-\underline{U}_A^o)}}}{1+\sqrt{\frac{\epsilon_B U_B}{\epsilon_A(\overline{U}_A^o-\underline{U}_A^o)}}} < \frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A\left(\overline{\pi}_A^E+\delta(\overline{\pi}_A^m-\overline{\pi}_A^E-\underline{\pi}_A^I)-\pi_A^*\right)}}}{1+\sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A\left(\overline{\pi}_A^E+\delta(\overline{\pi}_A^m-\overline{\pi}_A^E-\underline{\pi}_A^I)-\pi_A^*\right)}}}.$$

Since the LHS and RHS of this expression are decreasing in the term under the square root we have

$$(\overline{U}_A^o-\underline{U}_A^o)\pi_B < \left(\overline{\pi}_A^E+\delta(\overline{\pi}_A^m-\overline{\pi}_A^E-\underline{\pi}_A^I)-\pi_A^*\right)U_B.$$

Which we rewrite as

$$\pi_B < \frac{\overline{\pi}_A^E-\pi_A^*}{\overline{U}_A^o-\underline{U}_A^o}U_B + \frac{\delta(\overline{\pi}_A^m-\overline{\pi}_A^E-\underline{\pi}_A^I)}{\overline{U}_A^o-\underline{U}_A^o}U_B.$$

Using the expression for surplus given in Section 3, and for profits given in Section 6.2.1 and 6.2.2 and rewriting we find

$$\pi_B < \frac{2((n+1)^2\overline{s}-(n^2-1)\underline{s})}{(n+2)^2(\overline{s}+\underline{s})}U_B + \frac{2\delta(2\underline{s}-\overline{s})\left[(3n^2+6n+2)\overline{s}-(4n^2+6n)\underline{s}\right]}{(n+1)^2(n+2)^2(\overline{s}^2-\underline{s}^2)}U_B.$$

□

Proof of Proposition 3′

(i) The merger occurs if the joint profits are higher than the sum of the individual competitive

[18] We restrict attention to investment levels that are interior. $\hat{x}_o$ is interior when $\frac{\epsilon_A\epsilon_B}{(1+\epsilon_A)^2} < \frac{U_B}{\overline{U}_A^o-\underline{U}_A^o} < \frac{(1+\epsilon_B)^2}{\epsilon_A\epsilon_B}$. $\hat{x}_E$ is interior when $\frac{\epsilon_A\epsilon_B}{(1+\epsilon_A)^2} < \frac{\pi_B}{\overline{\pi}_A^E-\pi_A^*} < \frac{(1+\epsilon_B)^2}{\epsilon_A\epsilon_B}$.

[19] We restrict to investment levels that are interior. $\tilde{x}_E$ is interior when $\frac{\epsilon_A\epsilon_B}{(1+\epsilon_A)^2} < \frac{\pi_B}{\overline{\pi}_A^E+\delta(\overline{\pi}_A^m-\overline{\pi}_A^E-\underline{\pi}_A^I)-\pi_A^*} < \frac{(1+\epsilon_B)^2}{\epsilon_A\epsilon_B}$.

profits, or:

$$\overline{\pi}_A^E - \pi_A^* < \overline{\pi}_A^E + \delta(\overline{\pi}_A^m - \overline{\pi}_A^E - \underline{\pi}_A^I) - \pi_A^*.$$

Note that the same condition ensures that $\hat{x}_E < \tilde{x}_E$ in part (ii) of the proof.

(ii) $\hat{x}_E < \tilde{x}_E$ if and only if

$$\frac{1+\epsilon_B - \epsilon_A\sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A(\overline{\pi}_A^E - \pi_A^*)}}}{1+\sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A(\overline{\pi}_A^E - \pi_A^*)}}} < \frac{1+\epsilon_B - \epsilon_A\sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A\left(\overline{\pi}_A^E + \delta(\overline{\pi}_A^m - \overline{\pi}_A^E - \underline{\pi}_A^I) - \pi_A^*\right)}}}{1+\sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A\left(\overline{\pi}_A^E + \delta(\overline{\pi}_A^m - \overline{\pi}_A^E - \underline{\pi}_A^I) - \pi_A^*\right)}}}.$$

So

$$\overline{\pi}_A^E - \pi_A^* < \overline{\pi}_A^E + \delta(\overline{\pi}_A^m - \overline{\pi}_A^E - \underline{\pi}_A^I) - \pi_A^*.$$

Dropping the common terms gives

$$0 < \delta\left(\overline{\pi}_A^m - \overline{\pi}_A^E - \underline{\pi}_A^I\right).$$

Using the expression for profits given in Sections 6.2.1 and 6.2.2 we obtain

$$0 < \frac{(n(\overline{s} - \underline{s}) + \underline{s})^2}{2(n+1)^2} - \frac{(n(\overline{s} - \underline{s}) + \overline{s})^2 + (2\underline{s} - \overline{s})^2}{2(n+2)^2}.$$

Rewriting gives

$$0 < (-3n^2 - 6n - 2)\overline{s}^2 + (10n^2 + 18n + 4)\overline{s}\underline{s} + (-8n^2 - 12n)\underline{s}^2.$$

Solving we find

$$\frac{4n^2 + 6n}{3n^2 + 6n + 2}\underline{s} < \overline{s} < 2\underline{s}.$$

(iii) Since $\tilde{x}$ is increasing in δ we can conclude that the difference $\tilde{x} - \hat{x}$ is also increasing in δ. $\square$

Proof of Proposition 5′

Suppose that $\frac{4n^2+6n}{3n^2+6n+2}\underline{s} < \overline{s} < 2\underline{s}$, in which case, $\hat{x}_E < \tilde{x}_E$ and therefore the expected consumer surplus in market B is higher if acquisitions are prohibited.

$$\frac{1-\tilde{x}_E}{1-\tilde{x}_E+\epsilon_B}U_B < \frac{1-\hat{x}_E}{1-\hat{x}_E+\epsilon_B}U_B. \tag{22}$$

We now show that the expected consumer surplus in market A is also higher if acquisitions are prohibited. For this, we need to show that

$$\frac{\tilde{x}_E}{\tilde{x}_E+\epsilon_A}\overline{U}_A^m + \frac{\epsilon_A}{\tilde{x}_E+\epsilon_A}\underline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E+\epsilon_A}\overline{U}_A + \frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\underline{U}_A. \tag{23}$$

We can rewrite this as:

$$\left(\frac{\hat{x}_E}{\hat{x}_E+\epsilon_A}+\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}-\frac{\epsilon_A}{\tilde{x}_E+\epsilon_A}\right)\overline{U}_A^m+\frac{\epsilon_A}{\tilde{x}_E+\epsilon_A}\underline{U}_A^m<\frac{\hat{x}_E}{\hat{x}_E+\epsilon_A}\overline{U}_A+\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\underline{U}_A.$$

We have $\overline{U}_A^m < \overline{U}_A$ so:

$$\left(\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}-\frac{\epsilon_A}{\tilde{x}_E+\epsilon_A}\right)\overline{U}_A^m+\frac{\epsilon_A}{\tilde{x}_E+\epsilon_A}\underline{U}_A^m<\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\underline{U}_A.$$

Rewriting gives:

$$\frac{\epsilon_A}{\tilde{x}_E+\epsilon_A}\left(\underline{U}_A^m-\overline{U}_A^m\right)+\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\left(\overline{U}_A^m-\underline{U}_A\right)<0. \tag{24}$$

In market A we can rank consumer surplus in two different ways depending on the quality differences.

1. For all $\frac{4n^2+6n}{3n^2+6n+2} < \frac{\overline{s}}{\underline{s}} < \frac{n+3}{n+2}$ we have $\underline{U}_A^m < \overline{U}_A^m < \underline{U}_A < \overline{U}_A$.[20]
2. For all $\frac{n+3}{n+2} < \frac{\overline{s}}{\underline{s}} < 2$ we have $\underline{U}_A^m < \underline{U}_A < \overline{U}_A^m < \overline{U}_A$

Therefore, the first term is always negative since $\underline{U}_A^m < \overline{U}_A^m$. The second term is also negative when $\frac{4n^2+6n}{3n^2+6n+2} < \frac{\overline{s}}{\underline{s}} < \frac{n+3}{n+2}$ in which case (24) holds.

When $\frac{\overline{s}}{\underline{s}} \in \left(\frac{n+3}{n+2}, 2\right)$ because $\underline{U}_A < \overline{U}_A^m$, similar to the proof of Proposition 5, we write (24) as follows:

$$\frac{\hat{x}_E+\epsilon_A}{\tilde{x}_E+\epsilon_A}>\frac{\overline{U}_A^m-\underline{U}_A}{\overline{U}_A^m-\underline{U}_A^m}. \tag{25}$$

Similar to Proposition 5(ii) if we show that the smallest value achieved by the function on the LHS is greater than the highest value achieved by the function on the RHS, then the proof is complete. Denote the LHS by $f(n,\delta,\frac{\overline{s}}{\underline{s}},z)$ where $z=\frac{\pi_B\epsilon_B\beta^2}{\epsilon_A\alpha^2}$ as before.[21] Similar to the proof of Proposition 5, we set $\delta=1$ and take the limit $z\to\infty$ $(\epsilon_A\to 0)$ which is

$$\sqrt{\frac{(\overline{s}-\underline{s})\left[(n+1)^2(\overline{s}-\underline{s})+2(n+1)\underline{s}\right](n+1)^2}{(n(\overline{s}-\underline{s})+\underline{s})^2(n+2)^2-(n+1)^2((2\underline{s}-\overline{s})^2+\underline{s}^2)}}.$$

Next we solve for $\frac{\overline{s}}{\underline{s}}$ that minimizes this expression on the relevant interval and substitute it back in order to obtain the smallest possible value this limit takes at the critical $\frac{\overline{s}}{\underline{s}}$ as a function of n. We obtain

$$\frac{\hat{x}_E+\epsilon_A}{\tilde{x}_E+\epsilon_A}>\sqrt{\frac{2(n+1)^3}{n(n+2)(5+3n)-\sqrt{(n+3)(n^2-2)\left(2+n(n+2)(5+n)\right)}}}.$$

[20]Note that for $n>3$, this region is irrelevant because $\frac{n+3}{n+2}<\frac{4n^2+6n}{3n^2+6n+2}$. In this case only the second ordering of consumer surplus applies.

[21]Note, $f(n,\delta,\frac{\overline{s}}{\underline{s}},z)$ is decreasing in δ and in z, similar to analysis in Proposition 5.

The RHS of (25) expression is given by[22]

$$\frac{(\overline{s}+(n-1)\underline{s})^2\,(n+2)^2-(n+1)^4\underline{s}^2}{(n+2)^2\left((\overline{s}+(n-1)\underline{s})^2-n^2\underline{s}^2\right)}.$$

Evaluating this expression at the upper bound of the interval $\frac{\overline{s}}{\underline{s}}=2$ we obtain

$$\frac{\overline{U}_A^m-\underline{U}_A}{\overline{U}_A^m-\underline{U}_A^m}<\frac{(n+1)^2\,(2n+3)}{(n+2)^2\,(2n+1)}.$$

Finally, it can be easily verified that the difference between these two functions is positive for all $n\geq 2$. Hence, we have shown that for all $n\geq 2$ the smallest value achieved by the function on the LHS is greater than the highest value achieved by the function on the RHS. This completes the proof. □

Proof of Proposition 6

We restrict attention to investment levels that are interior as specified in Lemmas 1 and 4. $\hat{x}_o<\hat{x}_m$ if and only if

$$\frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B U_B}{\epsilon_A(\overline{U}_A^o-\underline{U}_A^o)}}}{1+\sqrt{\frac{\epsilon_B U_B}{\epsilon_A(\overline{U}_A^o-\underline{U}_A^o)}}}<\frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A(\overline{\pi}_A^m-\underline{\pi}_A^m)}}}{1+\sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A(\overline{\pi}_A^m-\underline{\pi}_A^m)}}}.$$

As before this is equivalent to

$$(\overline{U}_A^o-\underline{U}_A^o)\pi_B<(\overline{\pi}_A^m-\underline{\pi}_A^m)U_B.$$

Using the expression for surplus given in Section 3, and for profits given in Section 4.1 and Section 5.1 we obtain

$$\frac{\alpha^2(\overline{s}^2-\underline{s}^2)}{4\beta^2}\pi_B<\frac{\alpha^2(\overline{s}^2-\underline{s}^2)}{8\beta^2}U_B.$$

Solving we find

$$\pi_B<\frac{U_B}{2}.$$

□

Proof of Proposition 7

We restrict attention to investment levels that are interior as specified in Lemmas 2 and 4. $\hat{x}_m<\hat{x}_E$ if and only if

$$\frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A(\overline{\pi}_A^m-\underline{\pi}_A^m)}}}{1+\sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A(\overline{\pi}_A^m-\underline{\pi}_A^m)}}}<\frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A(\overline{\pi}_A^E-\pi_A^*)}}}{1+\sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A(\overline{\pi}_A^E-\pi_A^*)}}}.$$

[22]Note that the RHS of (25) is increasing in $\frac{\overline{s}}{\underline{s}}$ similar to analysis in Proposition 5.

So

$$\overline{\pi}_A^m - \underline{\pi}_A^m < \overline{\pi}_A^E - \pi_A^*.$$

Using the expression for profits given in Section 4.1 and Section 5.1 we obtain

$$\frac{\alpha^2(\overline{s}^2 - \underline{s}^2)}{8\beta^2} < \frac{2\alpha^2\overline{s}(\overline{s} - \underline{s})}{9\beta^2}.$$

Simplifying gives

$$0 < (7\overline{s} - 9\underline{s})(\overline{s} - \underline{s}).$$

Solving we find

$$\frac{9}{7}\underline{s} < \overline{s} < 2\underline{s}.$$

Therefore, $\hat{x}_E < \hat{x}_m$ when $\underline{s} < \overline{s} < \frac{9}{7}\underline{s}$. □

Proof of Proposition 9

(i) Suppose now that $\frac{9}{7}\underline{s} < \overline{s} < 2\underline{s}$, in which case, $\hat{x}_m < \hat{x}_E$ and therefore the expected consumer surplus in market B is lower if acquisitions are prohibited.

$$\frac{1 - \hat{x}_E}{1 - \hat{x}_E + \epsilon_B} U_B < \frac{1 - \hat{x}_m}{1 - \hat{x}_m + \epsilon_B} U_B. \tag{26}$$

To show the result, we start by noticing that, for $\overline{s} < 2\underline{s}$, we have $\overline{U}_A^m < \overline{U}_A$ so

$$\frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\overline{U}_A^m = \left(\frac{\hat{x}_m}{\hat{x}_m + \epsilon_A} + \frac{\epsilon_A}{\hat{x}_m + \epsilon_A} - \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\right)\overline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\overline{U}_A.$$

Further, because $\underline{U}_A^m < \overline{U}_A^m$,

$$\frac{\hat{x}_m}{\hat{x}_m + \epsilon_A}\overline{U}_A^m + \left(\frac{\epsilon_A}{\hat{x}_m + \epsilon_A} - \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\right)\underline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\overline{U}_A.$$

Furthermore, because $\underline{U}_A^m < \underline{U}_A$, we can write

$$\frac{\hat{x}_m}{\hat{x}_m + \epsilon_A}\overline{U}_A^m + \left(\frac{\epsilon_A}{\hat{x}_m + \epsilon_A} - \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\right)\underline{U}_A^m + \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\underline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\overline{U}_A + \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\underline{U}_A.$$

Simplifying gives:

$$\frac{\hat{x}_m}{\hat{x}_m + \epsilon_A}\overline{U}_A^m + \frac{\epsilon_A}{\hat{x}_m + \epsilon_A}\underline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\overline{U}_A + \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\underline{U}_A, \tag{27}$$

which means that the expected consumer surplus in market A is higher if acquisitions are prohibited.

Because the expected consumer surplus is a continuous function of U_B, putting together (26) and (27) implies the result.

(ii) Suppose that $\overline{s} < \frac{9}{7}\underline{s}$, in which case, $\hat{x}_E < \hat{x}_m$. To show that $\mathbb{E}U(\hat{x}_m) < \mathbb{E}U(\hat{x}_E)$ we start by noticing that, given that $\overline{s} < \frac{9}{7}\underline{s}$, we have $\overline{U}_A^m < \underline{U}_A$. Therefore,

$$\left(\frac{\hat{x}_m}{\hat{x}_m + \epsilon_A} + \frac{\epsilon_A}{\hat{x}_m + \epsilon_A}\right)\overline{U}_A^m < \left(\frac{\hat{x}_E}{\hat{x}_E + \epsilon_A} + \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\right)\underline{U}_A.$$

Further, because $\underline{U}_A^m < \overline{U}_A^m$ and $\underline{U}_A < \overline{U}_A$, we then have that

$$\frac{\hat{x}_m}{\hat{x}_m + \epsilon_A}\overline{U}_A^m + \frac{\epsilon_A}{\hat{x}_m + \epsilon_A}\underline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\overline{U}_A + \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\underline{U}_A. \tag{28}$$

Furthermore, because $\hat{x}_E < \hat{x}_m$, it holds that

$$\frac{1 - \hat{x}_m}{1 - \hat{x}_m + \epsilon_B}U_B < \frac{1 - \hat{x}_E}{1 - \hat{x}_E + \epsilon_B}U_B. \tag{29}$$

Combining (28) and (29), the result follows. □

Proof of Proposition 6′

$\hat{x}_o < \hat{x}_m$ if and only if [23]

$$(\overline{U}_A^o - \underline{U}_A^o)\pi_B < (\overline{\pi}_A^m - \pi_A^{*m})U_B.$$

Using the expression for surplus given in Section 3, and for profits given in Section 6.2.2 we obtain

$$\frac{\alpha^2(\overline{s}^2 - \underline{s}^2)}{4\beta^2}\pi_B < \frac{\alpha^2(n^2(\overline{s} - \underline{s})^2 + 2n\underline{s}(\overline{s} - \underline{s}))}{2(n+1)^2\beta^2}U_B.$$

Solving we find

$$\pi_B < \frac{2n(n(\overline{s} - \underline{s}) + 2\underline{s})}{(n+1)^2(\overline{s + \underline{s}})}U_B.$$

□

Proof of Proposition 9′

(i) Suppose now that $\frac{4n^2+6n-1}{2n^2+4n+1}\underline{s} < \overline{s} < 2\underline{s}$, in which case, $\hat{x}_m < \hat{x}_E$ and therefore the expected consumer surplus in market B is lower if acquisitions are prohibited.

$$\frac{1 - \hat{x}_E}{1 - \hat{x}_E + \epsilon_B}U_B < \frac{1 - \hat{x}_m}{1 - \hat{x}_m + \epsilon_B}U_B. \tag{30}$$

To show the result, we start by noticing that, for $\overline{s} < 2\underline{s}$, we have $\overline{U}_A^m < \overline{U}_A$ so

$$\frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\overline{U}_A^m = \left(\frac{\hat{x}_m}{\hat{x}_m + \epsilon_A} + \frac{\epsilon_A}{\hat{x}_m + \epsilon_A} - \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\right)\overline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\overline{U}_A.$$

[23] We restrict to investment levels that are interior. $\hat{x}_o$ is interior when $\frac{\epsilon_A\epsilon_B}{(1+\epsilon_A)^2} < \frac{U_B}{\overline{U}_A^o - \underline{U}_A^o} < \frac{(1+\epsilon_B)^2}{\epsilon_A\epsilon_B}$. $\hat{x}_m$ is interior when $\frac{\epsilon_A\epsilon_B}{(1+\epsilon_A)^2} < \frac{\pi_B}{\overline{\pi}_A^m - \pi_A^{*m}} < \frac{(1+\epsilon_B)^2}{\epsilon_A\epsilon_B}$.

Further, because $\underline{U}_A^m < \overline{U}_A^m$,

$$\frac{\hat{x}_m}{\hat{x}_m + \epsilon_A}\overline{U}_A^m + \left(\frac{\epsilon_A}{\hat{x}_m + \epsilon_A} - \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\right)\underline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\overline{U}_A.$$

Furthermore, because $\underline{U}_A^m < \underline{U}_A$, we can write

$$\frac{\hat{x}_m}{\hat{x}_m + \epsilon_A}\overline{U}_A^m + \left(\frac{\epsilon_A}{\hat{x}_m + \epsilon_A} - \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\right)\underline{U}_A^m + \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\underline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\overline{U}_A + \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\underline{U}_A.$$

Simplifying gives:

$$\frac{\hat{x}_m}{\hat{x}_m + \epsilon_A}\overline{U}_A^m + \frac{\epsilon_A}{\hat{x}_m + \epsilon_A}\underline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\overline{U}_A + \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\underline{U}_A, \tag{31}$$

which means that the expected consumer surplus in market A is higher if acquisitions are prohibited.

Because the expected consumer surplus is a continuous function of U_B, putting together (30) and (31) implies the result.

(ii) Suppose that $\overline{s} < \frac{4n^2+6n-1}{2n^2+4n+1}\underline{s}$, in which case by Proposition 7′ $\hat{x}_E < \hat{x}_m$ and therefore the expected consumer surplus in market B is higher if acquisitions are prohibited.

$$\frac{1 - \hat{x}_m}{1 - \hat{x}_m + \epsilon_B}U_B < \frac{1 - \hat{x}_E}{1 - \hat{x}_E + \epsilon_B}U_B. \tag{32}$$

We now show that the expected consumer surplus in market A is also higher if acquisitions are prohibited. For this, we need to show that

$$\frac{\hat{x}_m}{\hat{x}_m + \epsilon_A}\overline{U}_A^m + \frac{\epsilon_A}{\hat{x}_m + \epsilon_A}\underline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\overline{U}_A + \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\underline{U}_A. \tag{33}$$

We can rewrite this as:

$$\left(\frac{\hat{x}_E}{\hat{x}_E + \epsilon_A} + \frac{\epsilon_A}{\hat{x}_E + \epsilon_A} - \frac{\epsilon_A}{\hat{x}_m + \epsilon_A}\right)\overline{U}_A^m + \frac{\epsilon_A}{\hat{x}_m + \epsilon_A}\underline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\overline{U}_A + \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\underline{U}_A.$$

We have $\overline{U}_A^m < \overline{U}_A$ so:

$$\left(\frac{\epsilon_A}{\hat{x}_E + \epsilon_A} - \frac{\epsilon_A}{\hat{x}_m + \epsilon_A}\right)\overline{U}_A^m + \frac{\epsilon_A}{\hat{x}_m + \epsilon_A}\underline{U}_A^m < \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\underline{U}_A.$$

Rewriting gives:

$$\frac{\epsilon_A}{\hat{x}_m + \epsilon_A}\left(\underline{U}_A^m - \overline{U}_A^m\right) + \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\left(\overline{U}_A^m - \underline{U}_A\right) < 0. \tag{34}$$

In market A we can rank consumer surplus in two different ways depending on the quality differences.[24]

[24] Note that $\frac{n+3}{n+2} < \frac{4n^2+6n-1}{2n^2+4n+1}$ for all $n \geq 2$.

1. For all $1 < \frac{\overline{s}}{\underline{s}} < \frac{n+3}{n+2}$ we have $\underline{U}_A^m < \overline{U}_A^m < \underline{U}_A < \overline{U}_A$

2. For all $\frac{n+3}{n+2} < \frac{\overline{s}}{\underline{s}} < \frac{4n^2+6n-1}{2n^2+4n+1}$ we have $\underline{U}_A^m < \underline{U}_A < \overline{U}_A^m < \overline{U}_A$

Therefore, the first term is always negative since $\underline{U}_A^m < \overline{U}_A^m$. The second term is also negative when $1 < \frac{\overline{s}}{\underline{s}} < \frac{n+3}{n+2}$ in which case (34) holds.

When $\frac{n+3}{n+2} < \frac{\overline{s}}{\underline{s}} < \frac{4n^2+6n-1}{2n^2+4n+1}$ because $\underline{U}_A < \overline{U}_A^m$ we write (34) as

$$\frac{\hat{x}_E + \epsilon_A}{\hat{x}_m + \epsilon_A} > \frac{\overline{U}_A^m - \underline{U}_A}{\overline{U}_A^m - \underline{U}_A^m}. \tag{35}$$

Similar to Proposition 5(ii) if we show that the smallest value achieved by the function on the LHS is greater than the highest value achieved by the function on the RHS, we can conclude that $LHS\left(\frac{\overline{s}}{\underline{s}}\right) > RHS\left(\frac{\overline{s}}{\underline{s}}\right)$for all $\frac{\overline{s}}{\underline{s}} \in \left(\frac{n+3}{n+2}, \frac{4n^2+6n-1}{2n^2+4n+1}\right)$. Then the proof is complete.

Denote the LHS by $f(n, \frac{\overline{s}}{\underline{s}}, z)$ where $z = \frac{\pi_B \epsilon_B \beta^2}{\epsilon_A \alpha^2}$ as before.[25] Similar to the proof of Proposition 5′, we take the limit of $f(n, \frac{\overline{s}}{\underline{s}}, z)$ when $z \to \infty$ ($\epsilon_A \to 0$) which is

$$\sqrt{\frac{\left[(n+1)^2\left(\overline{s} - \underline{s}\right) + 2(n+1)\underline{s}\right](n+1)^2}{\left[n^2\left(\overline{s} - \underline{s}\right) + 2n\underline{s}\right](n+2)^2}}.$$

It is easy to verify that this expression is strictly increasing in $\frac{\overline{s}}{\underline{s}}$. Next, we substitute $\frac{\overline{s}}{\underline{s}} = \frac{n+3}{n+2}$ to obtain the smallest possible value this limit takes as a function of n on the relevant interval. We obtain

$$\frac{\hat{x}_E + \epsilon_A}{\hat{x}_m + \epsilon_A} > \sqrt{\frac{\left[(n+1)^2\left(\frac{1}{n+2}\right) + 2(n+1)\right](n+1)^2}{\left[n^2\left(\frac{1}{n+2}\right) + 2n\right](n+2)^2}}.$$

The RHS of expression (35) is given by[26]

$$\frac{\left(\overline{s} + (n-1)\underline{s}\right)^2 (n+2)^2 - (n+1)^4 \underline{s}^2}{(n+2)^2\left(\left(\overline{s} + (n-1)\underline{s}\right)^2 - n^2\underline{s}^2\right)}.$$

Evaluating this expression at the upper bound of the interval $\frac{\overline{s}}{\underline{s}} = \frac{4n^2+6n-1}{2n^2+4n+1}$ we obtain

$$\frac{\overline{U}_A^m - \underline{U}_A}{\overline{U}_A^m - \underline{U}_A^m} < \frac{\left(\frac{4n^2+6n-1}{2n^2+4n+1} + (n-1)\right)^2 (n+2)^2 - (n+1)^4}{(n+2)^2\left(\left(\frac{4n^2+6n-1}{2n^2+4n+1} + (n-1)\right)^2 - n^2\right)}$$

Finally, it can be verified that the difference between these two functions is positive for all $n \geq 2$. Hence, we have shown that for all $n \geq 2$ the smallest value achieved by the function on the LHS

[25]Note, $f(n, \frac{\overline{s}}{\underline{s}}, z)$ is decreasing in z, similar to analysis in Proposition 5.

[26]Note that the RHS of expression (35) is increasing in $\frac{\overline{s}}{\underline{s}}$ similar to analysis in Proposition 5.

is greater than the highest value achieved by the function on the RHS. This completes the proof. □

Proof of Proposition 1″

$\hat{x}_o < \hat{x}_E$ if and only if[27]

$$\frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B U_B}{\epsilon_A(\overline{U}_A^o-\underline{U}_A^o)}}}{1+\sqrt{\frac{\epsilon_B U_B}{\epsilon_A(\overline{U}_A^o-\underline{U}_A^o)}}} < \frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A(\overline{\pi}_A^m-\pi_A^*)}}}{1+\sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A(\overline{\pi}_A^m-\pi_A^*)}}}.$$

So

$$(\overline{U}_A^o-\underline{U}_A^o)\pi_B < (\overline{\pi}_A^m-\pi_A^*)U_B.$$

Using the expression for surplus given in Section 3, and for profits given in Section 4.1 and Section 5.1 we obtain

$$\frac{\alpha^2(\overline{s}^2-\underline{s}^2)}{4\beta^2}\pi_B < \frac{\alpha^2(9\overline{s}^2-4\underline{s}^2)}{72\beta^2}U_B.$$

Solving we find

$$\pi_B < \frac{9\overline{s}^2-4\underline{s}^2}{18(\overline{s}^2-\underline{s}^2)}U_B.$$

□

Proof of Proposition 2″

$\hat{x}_o < \tilde{x}_E$ if and only if[28]

$$\frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B U_B}{\epsilon_A(\overline{U}_A^o-\underline{U}_A^o)}}}{1+\sqrt{\frac{\epsilon_B U_B}{\epsilon_A(\overline{U}_A^o-\underline{U}_A^o)}}} < \frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A\left(\overline{\pi}_A^m-\pi_A^*-\delta(\underline{\pi}_A^m-2\pi_A^*)\right)}}}{1+\sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A\left(\overline{\pi}_A^m-\pi_A^*-\delta(\underline{\pi}_A^m-2\pi_A^*)\right)}}}.$$

So

$$(\overline{U}_A^o-\underline{U}_A^o)\pi_B < \left(\overline{\pi}_A^m-\pi_A^*-\delta(\underline{\pi}_A^m-2\pi_A^*)\right)U_B.$$

Using the expression for surplus given in Section 3, and for profits given in Section 4.1 and Section 5.1 we obtain

$$\frac{\alpha^2(\overline{s}^2-\underline{s}^2)}{4\beta^2}\pi_B < \frac{\alpha^2\left(9\overline{s}^2-(4+\delta)\underline{s}^2\right)}{72\beta^2}U_B.$$

Solving we find

$$\pi_B < \frac{9\overline{s}^2-(4+\delta)\underline{s}^2}{18(\overline{s}^2-\underline{s}^2)}U_B.$$

□

Proof of Proposition 3″

[27] We restrict to investment levels that are interior. $\hat{x}_o$ is interior when $\frac{\epsilon_A\epsilon_B}{(1+\epsilon_A)^2} < \frac{U_B}{\overline{U}_A^o-\underline{U}_A^o} < \frac{(1+\epsilon_B)^2}{\epsilon_A\epsilon_B}$. $\hat{x}_E$ is interior when $\frac{\epsilon_A\epsilon_B}{(1+\epsilon_A)^2} < \frac{\pi_B}{\overline{\pi}_A^m-\pi_A^*} < \frac{(1+\epsilon_B)^2}{\epsilon_A\epsilon_B}$.

[28] We restrict to investment levels that are interior. $\tilde{x}_E$ is interior when $\frac{\epsilon_A\epsilon_B}{(1+\epsilon_A)^2} < \frac{\pi_B}{\overline{\pi}_A^E-\pi_A^*-\delta(\underline{\pi}_A^m-2\pi_A^*)} < \frac{(1+\epsilon_B)^2}{\epsilon_A\epsilon_B}$.

$\tilde{x}_E < \hat{x}_E$ if and only if

$$\frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A\left(\overline{\pi}_A^m-\pi_A^*-\delta(\underline{\pi}_A^m-2\pi_A^*)\right)}}}{1+\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A\left(\overline{\pi}_A^m-\pi_A^*-\delta(\underline{\pi}_A^m-2\pi_A^*)\right)}}} < \frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A(\overline{\pi}_A^m-\pi_A^*)}}}{1+\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A(\overline{\pi}_A^m-\pi_A^*)}}}.$$

So

$$\overline{\pi}_A^m-\pi_A^*-\delta(\underline{\pi}_A^m-2\pi_A^*) < \overline{\pi}_A^m-\pi_A^*.$$

Dropping the common terms gives

$$\delta\left(2\pi_A^*-\underline{\pi}_A^m\right) < 0.$$

Using the expression for profits given in Section 4.1 and Section 5.1 we obtain

$$\frac{\alpha^2(-\underline{s}^2)}{72\beta^2} < 0.$$

□

Proof of Proposition 5″

Suppose now that $2\underline{s} < \overline{s}$, in which case, $\tilde{x}_E < \hat{x}_E$ and therefore the expected consumer surplus in market B is lower if acquisitions are prohibited.

$$\frac{1-\hat{x}_E}{1-\hat{x}_E+\epsilon_B}U_B < \frac{1-\tilde{x}_E}{1-\tilde{x}_E+\epsilon_B}U_B. \tag{36}$$

We have

$$\left(\frac{\tilde{x}_E}{\tilde{x}_E+\epsilon_A}+\frac{\epsilon_A}{\tilde{x}_E+\epsilon_A}-\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\right)\overline{U}_A^m = \frac{\hat{x}_E}{\hat{x}_E+\epsilon_A}\overline{U}_A^m.$$

Further, because $\underline{U}_A^m < \overline{U}_A^m$ and $\tilde{x}_E < \hat{x}_E$

$$\frac{\tilde{x}_E}{\tilde{x}_E+\epsilon_A}\overline{U}_A^m+\left(\frac{\epsilon_A}{\tilde{x}_E+\epsilon_A}-\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\right)\underline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E+\epsilon_A}\overline{U}_A^m.$$

Furthermore, because $\underline{U}_A^m < \underline{U}_A$, we can write

$$\frac{\tilde{x}_E}{\tilde{x}_E+\epsilon_A}\overline{U}_A^m+\left(\frac{\epsilon_A}{\tilde{x}_E+\epsilon_A}-\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\right)\underline{U}_A^m+\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\underline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E+\epsilon_A}\overline{U}_A^m+\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\underline{U}_A.$$

Simplifying gives:

$$\frac{\tilde{x}_E}{\tilde{x}_E+\epsilon_A}\overline{U}_A^m+\frac{\epsilon_A}{\tilde{x}_E+\epsilon_A}\underline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E+\epsilon_A}\overline{U}_A^m+\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\underline{U}_A, \tag{37}$$

which means that the expected consumer surplus in market A is higher if acquisitions are prohibited. Because the expected consumer surplus is a continuous function of U_B, putting together (36) and (37) implies the result. □

Proof of Proposition 7′′

$\hat{x}_m < \hat{x}_E$ if and only if[29]

$$\frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A(\overline{\pi}_A^m-\underline{\pi}_A^m)}}}{1+\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A(\overline{\pi}_A^m-\underline{\pi}_A^m)}}} < \frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A(\overline{\pi}_A^m-\pi_A^*)}}}{1+\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A(\overline{\pi}_A^m-\pi_A^*)}}}.$$

So

$$\overline{\pi}_A^m-\underline{\pi}_A^m<\overline{\pi}_A^m-\pi_A^*.$$

Using the expression for profits given in Section 4.1 and Section 5.1 we obtain

$$\frac{\alpha^2(\overline{s}^2-\underline{s}^2)}{8\beta^2}<\frac{\alpha^2(9\overline{s}^2-4\underline{s}^2)}{72\beta^2}.$$

Solving we find

$$0<5\overline{s}^2.$$

□

Proof of Proposition 9′′

Suppose now that $2\underline{s}<\overline{s}$, in which case, $\hat{x}_m<\hat{x}_E$ and therefore the expected consumer surplus in market B is lower if acquisitions are prohibited.

$$\frac{1-\hat{x}_E}{1-\hat{x}_E+\epsilon_B}U_B<\frac{1-\hat{x}_m}{1-\hat{x}_m+\epsilon_B}U_B. \tag{38}$$

We have

$$\frac{\hat{x}_E}{\hat{x}_E+\epsilon_A}\overline{U}_A^m=\left(\frac{\hat{x}_m}{\hat{x}_m+\epsilon_A}+\frac{\epsilon_A}{\hat{x}_m+\epsilon_A}-\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\right)\overline{U}_A^m.$$

Further, because $\underline{U}_A^m<\overline{U}_A^m$ and $\hat{x}_m<\hat{x}_E$

$$\frac{\hat{x}_m}{\hat{x}_m+\epsilon_A}\overline{U}_A^m+\left(\frac{\epsilon_A}{\hat{x}_m+\epsilon_A}-\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\right)\underline{U}_A^m<\frac{\hat{x}_E}{\hat{x}_E+\epsilon_A}\overline{U}_A^m.$$

Furthermore, because $\underline{U}_A^m<\underline{U}_A$, we can write

$$\frac{\hat{x}_m}{\hat{x}_m+\epsilon_A}\overline{U}_A^m+\left(\frac{\epsilon_A}{\hat{x}_m+\epsilon_A}-\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\right)\underline{U}_A^m+\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\underline{U}_A^m<\frac{\hat{x}_E}{\hat{x}_E+\epsilon_A}\overline{U}_A^m+\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\underline{U}_A.$$

Simplifying gives:

$$\frac{\hat{x}_m}{\hat{x}_m+\epsilon_A}\overline{U}_A^m+\frac{\epsilon_A}{\hat{x}_m+\epsilon_A}\underline{U}_A^m<\frac{\hat{x}_E}{\hat{x}_E+\epsilon_A}\overline{U}_A^m+\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\underline{U}_A, \tag{39}$$

[29]We restrict to investment levels that are interior. $\hat{x}_m$ is interior when $\frac{\epsilon_A\epsilon_B}{(1+\epsilon_A)^2}<\frac{1}{\overline{\pi}_A^m-\underline{\pi}_A^m}\pi_B<\frac{(1+\epsilon_B)^2}{\epsilon_A\epsilon_B}$. $\hat{x}_E$ is interior when $\frac{\epsilon_A\epsilon_B}{(1+\epsilon_A)^2}<\frac{1}{\overline{\pi}_A^m-\pi_A^*}\pi_B<\frac{(1+\epsilon_B)^2}{\epsilon_A\epsilon_B}$.

which means that the expected consumer surplus in market A is higher if acquisitions are prohibited. Because the expected consumer surplus is a continuous function of U_B, putting together (38) and (39) implies the result. □

Proof of Proposition 1'''

$\hat{x}_o < \hat{x}_E$ if and only if[30]

$$\frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B U_B}{\epsilon_A(\overline{U}_A^o-\underline{U}_A^o)}}}{1+\sqrt{\frac{\epsilon_B U_B}{\epsilon_A(\overline{U}_A^o-\underline{U}_A^o)}}} < \frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A\overline{\pi}_A^E}}}{1+\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A\overline{\pi}_A^E}}}.$$

So

$$(\overline{U}_A^o-\underline{U}_A^o)\pi_B < \overline{\pi}_A^E U_B.$$

Using the expression for surplus given in Section 3, and for profits given in Section 4.1 we obtain

$$\frac{\alpha^2(\overline{s}^2-\underline{s}^2)}{4\beta^2}\pi_B < \frac{\alpha^2(2\overline{s}-\underline{s})^2}{18\beta^2}U_B.$$

Which can be rewritten as

$$\pi_B < \frac{2(2\overline{s}-\underline{s})^2}{9(\overline{s}^2-\underline{s}^2)}U_B.$$

□

Proof of Proposition 2'''

$\hat{x}_o < \tilde{x}_E$ if and only if[31]

$$\frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B U_B}{\epsilon_A(\overline{U}_A^o-\underline{U}_A^o)}}}{1+\sqrt{\frac{\epsilon_B U_B}{\epsilon_A(\overline{U}_A^o-\underline{U}_A^o)}}} < \frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A\left(\overline{\pi}_A^E+\delta(\overline{\pi}_A^m-\overline{\pi}_A^E-\underline{\pi}_A^I)\right)}}}{1+\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A\left(\overline{\pi}_A^E+\delta(\overline{\pi}_A^m-\overline{\pi}_A^E-\underline{\pi}_A^I)\right)}}}.$$

So

$$(\overline{U}_A^o-\underline{U}_A^o)\pi_B < \left(\overline{\pi}_A^E+\delta(\overline{\pi}_A^m-\overline{\pi}_A^E-\underline{\pi}_A^I)\right)U_B.$$

Using the expression for surplus given in Section 3, and for profits given in Section 4.1 and Section 5.1 we obtain

$$\frac{\alpha^2(\overline{s}^2-\underline{s}^2)}{4\beta^2}\pi_B < \frac{\alpha^2\left(4(2\overline{s}-\underline{s})^2+\delta(-11\overline{s}^2-20\underline{s}^2+32\overline{s}\underline{s})\right)}{72\beta^2}U_B.$$

Solving we find

$$\pi_B < \frac{4(2\overline{s}-\underline{s})^2+\delta(-11\overline{s}^2-20\underline{s}^2+32\overline{s}\underline{s})}{18(\overline{s}^2-\underline{s}^2)}U_B.$$

[30] We restrict to investment levels that are interior. $\hat{x}_o$ is interior when $\frac{\epsilon_A\epsilon_B}{(1+\epsilon_A)^2} < \frac{U_B}{\overline{U}_A^o-\underline{U}_A^o} < \frac{(1+\epsilon_B)^2}{\epsilon_A\epsilon_B}$. $\hat{x}_E$ is interior when $\frac{\epsilon_A\epsilon_B}{(1+\epsilon_A)^2} < \frac{\pi_B}{\overline{\pi}_A^E} < \frac{(1+\epsilon_B)^2}{\epsilon_A\epsilon_B}$.

[31] We restrict to investment levels that are interior. $\tilde{x}_E$ is interior when $\frac{\epsilon_A\epsilon_B}{(1+\epsilon_A)^2} < \frac{\pi_B}{\overline{\pi}_A^E+\delta(\overline{\pi}_A^m-\overline{\pi}_A^E-\underline{\pi}_A^I)} < \frac{(1+\epsilon_B)^2}{\epsilon_A\epsilon_B}$.

□

Proof of Proposition 3‴

$\hat{x}_E < \tilde{x}_E$ if and only if

$$\frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A\overline{\pi}_A^E}}}{1+\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A\overline{\pi}_A^E}}} < \frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A\left(\overline{\pi}_A^E+\delta(\overline{\pi}_A^m-\overline{\pi}_A^E-\underline{\pi}_A^I)\right)}}}{1+\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A\left(\overline{\pi}_A^E+\delta(\overline{\pi}_A^m-\overline{\pi}_A^E-\underline{\pi}_A^I)\right)}}}.$$

So

$$\overline{\pi}_A^E < \overline{\pi}_A^E + \delta(\overline{\pi}_A^m - \overline{\pi}_A^E - \underline{\pi}_A^I).$$

Dropping the common terms gives

$$0 < \delta\left(\overline{\pi}_A^m - \overline{\pi}_A^E - \underline{\pi}_A^I\right).$$

Using the expression for profits given in Section 4.1 and Section 5.1 we obtain

$$0 < \frac{\alpha^2(-11\overline{s}^2 - 20\underline{s}^2 + 32\overline{s}\underline{s})}{72\beta^2}.$$

Simplifying gives

$$0 < -11\overline{s}^2 - 20\underline{s}^2 + 32\overline{s}\underline{s}.$$

Solving we find this holds for all $\overline{s} \in [\underline{s}, 2\underline{s})$. □

Proof of Proposition 5‴

Recall by Proposition 3‴ we have $\tilde{x}_E > \hat{x}_E$ for all $\underline{s} < \overline{s} < 2\underline{s}$. Therefore, the expected consumer surplus in market B is higher if acquisitions are prohibited.

$$\frac{1-\tilde{x}_E}{1-\tilde{x}_E+\epsilon_B}U_B < \frac{1-\hat{x}_E}{1-\hat{x}_E+\epsilon_B}U_B \tag{40}$$

The expected consumer surplus in market A is higher if acquisitions are prohibited when:

$$\frac{\tilde{x}_E}{\tilde{x}_E+\epsilon_A}\overline{U}_A^m + \frac{\epsilon_A}{\tilde{x}_E+\epsilon_A}\underline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E+\epsilon_A}\overline{U}_A + \frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\underline{U}_A^m \tag{41}$$

We can rewrite this as

$$\frac{(\tilde{x}_E-\hat{x}_E)\,\epsilon_A}{\hat{x}_E\,(\tilde{x}_E+\epsilon_A)} < \frac{\overline{U}_A-\overline{U}_A^m}{\overline{U}_A^m-\underline{U}_A^m}.$$

Numerical simulations show that depending on parameter values this inequality can be satisfied or not.

If $\frac{(\tilde{x}_E-\hat{x}_E)\epsilon_A}{\hat{x}_E(\tilde{x}_E+\epsilon_A)} > \frac{\overline{U}_A-\overline{U}_A^m}{\overline{U}_A^m-\underline{U}_A^m}$, the expected consumer surplus in market A is lower, when acquisitions are prohibited. Then, because the expected consumer surplus is a continuous function of U_B, for sufficiently small U_B we obtain that prohibition of acquisitions results in a decrease in consumer surplus.[32]

If $\frac{(\tilde{x}_E-\hat{x}_E)\epsilon_A}{\hat{x}_E(\tilde{x}_E+\epsilon_A)} < \frac{\overline{U}_A-\overline{U}_A^m}{\overline{U}_A^m-\underline{U}_A^m}$, the expected consumer surplus in market A is higher, when acquisitions are prohibited. Combining this with the impact of acquisitions on consumer surplus in market B, we obtain that prohibition of acquisitions results in an increase in consumer surplus for this range of parameters. □

Proof of Proposition 7‴

$\hat{x}_m < \hat{x}_E$ if and only if[33]

$$\frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A(\overline{\pi}_A^m-\underline{\pi}_A^m)}}}{1+\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A(\overline{\pi}_A^m-\underline{\pi}_A^m)}}} < \frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A\overline{\pi}_A^E}}}{1+\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A\overline{\pi}_A^E}}}.$$

So

$$\overline{\pi}_A^m-\underline{\pi}_A^m < \overline{\pi}_A^E.$$

Using the expression for profits given in Section 4.1 and Section 5.1 we obtain

$$\frac{\alpha^2(\overline{s}^2-\underline{s}^2)}{8\beta^2} < \frac{\alpha^2(2\overline{s}-\underline{s})^2}{18\beta^2}.$$

Simplifying gives

$$0 < 7\overline{s}^2-16\overline{s}\underline{s}+13\underline{s}^2.$$

This holds for all $\underline{s} < \overline{s} < 2\underline{s}$. □

Proof of Proposition 9‴

Suppose now that $\underline{s} < \overline{s} < 2\underline{s}$, in which case, $\hat{x}_m < \hat{x}_E$ and therefore the expected consumer surplus in market B is lower if acquisitions are prohibited.

$$\frac{1-\hat{x}_E}{1-\hat{x}_E+\epsilon_B}U_B < \frac{1-\hat{x}_m}{1-\hat{x}_m+\epsilon_B}U_B. \tag{42}$$

To show the result, we start by noticing that, for $\overline{s} < 2\underline{s}$, we have $\overline{U}_A^m < \overline{U}_A$ so

$$\frac{\hat{x}_E}{\hat{x}_E+\epsilon_A}\overline{U}_A^m = \left(\frac{\hat{x}_m}{\hat{x}_m+\epsilon_A}+\frac{\epsilon_A}{\hat{x}_m+\epsilon_A}-\frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\right)\overline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E+\epsilon_A}\overline{U}_A.$$

[32]Note that this increase in consumer surplus is only possible in Region IV.

[33]We restrict to investment levels that are interior. $\hat{x}_m$ is interior when $\frac{\epsilon_A\epsilon_B}{(1+\epsilon_A)^2} < \frac{\pi_B}{\overline{\pi}_A^m-\underline{\pi}_A^m} < \frac{(1+\epsilon_B)^2}{\epsilon_A\epsilon_B}$. $\hat{x}_E$ is interior when $\frac{\epsilon_A\epsilon_B}{(1+\epsilon_A)^2} < \frac{\pi_B}{\overline{\pi}_A^E} < \frac{(1+\epsilon_B)^2}{\epsilon_A\epsilon_B}$.

Further, because $\underline{U}_A^m < \overline{U}_A^m$,

$$\frac{\hat{x}_m}{\hat{x}_m + \epsilon_A}\overline{U}_A^m + \left(\frac{\epsilon_A}{\hat{x}_m + \epsilon_A} - \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\right)\underline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\overline{U}_A.$$

Furthermore, we can write

$$\frac{\hat{x}_m}{\hat{x}_m + \epsilon_A}\overline{U}_A^m + \left(\frac{\epsilon_A}{\hat{x}_m + \epsilon_A} - \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\right)\underline{U}_A^m + \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\underline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\overline{U}_A + \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\underline{U}_A^m.$$

Simplifying gives:

$$\frac{\hat{x}_m}{\hat{x}_m + \epsilon_A}\overline{U}_A^m + \frac{\epsilon_A}{\hat{x}_m + \epsilon_A}\underline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E + \epsilon_A}\overline{U}_A + \frac{\epsilon_A}{\hat{x}_E + \epsilon_A}\underline{U}_A^m, \tag{43}$$

which means that the expected consumer surplus in market A is higher if acquisitions are prohibited. Because the expected consumer surplus is a continuous function of U_B, putting together (42) and (43) implies the result. □

Proof of Proposition 1''''

$\hat{x}_o < \hat{x}_E$ if and only if[34]

$$\frac{1 + \epsilon_B - \epsilon_A\sqrt{\frac{\epsilon_B U_B}{\epsilon_A(\overline{U}_A^o - \underline{U}_A^o)}}}{1 + \sqrt{\frac{\epsilon_B U_B}{\epsilon_A(\overline{U}_A^o - \underline{U}_A^o)}}} < \frac{1 + \epsilon_B - \epsilon_A\sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A(\overline{\pi}_A^E - \pi_A^*)}}}{1 + \sqrt{\frac{\epsilon_B \pi_B}{\epsilon_A(\overline{\pi}_A^E - \pi_A^*)}}}.$$

This is equivalent to

$$(\overline{U}_A^o - \underline{U}_A^o)\pi_B < (\overline{\pi}_A^E - \pi_A^*)U_B.$$

Using the expression for surplus and profits given in Section 8.3 we obtain

$$\frac{\alpha^2}{\beta^2}\left[\frac{\overline{s}^2 + \underline{s}^2 - \sigma\overline{s}\underline{s} - \underline{s}^2(2-\sigma)}{4-\sigma^2}\right]\pi_B < \frac{\alpha^2}{\beta^2}\left[\frac{2(4\overline{s} - \sigma\underline{s})^2 - 2\underline{s}^2(4-\sigma)^2}{(16-\sigma^2)^2}\right]U_B.$$

Solving we find

$$\pi_B < \frac{16(2(\overline{s} + \underline{s}) - \sigma\underline{s})}{(\overline{s} + \underline{s}(1-\sigma))(4+\sigma)^2}U_B.$$

□

Proof of Proposition 2''''

[34] $\hat{x}_o$ is interior when $\frac{\epsilon_A\epsilon_B}{(1+\epsilon_A)^2} < \frac{U_B}{\overline{U}_A^o - \underline{U}_A^o} < \frac{(1+\epsilon_B)^2}{\epsilon_A\epsilon_B}$. $\hat{x}_E$ is interior when $\frac{\epsilon_A\epsilon_B}{(1+\epsilon_A)^2} < \frac{\pi_B}{\overline{\pi}_A^E - \pi_A^*} < \frac{(1+\epsilon_B)^2}{\epsilon_A\epsilon_B}$.

$\hat{x}_o < \tilde{x}_E$ if and only if[35]

$$\frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B U_B}{\epsilon_A(\overline{U}^o_A-\underline{U}^o_A)}}}{1+\sqrt{\frac{\epsilon_B U_B}{\epsilon_A(\overline{U}^o_A-\underline{U}^o_A)}}} < \frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A\left(\overline{\pi}^E_A+\delta(\overline{\pi}^m_A-\overline{\pi}^E_A-\underline{\pi}^I_A)-\pi^*_A-\delta(\underline{\pi}^m_A-2\pi^*_A)\right)}}}{1+\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A\left(\overline{\pi}^E_A+\delta(\overline{\pi}^m_A-\overline{\pi}^E_A-\underline{\pi}^I_A)-\pi^*_A-\delta(\underline{\pi}^m_A-2\pi^*_A)\right)}}}.$$

Since the LHS and RHS of this expression are decreasing in the term under the square root we have

$$(\overline{U}^o_A-\underline{U}^o_A)\pi_B < \left(\overline{\pi}^E_A+\delta(\overline{\pi}^m_A-\underline{\pi}^m_A-\overline{\pi}^E_A-\underline{\pi}^I_A+2\pi^*_A)-\pi^*_A\right)U_B.$$

Which we rewrite as

$$\pi_B < \frac{\overline{\pi}^E_A-\pi^*_A}{\overline{U}^o_A-\underline{U}^o_A}U_B + \frac{\delta(\overline{\pi}^m_A-\underline{\pi}^m_A-\overline{\pi}^E_A-\underline{\pi}^I_A+2\pi^*_A)}{\overline{U}^o_A-\underline{U}^o_A}U_B.$$

Using the expression for surplus and profits given in Section 8.3 and rewriting we find

$$\pi_B < \frac{16(2(\overline{s}+\underline{s})-\sigma\underline{s})}{(\overline{s}+\underline{s}(1-\sigma))(4+\sigma)^2}U_B + \frac{\delta\left[\overline{s}(16+5\sigma^2)-\underline{s}(\sigma(32+\sigma^2)-(16+5\sigma^2))\right]}{2(\overline{s}+\underline{s}(1-\sigma))(16-\sigma^2)^2}U_B.$$

□

Proof of Proposition 3''''

(*i*) $\tilde{x}_E < \hat{x}_E$ if and only if

$$\frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A\left(\overline{\pi}^E_A+\delta(\overline{\pi}^m_A-\overline{\pi}^E_A-\underline{\pi}^I_A)-\pi^*_A-\delta(\underline{\pi}^m_A-2\pi^*_A)\right)}}}{1+\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A\left(\overline{\pi}^E_A+\delta(\overline{\pi}^m_A-\overline{\pi}^E_A-\underline{\pi}^I_A)-\pi^*_A-\delta(\underline{\pi}^m_A-2\pi^*_A)\right)}}} < \frac{1+\epsilon_B-\epsilon_A\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A(\overline{\pi}^E_A-\pi^*_A)}}}{1+\sqrt{\frac{\epsilon_B\pi_B}{\epsilon_A(\overline{\pi}^E_A-\pi^*_A)}}}.$$

Which is equivalent to

$$\overline{\pi}^E_A+\delta(\overline{\pi}^m_A-\overline{\pi}^E_A-\underline{\pi}^I_A)-\pi^*_A-\delta(\underline{\pi}^m_A-2\pi^*_A) < \overline{\pi}^E_A-\pi^*_A.$$

Dropping the common terms gives

$$\delta\left(\overline{\pi}^m_A-\underline{\pi}^m_A-\overline{\pi}^E_A-\underline{\pi}^I_A+2\pi^*_A\right) < 0.$$

Using the expression for profits given in Section 8.3 and simplifying we obtain

$$\overline{s}^2(16+5\sigma^2)-\overline{s}\underline{s}(32\sigma+\sigma^3)+\underline{s}^2(-16+32\sigma-5\sigma^2+\sigma^3) < 0.$$

Solving for the range of parameters $\sigma\in(0,2)$ we find

$$\left(-1+\frac{32\sigma+\sigma^3}{16+5\sigma^2}\right)\underline{s} < \overline{s} < \underline{s}.$$

[35] $\tilde{x}_E$ is interior when $\frac{\epsilon_A\epsilon_B}{(1+\epsilon_A)^2} < \frac{\pi_B}{\overline{\pi}^E_A+\delta(\overline{\pi}^m_A-\overline{\pi}^E_A-\underline{\pi}^I_A)-\pi^*_A-\delta(\underline{\pi}^m_A-2\pi^*_A)} < \frac{(1+\epsilon_B)^2}{\epsilon_A\epsilon_B}$.

This never holds so we always have $\hat{x}_E < \tilde{x}_E$.

(ii) When $\hat{x} < \tilde{x}$, $\tilde{x}$ is increasing in δ. Hence $\tilde{x} - \hat{x}$ is also increasing in δ. □

Proof of Proposition 5''''

Suppose that $\underline{s} < \overline{s} < \frac{2}{\sigma}\underline{s}$, in which case, $\hat{x}_E < \tilde{x}_E$ and therefore the expected consumer surplus in market B is higher if acquisitions are prohibited.

$$\frac{1-\tilde{x}_E}{1-\tilde{x}_E+\epsilon_B}U_B < \frac{1-\hat{x}_E}{1-\hat{x}_E+\epsilon_B}U_B. \tag{44}$$

The expected consumer surplus in market A is higher if acquisitions are prohibited when:

$$\frac{\tilde{x}_E}{\tilde{x}_E+\epsilon_A}\overline{U}_A^m + \frac{\epsilon_A}{\tilde{x}_E+\epsilon_A}\underline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E+\epsilon_A}\overline{U}_A + \frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\underline{U}_A. \tag{45}$$

We can rewrite this as:

$$\left(\frac{\hat{x}_E}{\hat{x}_E+\epsilon_A} + \frac{\epsilon_A}{\hat{x}_E+\epsilon_A} - \frac{\epsilon_A}{\tilde{x}_E+\epsilon_A}\right)\overline{U}_A^m + \frac{\epsilon_A}{\tilde{x}_E+\epsilon_A}\underline{U}_A^m < \frac{\hat{x}_E}{\hat{x}_E+\epsilon_A}\overline{U}_A + \frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\underline{U}_A.$$

We have $\overline{U}_A^m < \overline{U}_A$ so:

$$\left(\frac{\epsilon_A}{\hat{x}_E+\epsilon_A} - \frac{\epsilon_A}{\tilde{x}_E+\epsilon_A}\right)\overline{U}_A^m + \frac{\epsilon_A}{\tilde{x}_E+\epsilon_A}\underline{U}_A^m < \frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\underline{U}_A.$$

Rewriting gives:

$$\frac{\epsilon_A}{\tilde{x}_E+\epsilon_A}\left(\underline{U}_A^m - \overline{U}_A^m\right) + \frac{\epsilon_A}{\hat{x}_E+\epsilon_A}\left(\overline{U}_A^m - \underline{U}_A\right) < 0. \tag{46}$$

In market A we can rank consumer surplus in two different ways depending on the quality differences.

1. For all $1 < \frac{\overline{s}}{\underline{s}} < \frac{\sigma}{2} + \frac{\sqrt{\sigma^4-8\sigma^3-20\sigma^2+32\sigma+64}}{2\sigma+8}$ we have $\underline{U}_A^m < \overline{U}_A^m < \underline{U}_A < \overline{U}_A$.
2. For all $\frac{\sigma}{2} + \frac{\sqrt{\sigma^4-8\sigma^3-20\sigma^2+32\sigma+64}}{2\sigma+8} < \frac{\overline{s}}{\underline{s}} < \frac{2}{\sigma}$ we have $\underline{U}_A^m < \underline{U}_A < \overline{U}_A^m < \overline{U}_A$.

Therefore, the first term is negative since $\underline{U}_A^m < \overline{U}_A^m$. The second term is also negative when $1 < \frac{\overline{s}}{\underline{s}} < \frac{\sigma}{2} + \frac{\sqrt{\sigma^4-8\sigma^3-20\sigma^2+32\sigma+64}}{2\sigma+8}$ in which case (46) holds.

When $\frac{\overline{s}}{\underline{s}} \in (\frac{\sigma^2+4\sigma+\sqrt{\sigma^4-8\sigma^3-20\sigma^2+32\sigma+64}}{2\sigma+8}, \frac{2}{\sigma})$,[36] because $\underline{U}_A < \overline{U}_A^m$, similar to the proof of Proposition 5, we write (46) as follows:

$$\frac{\hat{x}_E+\epsilon_A}{\tilde{x}_E+\epsilon_A} > \frac{\overline{U}_A^m - \underline{U}_A}{\overline{U}_A^m - \underline{U}_A^m}. \tag{47}$$

Similar to Proposition 5(ii) if we show that the smallest value achieved by the function on the LHS is greater than the highest value achieved by the function on the RHS, then the proof is complete.

[36] This holds only for $\sigma < 1.4681$. Hence, this second ordering is irrelevant when $\sigma > 1.4681$.

Denote the LHS by $f(\sigma, \delta, \frac{\overline{s}}{\underline{s}}, z)$ where $z = \frac{\pi_B \epsilon_B \beta^2}{\epsilon_A \alpha^2}$ as before.[37] Similar to the proof of Proposition 5, we set $\delta = 1$ and take the limit $z \to \infty$ ($\epsilon_A \to 0$) which is

$$4\sqrt{\frac{2(4-\sigma^2)(2\overline{s} + (2-\sigma)\,\underline{s})}{(272-59\sigma^2)\overline{s} + (272-59\sigma^2-160\sigma+31\sigma^3)\underline{s}}}.$$

Next, since this expression is decreasing in $\frac{\overline{s}}{\underline{s}}$ on the relevant interval, we substitute $\frac{\overline{s}}{\underline{s}} = \frac{2}{\sigma}$ to obtain the smallest possible value this limit takes as a function of σ. We obtain

$$\frac{\hat{x}_E + \epsilon_A}{\tilde{x}_E + \epsilon_A} > 4\sqrt{\frac{2(4-\sigma^2)(\frac{4}{\sigma} + 2 - \sigma)}{(272-59\sigma^2)\frac{2}{\sigma} + (272-59\sigma^2-160\sigma+31\sigma^3)}}.$$

The RHS of (47) is given by[38]

$$\frac{(\sigma+2)\left[\left(\overline{s}^2 + \underline{s}^2 - \sigma\overline{s}\underline{s}\right)(\sigma+4)^2 - 4(\sigma+2)\underline{s}^2(4-\sigma^2)\right]}{\left[\left(\overline{s}^2 + \underline{s}^2 - \sigma\overline{s}\underline{s}\right)(\sigma+2) - \underline{s}^2(4-\sigma^2)\right](\sigma+4)^2}$$

Evaluating this expression at the upper bound of the interval $\frac{\overline{s}}{\underline{s}} = \frac{2}{\sigma}$ we obtain

$$\frac{\overline{U}_A^m - \underline{U}_A}{\overline{U}_A^m - \underline{U}_A^m} < \frac{(\sigma+2)\left((\sigma+4)^2 - 4\sigma^2(\sigma+2)\right)}{\left((\sigma+2)-\sigma^2\right)(\sigma+4)^2}$$

Next, we analyze the difference between the functions $4\sqrt{\frac{2(4-\sigma^2)(\frac{4}{\sigma}+2-\sigma)}{(272-59\sigma^2)\frac{2}{\sigma}+(272-59\sigma^2-160\sigma+31\sigma^3)}}$ and $\frac{(\sigma+2)\left((\sigma+4)^2-4\sigma^2(\sigma+2)\right)}{((\sigma+2)-\sigma^2)(\sigma+4)^2}$ and obtain that on the interval $\sigma \in (0, 1.4681)$ the latter intersects the former from above at $\sigma \approx 0.52$. Hence, we have shown that for $0.52 < \sigma < 1.4681$ the smallest value achieved by the function on the LHS is greater than the highest value achieved by the function on the RHS. For $\sigma < 0.52$ we cannot use the same approach based on comparison of the upper and lower bound. However, we have checked numerically that this inequality holds.

□

[37] Note, $f(\sigma, \delta, \frac{\overline{s}}{\underline{s}}, z)$ is decreasing in δ and decreasing in z, similar to analysis in Proposition 5.

[38] Note that the RHS of (47) is increasing in $\frac{\overline{s}}{\underline{s}}$ similar to analysis in Proposition 5.

References

Arrow, K. (1962). Economic welfare and the allocation of resources for invention. In Nelson, R., editor, *The rate and direction of economic activities: economic and social factors*, pages 609–626. NBER.

Bourreau, M., Jullien, B., and Lefouili, Y. (2018). Mergers and demand-enhancing innovation. TSE Working Paper 18-907.

Bryan, K. A. and Hovenkamp, E. (2020a). Antitrust limits on startup acquisitions. *Review of Industrial Organization*, 56(4):615–636.

Bryan, K. A. and Hovenkamp, E. (2020b). Startup acquisitions, error costs, and antitrust policy. *The University of Chicago Law Review*, 87(2):331–356.

Cabral, L. (2018). Standing on the shoulders of dwarfs: Dominant firms and innovation incentives. CEPR Discussion Paper No. DP13115.

Cabral, L. (2020). Merger policy in digital industries. *Information Economics and Policy*, forthcoming.

Chen, Y. and Schwartz, M. (2013). Product innovation incentives: Monopoly vs. competition. *Journal of Economics Management Strategy*, 3(22):513–528.

Crémer, J., de Montjoye, Y., and Schweitzer, H. (2019). *Competition Policy for the Digital Era*. European Commission, Directorate-General for Competition.

Cunningham, C., Ederer, F., and Ma, S. (2019). Killer acquisitions. *Journal of Political Economy*, forthcoming.

Deneckere, R. and Davidson, C. (1985). Incentives to form coalitions with bertrand competition. *The RAND Journal of Economics*, pages 473–486.

Farrell, J. and Shapiro, C. (1990). Horizontal mergers: an equilibrium analysis. *The American Economic Review*, 80(1):107–126.

Federico, G., Langus, G., and Valletti, T. (2018). Horizontal mergers and product innovation. *International Journal of Industrial Organization*, 61:590 –612.

Fumagalli, C., Motta, M., and Tarantino, E. (2020). Shelving or developing? the acquisition of potential competitors under financial constraints. CEPR Discussion Paper No. DP15113.

Furman, J., Fletcher, A., Marsden, P., Coyle, D., and McAuley, D. (2019). *Unlocking Digital Competition*. Report of the Digital Competition Expert Panel.

Gilbert, R. J. (2019). Competition, mergers, and R&D diversity. *Review of Industrial Organization*, 54(3):465–484.

Gilbert, R. J. and Katz, M. L. (2021). Dynamic merger policy and pre-merger investment: Equilibrium product choice by an entrant. Unpublished manuscript.

Greenstein, S. and Ramey, G. (1998). Market structure, innovation and vertical product differentiation. *International Journal of Industrial Organization*, 3(16):285–311.

Gurman, M. (2020). Apple buys self-learning AI video company to improve apps.

Hollenbeck, B. (2020). Horizontal mergers and innovation in concentrated industries. *Quantitative Marketing and Economics*, (18):1–37.

Kamepalli, S. K., Rajan, R., and Zingales, L. (2020). Kill zone. Working Paper 27146, National Bureau of Economic Research.

Katz, M. L. (2020). Big tech mergers: Innovation, competition for the market, and the acquisition of emerging competitors. *Information Economics and Policy*, forthcoming.

Klemperer, P. and Padilla, A. J. (1997). Do firms' product lines include too many varieties? *The RAND Journal of Economics*, 28(3):472–488.

Krieger, J., Li, D., and Papanikolaou, D. (2017). Developing novel drugs. Technical report. Available at SSRN: `https://ssrn.com/abstract=3095246`.

Letina, I. (2016). The road not taken: competition and the R&D portfolio. *The RAND Journal of Economics*, 47(2):433–460.

Letina, I., Schmutzler, A., and Seibel, R. (2020). Killer acquisitions and beyond: policy effects on innovation strategies. CEPR Discussion Paper No. DP15167.

McLaughlin, D. (2020). Big tech goes on shopping spree, brushing off antitrust scrutiny.

Moraga-González, J. L., Motchenkova, E., and Nevrekar, S. (2019). Mergers and innovation portfolios. CEPR Discussion Paper No. DP 14188.

Moraga-González, J. L. and Petrikaitė, V. (2013). Search costs, demand-side economies, and the incentives to merge under Bertrand competition. *The RAND Journal of Economics*, 44(3):391–424.

Motta, M. and Peitz, M. (2020). Big tech mergers. *Information Economics and Policy*, forthcoming.

Motta, M. and Shelegia, S. (2021). Copying, acquiring and the "kill zone". Working Paper, Department of Economics and Business, Universitat Pompeu Fabra.

Motta, M. and Tarantino, E. (2017). The effect of horizontal mergers, when firms compete in prices and investments. Working Paper, Department of Economics and Business, Universitat Pompeu Fabra.

OECD (2020). Start-ups, killer acquisitions and merger control, `http://www.oecd.org/daf/competition/start-ups-killer-acquisitions-and-merger-control-2020.pdf`.

Rasmusen, E. (1988). Entry for buyout. *Journal of Industrial Economics*, 36(3):281–299.

Salant, S. W., Switzer, S., and Reynolds, R. J. (1983). Losses from horizontal merger: the effects of an exogenous change in industry structure on Cournot-Nash equilibrium. *The Quarterly Journal of Economics*, 98(2):185–199.

Scott-Morton, F., Bouvier, P., Ezrachi, A., Jullien, B., Katz, R., Kimmelman, G., Melamed, D., and Morgenstern, J. (2019). *Committee for the Study of Digital Platforms. Report.* Stigler Center for the Study of the Economy and the State.

Sutton, J. (1997). One smart agent. *The Rand Journal of Economics*, 28(4):605–628.

Sutton, J. (2001). *Technology and Market Structure: Theory and History.* MIT press.

Williamson, O. E. (1968). Economies as an antitrust defense: The welfare tradeoffs. *The American Economic Review*, 58(1):18–36.