Costs and Benefits of Dynamic Trading in a Lemons Market

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Abstract

We study a dynamic market with asymmetric information that creates the lemons problem. We compare efficiency of the market under different assumptions about the timing of trade. We identify positive and negative aspects of dynamic trading, describe the optimal market design under regularity conditions and show that continuous-time trading can be always improved upon.

1 Introduction

Consider liquidity-constrained owners who would like to sell assets to raise capital for profitable new opportunities. Adverse selection, as in Akerlof (1970), means that if owners have private information about value trade will be inefficient, even in competitive markets. In this paper we show how that inefficiency is affected by market design in terms of when the sellers can trade.

In Akerlof (1970) the seller makes only one decision: to sell the asset or not. However, in practice, if the seller does not sell immediately, there are often future opportunities to trade. Delayed trade can be used by the market as a screen to separate low value assets (those that sellers are more eager to sell) from high-value assets. As we show in this paper, dynamic trading creates costs and benefits for overall market efficiency. On the positive side, the screening via costly delay increases in some instances overall liquidity of the market: more

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types eventually trade in a dynamic trading market than in the static/restricted trading market. On the negative side, future opportunities to trade reduce the amount of early trade, making the adverse selection problem worse. There are two related reasons. First, keeping the time 0 price fixed, after a seller decides to reject it, buyers update positively about the value of the asset and hence the future price is higher. That makes it desirable for some seller types to wait. Second, the types who wait are a better-than-average selection of the types were supposed to trade at time 0 in a static model and hence this additional adverse selection reduces p_0. In turn, even more types wait, reducing efficiency further.

We study different ways of designing the market in terms of picking the times when the market opens. For example, we compare efficiency of a continuously opened market to a design in which the seller can trade only once at $t = 0$ and otherwise has to wait until the type is revealed at some T (we allow the asymmetric information to be short-lived, $T < \infty$, as well as fully persistent, $T = \infty$).[1]

We motivate our analysis by an example with linear valuations (the value to buyers is a linear function of the seller value) and uniform distribution of seller types. We show that the market with restricted trading opportunities (allowing trades only at $t = 0$ and at T) is on average more efficient than a market with continuous-time trading. In fact, for large T the deadweight loss caused by adverse selection is three times as large if continuous trading is allowed. It may appear that preventing costly screening/signaling could always welfare-improving as in the education signaling models (Spence 1973). Via a different example we show that this is not always true: since in a market for lemons immediate efficient trade is not possible, in some situations screening via costly delay can help welfare.

Our first main result (Proposition 4) is that under fairly standard regularity conditions, restricting the seller to have only one, immediate trading opportunity until information arrives, generates higher expected gains from trade than any other market design that allows the seller to trade more than once. Moreover, sometimes it is even beneficial to delay when the information arrives to reduce adverse selection further. The second main result (Proposition 5) is that even without the regularity conditions, we can always improve upon a continuous trading market design. In particular, we show that introducing a "lock-up" period, that is allowing the seller to trade at $t = 0$ and then closing the market for an appropriate time window, followed by continuous trading, is welfare improving.

We then consider an alternative design: what if market is opened continuously until some time interval before information arrives? We show that this design has qualitatively different

[1] In Section 6.1 we consider information arriving at random time.

consequences than the "lock-up" period in which market is closed after the initial trade opportunity at $t = 0$. The reason is that closing the market before T creates an additional endogenous market closure. If the last opportunity to trade before T is at t^*, in equilibrium there is an additional time interval (t^{**}, t^*) such that nobody trades even though trades are allowed. The intuition is that failing to trade at t^* implies a strictly positive delay cost for the seller and as a result an atom of types trades at t^*. That reduction in adverse selection allows the buyers to offer a good price. In turn, waiting for this good price makes the adverse selection right before t^* so extreme that the market freezes. This additional delay cost can completely undo the efficiency gains that accrue at t^* - we argue that such short closures have a very small impact on total welfare and that the overall effect can be either positive or negative.

Next we discuss how our findings can be applied to inform government policy. When information frictions get really bad, the government may consider a direct intervention (beyond trying to regulate the dynamic trading). We have seen several of these interventions during the recent financial crisis. For example, the government could guarantee a certain value of traded assets (this was done with the debt issues by several companies and as part of some of the takeover deals of financially distressed banks). Alternatively, the government could directly purchase some of the assets (for example, real estate loan portfolios from banks as has been done in Ireland and is being discussed as a remedy for the Spanish banking crisis).

We point out an important equilibrium effect that seems to be absent of many public discussions about such government bailouts. It is not just the banks that participate in the asset buyback or debt guarantee programs that benefit from the government's intervention. The whole financial sector benefits because liquidity is restored to markets. As a result, non-lemons manage to realize higher gains from trade thanks to the intervention. We relate our findings to the recent work by Philippon and Skreta (2012) and Tirole (2012). We argue that unlike in their static-market analysis, the government can improve welfare by a comprehensive intervention which involves not only assets buy-backs but also restricts the post-intervention private markets. Finally, we point out that expectation of an asset buy-back (or any other intervention that leads to an atom of types trading) in the near term may drastically reduce liquidity as in the "late closure" market design, partially undermining the benefits of that intervention.

1.1 Related Literature

Our paper is related to literature on dynamic markets with adverse selection. The closest paper is Janssen and Roy (2002) who study competitive equilibria in a market that opens at a fixed frequency (and long-lived private information, $T = \infty$). In equilibrium prices increase over time and eventually every type trades. They point out that the outcome is still inefficient even as per-period discounting disappears (which is equivalent to taking a limit to continuous trading in our model) since trade suffers from delay costs even in the limit. They do not ask market design questions as we do in this paper (for example, what is the optimal frequency of opening the market). Yet, we share with their model the observation that dynamic trading with $T = \infty$ leads to more and more types trading over time. For other papers on dynamic signaling/screening with a competitive market see Noldeke and van Damme (1990), Swinkels (1999), Kremer and Skrzypacz (2007) and Daley and Green (2011). While we share with these papers an interest in dynamic markets with asymmetric information, none of these papers focuses on market design questions.

From the mechanism-design perspective, a closely related paper is Samuelson (1984). It characterizes a welfare-maximizing mechanism in the static model subject to no-subsidy constraints. When $T = \infty$, this static mechanism design is mathematically equivalent to a dynamic mechanism design since choosing probabilities of trade is analogous to choosing delay. Therefore our proof of Proposition 4 uses the same methods as Samuelson (1984).

As we mentioned already, our paper is also related to Philippon and Skreta (2012) and Tirole (2012) who study mechanism design (i.e. government interventions) in the presence of a market ("competitive fringe"). Our focus is on a different element of market design, but we also discuss how these two approaches can be combined.

Our analysis can be described as "design of timing" in the sense that we compare equilibrium outcomes for markets/games that differ in terms of the time when players move. That is related in spirit to Damiano, Li and Suen (2012), who study optimal delay in committee decisions where the underlying game resembles a war of attrition.

A different design question for dynamic markets with asymmetric information is asked in Hörner and Vieille (2009), Kaya and Liu (2012), Kim (2012) and Fuchs, Öry and Skrzypacz (2012). These papers take the timing of the market as given (a fixed frequency) and ask how information about past rejected offers affects efficiency of trade. It is different from our observation in Remark 2 since this is about observability of accepted rather than rejected offers.

Finally, there is also a recent literature on adverse selection with correlated values in models with search frictions (among others, Guerrieri, Shimer and Wright (2010), Guerrieri and Shimer (2011) and Chang (2012)). Rather than having just one market in which different quality sellers sell at different times, the separation of types in these models is achieved because market differ in market tightness with the property that in a market with low prices a seller can find a buyer very quickly and in a market with high prices it takes a long time to find a buyer. Low-quality sellers which are more eager to sell quickly self-select into the low price market while high quality sellers are happy to wait longer in the high price market. One can relate our design questions to a search setting by studying the efficiency consequences of closing certain markets (for example, using a price ceiling). This would roughly correspond to closing the market after some time in our setting.

2 The Model

As in the classic market for lemons, a potential seller owns one unit of an indivisible asset. When the seller holds the asset, it generates for him a revenue stream $c \in [0,1]$ that is private information of the seller. c is drawn from a distribution $F(c)$, which is common knowledge, atomless and has a continuous, strictly positive density $f(c)$.

There is a competitive market of potential buyers. Each buyer values the asset at $v(c)$ which is strictly increasing, twice continuously differentiable, and satisfies $v(c) > c$ for all $c < 1$ (i.e. common knowledge of gains from trade) and $v(1) = 1$ (i.e. no gap on the top). These assumptions imply that in the static Akerlof (1970) problem some but not all types trade in equilibrium.[2]

Time is $t \in [0,T]$ and we consider different market designs in which the market is opened in different moments in that interval. We start the analysis with two extreme market designs: "infrequent trading" (or "restricted trading") in which the market is opened only twice at $t \in \{0,T\}$ and "continuous trading" in which the market is opened in all $t \in [0,T]$. Let $\Omega \subseteq [0,T]$ denote the set of times that the market is open (we assume that at the very least $\{0,T\} \subset \Omega$).

Every time the market is opened, there is a market price at which buyers are willing to trade and the seller either accepts it (which ends the game) or rejects. If the price is rejected the game moves to the next time the market is opened. If no trade takes place by time T the type of the seller is revealed and the price in the market is $v(c)$, at which all seller types trade.

[2] Assuming $v(1) = 1$ allows us not to worry about out-of-equilibrium beliefs after a history where all seller types are supposed to trade but trade did not take place. We discuss this assumption further in Section 6.3.

All players discount payoffs at a rate r and we let $\delta = e^{-rT}$. The values c and $v(c)$ are normalized to be in total discounted terms. If trade happens at time t at a price p_t, the seller's payoff is

$$\left(1 - e^{-rt}\right) c + e^{-rt} p_t$$

and the buyer's payoff is

$$e^{-rt} \left(v(c) - p_t\right)$$

A *competitive equilibrium* is a pair of functions $\{p_t, k_t\}$ for $t \in \Omega$ where p_t is the market price at time t and k_t is the highest type of the seller that trades at time t.[3] These functions satisfy:

(1) Zero profit condition: $p_t = E\left[v(c) \mid c \in [k_{t-}, k_t]\right]$ where k_{t-} is the cutoff type at the previous time the market is open before t (with $k_{t-} = 0$ for the first time the market is opened)[4]

(2) Seller optimality: given the process of prices, each seller type maximizes profits by trading according to the rule k_t.

(3) Market Clearing: in any period the market is open, the price is at least $p_t \geq v(k_{t-})$.

Conditions (1) and (2) are standard. Condition (3) deserves a bit of explanation. We justify it by a market clearing reasoning: suppose the asset was offered at a price $p_t < v(k_{t-})$ at time t. Then, since all buyers believe that the value of the good is at least $v(k_{t-})$, they would all demand it. Demand could not be equal to supply, the market could not clear. This condition removes some trivial multiplicity of equilibria, for example $(p_t, k_t) = (0, 0)$ for all periods (i.e. no trade and very low prices) satisfy the first two conditions. Condition (3) is analogous to the condition (iv) in Janssen and Roy (2002) and is weaker than the No Unrealized Deals condition in Daley and Green (2011) (see Definition 2.1 there; since they study the gap case, they need a stronger condition to account for out-of-equilibrium beliefs).

We assume that all market participants publicly observe all the trades. Hence, once a buyer obtains the asset, if he tries to put it back on the market, the market makes a correct inference about c based on the history. Since we assume that all buyers value the asset the same, there would not be any profitable re-trading of the asset (after the initial seller transacts) and hence we ignore that possibility in our model (however, see Remark 2).

[3] Since we know that the skimming property holds in this environment it is simpler to directly define the competitive equilibrium in terms of cutoffs.

[4] In continuous time we use a convention $k_{t-} = \lim_{s \uparrow t} k_s$, and $E\left[v(c) \mid c \in [k_{t-}, k_t]\right] = \lim_{s \uparrow t} E\left[v(c) \mid c \in [k_s, k_t]\right]$ and $v(k_{t-}) = \lim_{s \uparrow t} v(k_s)$. If $k_t = k_{t-}$ then the condition is $p_t = v(k_t)$.

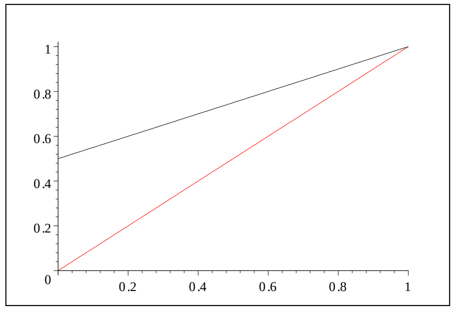

Figure 1: Gains from trade in the benchmark example.

3 Motivating Example

Before we present the general analysis of the problem, consider the following example. Assume c is distributed uniformly over $[0,1]$ and $v(c) = \frac{1+c}{2}$, as illustrated in Figure 1.

We compare two possible market designs. First, infrequent trading, that is $\Omega_I = \{0, T\}$. Second, continuous trading, $\Omega_C = [0, T]$.

Remark 1 *In this paper we analyze competitive equilibria. In this leading example it is possible to write a game-theoretic version of the model allowing two buyers to make public offers every time the market is open. If we write the model having* $\Omega = \{0, \Delta, 2\Delta, ..., T\}$ *then we can show that there is a unique Perfect Bayesian Equilibrium for every* T *and* $\Delta > 0$. *When* $\Delta = T$ *then the equilibrium coincides with the equilibrium in the infrequent trading market we identify below. Moreover, taking the sequence of equilibria as* $\Delta \to 0$, *the equilibrium path converges to the competitive equilibrium we identify for the continuous trading design. In other words, the equilibria we describe in this section have a game-theoretic foundation.*

Infrequent Trading The infrequent trading market design corresponds to the classic market for lemons as in Akerlof (1970). The equilibrium in this case is described by a price p_0 and a cutoff k_0 that satisfy that the cutoff type is indifferent between trading at $t = 0$ and waiting till T:

$$p_0 = (1 - \delta) k_0 + \delta \frac{1 + k_0}{2}$$

and that the buyers break even on average:

$$p_0 = E[v(c) | c \leq k_0]$$

The solution is $k_0 = \frac{2-2\delta}{3-2\delta}$ and $p_0 = \frac{4-3\delta}{6-4\delta}$. The expected gains from trade are

$$S_I = \int_0^{k_0} (v(c) - c)\, dc + \delta \int_{k_0}^{1} (v(c) - c)\, dc = \frac{4\delta^2 - 11\delta + 8}{4(2\delta - 3)^2}$$

Continuous Trading The above outcome cannot be sustained in equilibrium if there are multiple occasions to trade before T. If at $t = 0$ types below k_0 trade, the next time the market opens price would be at least $v(k_0)$. If so, types close to k_0 would be strictly better off delaying trade. As a result, for any set Ω richer than Ω_I, in equilibrium there is less trade in period 0.

If we look at the case of continuous trading, $\Omega_C = [0, T]$, then the equilibrium with continuous trade is a pair of two processes $\{p_t, k_t\}$ that satisfy:

$$\begin{aligned} p_t &= v(k_t) \\ r(p_t - k_t) &= \dot{p}_t \end{aligned}$$

The intuition is as follows. Since the process k_t is continuous, the zero profit condition is that the price is equal to the value of the current cutoff type. The second condition is the indifference of the current cutoff type between trading now and waiting for a dt and trading at a higher price. These conditions yield a differential equation for the cutoff type

$$r(v(k_t) - k_t) = v'(k_t)\, \dot{k}_t$$

with the boundary condition $k_0 = 0$. In our example it has a simple solution:

$$k_t = 1 - e^{-rt}.$$

The total surplus from continuous trading is

$$S_C = \int_0^T e^{-rt} (v(k_t) - k_t)\, \dot{k}_t dt + e^{-rT} \int_{k_T}^{1} (v(c) - c)\, dc = \frac{1}{12}\left(2 + \delta^3\right).$$

Comparing Infrequent and Continuous trading The graph below (left) compares the dynamics of trade (prices and cutoffs) in these two settings for $T = \infty$. The dashed line at 2/3 is the equilibrium price and cutoff when there is only one opportunity to trade. With continuous trading the cutoff starts at zero and gradually rises towards one.

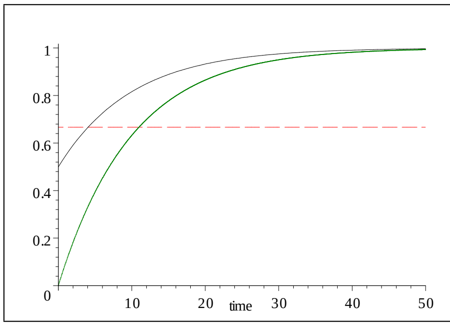

Figure 2: Trade Dynamics

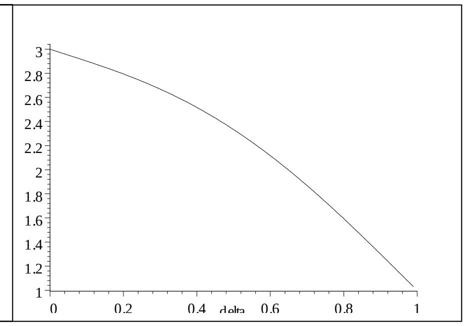

Figure 3: Efficiency

How do gains from trade compare in these two cases? Figure 3 shows the ratio $\frac{S_{FB}-S_C}{S_{FB}-S_I}$ where S_{FB} is the trade surplus if trade was efficient, while S_I and S_C are the trade surpluses computed above. The ratio represents the relative efficiency loss from adverse selection in these two markets:

- When $\delta \to 0$ (i.e. as $rT \to \infty$, the private information is long-lived) we get $\frac{S_{FB}-S_C}{S_{FB}-S_I} \to 3$ so the efficiency loss with continuous trading is three times higher than with infrequent trading.
- When $\delta \to 1$ (i.e. $T \to 0$ so the private information is very short-lived), the organization of the market does not matter since even by waiting till T players can achieve close to full efficiency in either case.

What affects relative efficiency of the two market designs? The trade-off is as follows. Committing to only one opportunity to trade generates a big loss of surplus if players do not reach an agreement in the current period. This clearly leaves a lot of unrealized gains from trade. But it is this inefficiency upon disagreement that helps overcome the adverse selection problem and increases the amount of trade in the initial period. Continuous trading on the other hand does not provide many incentives to trade in the current period since a seller suffers a negligible loss of surplus from delay. This leads to an equilibrium with smooth trading over time. While the screening of types via delay is costly, the advantage is that eventually (if T is large enough) more types trade. In determining which trading environment is more efficient on average, one has to weight the cost of delaying trade with low types with the advantage of eventually trading with more types.

3.1 Can Continuous Trading be Better?

Our example above demonstrates a case of $v(c)$ and $F(c)$ such that for every T the infrequent trading market is more efficient than the continuous trading market. Furthermore, the greater T, the greater the efficiency gains from using infrequent trading. Is it a general phenomenon? The answer is no:

Proposition 1 *There exist $v(c)$ and $F(c)$ such that for T large enough the continuous trading market generates more gains from trade than the infrequent trading market*

The example used in this proof (omitted proofs are in the Appendix) illustrates what is needed for the continuous trading market to dominate the infrequent one: we need a large mass at the bottom of the distribution, so that the infrequent trading market gets "stuck" with these types, while under continuous trading these types trade quickly, so the delay costs for these types are small. Additionally, we need some mass of higher types that would be reached in the continuous trading market after some time, generating additional surplus. Alternatively, if $v(c) - c$ were not decreasing, even for uniform distribution of c the continuously open market could be more efficient since the delay costs to efficiency of trade with the low types could be small compared to the gains from eventual trading with the high types if the market is opened more often. We formalize these intuitions below.

4 Optimality of Restricting Trading Opportunities

We now return to the general model. We first describe the equilibrium with continuous trading opportunities:

Proposition 2 (Continuous trading) *For $\Omega_C = [0, T]$ a competitive equilibrium (unique up to measure zero of times) is the unique solution to:*

$$\begin{aligned} p_t &= v(k_t) \\ k_0 &= 0 \\ r(v(k_t) - k_t) &= v'(k_t)\,\dot{k}_t \end{aligned} \tag{1}$$

Proof. First note that our requirement $p_t \geq v(k_{t-})$ implies that there cannot be any atoms of trade, i.e. that k_t has to be continuous. Suppose not, that at time s types $[k_{s-}, k_s]$ trade with $k_{s-} < k_s$. Then at time $s + \varepsilon$ the price would be at least $v(k_s)$ while at s the price would be strictly smaller to satisfy the zero-profit condition. But then for small ε types close to k_s would be better off not trading at s, a contradiction. Therefore we are left with processes such that k_t is continuous and $p_t = v(k_t)$. For k_t to be strictly increasing over time we need that $r(p_t - k_t) = \dot{p}_t$ for almost all t: if price was rising faster, current cutoffs would like to wait, a contradiction. If prices were rising slower over any time interval starting at s, there would be an atom of types trading at s, another contradiction. So the only remaining possibility is that $\{p_t, k_t\}$ are constant over some interval $[s_1, s_2]$. Since the price at s_1 is $v\left(k_{s_{1-}}\right)$ and the price at s_2 is $v(k_{s_2})$, we would obtain a contradiction that there is no atom of trade in equilibrium. In particular, if $p_{s_1} = p_{s_2}$ (which holds if and only if $k_{s_{1-}} = k_{s_1} = k_{s_2}$) then there exist types $k > k_{s_1}$ such that

$$v(k_{s_1}) > \left(1 - e^{r(s_2 - s_1)}\right) k + e^{r(s_2 - s_1)} v(k_{s_1})$$

and these types would strictly prefer to trade at $t = s_1$ than to wait till s_2, a contradiction again. ■

On the other extreme, with infrequent trading, Ω_I, the equilibrium is:[5]

Proposition 3 (Infrequent/Restricted Trading) *For $\Omega_I = \{0, T\}$ there exists a competitive equilibrium $\{p_0, k_0\}$. Equilibria are a solution to:*

$$p_0 = E[v(c) | c \in [0, k_0]] \tag{2}$$

$$p_0 = \left(1 - e^{-rT}\right) k_0 + e^{-rT} v(k_0) \tag{3}$$

If $\frac{f(c)}{F(c)}(v(c) - c) - \frac{\delta}{1-\delta} v'(c)$ is strictly decreasing, the equilibrium is unique.

4.1 General Market Designs

So far we have compared only the continuous trading market with the infrequent trading. But one can imagine many other ways to organize the market. For example, the market could clear every day; or every $\Delta \in (0, T)$. Or the market could be opened at 0, then closed for some time interval Δ and then be opened continuously. Or, the market could start being opened continuously and close some Δ before T (i.e. at $t = T - \Delta$). In this section we consider some of these alternative timings.

[5] The infrequent trading model is the same as the model in Akerlof (1970) if $T = \infty$. Even with $T < \infty$ the proof of existence and inefficiency of the equilibrium is standard so we leave it to the appendix.

4.1.1 When Infrequent Trading is Optimal

We start with providing a sufficient condition for the infrequent trading to dominate all these other possible designs:

Proposition 4 *If* $\frac{f(c)}{F(c)}\frac{v(c)-c}{1-\delta+\delta v'(c)}$ *and* $\frac{f(c)}{F(c)}\left(v\left(c\right)-c\right)$ *are decreasing,*[6] *then infrequent trading,* $\Omega_I = \{0, T\}$ *, generates higher expected gains from trade than any other market design.*

Proof. We use mechanism design to establish the result. Consider the following relaxed problem. There is a mechanism designer who chooses a direct revelation mechanism that maps reports of the seller to a probability distribution over times he trades and to transfers from the buyers to the mechanism designer and from the designer to the seller. The constraints on the mechanism are: incentive compatibility for the seller (to report truthfully); individual rationality for the seller and buyers (the seller prefers to participate in the mechanism rather than wait till T and get $v\left(c\right)$ and the buyers do not lose money on average); and that the mechanism designer does not lose money on average. Additionally, we require that the highest type, $c = 1$, does not trade until T (as in any equilibrium he does not).

For every game with a fixed Ω, the equilibrium outcome can be replicated by such a mechanism, but not necessarily vice versa, since if the mechanism calls for the designer cross-subsidizing buyers across periods, it cannot be replicated by a competitive equilibrium.

Within this class of direct mechanisms we characterize one that maximizes ex-ante expected gains from trade. We then show that under the conditions in the proposition, infrequent trading replicates the outcome of the best mechanism and hence any other market design generates lower expected gains from trade.

A general direct revelation mechanism can be described by 3 functions $x\left(c\right), y\left(c\right)$ and $P\left(c\right)$, where $y\left(c\right)$ is the probability that the seller will not trade before information is released, $x\left(c\right)$ is the discounted probability of trade over all possible trading times and $P\left(c\right)$ is the transfer received by the seller conditional on trading before information is released.[7] Note that $y\left(c\right) \in [0, 1]$ but $x\left(c\right) \in [\delta, 1]$.

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[6] A sufficient condition is that $v''\left(c\right) \geq 0$ and $\frac{f(c)}{F(c)}\left(v\left(c\right)-c\right)$ is decreasing.

[7] Letting $G_t\left(c\right)$ denote for a given type the distribution function over the times of trade:

$$x\left(c\right) = \int_0^T e^{-rt} dG_t\left(c\right).$$

The seller's value function in the mechanism is:

$$\begin{aligned} U(c) &= y(c)\left[(1-\delta)c+\delta v(c)\right]+(1-y(c))\left[P(c)+(1-x(c))c\right] && (4)\\ &= \max_{c'} y(c')\left[(1-\delta)c+\delta v(c)\right]+(1-y(c'))\left[P(c')+(1-x(c'))c\right] && (5)\end{aligned}$$

Using the envelope theorem:[8]

$$\begin{aligned} U'(c) &= y(c)\left[(1-\delta)+\delta v'(c)\right]+(1-y(c))(1-x(c))\\ &= \delta y(c)(v'(c)-1)+1-x(c)(1-y(c))\end{aligned}$$

Let $V(c)=\delta v(c)+(1-\delta)c$ be the no-trade surplus, so:

$$\begin{aligned} U'(c)-V'(c) &= \delta y(c)(v'(c)-1)+1-x(c)(1-y(c))-(\delta v'(c)+(1-\delta))\\ &= (1-y(c))(-x(c)-\delta(v'(c)-1))\end{aligned}$$

As a result, we can write the expected seller's gains from trade as a function of the allocations $x(c)$ and $y(c)$ only:

$$\begin{aligned} S &= \int_0^1 (U(c)-V(c))f(c)\,dc\\ &= (U(c)-V(c))F(c)\,|_{c=0}^{c=1}-\int_0^1 (U'(c)-V'(c))F(c)\,dc\\ &= \int_0^1 (1-y(c))\left[x(c)-\delta(1-v'(c))\right]F(c)\,dc && (6)\end{aligned}$$

Clearly, the mechanism designer will leave the buyers with no surplus (since he could use it to increase efficiency of trade) and so maximizing S is the designer's objective (see Samuelson 1984). That also means that the no-losses-on-average constraint is:

$$\int_0^1 (1-y(c))(x(c)v(c)-P(c))f(c)\,dc \geq 0$$

[8] This derivative exists almost everywhere and hence we can use the integral-form of the envelope formula, (6).

From the expression for $U(c)$ we have

$$\begin{aligned} U(c) - y(c)\left[(1-\delta)c + \delta v(c)\right] - (1-y(c))(1-x(c))c &= (1-y(c))P(c) \\ U(c) - V(c) + (1-y(c))(\delta(v(c)-c) + x(c)c) &= (1-y(c))P(c) \end{aligned}$$

So the constraint can be re-written as a function of the allocations alone (where the last term expands as in (6)):

$$\int_0^1 (1-y(c))(x(c)-\delta)(v(c)-c) f(c)\,dc - \int_0^1 (U(c)-V(c)) f(c)\,dc \geq 0 \tag{7}$$

We now optimize (6) subject to (7), ignoring necessary monotonicity constraints on $x(c)$ and $y(c)$ that assure that reporting c truthfully is incentive compatible (we check later that they are satisfied in the solution).

The derivatives of the Lagrangian with respect to $x(c)$ and $y(c)$ are:

$$\begin{aligned} L_x(c) &= (1-y(c))\left[F(c) + \Lambda\left((v(c)-c)f(c) - F(c)\right)\right] \\ -L_y(c) &= (x(c) - \delta(1-v'(c)))F(c) + \Lambda\left[(x(c)-\delta)(v(c)-c)f(c) - (x(c)-\delta(1-v'(c)))F(c)\right] \end{aligned}$$

where $\Lambda > 0$ is the Largrange multiplier.

Consider $L_x(c)$ first. Note that $\left[F(c) + \Lambda\left((v(c)-c)f(c) - F(c)\right)\right]$ is positive for $c = 0$. We know that in the optimal solution it has to be negative for $c = 1$, since otherwise we could achieve efficiency without subsidizing the mechanism and it is not possible. Hence, $\Lambda > 1$. Suppose $\frac{f(c)}{F(c)}(v(c)-c)$ is decreasing, which is one of the conditions in the proposition. Let c^* be a solution to $1 - \frac{f(c)}{F(c)}(v(c)-c) = \frac{1}{\Lambda}$. Then the second term in $L_x(c)$ changes sign once at c^*. An optimal $x(c)$ is therefore:

$$x(c) = \begin{cases} 1 \text{ if } c \leq c^* \\ \delta \text{ if } c > c^* \end{cases}$$

Now consider $-L_y(c)$. For all $c \leq c^*$, using the optimal $x(c)$, it simplifies to:

$$-L_y(c) = (1-\delta+\delta v'(c))F(c) + \Lambda\left[(1-\delta)(v(c)-c)f(c) - (1-\delta+\delta v'(c))F(c)\right] \text{ for } x(c) = 1$$

If $\frac{f(c)}{F(c)} \frac{v(c)-c}{1+\frac{\delta}{(1-\delta)}v'(c)}$ is decreasing in c, which is one of the conditions in the proposition, $L_y(c)$ changes sign once in this range. It is negative for $c \leq c^{**}$ and positive for $c > c^{**}$, where $c^{**} < c^*$ is a solution to $\frac{f(c)}{F(c)}(v(c)-c) = \left(1 - \frac{1}{\Lambda}\right)\left(1 + \frac{\delta}{(1-\delta)}v'(c)\right)$. Therefore the optimal

$y(c)$ in this range is

$$y(c) = \begin{cases} 0 \text{ if } c \leq c^{**} \\ 1 \text{ if } c > c^{**} \end{cases}$$

For $c > c^*$, using the optimal $x(c)$, the derivative $L_y(c)$ simplifies to

$$L_y(c) = -(1-\Lambda)\,\delta v'(c)\,F(c) \text{ for } x(c) = \delta$$

since $\Lambda > 1$, this is positive and the optimal $y(c)$ is equal to 1. That finishes the description of the optimal allocations in the relaxed problem: there exists a c^* such that types below c^* trade immediately and types above it wait till after information is revealed at T. The higher the c^* the higher the gains from trade. The largest c^* that satisfies the constraint is the largest solution of:

$$E[v(c)\,|c \leq c^*] = (1-\delta)\,c^* + \delta v(c^*)$$

since the LHS is the IR constraint of the buyers and the RHS is the IR constraint of the c^* seller. This is also the equilibrium condition in a market with design $\Omega = \{0, T\}$, so that equilibrium implements the solution to the relaxed problem. ■

The condition in the proposition is similar to the standard condition in optimal auction theory/pricing theory that the virtual valuation/marginal revenue curve be monotone. In particular, think about a static problem of a monopsonist buyer choosing a cutoff (or a probability to trade, $F(c)$), by making a take-it-or-leave-it offer equal to $P(c) = (1-\delta)\,c + \delta v(c)$. In that problem $\frac{f(c)}{F(c)}\frac{v(c)-c}{1-\delta+\delta v'(c)}$ decreasing guarantees that the marginal profit crosses zero exactly once.[9] In our relaxed mechanism design problem this condition appears as a bang-for-the-buck formula that captures how much gain from trade we can get from a type (the numerator) to the information rents we need to give him.

Our proof considers a relaxed mechanism design problem with a market maker who could cross-subsidize buyers buying in different periods and who has to break even only on average. For $T = \infty$, this is a problem analyzed in Samuelson (1984).[10] Samuelson (1984) shows that this problem has a solution that has at most two steps. That is, for any $v(c)$ and $F(c)$, the optimal solution is characterized by two cutoffs, c_1 and c_2, such that types $c \in [0, c_1]$ trade at time $t = 0$, types $c \in [c_1, c_2]$ trade at some time $t^* > 0$ and all the higher types do not trade at all. In some cases $c_1 = c_2$ (and our proposition has sufficient conditions for it). In that

[9] The FOC of the monopolist problem choosing c is: $(1-\delta)\,f(c)\,(v(c)-c) - F(c)\,((1-\delta)+\delta v'(c)) = 0$.

[10] A slight difference is that he is studying a static mechanism design. However, when $T = \infty$ the two problems are mathematically equivalent since time discounting and probability of trade enter the utilities of all players the same way.

case the solution to the relaxed problem can be implemented by a competitive equilibrium and hence in all these cases the Ω_I design is the most efficient. However, if $c_1 \neq c_2$, then, as shown in Samuelson (1984), the mechanism designer makes money on the trades at $t = 0$ and loses money on the trades at t^*. That allocation cannot be implemented by a competitive equilibrium for any Ω.

It is an open question how to solve for the optimal Ω in case the solution to the relaxed problem calls for trade in more than one period. The difficulty is that the constraints on the mechanism are then endogenous. A mechanism that calls for a set of types to trade at time t has to have a price equal to the average $v(c)$ across these types. Hence, as Ω changes (or the range of the allocation function changes), the set of constraints changes as well.

Remark 2 *One way to implement $\Omega_I = \{0, T\}$ in practice may be via an extreme anonymity of the market. In our model we have assumed that the initial seller of the asset can be told apart in the market from buyers who later become secondary sellers. However, if the trades are completely anonymous, even if $\Omega \neq \{0, T\}$, the equilibrium outcome would coincide with the outcome for Ω_I. The reason is that the price can never go up since otherwise the early buyers of the low-quality assets would resell them at the later markets.*
Such extreme anonymity may not be feasible in some markets (for example, IPO's), or not practical for reasons outside the model. Yet, it may be feasible in some situations. For example, a government as a part of an intervention aimed at improving efficiency of the market may create a trade platform in which it would act as a broker who anonymizes trades and traders.

4.1.2 Closing the Market Briefly after Initial Trade.

Even if the condition in Proposition 4 does not hold and we cannot find the optimal Ω, we can show that under very general conditions it is possible to improve upon the continuous trading market.

In particular, consider the design $\Omega^{EC} \equiv \{0\} \cup [\Delta, T]$: there is trade at $t = 0$, then the market is closed till $\Delta > 0$ and then it is opened continuously till T. We call this design "early closure". We show that there always exists a small delay that improves upon continuous trading:

Proposition 5 *For every r, T, $F(c)$, and $v(c)$, there exists $\Delta > 0$ such that the early closure market design $\Omega^{EC} = \{0\} \cup [\Delta, T]$ yields higher gains from trade than the continuous trading design $\Omega_C = [0, T]$.*

Proof. To establish that early closure increases efficiency of trade we show an even stronger result: that for small Δ with Ω^{EC} there is more trade at $t=0$ than with Ω_C by $t=\Delta$. Let k_Δ^{EC} be the highest type that trades at $t=0$ when the design is Ω^{EC}. Let k_Δ^C the equilibrium cutoff at time Δ in design Ω_C. Then the stronger claim is that for small Δ, $k_\Delta^C < k_\Delta^{EC}$. Since $\lim_{\Delta\to 0} k_\Delta^{EC} = \lim_{\Delta\to 0} k_\Delta^C = 0$ (for k_Δ^{EC} see discussion in Step 1 below). So it is sufficient for us to rank:

$$\lim_{\Delta\to 0}\frac{\partial k_\Delta^{EC}}{\partial \Delta} \text{ vs. } \lim_{\Delta\to 0}\frac{\partial k_\Delta^{C}}{\partial \Delta}$$

Step 1: Characterizing $\lim_{\Delta\to 0}\frac{\partial k_\Delta^{EC}}{\partial \Delta}$.

Consider Ω^{EC}. When the market reopens at $t=\Delta$ the market is continuously open from then on. Hence, the equilibrium in the continuation game is the same as the equilibrium characterized in Proposition (2) albeit with a different starting lowest type. Namely, for $t \geq \Delta$

$$\begin{aligned} p_t &= v(k_t) \\ r(v(k_t)-k_t) &= v'(k_t)\dot{k}_t \end{aligned}$$

with a boundary condition:

$$k_\Delta = k_\Delta^{EC}.$$

The break even condition for buyers at $t=0$ implies:

$$p_0 = E\left[v(k)\,|k\in\left[0,k_\Delta^{EC}\right]\right]$$

and type k_Δ^{EC} must be indifferent between trading at this price at $t=0$ or for $p_\Delta = v\left(k_\Delta^{EC}\right)$ at $t=\Delta$:

$$v\left(k_\Delta^{EC}\right) - p_0 = \left(1-e^{-r\Delta}\right)\left(v\left(k_\Delta^{EC}\right)-k_\Delta^{EC}\right)$$

For small Δ , $E\left[v(c)\,|c\leq k_\Delta^{EC}\right] \approx \frac{v\left(k_\Delta^{EC}\right)}{2}$ so the benefit of waiting is approximately $\frac{v\left(k_\Delta^{EC}\right)}{2}$ while the cost is approximately $rTv(0)$ so k_Δ^{EC} for small T solves approximately

$$\frac{v\left(k_\Delta^{EC}\right)}{2} \approx rTv(0)$$

and more precisely:

$$\lim_{\Delta\to 0}\frac{\partial k_\Delta^{EC}}{\partial\Delta} = \frac{2rv(0)}{v'(0)}$$

Step 2: Characterizing $\lim_{\Delta\to 0}\frac{\partial k_\Delta^C}{\partial \Delta}$.

Consider Ω^C. Since k_t is defined by the differential equation

$$r\left(v\left(k_t\right)-k_t\right)=v'\left(k_t\right)\dot{k}_t,$$

for small Δ :

$$k_\Delta^C \approx rT\frac{v\left(0\right)}{v'\left(0\right)},$$

and more precisely:

$$\lim_{\Delta\to 0}\frac{\partial k_\Delta^C}{\partial \Delta}=\frac{rv\left(0\right)}{v'\left(0\right)}.$$

Summing up steps 1 and 2, we have:

$$\lim_{\Delta\to 0}\frac{\partial k_\Delta^{EC}}{\partial \Delta}=2\lim_{\Delta\to 0}\frac{\partial k_\Delta^C}{\partial \Delta}$$

which implies the claim. ■

A closely related result is that when the private information is short lived, closing the market after the initial trade and waiting until the information arrives dominates continuous trading:

Corollary 1 *For every* r, $v\left(c\right)$, *and* $F\left(c\right)$ *there exists a* $T^* > 0$ *such that for all* $T \leq T^*$ *the infrequent trading market design generates higher expected gains from trade than the continuous trading design.*

The proof is analogous to the proof of the previous Proposition by noting that in either situation: $\Omega^{EC}=\{0\}\cup[\Delta,T]$ or $\Omega_I=\{0,T=\Delta\}$ the cutoff type trading at time 0 chooses between p_0 and price $v\left(k_0\right)$. In case information is revealed at T this is by assumption that the market is competitive at T. In case the market is open continuously after the early closure it is by our observation that the continuation equilibrium has smooth screening of types so the first price after closure is $p_\Delta = v\left(k_\Delta^{EC}\right)$.

4.1.3 Closing the Market Briefly before Information Arrives

The final design we consider is the possibility of keeping the market opened continuously from $t=0$ till $T-\Delta$ and then closing it till T. Such a design seems realistic and in some practical situations may be easier to implement than Ω^{EC} because it may be easier to determine when some private information is expected to arrive (i.e. when $t=T$) than when it is that the seller of the asset is hit by liquidity needs (i.e. when $t=0$).

The comparison of this "late closure" market with the continuous trading market is much more complicated than in the previous section for two related reasons. First, if the market is closed from $T-\Delta$ to T, there will be an atom of types trading at $T-\Delta$. As a result, there will be a "quiet period" before $T-\Delta$: there will be some time interval $[t^*, T-\Delta]$ such that despite the market being open, there will be no types that trade on the equilibrium path in that time period. The equilibrium outcome until t^* is the same in the "late closure" as in the continuous trading design, but diverges from that point on. That brings the second complication: starting at time t^*, the continuous trading market benefits from some types trading earlier than in the "late closure" market. Therefore it is not sufficient to show that by T there are more types that trade in the late closure market. We actually have to compare directly the total surplus generated between t^* and T. These two complications are not present when we consider the "early closure" design since there is no t^* before $t=0$ for the earlier trade to be affected by the early closure.

The equilibrium in the "late closure" design is as follows. Let $p^*_{T-\Delta}, k^*_{T-\Delta}$ and t^* be a solution to the following system of equations:

$$
\begin{aligned}
E\left[v\left(c\right)|c \in \left[k_{t^*}, k_{T-\Delta}\right]\right] &= p_{T-\Delta} \qquad (8) \\
\left(1-e^{-r\Delta}\right)k_{T-\Delta} + e^{-r\Delta}v\left(k_{T-\Delta}\right) &= p_{T-\Delta} \qquad (9) \\
\left(1-e^{-r(T-\Delta-t^*)}\right)k_{t^*} + e^{-r(T-\Delta-t^*)}p_{T-\Delta} &= v\left(k_{t^*}\right) \qquad (10)
\end{aligned}
$$

where the first equation is the zero-profit condition at $t=T-\Delta$, the second equation is the indifference condition for the highest type trading at $T-\Delta$ and the last equation is the indifference condition of the lowest type that reaches $T-\Delta$, who chooses between trading at t^* and at $T-\Delta$. The equilibrium for the late closure market is then:
1) at times $t \in [0, t^*]$, (p_t, k_t) are the same as in the continuous trading market
2) at times $t \in (t^*, T-\Delta)$, $(p_t, k_t) = (v(k_{t^*}), k_{t^*})$
3) at $t = T-\Delta$, $(p_t, k_t) = \left(p^*_{T-\Delta}, k^*_{T-\Delta}\right)$

Condition (10) guarantees that given the constant price at times $t \in (t^*, T-\Delta)$ it is indeed optimal for the seller not to trade. There are other equilibria that differ from this equilibrium in terms of the prices in the "quiet period" time: any price process that satisfies in this time period

$$\left(1-e^{-r(T-\Delta-t)}\right)k_{t^*} + e^{-r(T-\Delta-t)}p_{T-\Delta} \geq p_t \geq v\left(k_{t^*}\right)$$

satisfies all our equilibrium conditions. Yet, all these paths yield the same equilibrium outcome in terms of trade and surplus (of course, the system $(8) - (10)$ may have multiple solutions that would have different equilibrium outcomes).

Despite this countervailing inefficiency, for our leading example:

Proposition 6 *Suppose* $v(c) = \frac{1+c}{2}$ *and* $F(c) = c$. *For every* r *and* T *there exists a* $\Delta > 0$ *such that the "late closure" market design,* $\Omega^{LC} = [0, T - \Delta] \cup \{T\}$, *generates higher expected gains from trade than the continuous trading market,* Ω_C. *Yet, the gains from late closure are smaller than the gains from early closure.*

The proof is in the appendix. It shows third-order gains of welfare from the late closure (while the gains from early closure are first-order). Figure 4.1.3 below illustrates the reason the gains from closing the market are smaller relative to when the market is closed at time zero. The bottom two lines show the evolution of the cutoff type in Ω_C (continuous curve) and in Ω^{LC} (discontinuous at $t = T - \Delta = 0.9$). The top two lines show the corresponding path of prices. The gains from bringing forward trades that occur when the market is exogenously closed in $t \in (9, 10)$ (i.e. the jump in types at $t = 0.9$) are partially offset by the delay of types in the endogenous quiet period $t \in (8.23, 9)$. If we close the market for $t \in (0, \Delta)$ instead, there is no loss from some types postponing trade because there is no time before 0.

The intuition why the gains (if any) are in general very small is that we prove that the endogenous quiet period is on the same order as Δ. The reasoning in Proposition 5 implies that the jump in types at time $T - \Delta$ is approximately twice as large as the continuous increase in the cutoff when the market is opened continuously over a time interval of length Δ. Putting these two observations together implies that the final cutoff at time T is approximately (using a first-order approximation in Δ) the same for these two designs, as seen in Figure 4.1.3. Hence, any welfare effects are tiny.

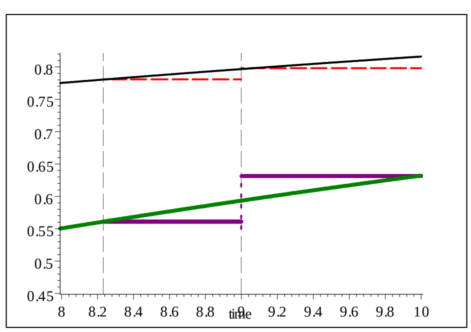

Figure 4: Late Closure

$$T = 10 \quad \Delta = 1 \qquad r = 0.1 \quad v(c) = \frac{c+1}{2} \quad F(c) = c$$

Given our results so far showing the benefits of restricting opportunities to trade, one might speculate that the optimal Ω may not contain any continuous-trading intervals but instead be characterized by a discrete grid of trading times $\Omega = \{0, \Delta_1, \Delta_2, \Delta_3, ..., T\}$. We do not know how to prove or disprove this claim without any restrictions on $v(c)$ and $f(c)$.

What we can show is that there are cases when some restrictions to continuous trading, even small, can reduce welfare. An example of such a situation is $f(c) = 2 - 2c$ and $v(c) = c + 1$. In this case, by direct calculations we can show that "late closure" reduces expected gains from trade. The intuition is that even though the gains from trade are constant across all types, since $f(c)$ is decreasing, the distribution assigns a higher weight to the types that delay in the endogenous "quiet period" than to the types that speed up thanks to the closure.

5 Implications for Asset Purchases by the Government

Market failure due to information frictions sometimes calls for government intervention. During the recent financial crisis several markets effectively shut down or became extremely illiquid. One of the main reasons cited for this was the realization by market players that the portfolios of asset backed securities that banks held were not all investment grade as initially thought. Potential buyers of these securities which used to trade them without much concern suddenly became very apprehensive of purchasing these assets for the potential risk of buying a lemon. The Treasury and the Federal Reserve tried many different things to restore liquidity into the markets. Some of the measures were aimed at providing protection against downside risk via guarantees effectively decreasing the adverse selection problem or by removing the most toxic assets from the banks' balance sheets (for example, via the TARP I and II programs or central banks' acceptance of toxic assets as collateral).

Our model provides a natural framework to study the potential role for government. To illustrate consider the case in which if $v(c) = \gamma c$ for $2 > \gamma > 1$ and $F(c) = c$.[11] Then for all Ω the unique equilibrium is for there never to be any trade before the information is revealed.

[11] This model arises for example if the seller has a higher discount rate than the buyers.

So the market is completely illiquid and no gains from trade are realized. The government could intervene in this market by making an offer $p_g > 0$ to buy any asset sellers are willing to sell at that price (these programs are by and large voluntary).

In this example, the average quality of these assets will be $\frac{p_g}{2}$ and hence the government would lose money on them. On the bright side is that once the toxic assets have been removed from the market and the remaining distribution is truncated to $c \in \left[\frac{p_g}{\gamma}, 1\right]$ now even if $\Omega = [0_+, T]$ buyers would be willing to start making offers again.

We want to make two observations about this intervention. First, In the post-intervention, continuously-opened market the liquidity is characterized by (1) which in this example simplifies to:

$$rk_t \frac{\gamma - 1}{\gamma} = \dot{k}_t$$

Therefore, the larger the initial intervention, the faster the trade in the free market afterwards. Second, this government intervention benefits not only the direct recipients of government funds but also all other sellers since by reducing the adverse selection problem in the market they will now have an opportunity trade with a private counterparty.

Optimal government interventions in very similar (though richer) models have been studied recently by Philippon and Skreta (2012) and Tirole (2012). In these papers the government offers financing to firms having an investment opportunity and it is secured by assets that the firms have private information about. That intervention is followed by a static competitive market in which firms that did not receive funds from the government can trade privately. This creates a problem of "mechanism design with a competitive fringe" as named by Philippon and Skreta (2012).

The setup in these two papers can be roughly mapped to ours if we assume $v(c) - c = \gamma$.[12] Our paper directly applies to section II (Buybacks only) in Tirole (2012), but we believe that the following observations apply more broadly.

Both papers show that the total surplus cannot be improved by the government shutting down private markets: see Proposition 2 in Tirole (2012) and Theorem 2 in Philippon and Skreta (2012). Since the post-intervention market creates endogenous IR constraints for the agents participating in the government program, making it less attractive could make it easier for the government to intervene. However, these two papers argue that this is never a good idea.

[12] In both papers there is an additional restriction that the seller needs to raise a minimal amount of money to make a profitable investment, which is the source of gains from trade. That additional aspect does not change our conclusions.

Our results show that taking into account the dynamic nature of the market changes this conclusion. In particular, the assumptions in Tirole (2012) satisfy the assumptions in Proposition 4 (he assumes $\frac{f(c)}{F(c)}$ is decreasing). Therefore it would be optimal for the government to concentrate post-intervention trades to be right after it and commit to shutting down the market afterwards. This could be achieved by organizing a market at $t = 0$, offering a subsidy to trades and announcing that all trades afterwards will be taxed. Alternatively, offering (partial) insurance for assets traded at a particular time window but not later. Finally, creating an anonymous exchange (see Remark 2) may be a practical solution.

Additionally, our analysis of the late closure suggests that if the market expects the government may run a program of that nature in near future, the market may close endogenously, even if trade would continue if no such intervention is expected. The reasoning is the same: if a non-trivial fraction of seller types participate in the government transaction, the post-intervention price is going to be strictly higher than the current cutoff's $v(k_t)$ and hence there are no trades that could be profitable for both sides.

6 Discussion

In this section we explore a few extensions of the model.

6.1 Stochastic Arrival of Information

So far we have assumed that it is known that the private information is revealed at T. However, in some markets, even if the private information is short-lived, the market participants may be uncertain about the timing of its revelation. We now return to our motivating example to illustrate that trade-offs we have identified so far apply also to the stochastic duration case.

Let $v(c) = \frac{1+c}{2}$ and $F(c) = c$. Suppose that with a Poisson rate λ information arrives that publicly reveals seller's type. Upon arrival trade is efficient at $p = v(c)$. Analogously to what we have done before, let *infrequent trading* market mean that the seller can trade only at $t = 0$ (or after information arrives). Let *continuous trading* market mean that the seller can trade at any time.

In the infrequent trading market, the equilibrium (p_0, k_0) is determined by:

$$
\begin{aligned}
p_0 &= \frac{\lambda}{\lambda + r} v(k_0) + \frac{r}{\lambda + r} k_0 \\
p_0 &= E[v(c) | c \leq k_0]
\end{aligned}
$$

where the first equation is the indifference condition of the cutoff type and the second equation is the usual zero-profit condition. In our example we get

$$k_0 = \frac{2r}{3r+\lambda},\ p_0 = \frac{4r+\lambda}{6r+2\lambda}$$

In the continuous trading market the equilibrium is described by the same equations as in the deterministic T case (see Proposition 2), with a solution $k_t = 1 - e^{-rt}$. The intuition the equilibrium path of prices before information arrives is the same in the stochastic and deterministic arrival of information cases as follows. In the deterministic case, the effect of delaying trade by dt is that the price increases by $\dot{p}_t dt$. In the stochastic case, the price also increases, but additionally with probability λdt the news arrives. If so, the current cutoff type gets a price $v(k_t)$ instead of p_{t+dt}. However, since $p_t = v(k_t)$, price p_{t+dt} is only of order dt higher. Hence the additional term is on the order dt^2 and does not affect incentives to delay.

We now can compare the gains from trade. The total gains from trade in the infrequent trading market are:

$$S_I = \int_0^{k_0} (v(c) - c)\, dc + \frac{\lambda}{\lambda + r} \int_{k_0}^1 (v(c) - c)\, dc.$$

In the continuous trading market the gains are:

$$S_C = \int_0^{+\infty} \lambda e^{-\lambda t} \left(\int_0^{k_t} e^{-r\tau(c)} (v(c) - c)\, dc + e^{-rt} \int_{k_t}^1 (v(c) - c)\, dc \right) dt$$

where $\tau(c) = -\frac{\ln(1-c)}{r}$ is the time type c trades if there is no arrival before $\tau(c)$. Direct calculations yield:

$$S_0(z) - S_C(z) = \frac{1}{2}(z+3)^{-2} > 0$$

where $z \equiv \frac{\lambda}{r}$. So, for every λ, the infrequent trading market is more efficient than the continuous trading market.

6.2 Beyond Design of Ω : Affecting T

In this paper we analyze different choices of Ω. A natural question is what else could a market designer affect to improve the market efficiency. One such possibility is information structure, as we have discussed in Remark 2. There are of course other options for changing

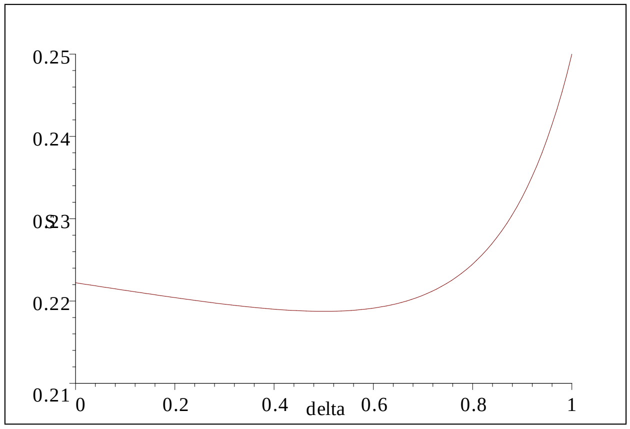

Figure 2: Surplus with infrequent trading as a function of T

information (for example, should past rejected offers be observed or not?), but that is beyond the scope of this paper.

Another possibility is affecting T. Clearly, if the market designer could make T very small, it would good for welfare since it would make the market imperfections short-lived. That may not be feasible though. Suppose instead that the designer could only increase T (for example, by making some verification take longer).[13] Surprisingly, it turns out that in some cases increasing T could improve efficiency. While it is never beneficial in the continuous-trading case (since it does not affect trade before T and only delays subsequent trades), it can help in other cases. To illustrate it, Figure 2 graphs the expected gains from trade in our leading example for $\Omega_I = \{0, T\}$ as a function of δ. It turns out that if and only if $e^{-rT} < \frac{1}{2}$, increasing T is welfare improving.

6.3 Common Knowledge of Gains from Trade

We have assumed that $v(0) > 0$ and $v(1) = 1$, that is, strictly positive gains from trade for the lowest type and no gains on the top. Can we relax these assumptions?

6.3.1 Role of $v(0) > 0$

If $v(0) = 0$ then Proposition (4) still applies. As we argued above, if the market is opened continuously, in equilibrium there is no trade before T (to see this note that the starting price would leave the lowest type with no surplus, so that type would always prefer to wait for a price increase). That does not need to be true for other Ω. For example, if $v(c) = \sqrt{c}$

[13] We thank Marina Halac for suggesting this question.

and $F(c) = c$, then for all T the conditions in Proposition (4) are satisfied. Therefore, $\Omega_I = \{0, T\}$ is welfare-maximizing and $\Omega_C = [0, T]$ is welfare-minimizing over all Ω; and if $\delta < \frac{2}{3}$ then the ranking is strict since there is some trade with Ω_I.

6.3.2 Role of $v(1) = 1$

The main reason we assume $v(1) = 1$ is that in this way we do not need to define equilibrium market prices after histories where the seller trades with probability 1. That is, when $v(1) = 1$, the highest type never trades in equilibrium no matter how large is T. This makes our definition of competitive equilibrium simpler than in Daley and Green (2011) (compare our condition (3) "Market Clearing" with Definition 2.1 there).

To illustrate how the freedom in selecting off-equilibrium-path beliefs can lead to a multiplicity of equilibria with radically different outcomes consider the following heuristic reasoning. Assume:

$$F(c) = c \ ; \ \ v(c) = c + s$$

Suppose that $\Omega = \{0, \Delta, 2\Delta, ..., T\}$ for $\Delta > 0$. Let $s > \frac{1}{2}$ so that in a static problem trade would be efficient.

Case 1: Assume that when an offer that all types accept on the equilibrium path is rejected, buyers believe the seller has the highest type, $c = 1$. That is, post-rejection price is $1 + s$. Then, taking a sequence of equilibria as $\Delta \to 0$, we can show that in the limit trade is smooth over time (no atoms) with:

$$\begin{aligned} p_t(k) &= v(k_t) \\ k_t &= rst \end{aligned}$$

On equilibrium path all types trade by:

$$\tau = \frac{1}{rs}$$

unless $\tau < T$. If the last offer, $p_\tau = 1 + s$ is rejected, the price stays constant after that, consistently with the beliefs and competition.

Case 2: Alternatively, assume that when an offer that all types accept on the equilibrium path is rejected, buyers do not update their beliefs. That is, after that history they believe the seller type is distributed uniformly over $[k_t, 1]$, where k_t is derived from the history of the game. In that case we can construct the following equilibrium for all $\Delta > 0$. At $t = 0$ there is an initial offer $p_0 = \frac{1}{2} + s$ and all types trade. If that initial offer is rejected, the buyers believe $c\tilde{}U[0,1]$ and continue to offer $p_t = p_0$ for all $t > 0$ (and again all types trade). This is indeed and equilibrium since the buyers break even at time zero (and at all future times given their beliefs) and no seller type is better off by rejecting the initial offer.

These equilibria are radically different in terms of efficiency: only the second one is efficient. It is beyond the scope of this paper to study in what situations or under what model extensions this multiplicity could be resolved and how. $v(1) > 1$ creates similar problems for large T even if immediate efficient trade is not possible. On the other hand, if the gap on top is small so that for a given T in equilibrium it is not possible that all types trade before T, then our analysis still applies.

7 Conclusions

In this paper we have analyzed a dynamic market with asymmetric information. Our two main results are that, first, under mild regularity conditions restricting trade to $\Omega_I = \{0, T\}$ dominates any other design. Second, even more generally, efficiency can be improved over continuous-time trading by the "early closure" design which after initial auction restricts additional trading for some interval of time. We discussed how these findings can inform government policy geared towards resolving market failures due to the lemons problem. Unlike the previous papers using a static model of the market, we argue that an intervention would be more successful if the government could at least partially restrict dynamic trading after the intervention. The bottom line is that we have identified a non-trivial cost to dynamic trading: it makes the adverse selection problem worse.

Many open questions remain. On a more technical note, it is an open question how to compute the optimal Ω if our regularity conditions do not hold. On a more practical note, in many markets the time the game actually starts is ill-defined and/or sellers arrive to the market at different times (as opposed to a whole market being hit by liquidity shocks as in the recent financial crisis). Even in the case of IPOs it is not clear how to define the first time the market considers the owners of the startup to delay an offering with the hope of affecting investors' beliefs (i.e. at which point signaling via delay starts). Moreover, one may be interested in embedding this model into a larger market with many sellers hit by liquidity needs at different times to gain additional insights about market design. Finally, in our model there were only two sources of signaling: delay and the exogenous signal that arrives only once. In many markets sellers may want to wait for multiple pieces of news to arrive over time before they agree to sell.

8 Appendix

Proof of Proposition 1. Consider a distribution that approximates the following: with probability ε c is drawn uniformly on $[0,1]$; with probability $\alpha(1-\varepsilon)$ it is uniform on $[0,\varepsilon]$; and with probability $(1-\alpha)(1-\varepsilon)$ it is uniform on $[c_1, c_1+\varepsilon]$ for some $c_1 > v(0)$. In other words, the mass is concentrated around 0 and c_1. Let $v(c) = \frac{1+c}{2}$ as in our example.

For small ε there exists $\alpha < 1$ such that

$$E\left[v(c) \,|c \leq c_1 + \varepsilon\right] < c_1$$

so that in the infrequent trading market trade will happen only with the low types. In particular, if α is such that

$$\alpha v(0) + (1-\alpha) v(c_1) < c_1$$

then as $\varepsilon \to 0$ and $T \to \infty$, the infrequent trading equilibrium price converges to $v(0)$ and the surplus converges to

$$\lim_{\varepsilon \to 0, T\to\infty} S_I = \alpha v(0) + (1-\alpha) c_1$$

The equilibrium path for the continuous trading market is independent of the distribution and hence

$$\begin{aligned} \lim_{\varepsilon \to 0, T\to\infty} S_C &= \alpha v(0) + (1-\alpha)\left[e^{-r\tau(c_1)} v(c_1) + \left(1 - e^{-r\tau(c_1)}\right) c_1\right] \\ &= \lim_{\varepsilon \to 0, T\to\infty} S_I + (1-\alpha)\left(e^{-r\tau(c_1)}\left(v(c_1) - c_1\right)\right) \end{aligned}$$

where $\tau(k)$ is the inverse of the function k_t. The last term is strictly positive for any $c_1 < v(c_1)$. In particular, with $v(c) = \frac{1+c}{2}$, $e^{-r\tau(c)} = (1-c)$ and $v(c_1) - c_1 = \frac{1}{2}(1-c_1)$, so

$$\lim_{\varepsilon \to 0, T\to\infty} S_C = \lim_{\varepsilon \to 0, T\to\infty} S_I + \frac{1}{2}(1-\alpha)(1-c_1)^2 .$$

■

Proof of Proposition 3. 1) **Existence**. The equilibrium conditions follow from the definition of equilibrium. To see that there exists at least one solution to (2) and (3) note that if we write the condition for the cutoff as:

$$E\left[v\left(c\right)|c\leq k_0\right]-\left(\left(1-e^{-rT}\right)k_0+e^{-rT}v\left(k_0\right)\right)=0 \tag{11}$$

then the LHS is continuous in k_0, it is positive at $k_0=0$ and negative at $k_0=1$. So there exists at least one solution. [14]

2) **Uniqueness**. To see that there is a unique solution under the two assumptions, note that the derivative of the LHS of (11) at any k is

$$\frac{f\left(k\right)}{F\left(k\right)}\left(v\left(k\right)-E\left[v\left(c\right)|c\leq k\right]\right)-\left(1-\delta\right)-\delta v'\left(k\right)$$

When we evaluate it at points where (11) holds, the derivative is

$$\frac{f\left(k\right)}{F\left(k\right)}\left(v\left(k\right)-k\right)\left(1-\delta\right)-\left(1-\delta\right)-\delta v'\left(k\right)$$

and that is by assumption decreasing in k.

Suppose that there are at least two solutions and select two: the lowest k_L and second-lowest k_H. Since k_L is the lowest solution, at that point the curve on the LHS of (11) must have a weakly negative slope (since the curve crosses zero from above). However, our assumption implies that curve has even strictly more negative slope at k_H. That leads to a contradiction since by assumption between $[k_L,k_H]$ the LHS is negative, so with this ranking of derivatives it cannot become 0 at k_H ■

Proof of Proposition 6**.**

In this case the equilibrium conditions $(8),(9)$ and (10) simplify to

$$\begin{aligned}\frac{1}{2}+\frac{k_{t^*}+k_{T-\Delta}}{4} &= p_{T-\Delta} &(6')\\ \left(1-e^{-r\Delta}\right)k_{T-\Delta}+\left(\frac{1}{2}+\frac{k_{T-\Delta}}{2}\right)e^{-r\Delta} &= p_{T-\Delta} &(7')\\ \left(1-e^{-r\Delta_2}\right)k_{t^*}+e^{-r\Delta_2}p_{T-\Delta} &= \frac{1}{2}+\frac{k_{t^*}}{2} &(8')\end{aligned}$$

where $\Delta_2=T-\Delta-t^*$.

[14] If there are multiple solutions, a game theoretic-model would refine some of them, see section 13.B of Mas-Colell, Whinston and Green (1995) for a discussion.

Solution of the first two equations is:

$$k_{T-\Delta} = \frac{k_{t^*} + 2 - 2e^{-r\Delta}}{3 - 2e^{-r\Delta}}$$

$$p_{T-\Delta} = \frac{1}{2}\left(\frac{2 - e^{-r\Delta}}{3 - 2e^{-r\Delta}} k_{t^*} + \frac{4 - 3e^{-r\Delta}}{3 - 2e^{-r\Delta}}\right)$$

Substituting the price to the last condition yields

$$\left(1 - e^{-r\Delta_2}\right) k_{t^*} + e^{-r\Delta_2}\left(\frac{1}{2}\left(\frac{2 - e^{-r\Delta}}{3 - 2e^{-r\Delta}} k_{t^*} + \frac{4 - 3e^{-r\Delta}}{3 - 2e^{-r\Delta}}\right)\right) = \frac{1}{2} + \frac{k_{t^*}}{2}$$

which can be solved for Δ_2 independently of k_{t^*} (given our assumptions about $v(c)$ and $F(c)$).

$$r\Delta_2 = -\ln \frac{3 - 2e^{-r\Delta}}{4 - 3e^{-r\Delta}}$$

Note that

$$\lim_{\Delta \to 0} \frac{\partial \Delta_2}{\partial \Delta} = \lim_{\Delta \to 0} \frac{\partial}{\partial \Delta} \frac{1}{r}\left(-\ln \frac{3 - 2e^{-r\Delta}}{4 - 3e^{-r\Delta}}\right) = 1$$

so Δ_2 is approximately equal to Δ.

In the continuous trading cutoffs follow $k_t = 1 - e^{-rt}$, $\dot{k}_t = re^{-rt}$. Normalize $T = 1$ (and rescale r appropriately). Then

$$k_{t^*} = 1 - e^{-r(1-\Delta-\Delta_2)} = 1 - \frac{4 - 3e^{-r\Delta}}{3 - 2e^{-r\Delta}} e^{r\Delta}\delta$$

$$\text{where } \delta = e^{-r} \text{ and}$$

$$t^* = 1 - \Delta - \Delta_2 = 1 - \Delta + \frac{1}{r}\ln\frac{3 - 2e^{-r\Delta}}{4 - 3e^{-r\Delta}}$$

We can now compare gains from trade in the two cases. The surplus starting at time t^* is (including discounting):

$$S_c(\Delta) = \int_{k_{t^*}}^{1-e^{-r}} e^{-r\tau(c)} \left(v(c) - c\right) dc + \delta \int_{1-e^{-r}}^{1} \left(v(c) - c\right) dc$$

$$= \int_{k_{t^*}}^{1-e^{-r}} (1-c)\left(\frac{1-c}{2}\right) dc + \delta \int_{1-e^{-r}}^{1} \left(\frac{1-c}{2}\right) dc$$

where we used $e^{-r\tau(c)} = 1 - c$.

$$\frac{\partial S_c(\Delta)}{\partial \Delta} = -\frac{\partial k_{t^*}}{\partial \Delta} \frac{(1-k_{t^*})^2}{2}$$

and since $\lim_{\Delta \to 0} \frac{\partial k_{t^*}}{\partial \Delta} = -2r\delta$ we get that

$$\lim_{\Delta \to 0} \frac{\partial S_c(\Delta)}{\partial \Delta} = r\delta^3$$

For the "late closure" market the gains from trade are

$$S_{LC}(\Delta) = e^{-r(1-\Delta)} \int_{k_{t^*}}^{k_{T-\Delta}} (v(c) - c)\, dc + e^{-r} \int_{k_{T-\Delta}}^{1} (v(c) - c)\, dc$$

after substituting the computed values for k_{t^*} and $k_{T-\Delta}$ it can be verified that

$$\lim_{\Delta \to 0} \frac{\partial S_{LC}(\Delta)}{\partial \Delta} = r\delta^3$$

which is the same as in the case of continuous market, so to the first approximation even conditional on reaching t^* the gains from trade are approximately the same in the two market designs.

We can compare the second derivatives:

$$\begin{aligned} \lim_{\Delta \to 0} \frac{\partial S_{LC}^2(\Delta)}{\partial \Delta^2} &= 3\delta^3 r^2 \\ \lim_{\Delta \to 0} \frac{\partial S_c^2(\Delta)}{\partial \Delta^2} &= 3\delta^3 r^2 \end{aligned}$$

and even these are the same. Finally, comparing third derivatives:

$$\begin{aligned} \lim_{\Delta \to 0} \frac{\partial S_{LC}^3(\Delta)}{\partial \Delta^3} &= 13r^3\delta^3 \\ \lim_{\Delta \to 0} \frac{\partial S_c^3(\Delta)}{\partial \Delta^3} &= 9r^3\delta^3 \end{aligned}$$

so we get that for small Δ, the "late closure" market generates slightly higher expected surplus, but the effects are really small. ■

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