Strategic Design under Uncertain Evaluations: Theory and Evidence from Design-Build Auctions

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January 27, 2014

Abstract

This paper investigates firms' competition over price and product design in the context of Design-Build (DB) procurement auctions. In a DB auction, bidders submit both a price and a design proposal for a public infrastructure project. Each reviewer for the auctioneer (the government) independently evaluates and assigns a score to every design bid. The bidder with the lowest price per design score wins, and receives its price bid upon completion of the project. Data on DB auctions from the Florida Department of Transportation reveal substantial disagreements in reviewers' evaluations of the same design. I argue that this discrepancy in evaluations is unknown to bidders at the time of making their bids. I develop and estimate a structural model of firm competition that incorporates uncertainty in design evaluations, introducing an element of *luck* in selecting the winner of a project. Based on the estimated model, I show that uncertain design evaluation induces more (respectively, less) competitive bidding by low (high) cost bidders, exacerbating uncertainty in both the winning price and design quality. I propose a second-price auction in which a bidder's design score determines the transfer amount, shutting down the impact of uncertain evaluations on bidders' behaviour through the winner selection process. I find that the alternative mechanism reduces the auctioneer's uncertainty in both the amount paid and the quality of the winning design by 40% and 60%, respectively.

*I am indebted to Victor Aguirregabiria for invaluable advice and generous guidance. I would also like to thank Ettore Damiano, Rahul Deb, Yao Luo, Robert McMillan, Xianwen Shi, Junichi Suzuki, Yuanyuan Wan, and the participants in CEPR/JIE, EARIE, and Jornadas de Economía Industrial for helpful comments. Thanks to Daniel Garcia and Andrea Pozzi for extensive discussions. All errors are mine. Email: hidenori.takahashi@utoronto.ca

1 Introduction

Design competitions in the face of uncertain evaluations are common in a variety of settings. A home owner may request several architects for a renovation proposal, but the architects may not know exactly how the home owner will evaluate their design or those of their rivals. In the context of large infrastructure projects, public procurements that involve billions of dollars often solicit designs from consulting firms, and those competing firms are also uncertain about how government officials will evaluate their designs of the infrastructure project.[1]

Prompted by these examples, this paper investigates the effect of a client's uncertain design evaluation on the decisions of a supplier competing for a contract to produce a customized product. Transactions involving customized products require a design of the end-product by the supplier before the product is manufactured, but suppliers typically do not know precisely what end-product their client would like.[2] Uncertainty in evaluations introduces an element of *luck* into design competitions, providing suppliers of different types with different incentives. Good designers face a lower likelihood of winning from increased uncertainty in evaluation while bad designers face a higher likelihood of winning a contract.[3] It is not clear how suppliers of different types will respond to a change in the degree of uncertainty in a client's evaluation of their design proposals, and thus the identification of this effect on suppliers' behaviour is an empirical question.

To address this issue, I study the Design-Build (DB) procurement auction used by many state departments of transportation in the U.S. and around the world.[4] In a DB auction, bidders compete over price and design to win a contract to implement an infrastructure project, ranging from road maintenance and bridge repair to building construction. Conversations with DB contractors reveal that uncertainty in design evaluation is a substantial concern among firms involved in DB auctions, and I use hand-collected data on DB auctions from the Florida Department of Transportation (FDOT) to investigate firms' competition in the face of uncertain design evaluations. Upon receiving price and design proposals, each reviewer employed by the FDOT independently evaluates and assigns a score to every design proposal. The quality score of a design proposal is then determined by the average

[1] Public Private Partnerships, which have surged in popularity, are also an example of public procurement that involves a design competition among consulting firms.

[2] While suppliers may communicate with a client to reduce uncertainty, the client may be unwilling to do so since repeated interactions can be very costly. Speed of delivery is often an important consideration in procurement.

[3] Note that a supplier faces uncertainty not only in the evaluation of its own design but also the evaluation of its opponents' designs.

[4] As of October 2010, there are 39 state departments of transportation that use DB, including California, Delaware, Georgia, Minnesota, etc. DB auctions are also common in other developed countries, including Canada, Japan, and Sweden.

reviewer's evaluation. The bidder with the lowest price per quality score ratio (PQR) wins the project, and receives its price bid upon the completion of the project.[5]

This paper makes several contributions, building on new evidence that documents the existence of wide disagreements in reviewers' evaluations of a given design proposal. Such uncertainty in design evaluations, which I refer to as *evaluation uncertainty*, has not been considered to date in the large auction literature. Indeed, to the best of my knowledge, there is virtually no previous work that has investigated the implications of uncertain design evaluations on supplier behaviour.

At the heart of the arguments, I develop an estimable model in which each bidder strategically chooses its price and design proposal in the face of uncertain design evaluations. The model allows for unobserved auction heterogeneity, and two sources of firm heterogeneity: variable costs that depend on the quality choice and fixed costs. Most importantly, the model deals with the problem that the econometrician does not observe actual design quality of a proposal. I propose a sequential method for estimating the degree of evaluation uncertainty together with the primitives of the model.

Based on the estimated model, I show that the main effect of evaluation uncertainty is to induce low (respectively, high) cost bidders to become more (less) competitive, exacerbating uncertainty in price and design bids. Consequently, the auction outcomes also become more uncertain from the auctioneer's standpoint. To reduce the auctioneer's uncertainty in auction outcomes, I propose a simple and dominant strategy implementable mechanism. The proposed mechanism shuts down the effect of uncertain evaluations on bidding incentives, and significantly mitigates the auctioneer's uncertainty in the amount paid and the quality of the winning design.

I distinguish two sources of heterogeneity in reviewers' evaluations: vertical reviewer heterogeneity and horizontal reviewer heterogeneity. Vertical reviewer heterogeneity comes from the fact that reviewers have different quality standards, which affect design scores of different bidders equally.[6] While vertical reviewer heterogeneity is rank-neutral, horizontal reviewer heterogeneity is not. For instance, if a reviewer happens to place a higher value on the appearance of a bridge than other reviewers, the bidder with a fancy design proposal may unexpectedly obtain a high score from the reviewer. While I argue that both forms of reviewer heterogeneity are uncertain to bidders when they make their bids, this paper focuses on the role of horizontal reviewer heterogeneity, which I interpret as evaluation uncertainty, mainly because vertical reviewer heterogeneity does not affect the rankings of design proposals,

[5]PQR is a winner selection rule used by many state departments of transportation, including Alaska, Michigan, North Carolina, and South Dakota.

[6]For example, reviewers' leniency in assigning a score may be captured by vertical reviewer heterogeneity since a lenient reviewer tends to give a high score to every design proposal.

and so is unlikely to affect bidders' incentives in a DB auction.[7]

I estimate the structural model in three steps. First, I use data on price bids, reviewers' evaluations, and observable characteristics together with a reduced-form model to estimate the degree of reviewer heterogeneity, both vertical and horizontal. I find that 18% of the total variation in reviewers' evaluations is due to horizontal reviewer heterogeneity. Second, I estimate the curvature of a bidder's cost function using the estimates obtained from the first step. Finally, I combine the estimates from the previous two steps with bidders' first-order optimality conditions to identify the distribution functions of bidders' costs nonparametrically.

The estimated model predicts that uncertain design evaluation raises uncertainty in the auction outcomes from the auctioneer's standpoint. A large amount of uncertainty in design evaluation implies that winner selection is heavily influenced by luck. Non-competitive bidders, characterized by a high PQR bid, benefit from noisy evaluations since the bidders would otherwise lose in the absence of reviewers' subjective judgments. Therefore, non-competitive bidders tend to become less competitive, and enjoy a higher payoff upon winning. For the same reason, competitive bidders are less likely to win due to increased evaluation uncertainty, and therefore competitive bidders tend to become more competitive to ensure they win. Consequently, greater bidders' uncertainty in evaluations leads to greater uncertainty for the auctioneer in the winning price and winning design quality. As a rise in a PQR bid implies that each unit of design quality is sold at a higher price, a bidder strategically substitutes its price offer for a higher design quality regardless of its type. Since a higher quality design costs the bidder more, the bidder offers its design at a higher price.

A large amount of uncertainty in auction outcomes may not be desirable from the auctioneer's standpoint. A low winning price may raise a chance of bankruptcy during the implementation of the project while a high winning price may make the procurement unaffordable if the auctioneer is budget constrained. In light of these issues, I propose a second-price auction in which a bidder's design score determines the transfer amount, shutting down the effect of evaluation uncertainty on bidders' behaviour. The alternative mechanism is simple and dominant strategy-implementable. Based on the estimated model, I find that the alternative mechanism reduces the auctioneer's uncertainty in both the amount paid and the quality of the winning design by 40% and 60%, respectively.[8]

This paper has an important implication for the market for customized products. Uncertain design

[7]In my sample, most of reviewers show up only once or twice in ten years and reviewers are not experienced, which partially explains the discrepancy in reviewers' evaluations. This observation is intriguing since the FDOT could reduce evaluation uncertainty by training reviewers through on-the-job training. Non-repeated use of reviewers could be rationalized as a precautionary measure against corruption between reviewers and bidders.

[8]The auctioneer's uncertainty is measured by the variances of the winning price and the winning design quality.

evaluations may lead to greater uncertainty for the client in the outcomes of design competitions due to the endogenous response of suppliers. The client can be more certain about the attributes of the end-product by committing to select the contractor based on an objective measure (e.g., price), and not letting his subjective judgment distort suppliers' behaviour.

This paper builds on and contributes to two main literatures: the literature on multi-attribute auctions, and the literature on the structural estimation of auction models. DB auctions are a particular type of a multi-attribute auction, also known as a scoring auction, in which bidders compete in attributes of an auctioned product. While first-price low-bid auctions rank bidders solely based on price, scoring auctions select the winner based on several attributes of a bid; the DB auction is a type of scoring auction where the winner is selected based on PQR.[9] Krasnokutskaya, Song, and Tang (2012) empirically investigate an auction environment in which the attributes-based winner selection rule is unknown to bidders.[10] In this paper, I argue that disclosing information about a buyer's preference may not be straightforward, and uncertainty associated with subjective evaluation is likely to remain after disclosing the selection rule whenever a design score is used as a means of selecting the winner.[11]

From a technical point of view, I add to the growing literature on multi-attribute auctions by developing an estimation approach in the case where some attributes are unobserved, precluding the standard inversion approach pioneered by Guerre, Perrigne, and Vuong (2000). As the econometrician observes only a few noisy quality signals for each design proposal, the actual design decision of each bidder cannot be identified from the data. Thus, I attempt to estimate structural parameters of the model by exploiting the distribution of bidders' design choices. I also take into account other relevant issues in estimation. Unobserved auction heterogeneity, observed by auction participants but unobserved by the econometrician, is particularly relevant in the context of the analysis here since the effect of evaluation uncertainty is local.[12] As emphasized in Krasnokutskaya (2011), ignoring the existence of unobserved auction heterogeneity overemphasizes the dispersion in the private information of bidders, which in turn leads the effect of uncertainty in design evaluation on bidders' behaviour to

[9] The literature on scoring auctions includes Athey and Levin (2001), Athey and Nekipelov (2012), Bajari *et al.* (2006), Bajari and Lewis (2011), Che (1993), Hanazono *et al.* (2012), and Nakabayashi and Hirose (2013), among others. There has been an extensive literature on structural estimation of first-price auction models, including Guerre, Perrigne, and Vuong (2000), Hendricks and Porter (1988), Laffont and Vuong (1993), Paarsch (1992), and Porter (1995).

[10] Krasnokutskaya, Song, and Tang (2012) make a distinction between multi-attribute auctions and scoring auctions based on whether the auctioneer's taste is observed or not. In this paper, I do not make this distinction and treat them synonymously.

[11] Nakabayashi and Hirose (2013) consider public procurement auctions in Japan in which quality is verifiable. However, many state departments of transportation assign reviewers to evaluate design proposals.

[12] i.e., reviewers' preference heterogeneity can switch the rankings of design proposals only when bidders' design proposals are close to each other in true design quality.

be underestimated.[13] Computation of an equilibrium is also challenging with evaluation uncertainty since uncertain evaluations preclude expressing bidders' probabilities of winning as a function of the primitives of the model, which is a common approach in the auction literature. To overcome this computational difficulty, I apply a Homotopy-based approach.

The rest of the paper is organized as follows. Section 2 provides institutional details about the DB auction process, and describes the data. Section 3 develops a structural model and derives comparative statics results. Section 4 describes the structural estimation procedure, and Section 5 presents the estimation results. Section 6 demonstrates the economic significance of the degree of horizontal reviewer heterogeneity in the data with and without bidders' endogenous responses to a change in auction environment. Section 7 concludes.

2 Institutional Details and Data

2.1 Design-Build Procurement Auction

Here I describe DB procurement process in detail, and explain some institutional details that become important from structural modeling perspective. Some questions to be answered in this section include: (i) Who are the reviewers and how are they selected? (ii) What are the necessary pre-qualification requirements to become a "bidder" in a DB auction? (iii) Do bidders know who would be reviewing their designs ex-ante? (iv) Would bidders have incentives to lobby reviewers to win auctions?

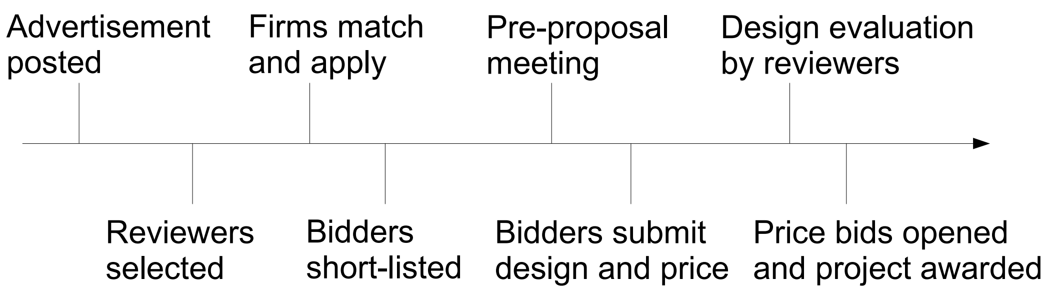
DB procedure can be decomposed into two consecutive stages, a pre-selection stage and a bidding stage. In the pre-selection stage of a DB procurement, the FDOT posts an advertisement on-line which lists information about the project location, description of work, criteria for evaluating a letter of interest, and technical qualification requirements. Then, reviewers are selected from a pool of the FDOT employees by a department secretary based on qualifications and availability. Meanwhile, an interested builder and a designer matches to form a DB firm.[14] The DB firm then writes a letter of interest to the FDOT. The appointed reviewers then evaluate the letter of interest based on the criteria described in the advertisement, which include past performance grades of builders and designers, DB experience, and current capacity of builders. Those DB firms that are judged as pre-qualified by

[13] I also allow bidders to have private information in variable cost and fixed cost of production to rationalize the observed bids' distribution as discussed in Asker and Cantillon (2008). Under PQR selection rule, a bidder's cost function cannot be linear in true design quality as linear cost function implies a non-smooth substitution between its price and design decisions. Thus, I allow for non-linearity in the cost function of bidders and estimate the curvature.

[14] A DB firm usually consists of a builder sub-contracting with a designer.

reviewers are short-listed and become a "bidder". There is no specific rule as to how many DB firms should be short-listed, and the number of short-listed bidders ranges from 2 to 5 in the sample.[15] The identities of these bidders are posted on-line and become common knowledge. The bidders then receive the request for proposal, which describes detailed specification of the project and design evaluation criteria.[16] Following the pre-selection stage, the bidders now enter the bidding stage. All the bidders and the reviewers meet in a mandatory pre-proposal meeting in which the reviewers provide proposal instructions and the scope of the project. Both design and price bids are usually due 1 to 2 months after the pre-proposal meeting, and the design and price bids need be sent to the FDOT in separate envelopes. Next, the reviewers independently evaluate each design proposal, and the quality score of a design is determined by the average reviewer's evaluation. Finally, the price bids are opened to determine the winner of the project based on PQR.

Figure 1: Timeline of Events in a DB Auction



The answers to some of the questions raised at the beginning of this subsection are clear from the above description of events. First, the reviewers are all employees of the FDOT. In addition, the appointed reviewers' compensation are salary based, and not based on each review task. Therefore, it is likely that reviewers' incentive to exert effort in reviewing tasks is not strong, and could potentially explain some of the variation in reviewers' evaluations. Second, past record is an important factor for a DB firm to be pre-qualified as a bidder. From a subset of DB records for which the identities of applicants are available, I find that the number of bidders significantly differs from the number of applicants. This observation is intriguing since if the FDOT simply wants to lower the winning PQR, decreasing competition through removing potential bidders would adversely affect the winning PQR.

[15]There was only one auction with one bidder in the original DB auction records sent by the FDOT, and this auction is not used in any part of the analyses.

[16]Design evaluation criteria vary across auctions. Some repeatedly observed evaluation criteria include warranty, innovative aspect of design, maintenance of traffic, construction methods, commitment to environmental protection, project schedule, etc.

An explanation for this fact is that the FDOT's objective is not only to lower the winning PQR, but also to avoid ex-post default after the contract is made. As is well known, renegotiation is prevalent in government procurement auctions.[17] If the FDOT wishes to avoid costly renegotiation and/or possible ex-post default, then the FDOT may choose to moderate competition through pre-screening process, leaving some rent to the winner of a project.[18] Lastly and most importantly, the bidders do observe the reviewers who evaluate their designs before bidding stage in a pre-proposal meeting. While the presence of pre-proposal meeting could imply that some of uncertainty is resolved ex-ante, it is not clear how much information a bidder possesses about the reviewers at the time of bidding. Knowing the identities of reviewers is meaningful to a bidder only if some pattern or tendency can be inferred from the reviewers' identities. The sample of DB projects shows that the majority of reviewers are appointed only once in a decade and thus, the bidders are unlikely to make an inference about reviewers' characteristics from their past evaluations.

Table 1: Number of Times Bidders and Reviewers Participate in Auctions

Participant	Mean	Std	Min	Max
Builder	3.61	3.68	1	19
Designer	3.05	3.23	1	18
Reviewer	1.68	1.68	1	16

[a] The sample contains 110 DB auctions procured between years 2000 and 2011.
[b] In total, there are 53 builders, 64 designers, and 250 reviewers in the sample.

The pre-proposal meeting also casts a doubt on bidders' incentives to lobby reviewers. Table 1 shows how frequently a particular reviewer is observed in the sample. On average, a reviewer shows up in less than two auctions and most reviewers show up only once in a decade. Thus, bidders play a one-shot game rather than a repeated game if they are to connect to a particular reviewer. While the one-shot nature of the game may not completely eliminate bidders' incentive to lobby reviewers, its scope can be significantly limited by reducing the benefit of establishing a reviewer specific connection. Further, it is difficult for a reviewer to ensure winning of a particular bidder in a DB auction. While the reviewer could increase the winning probability of a particular bidder by enlarging the gap in quality scores between the bidder and others, the reviewer cannot ensure that the bidder will win since he/she does not know how other reviewers will evaluate the design and what the price bids are.

[17] Theoretical analyses of procurement auctions with ex-post bankruptcy is found in Board (2007).
[18] For the analysis of incomplete contract in public procurement projects, see Bajari, Houghton, and Tadelis (2006).

In short, the reviewers from the FDOT have a weak incentive to exert effort, and have little experience in evaluating design proposals in DB auctions. Past performance is an important determinant that makes a DB firm pre-qualified, and the FDOT's objective is unlikely achieving the lowest possible PQR. While bidders meet with reviewers before deciding on their bids, knowing the identities of reviewers is unlikely to reduce uncertainty in reviewers' evaluations.

2.2 Data

I investigate a sample of DB auctions that took place between years 2000 and 2011 in Florida. Although DB is also a common practice in other states (e.g., Alaska, Pennsylvania, Minnesota, etc.), scoring rules and point systems differ across these state departments of transportation. Therefore, a single department of transportation, the FDOT, is chosen for consistency and auction record availability. In particular, records on design evaluations in other state departments of transportation are often aggregated and not preserved at the individual reviewer level.

The sample of DB auctions used in the analysis here is a result of selecting a subset of original DB auctions, and the selection procedure is as follows. First, I requested the FDOT for the records of all DB auctions that have been procured between years 2000 and 2011. The FDOT provided 152 auction records. Second, I manually compiled a dataset from the provided records, and all the auction records with only 1 bidder, or missing engineer's estimate of project cost, or with modified scoring rule, or missing reviewers' evaluations are excluded from the dataset. An engineer's estimate of project cost is an important control for project size heterogeneity across auctions in the data. Moreover, the original dataset contained a variant of DB auctions (DB' auctions) in which the scoring rule involves a time incentive component.[19] I do not consider DB' auctions here to maintain consistency in auction format.[20] Those auctions with no individual reviewer's evaluations are also excluded since the variation in reviewer level scores is the focus of the paper. The selected sample is complemented by bidder characteristics, which I obtained through web-scraping.

In total, 42 auctions (27% of the original auction records) are excluded from the sample and not used in any part of the data analysis. Consequently, I am left with 110 DB auctions with detailed information on design evaluations. Out of the excluded records, 28 auctions are removed for having DB' rule, 11 auctions are removed for not showing engineers' estimates, 2 auctions are removed for not showing individual reviewer level evaluation scores, and 1 auction is removed for having only one

[19]The variant of DB auctions is a combination of DB and A+B auction studied in Bajari and Lewis (2011).

[20]It turns out that DB' auctions have a larger horizontal reviewer heterogeneity than DB auctions on average, suggesting a selection of auction format by the FDOT.

participating bidder. I test whether the selected sample systematically differs from that of the entire raw sample in the distribution of quality scores. More specifically, I test the difference in the means of the removed sample and the selected sample. I find no evidence that quality scores systematically differ across the two sets of sample. While selection problems could still exist, I find no systematic differences in the distributions of quality scores.

Figure 2: Summary of Evaluation Scores on E7E10 Barge Canal Bridge Design Build Project

Team / Evaluator	Max Allowed	Cone & Graham / Jacobs MS	DH	JD	DK	LC	Category Total Total Possible	Johnson Bros. / GAI MS	DH	JD	DK	LC	Category Total Total Possible	PCL / HDR MS	DH	JD	DK	LC	Category Total Total Possible
Environmental Protection/ Committments	9	7	8	6	8	7	36 of 45	7	6	8	7	8	36 of 45	7	7	8	6	7	35 of 45
Maintainability	14	12	10	11	13	13	59 of 70	12	8	10	11	12	53 of 70	11	9	5	11	10	46 of 70
Schedule	9	8	6	8	8	8	38 of 45	7	8	6	5	7	33 of 45	8	7	9	9	8	41 of 45
Coordination	9	7	7	8	8	9	39 of 45	7	6	8	8	8	37 of 45	7	5	8	8	8	36 of 45
Quality Management Plan	9	7	7	5	9	9	37 of 45	6	6	6	5	6	29 of 45	6	8	4	8	7	33 of 45
Maintenance of Traffic	4	3	4	2	3	4	16 of 20	3	4	2	2	3	14 of 20	3	4	2	4	3	16 of 20
Aesthetics	4	3	4	3	3	4	17 of 20	3	2	3	3	4	15 of 20	3	2	2	2	3	12 of 20
Design and Geotech Svcs Investigation	14	12	10	6	14	13	55 of 70	12	10	10	9	13	54 of 70	11	8	6	12	11	48 of 70
Construction Methods	14	12	11	7	13	12	55 of 70	12	9	12	11	12	56 of 70	12	10	9	14	11	56 of 70
Permit Acquisition	14	12	11	12	13	13	61 of 70	12	10	13	12	13	60 of 70	12	11	13	13	12	61 of 70
Total Score	100	83	78	68	92	92	**413 of 500**	81	69	78	73	86	**387 of 500**	80	71	66	87	80	**384 of 500**
Ordinal		1	1	2	1	1		2	3	1	3	2		3	2	3	2	3	

Evaluators:
MS = Margaret Smith
DH = Dave Hoover
JD = Jose Danon
DK= Dwayne Kile
LC = Lynda Crescentini

Figure 2 is an actual record of design evaluations for a bridge construction project, and is one of the DB auctions with a large spread in design evaluations across reviewers in the sample. The first and second rows of the table shows the identity of 3 bidders and 5 reviewers (or evaluators), respectively. The first and second columns show 10 evaluation categories and weights. Each reviewer independently reviews each quality aspect of a design proposal, and assigns a score out of the category specific maximum score. Then, these scores are summed across all categories to obtain the total score of a design proposal, which is what I define as a reviewer's evaluation. These total scores are averaged across reviewers to determine the quality score of a bidder's design proposal. The three bidders are ranked by their PQR, and the bidder with the lowest PQR wins the project. A large variation in reviewers' evaluations can be easily verified in this auction. For example, the difference in evaluation scores assigned by JD and DK is 24 points for Cone & Graham/Jacob, which is 24% of the maximum allowable points. Also, JD ranks Cone & Graham/Jacob fifth and Johnson Bros./GAI third, while DK

Table 2: Summary Statistics of Key Variables

	Mean	Std	Min	Max	Obs
Winning PQR ($1,000 / score point)	193	266	3.125	1125	110
Winning Price ($1,000,000)	16.6	22.8	0.253	103	110
Winning Quality Score (score point)	86.3	5.57	69.7	95.5	110
# Bidders (# of bidders / auction)	3.12	0.534	2	5	110
# Reviewers (# of reviewers / auction)	3.82	0.800	3	6	110
PQR ($1,000 / score point)	249	346	3.12	1945	338
Price ($1,000,000)	20.9	28.6	0.253	142	338
Quality Score (score point)	84.4	6.17	63.3	97.0	338
Reviewer's Evaluation (score point)	84.3	8.19	38.6	100	1296

The summary statistics is calculated based on 110 DB auctions procured between years 2000 and 2011.

ranks Cone & Graham/Jacob first and Johnson Bros./GAI fourth.

Table 2 shows the summary statistics of the key variables. Prices are adjusted for inflation and expressed in 2011 USD. The average winning price is more than 16 million USD. Considering the fact that the average winning price in usual first-price low-bid auction is 7.4 million USD in Florida, DB auctions seem to be adopted for relatively large scale projects. A quality score is the average of quality scores across reviewers' evaluations, which is the weighted sum of category level scores.[21] Since the maximum quality scores vary across auctions, every quality score is standardized by its maximum possible score, and expressed out of 100 points.[22]

Table 3 shows how many winners received the non-highest design quality score, and how many winners bid the non-lowest price. It is readily seen that neither lowest price bidder nor highest quality score bidder always win in a DB auction. Indeed, the majority of the winners in DB auctions do not receive the highest quality score. In order to gain a sense about how much variation exists in price bids and reviewers' evaluations, consider the following simple decomposition of variance in the natural logarithm of price, reviewer's evaluation, and price per reviewer's evaluation. Table 4 shows the decomposition of variance in the above three variables into between-auction, within-auction-between-bidder, and within-bidder-between-reviewer. As prices do not vary within-bidders, within-bidder-between-reviewer variation in price per reviewer's evaluation is entirely driven by the variation in reviewers' evaluations.

[21] The weight for each category level score is the maximum cap on that category. Each evaluation category has an auction specific maximum cap.

[22] Note that rescaling of quality scores does not introduce any problem to my analysis here as the winner selection rule is price per quality score which is scale invariant.

Table 3: Distribution of Winning Price and Quality Score

	Lowest Price	Non-Lowest Price	Total
Highest Quality Score	38	19	57
	(34.5%)	(17.2%)	(51.8%)
Non-Highest Quality Score	51	2	53
	(46.3%)	(1.8%)	(48.1%)
Total	89	21	110
	(80.9%)	(19.1%)	(100%)

The above calculation is based on 110 DB auctions procured between years 2000 and 2011.

As the sample contains projects of different sizes, most of the variation in price per reviewer's evaluation is driven by between-auction variation. Conversely, the rest of within-auction variations are comparable in size. Note here that within-bidder-between-reviewer variation in reviewers' evaluations accounts for 69% of its total variance. Also, 17% of within-auction variation in price per reviewer's evaluation is explained by within-bidder-between-reviewer variation in evaluations.

Table 4: Variance Decomposition of Price Bids and Reviewers' Evaluations

Variables	Between-Auction	Within-Auction Between-Bidder	Within-Bidder Between-Reviewer
ln(Price)	1.43	0.158	
	(0.0971)	(0.00739)	
ln(Reviewer's Evaluation)	0.0389	0.0516	0.0779
	(0.00540)	(0.00400)	(0.00178)
ln(Price per Reviewer's Evaluation)	1.43	0.170	0.0779
	(0.0970)	(0.00842)	(0.00178)
Obs	110	338	1296

[a] Estimates of standard deviations at each level of the hierarchy are presented.
[b] Standard errors in parentheses

3 A Structural Model of a DB Auction

3.1 Model

In order to allow for endogenous response of bidders upon changing the auction environment, I construct a model that endogenizes a bidder's design quality choice and that incorporates noisy reviewers' evaluations of design proposals. Keeping bidders' behavior constant, an increase in evaluation uncertainty "flattens out" probability of winning function. That is, a more competitive bidder (who is characterized by a low PQR) experiences a lower probability of winning while a less competitive bidder (who is characterized by a high PQR) may win with a higher probability. The incentive effect of increased uncertainty on bidders' behaviour is heterogeneous across different types of bidders, and it is not clear what the equilibrium bidding strategies look like.

Descriptive statistics and institutional details provide important information on the structure of the game. First, a request for proposals lists pre-qualified bidders and thus, every bidder is aware of how many bidders participate in a DB auction. Second, every bidder is aware of how many reviewers are involved in a DB auction since every reviewer shows up in the pre-proposal meeting to explain the details of the project. Lastly, the FDOT decides on how many bidders and reviewers to participate in a DB auction, and thus the number of bidders and reviewers may contain information that is observed by the FDOT, but not by the econometrician.

Characterizing an equilibrium of a reasonable model that captures the key features of a DB auction is a difficult task. Auction models in a private value paradigm usually assume single private information, and proceed to characterize its unique equilibrium by forming a differential equation. Multiplicity in private information with non-linear PQR scoring rule and evaluation uncertainty makes the standard differential equation approach infeasible.[23] In particular, the source of difficulty in the characterization of equilibrium comes from the fact that a bidder faces three types of uncertainty in a DB auction: i) A bidder is uncertain about what its opponents' bids are, ii) how its design proposal will be evaluated, and iii) how its opponents' designs will be evaluated. Consequently, a bidder's probability of winning function becomes a complex object, and the model here is unable to guarantee uniqueness of equilibria. However, I show that a sensible (isotone pure strategy) equilibrium exists even with a possibly complex bidder's cost structure. As equilibrium uniqueness is not guaranteed, I apply a Homotopy-based approach to stay in a similar equilibrium when conducting my numerical exercise.

[23] Asker and Cantillon (2008) showed that it is possible to collapse multiple private information into a single pseudo-type with quasi-linear scoring rule. Hanazono *et al.* (2012) extends this result to PQR selection rule. However, evaluation uncertainty precludes this approach.

Consider $N \equiv |\mathcal{N}|$ risk neutral bidders in a DB auction where $\mathcal{N}$ denotes the set of bidders in a given auction. For the sake of notational simplicity, I suppress the auction subscript a in this section. Let $\{p_i, q_i^0\} \in \Re_+^2$ be the price bid, and the objective quality of the design proposed by bidder $i \in \mathcal{N}$, respectively. Also, let $b_i \equiv p_i/q_i^0$ be the objective PQR bid of bidder i, which is assumed to be responsive only within the support $[\underline{b}, \overline{b}] \subset \Re_+$, and the government rejects proposals outside the bounds. An interpretation of the boundedness assumption is that the government does not accept a bid that seems too good to be true, and also does not accept a bid that does not meet its cost.

The model introduces multiple private information into a bidder's cost structure to account for complexity in bidding strategies. For example, a bidder consisting of an efficient builder and an inefficient designer may take a low-price-low-quality strategy while a bidder consisting of an inefficient builder with an efficient designer may take a high-price-high-quality strategy, exploiting their comparative advantages. However, these two completely different types of bidders may end up with very similar objective PQR bids, and generating this sort of complexity in bidding strategies is difficult without multiple private cost information in a bidder's cost information. Indeed, the observed distribution of price and reviewers' evaluations in the data is difficult to rationalize without multiplicity in private cost information. Thus, I assume that the cost structure of a bidder consists of a variable cost (vc_i) of providing a quality design, and a fixed cost (fc_i) of delivering the project.

Now, define the ex-post payoff of a bidder i by:

$$\pi_i^{post} \quad = \quad \begin{cases} p_i - vc_i\, C(q_i^0) - fc_i & \text{if win} \\ 0 & \text{otherwise} \end{cases}$$

where $C(.)$ captures the cost of providing a quality project, which is assumed to be differentiable, strictly increasing (i.e., $C^{'}(.) > 0$), and convex (i.e., $C^{''}(.) \geq 0$). $C(.)$ is a part of the bidders' cost structure that is common to all bidders. In addition, $q\, C^{'}(q)/C(q)$ is assumed to be non-decreasing in q. That is, the marginal cost of providing a quality project is increasing faster than the average variable cost. This assumption guarantees the existence of an isotone equilibrium by excluding the possibility that a bidder wins by bidding infinitesimally small price and design quality. Later in specifying the econometric form of the model, I will allow for vc_i and fc_i to be partially observed by bidder i's rivals but I assume these two parameters are entirely private information of bidder i in this section without loss of generality.

Let $c_i \equiv \{vc_i, fc_i\} \in T_i \subset \Re_+^2$ and $\mathbf{c_{-i}}$ denote the vector of bidders' types in the auction excluding bidder i. I use bold cases to refer to a vector with i_{th} element corresponding to bidder i's choice or type (e.g., $\mathbf{b} \equiv [b_1,,,b_i,,,b_N]$, $\mathbf{c_{-i}} \equiv [c_1,,,c_{i-1},c_{i+1},,,c_N]$). Also, let $f(\mathbf{c_{-i}}|\ c_i)$ denote the joint distribution of $\mathbf{c_{-i}}$ conditional on the realization of bidder i's type, which is everywhere differentiable over its compact support. Now, define evaluation noise here as the total amount of subjectivity that a set of review committee introduces into a quality score of bidder i's design proposal. Also, define evaluation uncertainty as the degree of dispersion in evaluation noise. I assume that evaluation noise (w_i) generates realization of a quality score by multiplicatively affecting q_i^0, such that $q_i \equiv q_i^0\, w_i$. Let $F_w(\mathbf{w})$ be the joint distribution function of evaluation noise w_i. Then, define:

$$\begin{aligned} G(b_i|\mathbf{b_{-i}}) &= \int_{\mathbf{w}} \mathbf{1}\{\text{bidder } i \text{is the winner}\} dF_w(\mathbf{w}) \\ &= \int_{\mathbf{w}} \mathbf{1}\left\{\frac{p_i}{q_i^0 w_i} < \frac{p_j}{q_j^0 w_j}\ \forall\, j \neq i\right\} dF_w(\mathbf{w}) \\ &= \int_{\mathbf{w}} \mathbf{1}\left\{ln(b_i) - ln(w_i) < ln(b_j) - ln(w_j)\ \forall\, j \neq i\right\} dF_w(\mathbf{w}) \end{aligned}$$

I show in Appendix that a sufficient condition for the existence of an isotone pure strategy equilibrium is that $G(b_i|\mathbf{b_{-i}})$ be log-supermodular in $\mathbf{b}$, and this assumption is maintained throughout the rest of the paper.[24] Finally, a bidder i's objective function (or equivalently interim expected payoff) is defined as:

$$\pi_i^{int} \equiv \max_{p_i, q_i^0} \quad \tilde{G}_i(b_i)\, \pi_i^{post} \qquad s.t. \quad p_i/q_i^0 = b_i \in [\underline{b}, \overline{b}] \tag{1}$$

where $\tilde{G}_i(b_i) \equiv \int_{\mathbf{c_{-i}}} G(b_i|\mathbf{b_{-i}})\ f(\mathbf{c_{-i}}|c_i)\ d\mathbf{c_{-i}}$ and b_{-i} is a deterministic function of c_{-i}. It is clear that $\tilde{G}_i(b_i)$ depends on the distribution of evaluation noise and bidder i's opponents' strategies. Let $\{p_i^{BR}(c_i), q_i^{BR}(c_i)\}$ be a best response correspondence of bidder i of type c_i with an arbitrary belief about its opponents' strategies. A Bayesian Nash Equilibrium is a state in which every bidder's belief is consistent with best responses of its opponents. A Bayesian Nash Equilibrium is called "Pure" if every bidder's strategy is a deterministic function of its own type.

Definition 1. *A Pure Strategy Bayesian Nash Equilibrium consists of a profile of best response functions* $\{\mathbf{p^{BR}}(\mathbf{c}), \mathbf{q^{BR}}(\mathbf{c})\}$ *in which every bidder* $i \in \mathcal{N}$ *believes that its opponents behave according to* $\{\mathbf{p^{BR}_{-i}}(\mathbf{c_{-i}}), \mathbf{q^{BR}_{-i}}(\mathbf{c_{-i}})\}$.

Before proceeding, I show that this two-dimensional decision problem can be transformed into a

[24]With an assumption that $ln(w_i)$ follows Gumbel distribution, which is imposed later when parametrizing the model, $G(b_i|\mathbf{b_{-i}})$ collapses to Tullock contest success function. See Skaperdas (1996) for the formal axiomatization of contest success functions.

one-dimensional choice problem. Consider the constrained optimization problem in (1). The bidder chooses a price and objective design quality to maximize its expected profit but subject to the constraint that the objective PQR is equal to an arbitrary constant α. This constrained optimization problem has a unique solution and the values of price and objective design quality that solve this problem are given by the following closed form expressions:

$$p_i(\alpha) = \alpha\, C'^{-1}(\alpha/vc_i) \tag{2}$$

$$q_i^0(\alpha) = C'^{-1}(\alpha/vc_i) \tag{3}$$

where $C'^{-1}(.)$ is the inverse function of the derivative of the quality cost function $C(.)$.

Proposition 1. *For any given $b_i = \alpha \in [\underline{b}, \overline{b}]$, there is always a unique pair of $\{p_i, q_i^0\} \in \Re_+^2$ that maximizes i's interim expected payoff.*

Proof: See Appendix. The proposition establishes that p_i and q_i^0 are uniquely pinned down for any feasible b_i. It follows that the problem of a bidder can be rewritten as one-dimensional choice problem, such that:

$$\begin{aligned} &\max_{p_i, q_i^0} \quad \pi_i^{int} \qquad s.t. \quad p_i/q_i^0 = b_i \in [\underline{b}, \overline{b}] \\ = &\max_{b_i \in [\underline{b}, \overline{b}]} \quad \tilde{G}_i(b_i)\,(p_i(b_i) - vc_i\, C(q_i^0(b_i)) - fc_i) \end{aligned}$$

where $p_i(.)$ and $q_i^0(.)$ are the functions defined in (2) and (3), respectively. Let $\boldsymbol{\psi(c)}$ be a pure strategy profile in a Bayesian Nash Equilibrium in which:

$$\psi_i(c_i) = \arg\max_{b_i \in [\underline{b}, \overline{b}]} \quad \tilde{G}_i(b_i)\,(p_i(b_i) - vc_i\, C(q_i^0(b_i)) - fc_i) \qquad \forall \;\; i \in \mathcal{N}$$

Proposition 2. *There exists an isotone pure strategy equilibrium in which $\psi_i(c_i)$ is non-decreasing in each element of c_i $\forall\, i \in \mathcal{N}$.*

Proof: See Appendix. Let $\{\mathbf{p}^{\psi}(\mathbf{c}), \mathbf{q}^{\psi}(\mathbf{c})\}$ be the corresponding price and objective design quality strategy profiles in an isotone pure strategy equilibrium. If bidders' strategies are interior (which I assume for the rest of the paper), the first-order optimality condition from the above one-dimensional choice problem together with the ratio of (2) and (3) gives the following two equations:

$$\psi_i(c_i) = vc_i\, C^{'}(q_i^{\psi}(c_i)) \tag{4}$$

$$p_i^{\psi}(c_i) - vc_i\, C(q_i^{\psi}(c_i)) + \frac{\tilde{G}_i(\psi_i(c_i))}{\tilde{g}_i(\psi_i(c_i))} = fc_i \tag{5}$$

where $\tilde{g}_i(.) \equiv \partial\tilde{G}_i(.)/\partial p_i$. Condition (4) is derived from the ratio of (2) and (3), and determines the choice of price and design quality level for a given choice of objective PQR. Condition (5) is commonly seen in an auction model with private information in which the inverse hazard rate of the probability of winning function is simply the rent that bidder i earns upon winning. An interesting observation in (4) is that every bidder overproduces design quality relative to its efficient scale: the level of design quality that achieves its minimum average cost of production.[25] As a bidder's PQR determines the average revenue of a bidder and is chosen by the bidder, it always selects the level of PQR above its minimum average cost, leading to overproduction of design quality.[26]

Note also that (4) is independent of the distribution of evaluation noise. Therefore, if a bidder becomes less competitive (i.e., higher objective PQR) upon a change in the distribution of evaluation noise, then it will respond by producing a design of higher quality, and there is no way to decrease its price while increasing both its objective PQR and its design quality. Therefore, any change in the distribution of evaluation noise affects price, design quality, and PQR bids in the same direction, though this effect can be either positive or negative. That is, an increase in the degree of evaluation uncertainty may imply either an increase or a decrease in bidders' equilibrium strategies, but equilibrium price and quality choices cannot respond differently upon a change in evaluation uncertainty. If the effect of evaluation uncertainty is positive, then the effect is also positive on price and quality, i.e., the increase in price is greater proportionally than the increase in quality. Similarly, if the effect of evaluation uncertainty on equilibrium scores is negative, then the effect is also negative on price and quality.

Proposition 3. *Let τ be any parameter that affects the winner selection outcome, but does not have any effect on bidders' exogenous costs. Then, sign $\left(\frac{d\,\psi_i(c_i)}{d\,\tau}\right)$ = sign $\left(\frac{d\,p_i^{\psi}(c_i)}{d\,\tau}\right)$ = sign $\left(\frac{d\,q_i^{\psi}(c_i)}{d\,\tau}\right)$ = sign $\left(\frac{d\,\pi_i^{post}}{d\,\tau}\right)$. That is, a change in the distribution of evaluation noise induces positive co-movements in bidders' strategies and ex-post winning payoffs.*

[25] Condition (4) collapses to the average cost minimization condition if and only if $p_i^{\psi} = vc_iC(q_i^{\psi}) + fc_i$, but $p_i^{\psi} \geq vc_iC(q_i^{\psi}) + fc_i$ in any equilibrium as bidders always have a positive probability of winning due to evaluation noise.

[26] One can think of PQR as the price at which each unit of design quality is sold. Note here that the overproduction of design quality is not driven by evaluation noise. The PQR selection rule together with uncertainty in rivals' types are the sources of this prediction.

Proof: See Appendix. This result is intuitive as it suggests a necessary trade-off between the monetary cost and quality in purchasing a customized product in general. For instance, suppose that τ represents the number of participating bidders in an auction. An intense competition may lower the price of the product but it may also deteriorate the quality of the customized product, invoking race to the bottom.[27]

A particular implication of this proposition is when τ represents a measure of uncertainty in reviewers' evaluations. The intuition behind this proposition is straightforward. Suppose that a bidder wants to become less competitive upon an increase in evaluation uncertainty for some reason. An increase in its PQR bid implies that each unit of design quality is sold at a higher price, and therefore the bidder strategically substitute design quality for price, producing a design of higher quality and sell it at a higher price. Lastly, the increase in price per design quality increases its rent since a bidder always prefers a higher price per quality ex-post winning.

The above proposition has some important economic implications. First, there is no ex-post Pareto improvement through a change in the degree of evaluation uncertainty. If a client becomes worse off as a result of the increased uncertainty in design evaluation, then the winner is necessarily better off, and vice versa. Second, uncertainty in design evaluation does not necessarily decrease the level of design quality. Designers may strategically substitute design quality for price (or price for design quality).

3.2 Numerical Exercise

While a theoretical characterization of bidders' equilibrium behavior is difficult, a numerical exercise may shed a light on how bidders' incentives change with evaluation uncertainty. In particular, I illustrate how evaluation uncertainty affects bidding incentives of different types, and show the equilibrium effect of evaluation uncertainty on the distribution of price and design quality bids.[28] In addition, I demonstrate how ignoring multiplicity in private information would be reflected in the bidding strategies.

Throughout this numerical exercise, I assume that the distribution of evaluation noise follows the Type 1 Extreme Value distribution with evaluation uncertainty parameter τ. All the parameter values in this numerical exercise are set equal to the estimates that are obtained from a structural estimation of the model, which I describe later. Figure 3 below illustrates how evaluation uncertainty affects prob-

[27] While not shown in this paper, I find in a numerical exercise that increased competition actually lowers both price and design quality on average.

[28] For every numerical exercise, I use the distribution of estimated private cost information and structural parameters of the model, which I obtain in Section 5.

ability of winning keeping rivals' strategies constant. An increase in evaluation uncertainty flattens out the probability of winning function. In particular, there are two distinct effects of increased randomization in the winner selection process. The first effect is a level effect that is heterogeneous across different types of bidders. Suppose, for example, that there is an exogenous increase in evaluation uncertainty. Non-competitive bidders, characterized by a high PQR bid, expect to win the project with higher probability than before since these bidders have little chance of winning in the absence of reviewers' subjective judgments. Therefore, non-competitive bidders have an incentive to shade their PQR bids and enjoy a higher payoff upon winning. In contrast to non-competitive bidders, competitive bidders experience an exogenous decrease in their likelihood of winning, generating incentive to become more competitive to ensure winning. Therefore, the level effect generates a wider spread in the distribution of PQR bids and associated choice of price and design quality bids. The second effect is a slope effect that is common across different types of bidders. Since an increase in evaluation uncertainty lowers the marginal effect of lowering PQR bid on every bidder's probability of winning, the incentive to invest on winning is weakened, and consequently every bidder becomes less competitive. Therefore, increased randomization in winner selection generates a higher PQR bid (and associated choices of price and design quality bid) on average. Figure 2 illustrates the equilibrium effect of evaluation uncertainty on the distribution of price and design quality bids.

Since a bidder's strategy is a function of multiple private information, it is difficult to illustrate the above mentioned relationships in two-dimensional graphs. In order to isolate the effect of one cost component from the other, and to obtain ceteris paribus interpretation, the following numerical exercise is considered. First, I completely remove private information on variable costs, such that all bidders are aware of their opponents' degenerate variable costs. Second, I compute a monotone pure strategy equilibrium using a Homotopy-based approach. Lastly, the equilibrium strategies of different fixed cost types are plotted against the degree of evaluation uncertainty. By the same analogy, I conduct an exercise on variable costs by removing private information on fixed costs. The results of the numerical comparative statics results are shown in Figure 5.

Figure 3: The Effects of Uncertain Design Evaluation on Probability of Winning $\tilde{G}_i(b_i)$

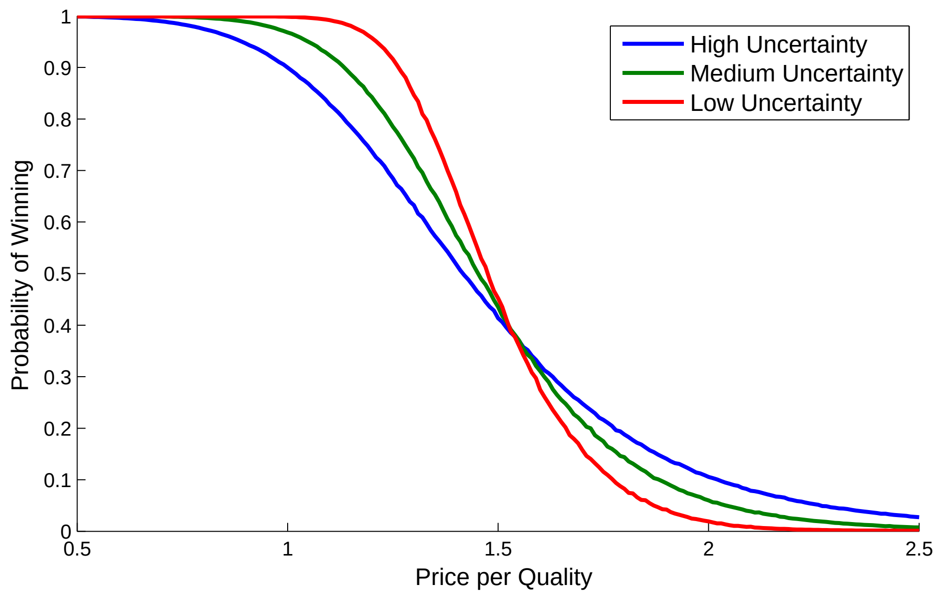
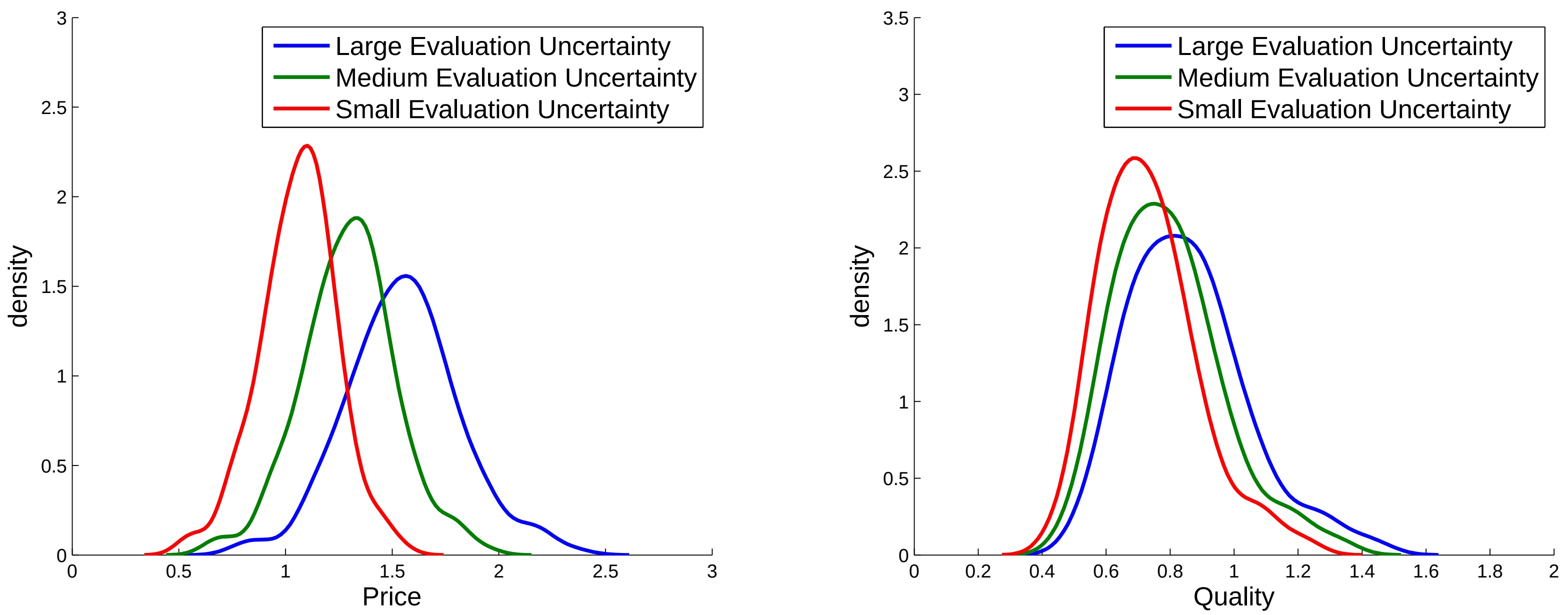


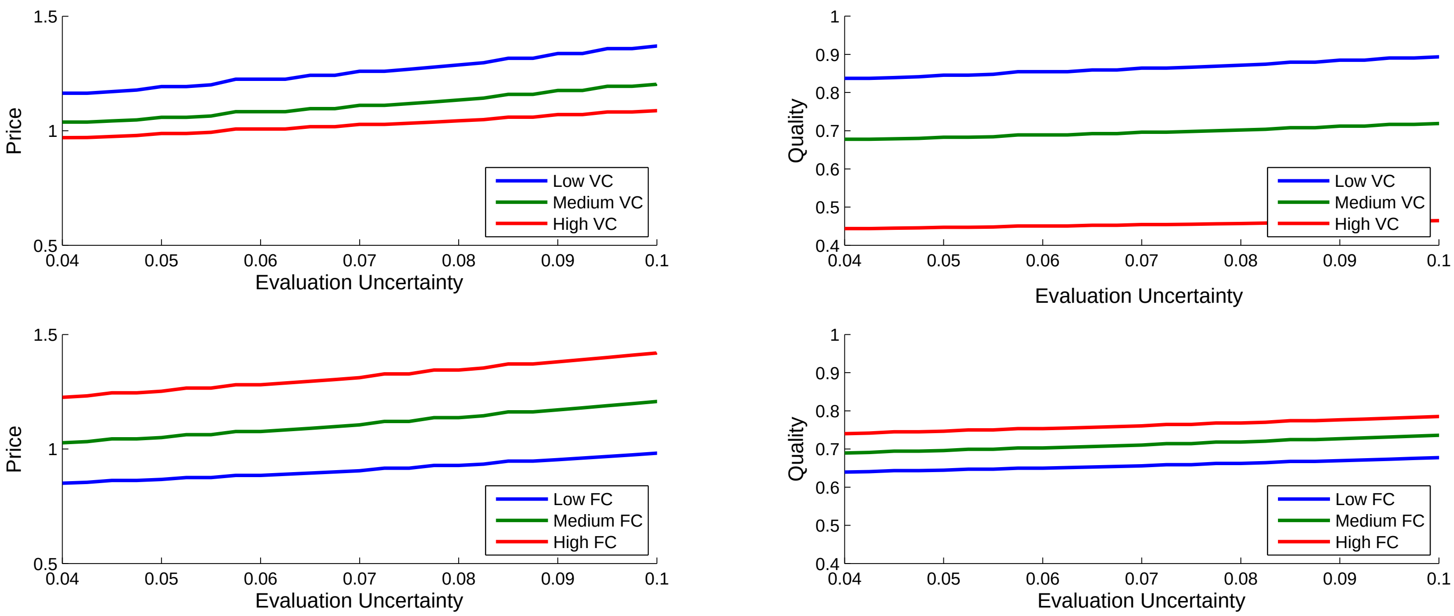
Figure 4: The Effects of Uncertain Design Evaluation on Equilibrium Distribution of Price and Design Quality Bids



The numerical exercise shows that bidders' strategies are monotonically increasing in evaluation uncertainty, indicating that evaluation uncertainty is costly monetary wise, but improves design quality.[29] Further, differences in bidders' equilibrium strategies are noticeable depending on how private information enters in a bidder's cost structure. For example, in the absence of private information on a bidder's variable cost, an efficient bidder (a bidder with low fixed cost private information) bids a lower price with lower design quality than inefficient bidders. Conversely, in the absence of private information on a bidder's fixed cost, an efficient bidder (a bidder with low variable cost private information) bids a higher price with higher design quality than inefficient bidders. Yet, price per quality is sorted in the order of efficiency levels in both scenarios. Another conspicuous difference is in the dispersion in strategies. More specifically, bidders compete in price when private information is in fixed costs while bidders compete in quality when private information is in variable costs. The numerical exercise indicates that bidders' strategies could differ substantially depending on their comparative advantages. That is, two bidders of same cost efficiency may achieve the same level of price per quality with differing pricing and designing strategies. If a bidder has a low fixed cost and a high variable cost, then its strategy is to bid a low price with low design quality as quality becomes expensive quickly. Conversely, if a bidder has a comparative advantage in variable cost, then it may exercise a high price and high quality strategy. The numerical exercise also reveals the importance of allowing for multiple private information in bidders' cost structure. In a single private cost information case, which corresponds to either one of the above scenarios, I would observe a strong positive correlation between price and design quality bids, which is not what I observe in the data. Thus, a reasonable model of DB auction needs to account for the low correlation between price and design quality bids in the data by introducing another source of variation, such as multiple private information.

[29] The picture does not change qualitatively when changing the values of common knowledge components of bidders' variable and fixed costs.

Figure 5: Equilibrium Bidding Strategies with Single Private Information



4 Estimation

Estimation of the model is challenging due to unobserved design decisions The econometrician only observes some noisy signals of design quality, i.e., reviewers' evaluations. Since pointwise-identification of the objective quality of a design proposal is infeasible, the probability of winning function $\tilde{G}$ (and also $\tilde{g}$) cannot be evaluated directly, precluding the standard inversion approach pioneered by Guerre, Perrigne, and Vuong (2000). Further, the estimation of quality cost function, $C(.)$, has to deal with endogeneity problem in the first-order optimality condition (4) without observing the actual design choices.

As is demonstrated in Krasnokutskaya (2011), public procurement auctions of infrastructure projects involve substantial amount of unobserved auction heterogeneity: auction characteristics that are observed to all participating bidders but unobserved to the econometrician. Ignoring the presence of unobserved auction heterogeneity exaggerates the dispersion in bidders' private information, which in turn underestimates the impact of evaluation uncertainty on bidders' behaviour. Evaluation noise affects the rankings of design proposals only if bidders' design proposals are similar in objective design quality. In other words, if two bidders clearly differ in the level of private cost information, then luck may not influence the winner of the auction. Asker (2010), Krasnokutskaya (2011), and Bajari and

Lewis (2011) have dealt with this problem in the context of first-price sealed-bid auctions, but it has yet to be extended to the case of multi-attribute auctions.

The rest of this section is organized as follows. First, I specify the model in an econometric form, and describe some assumptions needed to estimate the model. Second, I describe how I approximate the equilibrium strategies of bidders in a flexible functional form model taking into account a bidder's substition between price and design choices, which I call reduced-form (RF) factor model. Finally, using the estimates obtained from the RF factor model, I describe how I back out the structural parameters and the distribution of bidders' private information in the presence of unobserved auction heterogeneity.

4.1 Econometric Specification of Structural Model

The econometrician observes a random sample of A auctions, indexed by a, with the following information for each auction: price, $\{p_{ia} : i = 1, 2, , , N_a\}$, where i is the bidder subindex, and N_a is the number of bidders in auction a.[30] Each individual reviewer's evaluation of each bidder's design proposal, $\{q_{ria} : r = 1, 2, ..., R_a\}$, where r is the reviewer subindex, and R_a is the number of reviewers in auction a. There is also information about exogenous auction and bidder characteristics, Z_{ia}, which includes the engineer's estimate of the project cost, bidder i's capacity utilization, and the distance between project worksite and bidder i's nearest branch. It is assumed that these data have been generated from the equilibrium of the model presented in Section 3. I use these data to estimate the structural parameters/functions of the model: the probability distribution of costs c_{ia}, the cost function $C(.)$, and the probability distribution of reviewers' noise F_w. This subsection describes some assumptions on these structural functions for the estimation of the model.

The first assumption decomposes a bidder's variable and fixed costs of production into three components based on who possesses what information. While the theoretical model in Section 3 abstracts from this information asymmetry among bidders and the econometrician without loss of generality, empirical results can be significantly influenced by how a bidder's cost is decomposed. Assumption 1 below makes clear who observes which component of a particular bidder's cost.

Assumption 1. The variable and fixed costs of a bidder have the following form:

$$vc_{ia} = \exp\{Z_{ia}\beta_v + \theta_a^v + \varepsilon_{ia}^v\} \tag{6}$$

$$fc_{ia} = \exp\{Z_{ia}\beta_f + \theta_a^f + \varepsilon_{ia}^f\} \tag{7}$$

[30] As there is a large amount of heterogeneity in project size across auction, I normalize price bids by the engineer's estimate of project cost for each auction. Thus, p_{ia} denotes normalized price.

where Z_{ia} is a vector of auction and bidder characteristics which are observed to all bidders and the econometrician. β_v and β_f are parameters. $\theta_a = \{\theta_a^v, \theta_a^f\}$ are unobserved auction heterogeneity, observed by all participating bidders but unobserved to the econometrician, that are independently and identically distributed across auctions. $\varepsilon_{ia} = \{\varepsilon_{ia}^v, \varepsilon_{ia}^f\}$ are private information observed only by bidder i, independently and identically distributed across bidders and auctions. θ_a is a part of a bidder's cost component that is shared by all the bidders in the auction, and may capture project complexity that raises the cost of implementing the project, which is discovered by all the bidders in the auction as a result of preliminary investigation. ε_{ia} captures the (in)efficiency of bidder i. For example, ε_{ia} may capture an innovative idea that lowers the cost of delivering the project and/or the goodness of match between the builder and the designer. Under Assumption 1, the first-order optimality conditions (4) and (5) can be rewritten as follows:

$$\begin{aligned} ln(p_{ia}/q_{ia}^0) &= ln(C^{'}(q_{ia}^0)) + Z_{ia}\beta_v + \theta_a^v + \varepsilon_{ia}^v & (8) \\ p_{ia} - vc_{ia}C(q_{ia}^0) + \frac{\tilde{G}_{ia}(p_{ia}/q_{ia}^0)}{\tilde{g}_{ia}(p_{ia}/q_{ia}^0)} &= \exp\{Z_{ia}\beta_f + \theta_a^f + \varepsilon_{ia}^f\} & (9) \end{aligned}$$

Given small sample size, I adopt a semi-parametric approach to estimate the structural model. Assumption 2 parametrizes the distribution function of evaluation noise w_{ia}, and the shape of the quality cost function $C(.)$.

Assumption 2. Reviewers' noise, $ln(w_{ia})$ has the Type 1 Extreme Value distribution with dispersion parameter $\tau(R)$ and is independently distributed across bidders. The cost function $C(.)$ is a power function q^γ where $\gamma > 1$.

Under Assumptions 1 and 2, the first-order optimality conditions (8) and (9) simplify to the following form:

$$\begin{aligned} ln(p_{ia}) - \gamma\, ln(q_{ia}^0) &= ln(\gamma) + Z_{ia}\beta_v + \theta_a^v + \varepsilon_{ia}^v & (10) \\ (1 - 1/\gamma)\, p_{ia} + \frac{\tilde{G}_{ia}(p_{ia}/q_{ia}^0)}{\tilde{g}_{ia}(p_{ia}/q_{ia}^0)} &= \exp\{Z_{ia}\beta_f + \theta_a^f + \varepsilon_{ia}^f\} & (11) \end{aligned}$$

4.2 Separating the Sources of Variation in Equilibrium Strategies

Assumptions 1 and 2 have an implication for the estimation of the structural model, which is developed in the following Proposition 4. Let $\boldsymbol{X}_a$ represent the vector of auction characteristics and all participating bidders' characteristics. Let $\boldsymbol{\varepsilon_a}$ be the vector of all participating bidders' private information. The equilibrium strategies of bidders are functions of the vectors $(\boldsymbol{X_a}, \boldsymbol{\varepsilon_a})$ and unobserved auction heterogeneity θ_a. Proposition 4 establishes that both θ_a^v and θ_a^f show up multiplicatively in the equilibrium strategies.

Proposition 4. *Consider an isotone pure strategy equilibrium characterized by a set of first-order conditions* (4) *and* (5)*, which are denoted by* $\{\boldsymbol{p(X_a,\theta_a,\varepsilon_a)},\boldsymbol{q(X_a,\theta_a,\varepsilon_a)}\}$*. Then, the equilibrium strategies of bidder* i *can be written as:*

$$p(\boldsymbol{X}_a,\theta_a,\varepsilon_{ia}) \quad = \quad \exp\{\theta_a^f\}\, p(\boldsymbol{X}_a,0,\varepsilon_{ia}) \tag{12}$$

$$q(\boldsymbol{X}_a,\theta_a,\varepsilon_{ia}) \quad = \quad \exp\{\theta_a^f/\gamma-\theta_a^v/\gamma\}\, q(\boldsymbol{X}_a,0,\varepsilon_{ia}) \quad \forall\, i\in\mathcal{N} \tag{13}$$

Proof: See Appendix. Proposition 4 states that both θ_a^v and θ_a^f show up multiplicatively in bidders' strategies.

Using the standard terminology in structural econometrics, the Reduced-Form (RF) equations of this structural model are the equations that specifies the endogenous variables as functions only of the exogenous variables. Proposition 4 has an important implication for the RF equations of the model: the RF equations in logarithm are additively separable in a function of unobserved auction heterogeneity and a function of the rest of the cost components.

Note that the RF equation is given in the form of objective design quality, but the econometrician does not observe this variable, i.e., the econometrician only observes noisy signals of design quality. Assumption 3 establishes a link between noisy signals and objective design quality.

Assumption 3. A reviewer's evaluation of a bidder's design, q_{ria}, is noisy but unbiased measure of true quality q_{ia}^0. More specifically,

$$ln(q_{ria}) \quad = \quad ln(q_{ia}^0)+\zeta_{ria}$$

where the random variable $\zeta_{ria}=\mu_{ra}+u_{ria}$ represents reviewer heterogeneity: the sum of vertical and horizontal, repsectively. Both μ_{ra} and u_{ria} are independently and identically distributed with zero mean and a constant variance. Note that the decomposition of reviewer heterogeneity into vertical and horizontal heterogeneity allows for a positive correlation across bidders' design proposals that arises from reviewer specific characteristics: a reviewer's leniency in evaluating design proposals in general. A lenient (resp. stringent) reviewer may give out a high (resp. low) score to every design proposal due to his/her low (resp. high) quality standard. As opposed to vertical reviewer heterogeneity μ_{ra}, horizontal reviewer heterogeneity u_{ria} may capture a reviewer's preference heterogeneity. For example, some reviewers may prefer a fancy design to a simple design of a bridge.

Assumption 3 states that any disagreement among reviewers about the quality of a design is considered as unknown to bidders, but the part of design quality agreed among reviewers is completely known to bidders at the time of bidding. With this assumption, the first-order optimality conditions (12) and (13) can be rewritten in terms of observed dependent variables:

$$\begin{aligned} ln(p_{ia}) &= \theta_a^f + \alpha_p(Z_{ia}, Z_{-ia}, \varepsilon_{ia}) & (14) \\ ln(q_{ria}) &= \frac{1}{\gamma}\theta_a^f - \frac{1}{\gamma}\theta_a^v + \alpha_q(Z_{ia}, Z_{-ia}, \varepsilon_{ia}) + \zeta_{ria} & (15) \end{aligned}$$

where Z_{-ia} represents the observable characteristics of rival bidders. $\alpha_p(.)$ and $\alpha_q(.)$ are functions that depend on the primitives of the model but do not have a closed form expression because they depend on the equilibrium of the model.

Now, linearize the RF equations $\alpha_p(.)$ and $\alpha_q(.)$ in (14) and (15), such that:

$$\begin{aligned} ln(p_{ia}) &= \boldsymbol{X}_a\alpha_x^p + \theta_a^f + \alpha_v^p\varepsilon_{ia}^v + \alpha_f^p\varepsilon_{ia}^f & (16) \\ ln(q_{ria}) &= \boldsymbol{X}_a\alpha_x^q + \frac{1}{\gamma}\theta_a^f - \frac{1}{\gamma}\theta_a^v + \alpha_v^q\varepsilon_{ia}^v + \alpha_f^q\varepsilon_{ia}^f + \zeta_{ria} & (17) \end{aligned}$$

where $\boldsymbol{X}_a$ is the vector of observables (Z_{ia}, Z_{-ia}), and α_x^p, α_v^p, α_f^p, α_x^q, α_v^q, and α_f^q are parameters which are (unknown) functions of the structural parameters of the model. Note here that additive separability of unobserved auction heterogeneity θ_a from the rest of the cost components is predicted by the model and not a result of approximation. Further, the variation in price that is caused by unobserved auction heterogeneity is necessarily due to θ_a^f because θ_a^v does not enter (16). Moreover, additive separability of ζ_{ria} also follows from the model, and ζ_{ria} does not enter the price equation as bidders do not know how reviewers evaluate designs. Note also that consistency between the first-order conditions and RF equations imposes the following restrictions on the parameters of RF equations:

$$\begin{aligned} \alpha_v^p &= \gamma\,\alpha_v^q + 1 \\ \alpha_f^p &= \gamma\,\alpha_f^q \\ \beta_v &= \alpha_x^p - \gamma\,\alpha_x^q \end{aligned}$$

However, I am unable to distinguish the sources of variation caused by variable cost private information, ε_{ia}^v, and fixed cost private information, ε_{ia}^f. In addition, the model predicts a strong correlation between price and design quality choices across auctions, which is not supported by the data. Therefore, further assumptions are required to estimate the model.

Assumption 4.1: $\varepsilon^v_{ia} = -(1/\gamma)\, v_{ia}$ where v_{ia} is a latent factor that captures bidder heterogeneity in design quality.

Assumption 4.2: $\theta^v_a = \theta^f_a$, such that unobserved auction heterogeneity is one-dimensional.

Assumption 4.3: Design quality choice is independent of fixed cost private information in RF equation (17).

Under Assumptions 1 - 4, the first-order optimality conditions (10) and (11) collapse to:

$$ln(p_{ia}) - \gamma\, ln(q^0_{ia}) = ln(\gamma) + Z_{ia}\beta_v + \theta^f_a - \frac{v_{ia}}{\gamma} \tag{18}$$

$$(1 - 1/\gamma)\, p_{ia} + \frac{\tilde{G}_{ia}(p_{ia}/q^0_{ia})}{\tilde{g}_{ia}(p_{ia}/q^0_{ia})} = \exp\{Z_{ia}\beta_f + \theta^f_a + \varepsilon^f_{ia}\} \tag{19}$$

and the associated RF equations for the price and measured design quality in logarithm are:

$$ln(p_{ia}) = \boldsymbol{X}_a\beta_p + \eta_{p1}\, z_a + \eta_{p2}\, v_{ia} + \theta^f_a + e_{ia} \tag{20}$$

$$ln(q_{ria}) = \boldsymbol{X}_a\beta_q + \eta_{q1}\, z_a + \eta_{q2}\, v_{ia} + \mu_{ra} + u_{ria} \tag{21}$$

where $e_{ia} = \alpha^p_f \varepsilon^f_{ia}$, $\eta_{p2} = -(\alpha^q_v + \frac{1}{\gamma})$, and $\eta_{q2} = \frac{\alpha^q_v \alpha^q_f}{\gamma}$. z_a and v_{ia} are latent factors that capture the correlation between price and design quality across auction and within auction, respectively.[31] As any factor model requires normalization of latent factor (or corresponding coefficient parameters), I impose $z_a \sim iid(0,1)$ and $v_{ia} \sim iid(0,1)$. I denote the variance of variable j by σ_j. The set of parameters in the RF factor model is denoted by $\Theta \equiv \{\beta_p, \beta_q, \eta_{p1}, \eta_{p2}, \eta_{q1}, \eta_{q2}, \sigma_\mu, \sigma_u, \sigma_\theta, \sigma_e\}$.

Assumption 4.1 is required to estimate the structural parameter γ from the above reduced-factor (RF) model, and Assumption 4.2 is made to be consistent with the fact that there is very little across-auction variation in reviewers' evaluation scores in the data. Assumption 4.2 also allows for simple estimation of the distribution of unobserved auction heterogeneity from the RF factor model. Assumption 4.3 is required to back out the structural parameter γ as a function of factor loadings $\{\eta_{p2}, \eta_{q2}\}$ in (20) and (21).

A key part of the estimation approach here, and one of the contributions of this paper, is in the estimation of the RF factor model (20) and (21) in the first step, and the following subsection describes

[31] Under Assumption 4.2, z_a should not be present in the RF model, but it is there to show little between-auction correlation in price and design quality choices.

how this RF factor model is used to back out the structural parameters of the model. I also discuss identification of the RF factor model in the subsection following the description of the estimation steps.

The roadmap of the estimation steps are as follows. First, I approximate the equilibrium bidding strategies by RF factor model (20) and (21). The coefficients on the latent factors (i.e., factor loadings) capture the correlation between price and design quality bids that arises from bidders' cost minimization condition (18). This first stage also gives an estimate of horizontal reviewer heterogeneity σ_u and the distribution of unobserved auction heterogeneity θ_a^f. Second, using the estimates of factor loadings and the first-order condition (18), I estimate the evaluation uncertainty parameter $\tau(R)$ and the curvature of the quality cost function γ . Lastly, using the estimate of γ and the joint distribution of price and design quality strategies predicted by the RF factor model, I simulate the LHS of (19) to obtain the distribution of private cost information ε_{ia}^f. The following subsection provides a detailed description of each estimation step.

4.3 Structural Estimation

While I simplify the estimation of the distribution of w_{ia}, I do not impose any parametric assumption on the distribution of unobserved auction heterogeneity θ_a^f and private cost information ε_{ia}^f. I estimate the structural parameters $\tau(R)$, γ, the distribution of θ_a^f, and the distribution of ε_{ia}^f in the following three steps.

Step 1: Estimation of Reduced-Form Equations.

I estimate the parameters of the RF factor model Θ by Pseudo Maximum Likelihood with an assumption that all the latent random variables are drawn from a Normal distribution. The Normality assumption does not introduce inconsistency in estimating the variance parameters even if it is violated. See Gourieroux, Monfort, and Trognon (1984) for details. I denote the Maximum Likelihood estimates of Θ by $\hat{\Theta} \equiv \{\hat{\beta}_p, \hat{\beta}_q, \hat{\eta}_{p1}, \hat{\eta}_{p2}, \hat{\eta}_{q1}, \hat{\eta}_{q2}, \hat{\sigma}_\mu, \hat{\sigma}_u, \hat{\sigma}_\theta, \hat{\sigma}_e\}$.

Step 2: Estimation of Structural Parameters, $\tau(R)$ and γ.

The estimation of the factor model does not provide pointwise residual estimates of the reviewers' evaluation noise u_{ria}. However, using my estimate of the variance of u_{ria}, I obtain a consistent estimate of the variance of average evaluation noise. Remember that the average evaluation noise for a design is $w_{ia} = \frac{1}{R_a}\sum \exp\{u_{ria}\}$. Let $\tau(R)$ be the standard deviation of w_{ia} in an auction with R reviewers. I estimate this standard deviation using the following bootstrap method. Let $u'_{rk}: r = 1, 2, \ldots, R;\ k = 1, 2, \ldots, K$ be KR random draws from the estimated distribution of u_{ria}, where K is a very large number (i.e., I use $K = 10^5$). For every replication k, I calculate the mean evaluation noise, $\frac{1}{R}\sum_k \exp\{u'_{rk}\}$. Then, the estimator of $\tau(R)$ is the empirical standard deviation of mean evaluation noise calculated K times, such that:

$$\hat{\tau}(R) \equiv \frac{\sqrt{6}}{\pi}\left(\frac{\sum_k}{K}\left(ln\left(\frac{\sum_r \exp\{u'_{rk}\}}{R}\right)\right)^2\right)^{1/2}$$

Now, consider two distinct types of v_{ia} , and denote them by $v_{ia}^{(1)} > v_{ia}^{(2)}$. For any efficiency type of a bidder, the first-order condition (18) holds, such that:

$$ln(p_{ia}(v_{ia}^{(1)})) - \gamma\, ln(q_{ia}^0(v_{ia}^{(1)})) + \frac{1}{\gamma}\, v_{ia}^{(1)} = B_{ia} \tag{22}$$

$$ln(p_{ia}(v_{ia}^{(2)})) - \gamma\, ln(q_{ia}^0(v_{ia}^{(2)})) + \frac{1}{\gamma}\, v_{ia}^{(2)} = B_{ia} \tag{23}$$

where $p_{ia}(v)$ and $q_{ia}^0(v)$ explicitly indicates the dependency of bids on bidder's efficiency type in equilibrium, and $B_{ia} = ln(\gamma) + Z_{ia}\beta_v + \theta_a^f$ which is independent of v_{ia}. Now, by letting $v_{ia}^{(1)}$ approach $v_{ia}^{(2)}$, I obtain:

$$\frac{d\, ln(p_{ia})}{d\, v_{ia}} - \gamma\, \frac{d\, ln(q_{ia}^0)}{d\, v_{ia}} + \frac{1}{\gamma} = 0 \tag{24}$$

From the RF factor model, I obtain $\frac{dln(p_{ia})}{dv_{ia}} = \eta_{p2}$, and $\frac{dln(q_{ia}^0)}{dv_{ia}} = \frac{dln(q_{ria})}{dv_{ia}} = \eta_{q2}$. Therefore, substituting the counterpart estimates of $\{\eta_{p2}, \eta_{q2}\}$ into (24), I obtain:

$$\hat{\eta}_{p2} - \gamma\, \hat{\eta}_{q2} + \frac{1}{\gamma} = 0$$

Note here that the above approach does not suffer from an endogeneity problem that would arise if price was regressed on objective design quality (even if objective design quality was observed). I exploit the variation in price and design quality that arises from exogenous variation in bidder's variable costs, which is consistently estimated in the form of factor loadings. Thus, γ is identified by tracing out the cost minimization condition of different types. Finally, solving for γ gives the estimate $\hat{\gamma}$, such that:[32]

$$\hat{\gamma} = \frac{\hat{\eta}_{p2} + (\hat{\eta}_{p2}^2 + 4\hat{\eta}_{q2})^{1/2}}{2\hat{\eta}_{q2}}$$

Step 3: Estimation of Fixed Cost Components.

[32] The quadratic equation gives two distinct solutions. However, as $\gamma > 1$ and one of the solutions turn out to be negative, I use the positive solution of γ as my estimate.

The distribution of unobserved auction heterogeneity θ_a^f is estimated directly from the RF factor model (20), and the LHS of equation (11) is simulated using the estimated parameters of the covariance structure without unobserved auction heterogeneity (θ_a^f) to obtain the distribution of fixed cost private information ε_{ia}^f. More specifically, (i) I draw pseudo random variables p_{1k} and q_{1k} from the joint log-normal distribution, such that $[ln(p_{1k}), ln(q_{1k})]' \sim N(\mathbf{0}, \mathbf{\Omega})$ where $\mathbf{\Omega}$ is a 2×2 covariance matrix with elements $\Omega_{11} = \hat{\eta}_{p2}^2 + \hat{\sigma}_e$, $\Omega_{12} = \Omega_{21} = \hat{\eta}_{p2}\hat{\eta}_{q2}$, and $\Omega_{22} = \hat{\eta}_{q2}^2$. Define $b_{1k} \equiv p_{1k}/q_{1k}$. (ii) Numerically integrate over the distribution of opponents' strategies by repeatedly drawing , $\boldsymbol{b_{-1kl}} = [p_{2kl}/q_{2kl}, p_{3kl}/q_{3kl}, , , , , p_{Nkl}/q_{Nkl}]$ L times where $[ln(p_{jkl}), ln(q_{jkl})]' \sim N(\mathbf{0}, \mathbf{\Omega})$ $\forall$ $j \neq 1$ and $\forall$ $l = 1, 2, , , , , L$ where I set $L = 10^5$ to obtain the estimate of the probability of winning, such that:

$$\hat{G}(b_{1k}; N, R) \quad = \quad \frac{\sum_l}{L} \left(\frac{b_{1k}^{-\frac{1}{\hat{\tau}(R)}}}{b_{1k}^{-\frac{1}{\hat{\tau}(R)}} + \sum_{i=2}^{N} b_{ikl}^{-\frac{1}{\hat{\tau}(R)}}} \right)$$

Note that the simulated probability of winning $\hat{G}(b_{1k}; N, R)$ depends on the number of bidders, and the number of reviewers by construction. Similarly, the derivative of $\hat{G}(b_{1kd}; N, R)$ can be obtained by:

$$\hat{g}(b_{1k}; N, R) \quad = \quad -\frac{1}{p_{1k}\,\hat{\tau}(R)} \frac{\sum_l}{L} \left(\frac{b_{1k}^{-\frac{1}{\hat{\tau}(R)}}}{b_{1k}^{-\frac{1}{\hat{\tau}(R)}} + \sum_{i=2}^{N} b_{ikl}^{-\frac{1}{\hat{\tau}(R)}}} \right) \left(1 - \frac{b_{1k}^{-\frac{1}{\hat{\tau}(R)}}}{b_{1k}^{-\frac{1}{\hat{\tau}(R)}} + \sum_{i=2}^{N} b_{ikl}^{-\frac{1}{\hat{\tau}(R)}}} \right)$$

(iii) Obtain the simulated private cost, $\hat{\varepsilon}_{1k}$, by:

$$(1 - 1/\hat{\gamma})p_{1k} + \frac{\hat{G}(b_{1k}; N, R)}{\hat{g}(b_{1k}; N, R)} \quad = \quad \exp\{\hat{\varepsilon}_{1k}\} \tag{25}$$

Iterate through (i) to (iii) K times (where I set $K = 10^5$) to estimate the distribution of $\hat{\varepsilon}_{1k}$ using (25), which I interpret as the distribution of ε_{ia}^f. Note here that the distribution of $\hat{\varepsilon}_{1k}$, denoted by $\hat{F}_\varepsilon(.|N, R)$, depends on the number of bidders and reviewers. Therefore, I compute $\hat{F}_\varepsilon(.|N, R)$ for all possible combinations of N and R in the data.

4.4 Identification of Reduced-Form Factor Model

In order to estimate the degree of uncertainty from a bidder's point of view, I isolate the part of a reviewer's evaluations that bidders knew at the time of bidding from the part they did not. The variation in reviewers' evaluations caused by unknown (resp. known) components is defined as unknown (resp. known) heterogeneity. The RF factor model exploits the fact that bidders do not observe reviewers' evaluations of their designs at the time of bidding, and also reviewers do not observe bidders' price bids at the time of evaluation. In particular, the RF factor model assumes that disagreement among reviewers on the design quality of a proposal is unknown to the bidder, but the part of design quality that is agreed among reviewers is known to the bidder at the time of bidding.

Note here that the notion of design quality here is broad in the sense that the latent factor captures all the information that reviewers have about bidder i at the time of evaluation. That is, if all reviewers *agree* that a particular design proposal is of high quality, then the design is deemed of high quality. Therefore, a bidder's reputation or the impression that reviewers received from a particular bidder in the pre-proposal meeting can be regarded as a part of design quality as long as reviewers agree and a bidder knows what reviewers know about him.

Identification of parameters due to unknown heterogeneity (σ_μ and σ_u), and known heterogeneity (all the other parameters) comes from within-auction variation in price bids and reviewers' evaluations. More specifically, I obtain identification of variance components using the fact that the set of sample variances and covariances of prices and reviewers' evaluations converge (as the number of auctions becomes large) to the corresponding population variances and covariances by the law of large numbers. Define $\tilde{p}_{ia} = ln(p_{ia}) - \boldsymbol{X}_a\beta_p$ and $\tilde{q}_{ria} = ln(q_{ria}) - \boldsymbol{X}_a\beta_q$.

$$\begin{aligned}
&E[\tilde{p}_{ia}^2|\boldsymbol{X}_a] = \eta_{p1}^2 + \eta_{p2}^2 + \sigma_\theta + \sigma_e && E[\tilde{p}_{ia}\,\tilde{p}_{i'a}|\boldsymbol{X}_a] = \eta_{p1}^2 + \sigma_\theta \\
&E[\tilde{p}_{ia}\,\tilde{q}_{ria}|\boldsymbol{X}_a] = \eta_{p1}\,\eta_{q1} + \eta_{p2}\,\eta_{q2} && E[\tilde{p}_{ia}\,\tilde{q}_{ri'a}|\boldsymbol{X}_a] = \eta_{p1}\,\eta_{q1} \\
&E[\tilde{q}_{ria}\,\tilde{q}_{r'ia}|\boldsymbol{X}_a] = \eta_{q1}^2 + \eta_{q2}^2 && E[\tilde{q}_{ria}^2|\boldsymbol{X}_a] = \eta_{q1}^2 + \eta_{q2}^2 + \sigma_\mu + \sigma_u \\
&E[\tilde{q}_{ria}\,\tilde{q}_{ri'a}|\boldsymbol{X}_a] = \eta_{q1}^2 + \sigma_\mu && E[\tilde{q}_{ria}\,\tilde{q}_{r'i'a}|\boldsymbol{X}_a] = \eta_{q1}^2
\end{aligned} \tag{26}$$

Thus, a parameter of the covariance structure is identified if the parameter can be uniquely expressed as a function of the population second moments.

Proposition 5. *The vector of parameters in the RF factor model (20) and (21) is identified from the variances and covariances of the logarithm of price and reviewers' evaluations*

Proof: See Appendix. In order to illustrate how much of the variation in reviewers' evaluations is accounted for by horizontal reviewer heterogeneity, σ_u, I decompose RF quality equation (21) into variance components, such that:

$$Var(\tilde{q}_{ria}) \quad = \quad Var(\boldsymbol{X}_a\beta_q) + Var(\eta_{q1}z_a + \eta_{q2}v_{ia}) + Var(\mu_{ra}) + Var(u_{ria}) \tag{27}$$

Dividing (27) by $Var(\tilde{q}_{ria})$, I have:

$$\begin{aligned} 1 &= \frac{Var(\boldsymbol{X}_a\beta_q)}{Var(\tilde{q}_{ria})} + \frac{Var(\eta_{q1}z_a + \eta_{q2}v_{ia})}{Var(\tilde{q}_{ria})} + \frac{Var(\mu_{ra})}{Var(\tilde{q}_{ria})} + \frac{Var(u_{ria})}{Var(\tilde{q}_{ria})} \\ &\equiv \lambda_x^{(q)} + \lambda_v^{(q)} + \lambda_\mu^{(q)} + \lambda_u^{(q)} \end{aligned}$$

where $\lambda_j^{(q)}$ is the fraction of variance in $\tilde{q}_{ria}$ explained by component $j \in \{x, v, \mu, u\}$. Similarly, I decompose RF price equation (20) by:

$$\begin{aligned} 1 &= \frac{Var(\boldsymbol{X}_a\beta_p)}{Var(\tilde{p}_{ia})} + \frac{Var(\eta_{p1}z_a + \eta_{p2}v_{ia})}{Var(\tilde{p}_{ia})} + \frac{Var(\theta_a^f)}{Var(\tilde{p}_{ia})} + \frac{Var(e_{ia})}{Var(\tilde{p}_{ia})} \\ &\equiv \lambda_x^{(p)} + \lambda_v^{(p)} + \lambda_\theta^{(p)} + \lambda_e^{(p)} \end{aligned}$$

where $\lambda_\theta^{(p)}$ and $\lambda_e^{(p)}$ are the part of variation in price explained by θ_a^f and e_{ia}, respectively.

I estimate the parameters of the factor model $\Theta \equiv \{\beta_p, \beta_q, \eta_{p1}, \eta_{p2}, \eta_{q1}, \eta_{q2}, \sigma_\mu, \sigma_u, \sigma_\theta, \sigma_e\}$, denoted by $\hat{\Theta}$, by Pseudo Maximum Likelihood. See Appendix for estimation steps. While the variance co-variance structure of the factor model is identified without fully parametrizing the joint distribution of latent variables, I do so for the purpose of simplicity and efficiency. In particular, I assume that all the latent variables are normally distributed. The normality assumption does not introduce inconsistency of my estimator as long as latent variables are drawn from one of the linear exponential family of distributions (Gourieroux, Monfort, and Trognon, 1984).

5 Results

5.1 RF Factor Model

Table 5 shows estimates of Θ and λs. I estimate the factor model in logarithm of p_{ia} and q_{ria} in order to be consistent with the structural model. The vector of covariates includes the logarithm of distance between project work site and bidder's nearest branch, and utilization rate which is defined as backlog per capacity.[33] I include the mean of a bidder's rivals' characteristics, such as the logarithm of distance and utilization rate, since rivals' characteristics affect its decision about pricing and designing through strategic interaction. Likewise, I include the number of bidders participating in the auction to capture the effect of competition. Project type dummies controls for observed auction heterogeneity

[33] Backlog of a bidder is calculated as the total dollar value of all projects at hand at the time of bidding. I use all the projects that FDOT procured between years 2000 and 2012 to compute the backlog level of each bidder. Capacity of a bidder is calculated as the maximum backlog of the bidder in the sample.

(e.g., bridge, road, building, etc).

It turns out that the majority of the variation in reviewers' evaluations is explained by vertical reviewer heterogeneity ($\hat{\lambda}_{\mu}^{(q)} \approx 0.39$), and sizable horizontal reviewer heterogeneity ($\hat{\lambda}_{u}^{(q)} \approx 0.18$). Surprisingly, less than 45% ($\hat{\lambda}_{x}^{(q)} + \hat{\lambda}_{v}^{(q)} < 0.45$) of the total variation in reviewers' evaluations is explained by known heterogeneity. That is, excluding vertical reviewer heterogeneity, which preserves design rankings of proposals, more than 28% of the variation in evaluations are due to horizontal reviewer heterogeneity. Further, bidders' decisions about design quality is not strongly correlated with its pricing decision across auctions ($\hat{\eta}_{p1}$ and $\hat{\eta}_{q1}$ are statistically very insignificant). While bidders with low prices tend to bid designs of high quality score (shown in the estimates of $sign(\eta_{p2}) = -sign(\eta_{q2})$) the correlation is small and there seem to be a weak link between pricing and designing decisions. Another noticeable finding is the large estimate of $\lambda_{\theta}^{(p)}$. Approximately, 60% of the variation in price bids are generated by private information that are common across bidders but unknown to reviewers. Observables do not predict either price or reviewers' evaluations well, but the signs and economic significance of coefficients follow one's intuition. A 1% increase in the distance between bidder i's branch and the project worksite increase its price bid by 3%. Also, a one standard deviation increase in its own utilization rate would increase its price by 4.5%. As a longer distance to project worksite and a higher utilization rate are associated with higher cost of implementing a project, the signs (and the magnitudes) make intuitive sense. In addition, one standard deviation increase in the average rival's distance to work project worksite is predicted to increase its price bid 6.8%. Since an increase in its opponents' distance from project worksite makes opponents' uncompetitive, the bidder may want to increase its price to increase its rent. While an increase in a bidder's opponent's distance is negatively correlated with its pricing decision, the coefficient estimate is statistically insignificant at 10% level. Also, most of the covariates turn out to predict reviewers' evaluations imprecisely.

Table 5: Reduced-Form Factor Model: Estimation Results

	$ln(p_{ia})$	$ln(q_{ria})$
Project Size	$-.000721$	$-.0000418$
	(.0011)	(.00018)
ln(Distance)	.0309**	$-.00506$
	(.014)	(.0037)
ln(Rival Distance)	$-.0104$	$-.00733$
	(.024)	(.0056)
Utilization Rate	.115**	$-.0229$
	(.045)	(.011)
Rival Utilization Rate	.174**	.0236
	(.080)	(.016)
# of Bidders	.0353	.000435
	(.059)	(.56)
# of Reviewers	.0590	$-.00624$
	(.038)	(.0047)
$\hat{\eta}_{p1}$, $\hat{\eta}_{q1}$	0.0659	-0.0077
	(0.26)	(0.023)
$\hat{\eta}_{p2}$, $\hat{\eta}_{q2}$	-0.0759	0.0608***
	(0.26)	(0.0032)
$\hat{\sigma}_{\mu}$		0.0040***
		(0.0004)
$\hat{\sigma}_{u}$		0.0019***
		(0.0001)
$\hat{\sigma}_{\theta}$	0.0640**	
	(0.026)	
$\hat{\sigma}_{e}$	0.0179	
	(0.020)	
$\hat{\lambda}_x^{(p)}$, $\hat{\lambda}_x^{(q)}$	0.100	0.0507
$\hat{\lambda}_v^{(p)}$, $\hat{\lambda}_v^{(q)}$	0.0959	0.372
$\hat{\lambda}_\theta^{(p)}$, $\hat{\lambda}_\mu^{(q)}$	0.608	0.392
$\hat{\lambda}_e^{(p)}$, $\hat{\lambda}_u^{(q)}$	0.170	0.186
Project Type FE	Yes	Yes
Obs	338	1296

Standard errors in parentheses.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The variation in μ_{ra} turn out to be large, but the winner selection outcome is not strongly affected by μ_{ra} as the variation in μ_{ra} preserves the rankings of design proposals. Intuitively, a bidder who does not know whether reviewers are lenient or stringent may not care since the bidder knows the same set of reviewers will evaluate its opponents' design proposals. Thus, vertical reviewer heterogeneity is likely to have a weak effect on the winner selection outcome. As the incentive effect of vertical reviewer heterogeneity is weak, I abstract from μ_{ra} in the structural analysis.

At first glance, it is counter-intuitive that there is not much vertical differentiation in design quality. However, one might imagine quality differentiation in infrastructure design to be difficult in general, resulting in relatively small variation in quality across bidders' designs. For example, enhancing design quality of a simple road construction project may not be easy no matter how much resource is put in place. If bidders are aware that enhancing design quality is difficult, then bidders may compete in price. This supposition appears in the estimation result to some extent in the form of small (and negative) correlation between price and design quality. The small negative correlation between price bids and design quality could also be an indication that bidders have multiple private information, such that bidders may differ in terms of design efficiency and also delivery efficiency. Suppose for simplicity that those who are efficient in designing (delivering) always bid a design of high quality (a low price), and vice versa. Also, suppose that designing efficiency is not correlated with delivering efficiency. Then, bidders' design quality contains very little information about their pricing decision, resulting in the small correlation between price and design quality bids.

The large estimate of $\lambda_{\theta}^{(p)}$ suggests that ignoring unobserved auction heterogeneity would lead to an overestimation of σ_e as pointed out in Krasnokutskaya (2011) and Bajari, Houghton, and Tadelis (2006). A consistent estimation of σ_e is particularly relevant in the context of this paper since horizontal reviewer heterogeneity is likely to change the winner of a project by chance when bidders' objective PQR bids are more or less similar to each other. Therefore, overestimating the dispersion in private information of bidders will result in underestimation of the effect of horizontal reviewer heterogeneity on bidders' behaviour. Conversely, horizontal reviewer heterogeneity is expected to have a large effect on a bidder's behavior when the distribution of bidders' private information is dense. For example, suppose that the government succeeds in selecting efficient bidders by implementing a certain screening device. Although the government does not observe bidders' types, those selected bidders are on average efficient, and also similar in efficiency levels. Then, ignoring the adverse effect of horizontal reviewer heterogeneity may offset the effect of selecting efficient bidders. The more similar the population of bidders, the more relevant the issue of horizontal reviewer heterogeneity. I illustrate this point in the following section with a simple counter-factual exercise.

As I mentioned earlier, I find no significant correlation between the number of bidders and bidding strategies. Indeed, the effect of competition on pricing decision is positive. This observation is counter-intuitive at first glance. However, the number of participating bidders in a DB auction is chosen by the

FDOT. Therefore, if the FDOT observes complexity of an auctioned project, and if the FDOT tends to let more applicants participate into an auction for more complex project, then the effect of competition on pricing and designing decisions can be completely offset by project complexity. That is, bidders are on average inefficient at implementing a complex project, and therefore the effect of competition on bidders' behaviour is hidden by project complexity. For the same reason, correlation between the number of reviewers and bidders' behaviour can be hidden by project complexity.

5.2 Structural Estimation Results

Figure 6 shows the distribution of bidders' private cost information for varying number of bidders and reviewers. Increased competition and horizontal reviewer heterogeneity is associated with a right shift in the distribution of private cost information. That is, the more bidders or more reviewers there are, the more inefficient each bidder is on average. This finding is in line with the result obtained in the previous subsection. Suppose that the FDOT observes the degree of project complexity. Then, the FDOT may let more bidders participate in the auction process or assign more reviewers to the design evaluation process to keep the winning price low. However, since complex projects are costly in general, bidders tend to be inefficient on average. Thus, the effects of competition and horizontal reviewer heterogeneity is offset by the fact that bidders are on average inefficient in the auction for a complex project.

Table 6: Evaluation Uncertainty ($\tau(R)$) and Quality Cost Curvature (γ)

	$\hat{\tau}(3)$	$\hat{\tau}(4)$	$\hat{\tau}(5)$	$\hat{\tau}(6)$	$\hat{\gamma}$
Estimate	0.020^{***}	0.0174^{***}	0.0156^{***}	0.0143^{***}	3.48^{*}
Std. Error	(0.0006)	(0.00059)	(0.00052)	(0.00046)	(1.82)

[a] * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.
[b] The standard error of $\hat{\gamma}$ is computed using Delta Method.
[c] $\tau(R)$ captures the dispersion in evaluation noise, $ln(w_{ia})$, with R reviewers.
[d] The standard error of $\hat{\tau}(R)$ is computed using Bootstrap Method.

Figure 7 shows the distribution of equilibrium bids in an auction where all bidders' characteristics are set equal to the sample average. It is not clear from the figure that increased competition lead to competitive bids. That is, the competition effects on pricing and designing strategies are offset by the asymmetry in the distribution of private information. Therefore, the structural model here is consistent with the fact that the number of bidders and the number of reviewers are insignificantly correlated with both pricing and designing strategies in the RF factor model.

Figure 6: Distribution of Fixed Cost Private Information

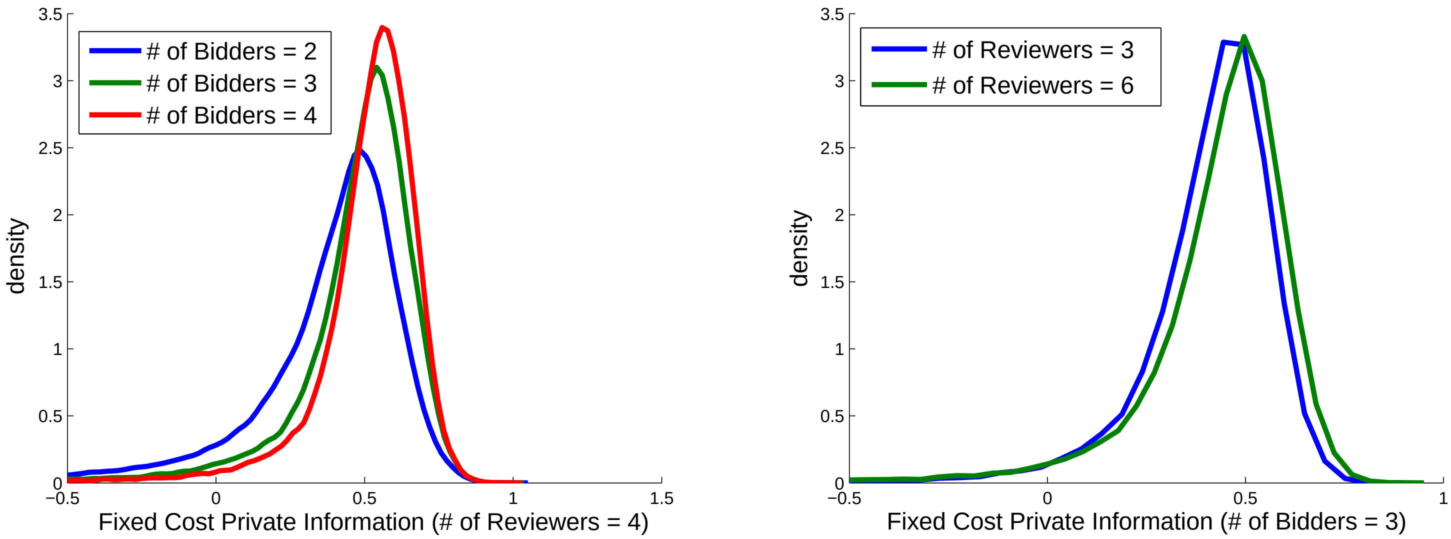
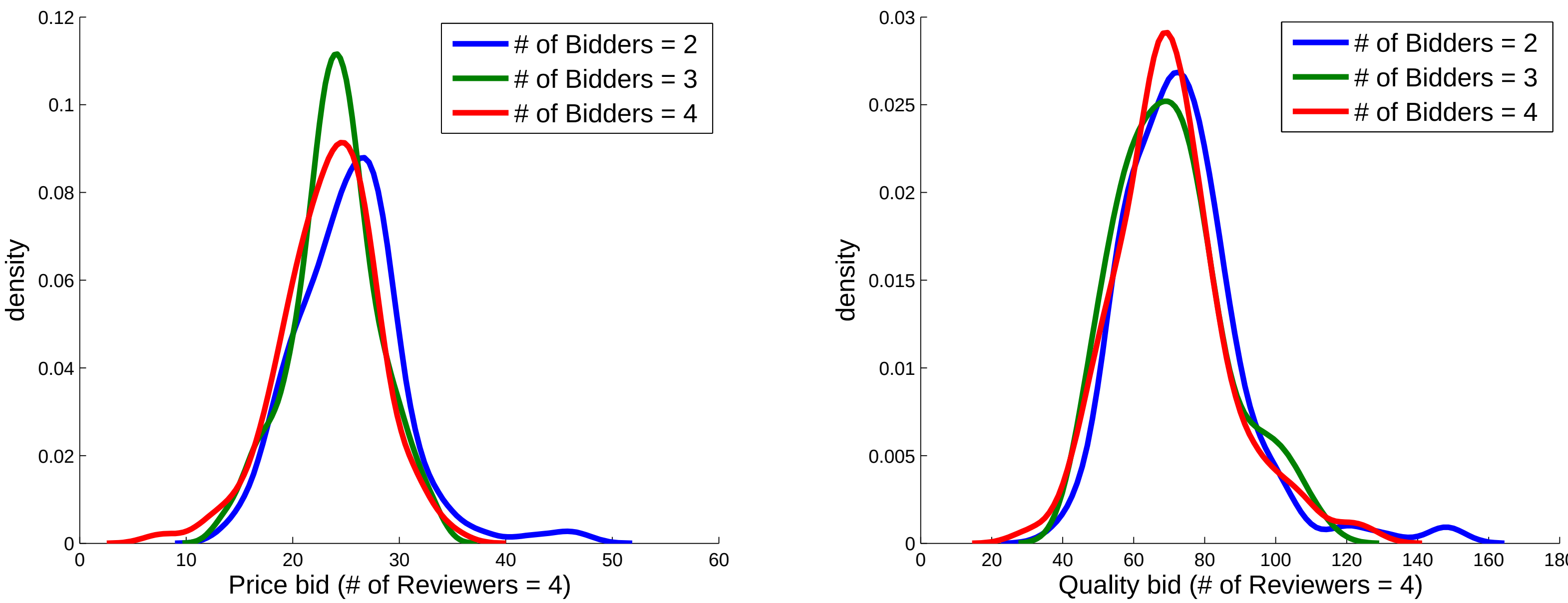


Figure 7: Equilibrium Distribution of Price and Design Quality Bids



6 Does Evaluation Uncertainty Matter?

I demonstrate the economic significance of evaluation uncertainty, and propose a way to circumvent the problem using an alternative auction mechanism in this section. More specifically, I show that, due to evaluation uncertainty, the auctioneer faces a significant amount of uncertainty in the auction outcomes: winning price and winning design quality. Keeping the strategy of bidders constant, evaluation noise can alter the winner of the project due to luck. As shown in Section 3, competitive (resp. non-competitive) bidders respond to greater evaluation uncertainty by becoming more (resp. less) competitive, widening the gap between bids. The above direct and behavioral effects together create a significant amount of uncertainty to the auctioneer in both the amount paid and the quality of winning design. I propose a simple second-price auction with the transfer amount contingent on design score as a mechanism to mitigate auctioneer's uncertainty in the auction outcomes.

6.1 Simulation without Endogenous Response of Bidders

In this subsection, I examine whether the estimated amount of horizontal reviewer heterogeneity, which I interpret as evaluation uncertainty, is large enough to affect the auction outcomes. To this end, I simulate the RF factor model with and without horizontal reviewer heterogeneity, and examine how the auction outcomes change depending on the auction environment. In conducting this experiment, I assume that bidders would not respond to a change in the auction environment. This assumption is strong and likely violated, but the objective here is to demonstrate the economic significance of horizontal reviewer heterogeneity in a simple manner.

The first experiment (*Experiment* 1) considers how the allocation of a project and the winning bids change solely due to uncertain design evaluations. More specifically, I simulate the distribution of percentage differences in winning price and winning design quality using the estimated parameters from the RF model. As a measure for misallocation of a project, I compute the probability that a bidder with non-lowest objective PQR wins, which I define as PWS. The set up of *Experiment* 2 is exactly the same as *Experiment* 1 except for the degree of dispersion in bidders' efficiency levels (i.e., σ_e). A lower σ_e implies that bids tend to cluster, and uncertain evaluations are likely to affect the allocation of a project when bids are close to each other. In both experiments, I consider a symmetric auction in which each bidder has the sample average characteristics, denoted by $\bar{X}$. For each experiment, every pseudo random variable is drawn from the distribution characterized by a vector of parameters $\tilde{\Theta}$, and I set $\tilde{\Theta} = \hat{\Theta}$ unless otherwise specified. Also, denote the hypothetical number of bidders and reviewers in this simulation exercise by N and R, respectively. Details of the simulation steps are found in Appendix.

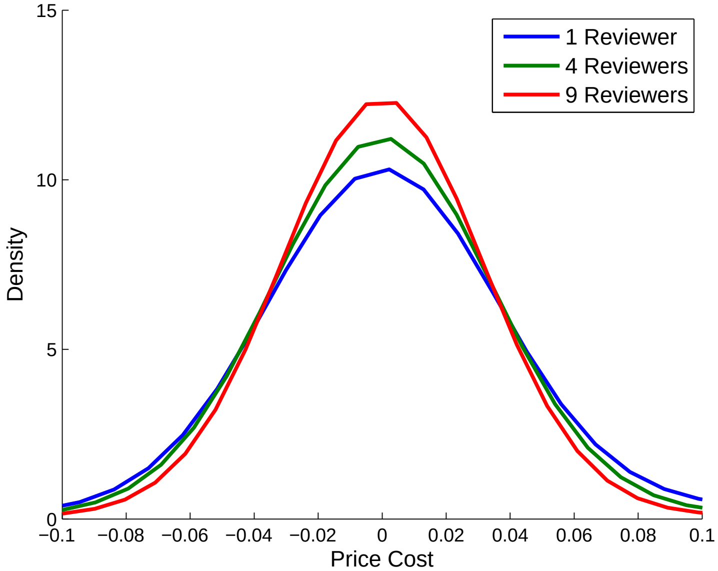
The results of the experiments are presented in Figure 8, Table 7, and Table 8. Each exercise shows that uncertain design evaluation is an economically relevant issue. In *Experiment* 1, I find that horizontal reviewer heterogeneity can generate up to 10% discrepancy in winning price and winning design quality. The estimated distribution of percentage differences in winning prices (resp. design quality) shows a standard deviation of 2.8% (resp. 2.4%) with 3 bidders and 1 reviewer, and standard deviation decreases to 1.4% (resp. 1.4%) with 9 reviewers. In addition, shows that evaluation noise is likely to select the bidder with the non-lowest objective price per quality when many bidders participate in an auction. With 5 bidders and 1 reviewer, 14% of DB projects would be awarded to a bidder of non-lowest objective price per quality. *Experiment* 2 indicates that the impact of uncertain evaluations is large when bidders are similar in their PQR bids.

Experiment 1 demonstrates that the effect of horizontal reviewer heterogeneity is economically sizable, but adding more reviewers to the review process has relatively small effect on the auction outcomes. This result comes from the fact that the effect of an additional reviewer on the degree of horizontal reviewer heterogeneity quickly dissipates as more reviewers participate, and completely removing horizontal reviewer heterogeneity by adding infinitely many reviewers would be prohibitively costly for the FDOT.

Experiment 2 illustrates the point that the winner is less likely the bidder with the lowest objective PQR bid when there are more bidders and/or less reviewers. Thus, the incentive effects of uncertain design evaluation on bidders' behaviour depend on the relative dispersion in the private information of bidders and evaluation noise. The less dispersed private information is, the larger the incentive effects of horizontal reviewer heterogeneity on bidders' behaviour.

The simulation exercise suggests that the degree of horizontal reviewer heterogeneity in the data is non-negligible. Yet, the behavior of bidders is kept constant in this exercise. In the numerical exercise of Section 3, I show that uncertainty in evaluations amplifies uncertainty in the auction outcomes: winning price and winning design quality. Thus, I conduct counterfactual experiments that take into account the bidders' endogenous response in the following section.

Figure 8: Distribution of Percentage Differences in Winning Price and Design Quality Relative to No Uncertainty Case



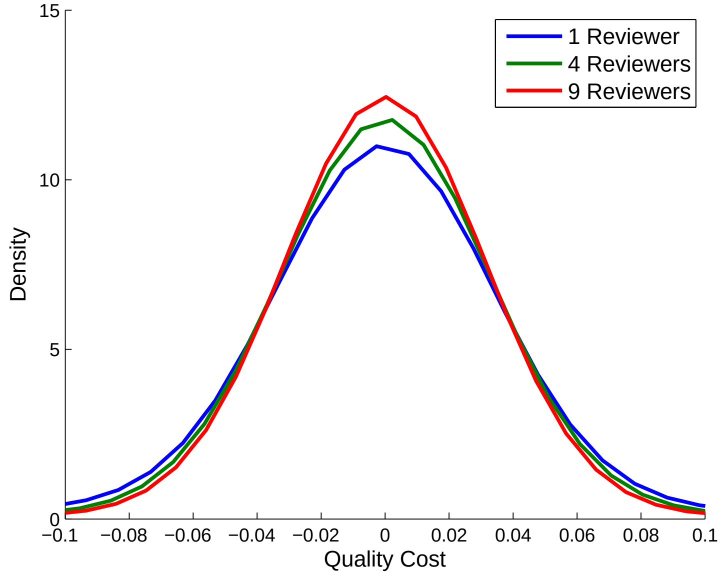


Table 7: Probability of Selecting a Bidder with Non-Lowest Objective PQR
($\tilde{\sigma}_e = \hat{\sigma}_e$)

# of Bidders (N) & Reviewers (R)	$R = 1$	$R = 2$	$R = 3$	$R = 4$	$R = 9$
$N = 2$	0.0729	0.0518	0.0425	0.0369	0.0246
$N = 3$	0.107	0.0765	0.0626	0.0544	0.0364
$N = 4$	0.128	0.0916	0.0754	0.0655	0.0439
$N = 5$	0.143	0.103	0.0847	0.0735	0.0493

Simulated probability based on the estimates of RF factor model is shown in the table.

Table 8: Probability of Selecting a Bidder with Non-Lowest Objective PQR
($\tilde{\sigma}_e = \frac{\hat{\sigma}_e}{2}$)

# of Bidders (N) & Reviewers (R)	$R = 1$	$R = 2$	$R = 3$	$R = 4$	$R = 9$
$N = 2$	0.106	0.0767	0.0630	0.0547	0.0366
$N = 3$	0.155	0.112	0.0928	0.0809	0.0542
$N = 4$	0.186	0.135	0.111	0.0974	0.0656
$N = 5$	0.207	0.151	0.125	0.109	0.0739

Simulated probability based on the estimates of RF factor model is shown in the table.

6.2 Simulation with Endogenous Response of Bidders

The more uncertainty in evaluation process, the more uncertainty in the auction outcomes. Previous section demonstrated this point under the assumption that bidders would not respond upon a reduction in the degree of evaluation uncertainty. This section considers some ways in which the auctioneer may deem to mitigate the uncertainty in the auction outcomes, taking into account endogenous responses of bidders using the model developed in Section 3. To this end, I consider two simple changes in the auction environment: an increase in the number of reviewers and a decrease in the weight assigned to design score in selecting the winner. As I show in the previous section, an increase in the number of reviewers lowers the degree of evaluation uncertainty. Alternatively, the auctioneer may reduce the weight assigned to design quality in determining which bidder to contract with. As the degree of evaluation uncertainty is likely strongly tied to the weight assigned to design score, reducing the weight may reduce uncertainty in the auction outcomes by mitigating evaluation uncertainty.

Consider a symmetric auction with three bidders where all the bidders in this auction have the sample average characteristics. The degree of evaluation uncertainty is given by $\hat{\tau}(R)$ where R is the number of reviewers in this hypothetical auction. The winner in this auction is the bidder with the lowest p/q^{ω} where $\omega > 0$, and the winner receives its price bid upon winning the project. Thus, the only difference between this counterfactual auction and the ordinal DB auction is the weight assigned to design score to determine the winner of the project.[34] Note here that all the propositions developed in the model in section 3 still holds under this generalized model. Moreover, I am able to compute a similar type of equilibrium in this counterfactual experiment since evaluation uncertainty ($\tau(R)$) and the relative weight assigned to design quality (ω) can be continuously changed. Intuitively, by slightly changing one parameter value at a time, I am able to compute an equilibrium that is close to the original equilibrium, and repeating this process precludes a shift in equilibrium, allowing me to trace out the equilibrium path for a variety of parameter values. This homotopy-based computation of equilibrium is important as there is no guarantee that the computed equilibrium is unique, and a potential shift in equilibrium is often a concern raised in a counterfactual experiment of models with multiple equilibria.

I find that adding five reviewers to an ordinary DB auction ($\omega = 1$) with four reviewers would winning price would only decrease by 1.3%, and winning design quality declines by 0.6%. The effect on the variance of winning price is somewhat larger than the effect on its expected value, and declines by 2.3% from the same change. The effect of increased weight assigned to design quality score has a large effect on auction outcomes, but there is always a trade-off between price and design quality.

[34]The selection rule in an actual DB auction is a special case where $\omega = 1$.

An increase in the weight assigned to design quality would imply an increase in both winning price and design quality. Another finding here is the complementarity between uncertainty and the weight assigned to design quality. For example, adding five reviewers into the normal DB auction (i.e., $\omega = 1$) with four reviewers would decrease the variance of winning price by 2.3% while the same change would reduce the variance of winning price by 4%. That is, the effect of uncertainty is amplified by the increase in the weight assigned to design quality in the winner selection process.

Table 9: Distributions of Winning Price and Winning Design Quality

	Mean		Variance	
$\omega = 0.8$	Price	Quality	Price	Quality
$R = 1$	16.20	52.54	8.20	116.0
$R = 4$	15.95	52.21	8.03	114.3
$R = 9$	15.75	51.94	7.82	113.1
	Mean		**Variance**	
$\omega = 1$	Price	Quality	Price	Quality
$R = 1$	19.88	64.59	17.39	193.7
$R = 4$	19.44	64.04	16.74	190.6
$R = 9$	19.18	63.64	16.33	186.8
	Mean		**Variance**	
$\omega = 1.2$	Price	Quality	Price	Quality
$R = 1$	25.69	75.45	37.31	279.3
$R = 4$	25.12	74.73	35.43	271.4
$R = 9$	24.68	74.20	34.01	266.6

ω is the weight assigned to design quality score in the winner selection process. $\omega = 1$ is the standard DB auction rule.

In general, I find that reducing auctioneer's uncertainty in the auction outcomes at a cost of deteriorating the expected outcomes in either type of the changes. While increased number of reviewers mitigates evaluation uncertainty, the effect of additional reviewers is not large. In addition, assigning many employees of the FDOT to the review task can be prohibitively costly. Contrary to a change in the number of reviewers, a reduction in the weight assgined to design score leads to a large decline in uncertainty about the auction outcomes. However, the weight reduction also causes a large change in the expected auction outcomes, and so it is not clear if the weight reduction is a prefered alternative auction mechanism. In the following subsection, I propose an alternative auction mechanism that significantly reduces the auctioneer's uncertainty in the auction outcomes without deteriorating the expected auction outcomes much.

6.3 Second-Price Auction with Design Score Contingent Transfer

I examine what would happen to the auction outcome if the auctioneer uses bidders' design scores by means of making trasnsfer rather than selecting the winner. In particular, I consider a second-price auction with design quality transfer in which the winner is selected solely based on the lowest submitted price, and the winner's design score determines the amount of transfer it receives. This auction mechanism is of interest as it shuts down the effect of evaluation uncertainty on bidders' behavior by construction. More specifically, neither a bidder's winning likelihood nor its ex-post payoff is affected by the design evaluations of its rivals, and thus bidders would not respond to the uncertainty in rivals' design evaluations. Furthermore, a bidder would also be non-responsive to uncertainty in its own design evaluations under risk neutrality assumption.

In this auction, bidders simultaneously bid on both price and design quality as is the case in a DB auction. However, the winner is solely determined by the lowest price bid, and design quality score, which is determined by the average reviewers' evaluation, determines the amount of transfer that a bidder receives upon winning the project. I denote the transfer function by $trans(q_i^0 w_i)$ where q_i^0 and w_i are objective design quality and evaluation noise as defined before. The winner receives the second lowest price, denoted by $p_{-i}^{(min)}$, plus $trans(q_i^0 w_i)$ upon winning. Now, let $E_w[.]$ and $E_{p^{(min)}}[.]$ denote the expectation operator over the distribution of w_i and $p_{-i}^{(min)}$, respectively. Also, denote the probability of winning function conditional on i's price bid by $Pr(win|p_i)$. The other variables are denoted as in section 3. Then, the interim expected payoff of bidder i of cost type $\{vc_i, fc_i\}$ is given by:

$$\pi_i^{int} = \max_{p_i, q_i^0} E_{p^{(min)}}[E_w[Pr(win|p_i)(p_{-i}^{(min)} + trans(q_i^0 w_i) - vc_i(q_i^0)^\gamma - fc_i)]]$$

Now, the bidder's profit function can be rewritten as follows with a log-linear transfer function, $trans(q_i^0 w_i) = \phi\, ln(q_i^0) + \phi\, ln(w_i)$ where ϕ is a constant. The log-linear transfer function shuts down the impact of uncertainty associated with its own design evaluation on its behavior under the risk neutrality assumption.

$$\pi_i^{int} = \max_{p_i, q_i^0} E_{p^{(min)}}[E_w[Pr(win|p_i)(p_{-i}^{(min)} + \phi\, ln(q_i^0) + \phi\, ln(w_i) - vc_i(q_i^0)^\gamma - fc_i)]]$$

The first-order optimality condition with respect to q_i^0 gives the equilibrium design quality choice $q_i^{(eqm)}$, such that:

$$q_i^{(eqm)} = \left(\frac{\phi}{\gamma\, vc_i}\right)^{\frac{1}{\gamma}} \tag{28}$$

which is independent of evaluation uncertainty. Define the expected total cost by $ETC_i = -(\phi\, ln(q_i^{(eqm)}) + \phi\, ln(w_i) - vc_i(q_i^{(eqm)})^{\gamma} - fc_i)$.[35] Note here that ETC_i is independent of p_i as design quality is chosen independent of price bid. Therefore, bidder i's problem collapses to:

$$\pi_i^{int} = \max_{p_i} E_{p^{(min)}}[Pr(win|p_i)(p_{-i}^{(min)} - ETC_i)]$$

which is equivalent to the ordinal second-price low-bid auction. Thus, the (weakly) dominant truth-telling strategy of a bidder is:

$$p_i^{(eqm)} = ETC_i \tag{29}$$

Therefore, the dominant strategy equilibrium of the game is characterized by (28) and (29) for any cost type of a bidder. Note here that a bidder's design quality choice is increasing in the transfer parameter ϕ while its pricing strategy is not necessarily increasing in ϕ since the transfer amount, $\phi\, ln(q_i^{(eqm)})$, can be negative. That is, a rise in ϕ may induce a lower price with a higher design quality for a certain range of values of ϕ, which was not the case under PQR rule by proposition 3.

In order to conduct this analysis, the linear transfer parameter, ϕ, is set so that the average fixed cost bidder minimizes the average total cost of implementing the project. To be more specific, consider the following problem. Let $q_{AC}(vc_i, fc_i)$ denote the choice of design quality that minimizes the average total cost of implementing the project for any arbitrary pair of variable cost and fixed cost. It is straightforward to obtain:

$$\begin{aligned} q_{AC}(vc_i, fc_i) &= \arg\min_{q_i^0} \frac{vc_i(q_i^0)^{\gamma} + fc_i}{q_i^0} \\ &= \left(\frac{fc_i}{(\gamma - 1)\, vc_i}\right)^{\frac{1}{\gamma}} \end{aligned}$$

Therefore, $\phi = \frac{\gamma}{\gamma-1}\bar{fc}$ where $\bar{fc}$ is the average fixed cost private information, induces the level of design quality that achieves the minimum average total cost for the average fixed cost bidder of any variable cost type.

[35] Expectation is taken over the distribution of evaluation noise w_i.

I simulate and obtain the distribution of winning price and design quality in this alternative auction, and compare the outcomes with the auction outcomes from DB auction for the case of three bidders in Table 10.

Table 10: Second Price Auction with Design Quality Transfer

	Mean		Variance	
	Price	Quality	Price	Quality
DB Auction	19.4	64.0	16.7	190
SPA with Quality Transfer	19.6	63.0	10.0	80.4

Winning price and winning design quality are shown under each auction mechanism.

SPA with design quality contingent transfer reduces the auctioneer's uncertainty in the auction outcomes without affecting the expected auction outcomes much. Variances of winning price and design quality decline by 40% and 60%, respectively. While 1% (resp. 1.5%) increase (resp. decrease) in the average winning price (resp. design quality) may not be negligible, the objective of the FDOT here is unlikely to minimize the expected auction outcomes as discussed in Section 2.[36] If the auctioneer wishes to avoid a large uncertainty in the auction outcomes, the alternative auction mechanism here may serve as a way to mitigate such uncertainty. Note here that the smaller dispersion in auction outcomes is not due to the second-price auction format since the second-price auction generates a larger dispersion than the first-price auction in winning price.[37] Also, the expected winning price and design quality will be the same under the first price auction by the revenue equivalence theorem.

The reduction in the uncertainty in the auction outcomes partly comes from the transfer coefficient ϕ that implements the efficient scale in the production of design quality by the average fixed cost type of a bidder. It is clear from (28) that bidder i with $fc_i < \bar{fc}$ (resp. $fc_i > \bar{fc}$) overproduces (resp. underproduce) design quality relative to its efficient scale, generating a smaller dispersion in design bids. As its price bid is a function of its design quality, a smaller dispersion in design quality leads to a smaller dispersion in price bids.

The small changes in the expected auction outcomes are intriguing at first glance. As discussed in Section 3, every bidder overproduces design quality in a DB auction while bidders are, on average, producing at their efficient scale under the alternative mechanism here. However, it is low fixed

[36]If the FDOT's objective is to minimize the expected objective PQR, it should let more bidders participate in each auction to encourage competition.

[37]The obtained result does not depend on any auction characteristics, either observed or unobserved, by proposition 4 of the paper.

cost bidders, who is likely to win the project, that overproduces design quality under the alternative mechanism, and production cost is reflected in the form of a higher price bid.

The main message here is not to suggest that the alternative mechanism is superior to the DB auction mechanism, and the results obtained in the analysis does not necessarily hold when there is a large amount of evaluation uncertainty in a DB auction.[38] However, if the auctioneer wishes to mitigate its uncertainty in the auction outcomes for the level of evaluation uncertainty present in the data without worsening the expected auction outcomes drastically, then it may consider implementing the proposed auction mechanism here.

7 Conclusion

In this paper, I have studied the effects of uncertain design evaluations on competing suppliers' behavior using a sample of Design-Build auctions from the Florida Department of Transportation. I document the presence of horizontal reviewer heterogeneity, which affects the rankings of bidders by introducing randomization into the winner selection process. I construct a structural model that incorporates uncertain evaluations of design proposals. The estimation results are consistent with the observed fact that both the number of bidders and reviewers have no effect on bidders' behavior on the surface. I also provide the first attempt in the literature of multi-attribute auctions at estimating structural parameters when some attributes of a bid are unobserved to the econometrician. The estimation results indicate that 18% of the total variation in reviewers' evaluations is due to horizontal reviewer heterogeneity. The economic significance of horizontal reviewer heterogeneity is demonstrated in a variety of simulation exercises. I find that an increase in horizontal reviewer heterogeneity not only generates a higher winning price and higher winning design quality on average, but also leads to a greater dispersion in these variables. I propose a simple second-price auction with transfer amount contingent on design score of a bidder, shutting down the impact of uncertain design evaluations on bidding incentives. I find, without worsening the expected auction outcomes, that the alternative mechanism reduces the auctioneer's uncertainty in both the amount paid and the quality of the winning design by 40% and 60%, respectively.

[38] In an extreme case in which there is infinite amount of evaluation uncertainty in a DB auction, every bidder bids at the upper bound $\bar{b}$, and there is no uncertainty in the outcome of PQR.

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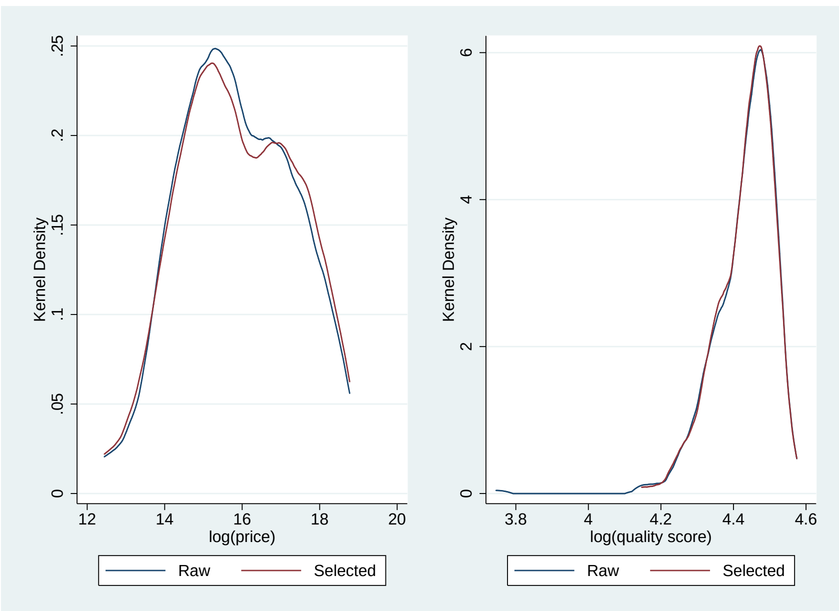
8 Appendix

8.1 Test of Sampling Selection Bias

Table 11: Test of Mean Difference between Removed and Selected Samples

	ln(Price)	ln(Quality Score)
Selcted	.368	.0141
	(1.45)	(1.04)
Constant	15.5	4.41
	(64.5)***	(344)***
R^2	0.0057	0.0029
Obs	374	381

where *Selected* = 1 if belong to selected sample, and 0 otherwise.



8.2 Definition of Observables

- Engineer's estimate of project cost: A proxy for the project size.
- Distance : Distance between project site and the closest branch of bidder.
- Opponent's Distance : Minimum distance of opponent bidders within an auction.
- Utilization Rate : A bidder's backlog per capacity. Backlog is defined as the total dollar value of projects ongoing at the time of bidding. Capacity of a bidder is defined as the maximum backlog during the period the sample is taken from. Backlog and capacity are calculated using all other types of auctions and DB auctions procured by the FDOT from 1999 to 2012.
- Opponent's Utilization Rate : The average of all opponent's utilization rate.
- Project Type : Projects are classified into Road, Bridge, Building, Mixed Project, Monitoring System Implementation, and Others

8.3 Proof of Proposition 1

Proof. Let $\{p_i(\alpha),\, q_i^0(\alpha)\}$ be the solutions to this constrained optimization problem of a bidder i, such that:

$$\begin{aligned}
\{p_i(\alpha),\, q_i^0(\alpha)\} &= \arg\max_{p_i, q_i^0} \;\; \pi_i^{int} \quad \text{subject to } \; p_i = \alpha\, q_i^0 \\
\Leftrightarrow \;\; \{\alpha\, q_i^0(\alpha),\, q_i^0(\alpha)\} &= \arg\max_{q_i^0} \;\; \int_{\mathbf{c_{-i}}} G(\alpha|\mathbf{b_{-i}})\, f(\mathbf{c_{-i}}|c_i)\, d\mathbf{c_{-i}}\, (\alpha q_i^0 - vc_i\, C(q_i^0) - fc_i) \\
\Leftrightarrow \;\; \{\alpha\, q_i^0(\alpha),\, q_i^0(\alpha)\} &= \arg\max_{q_i^0} \;\; \alpha\, q_i^0 - vc_i\, C(q_i^0) - fc_i \qquad (30)
\end{aligned}$$

The first-order necessary and sufficient condition w.r.t. q_i^0 gives:

$$\begin{aligned}
\alpha - vc_i\, C^{'}(q_i^0(\alpha)) &= 0 \\
\Rightarrow \;\; q_i^0(\alpha) &= C^{'-1}(\alpha/vc_i) \qquad (31) \\
\Rightarrow \;\; p_i(\alpha) &= \alpha\, C^{'-1}(\alpha/vc_i) \qquad (32)
\end{aligned}$$

□

8.4 Proof of Proposition 2: Existence of Isotone Equilibrium

I argue that $G(b_i|\mathbf{b_{-i}})\,(p_i(b_i)-vc_i\,C(q_i^0(b_i))-fc_i)$ is log-supermodular in $\mathbf{b}$ and $\mathbf{c}$. Then, apply theorems from McAdams (2003) to show existence of an isotone pure strategy equilibrium.

Proof. With the evaluation noise following the Type 1 Extreme Value distribution, $ln(G(b_i|\mathbf{b_{-i}})$ is supermodular in b_i and b_k $\forall\, k \neq i$, such that:

$$\begin{aligned} \frac{\partial^2 ln(G(b_i|\mathbf{b_{-i}}))}{\partial b_i\,\partial b_k} &= \frac{(b_i\,b_k)^{-\frac{1}{\tau}-1}}{\tau^2(\sum_j b_j^{-\frac{1}{\tau}})^2} \\ &> 0 \qquad \forall\;\; k \neq i \end{aligned} \tag{33}$$

Also, it is obvious that $G(b_i|\mathbf{b_{-i}})$ is independent of $\mathbf{c}$, and $\pi_i^{post} = p_i(b_i) - vc_i\,C(q_i^0(b_i)) - fc_i$ is independent of $\mathbf{b_{-i}}$. Thus, it suffices to show that π_i^{post} is log-supermodular in b_i and c_i. That is,

$$\begin{aligned} \frac{\partial^2 ln(\pi_i^{post})}{\partial b_i\,\partial fc_i} &= \frac{q_i^0(b_i)}{(b_i\,q_i^0(b_i) - vc_i\,C(q_i^0(b_i)) - fc_i)^2} \\ &> 0 \end{aligned} \tag{34}$$

Finally, using the fact that $C^{'-1'}(b_i/vc_i) = 1/C^{''}(q_i^0(b_i))$, I have:

$$\begin{aligned} & \frac{\partial^2 ln(\pi_i^{post})}{\partial b_i\,\partial vc_i} \gtrless 0 \\ \Leftrightarrow\quad & \frac{\frac{\partial q_i^0(b_i)}{\partial vc_i}(b_i\,q_i^0(b_i) - vc_i\,C(q_i^0(b_i)) - fc_i) - (\frac{\partial q_i^0(b_i)}{\partial vc_i}b_i - C(q_i^0(b_i)) - vc_i\,C(q_i^0(b_i))\frac{\partial q_i^0(b_i)}{\partial vc_i})}{(b_i\,q_i^0(b_i) - vc_i\,C(q_i^0(b_i)) - fc_i)^2} \gtrless 0 \\ \Leftrightarrow\quad & \frac{\frac{\partial q_i^0(b_i)}{\partial vc_i}(b_i\,q_i^0(b_i) - vc_i\,C(q_i^0(b_i)) - fc_i) + C(q_i^0(b_i))}{(b_i\,q_i^0(b_i) - vc_i\,C(q_i^0(b_i)) - fc_i)^2} \gtrless 0 \quad \because\; b_i = vc_i\,C^{'}(q_i^0(b_i)) \\ \Leftrightarrow\quad & -\frac{b_i}{vc_i^2\,C''(q_i^0(b_i))}(b_i\,q_i^0(b_i) - vc_i\,C(q_i^0(b_i)) - fc_i) + C(q_i^0(b_i)) \gtrless 0 \quad \because\; \frac{\partial q_i^0(b_i)}{\partial vc_i} = -\frac{b_i}{vc_i^2\,C''(q_i^0(b_i))} \end{aligned} \tag{35}$$

Since $fc_i > 0$, I have:

$$\begin{aligned} & -\frac{b_i}{vc_i^2\,C''(q_i^0(b_i))}(b_i\,q_i^0(b_i) - vc_i\,C(q_i^0(b_i)) - fc_i) + C(q_i^0(b_i)) \\ > \quad & -\frac{b_i}{vc_i^2\,C''(q_i^0(b_i))}(b_i\,q_i^0(b_i) - vc_i\,C(q_i^0(b_i))) + C(q_i^0(b_i)) \\ = \quad & -\frac{q_i^0(b_i)\,C^{'}(q_i^0(b_i))^2}{C''(q_i^0(b_i))} + \frac{C^{'}(q_i^0(b_i))C(q_i^0(b_i))}{C''(q_i^0(b_i))} + C(q_i^0(b_i)) \quad \because\; b_i = vc_i\,C^{'}(q_i^0(b_i)) \\ > \quad & 0 \quad \because\; \frac{\partial q\,C^{'}(q)/C(q)}{\partial q} > 0 \end{aligned} \tag{36}$$

Therefore, $G(b_i|\mathbf{b_{-i}})\pi_i^{post}$ is log-supermodular in $\mathbf{b}$ and $\mathbf{c}$. Existence of an isotone pure strategy equilibrium directly follows from McAdams (2003). □

8.5 Proof of Proposition 3

Proof. Consider equation (4). By implicitly differentiating with respect to τ, I have:

$$\frac{d\,ln(\psi_i(c_i))}{d\,\tau} \quad = \quad \frac{d\,ln(q_i^0(c_i))}{d\,\tau}\,\frac{C^{''}(q_i^{\psi}(c_i))}{C^{'}(q_i^{\psi}(c_i))} \tag{37}$$

which implies that $\frac{d\,q_i^{\psi}(c_i)}{d\,\tau}$ and $\frac{d\,\psi_i(c_i)}{d\,\tau}$ have the same sign. Also,

$$\frac{d\,p_i^{\psi}(c_i)}{d\,\tau} \quad = \quad \frac{d\,\psi_i(c_i)}{d\,\tau}q_i^{\psi}(c_i) + \frac{d\,q_i^{\psi}(c_i)}{d\,\tau}\psi_i(c_i) \tag{38}$$

Thus, both $p_i^{\psi}(c_i)$ and $q_i^{\psi}(c_i)$ have the same comparative statics sign as $\psi_i(c_i)$. Now, rewrite the fixed cost equation in (eq2), such that:

$$p_i^{\psi}(c_i) - vc_i\,C(q_i^{\psi}(c_i)) - fc_i = -\frac{\tilde{G}_i(\psi_i(c_i))}{\tilde{g}_i(\psi_i(c_i))} = \pi_i^{post} \tag{39}$$

By substituting equation (4) into (39), I have:

$$vc_i\,(q_i^{\psi}(c_i)\,C^{'}(q_i^{\psi}(c_i)) - \,C(q_i^{\psi}(c_i))) - fc_i \quad \equiv \quad \pi_i^{post} \tag{40}$$

As marginal cost is increasing faster than average variable cost (i.e., $q_i^{\psi}\,C^{'}(q_i^{\psi}) - C(q_i^{\psi})$ is increasing in q_i^{ψ}), π_i^{post} has the same comparative statics sign as q_i^{ψ}. Therefore,

$$sign\left(\frac{d\,\psi_i(c_i)}{d\,\tau}\right) = sign\left(\frac{d\,p_i^{\psi}(c_i)}{d\,\tau}\right) = sign\left(\frac{d\,q_i^{\psi}(c_i)}{d\,\tau}\right) = sign\left(\frac{d\,\pi_i^{post}}{d\,\tau}\right) \tag{41}$$

□

8.6 Proof of Proposition 4

Let $c_{ia}^0 \equiv \{\varepsilon_{ia}^v, \varepsilon_{ia}^f, 0, 0\}$ and $c_{ia} \equiv \{\varepsilon_{ia}^v, \varepsilon_{ia}^f, \theta_a^v, \theta_a^f\}$. I omit the vector of observables, X_{ia}, from the proof to avoid cluttering but the argument here goes through trivially with X_{ia}. Let $\boldsymbol{\{p(c^0),\ q(c^0)\}}$ be an isotone pure strategy equilibrium profile that satisfies the first-order optimality conditions of the bidder's problem. I show that a profile of strategy $\boldsymbol{\{p(c) = \varphi^p(\theta)\,p(c^0),\ q(c) = \varphi^q(\theta)\,q(c^0)\}}$ where

$\varphi^p(\theta) = \exp\{\theta_a^f\}$, and $\varphi^q(\theta) = \exp\{\theta_a^f/\gamma - \theta_a^v/\gamma\}$ are the unique functions that satisfy the first-order optimality conditions of the problem in the space of log linear functions.

Proof. Let $\varphi^p(\theta) = \exp\{t_1\theta_a^f + s_1\theta_a^v\}$ and $\varphi^q(\theta) = \{t_2\theta_a^f + s_2\theta_a^v\}$ where t_j and s_j for $j \in \{1,2\}$ are parameters. From the first-order optimality condition (10), I obtain:

$$t_1\theta_a^f + s_1\theta_a^v + ln(p_{ia}(c_{ia}^0)) \quad = \quad ln(\gamma) + \gamma ln(q_{ia}^0(c_{ia}^0)) + \gamma(t_2\theta_a^f + s_2\theta_a^v) + \theta_a^v + \varepsilon_{ia}^v \tag{42}$$

which yields $t_1 = \gamma t_2$ and $s_1 = 1 + \gamma s_2$.

Now, define a function $\tilde{g}_{ia}^p$, as follows:

$$\tilde{g}_{ia}^p(p_{ia}(c_{ia})/q_{ia}^0(c_{ia})) \quad = \quad p_{ia}(c_{ia})\tilde{g}_{ia}(p_{ia}(c_{ia})/q_{ia}^0(c_{ia})) \tag{43}$$

Then, it is clear that:

$$\begin{aligned}(1-1/\gamma)p_{ia}(c_{ia}) + \frac{\tilde{G}_{ia}(p_{ia}(c_{ia})/q_{ia}^0(c_{ia}))}{\tilde{g}_{ia}(p_{ia}(c_{ia})/q_{ia}^0(c_{ia}))} \quad &= \quad \left((1-1/\gamma) + \frac{\tilde{G}_{ia}(p_{ia}(c_{ia})/q_{ia}^0(c_{ia}))}{\tilde{g}_{ia}^p(p_{ia}(c_{ia})/q_{ia}^0(c_{ia}))}\right)p_{ia}(c_{ia}) \\ &= \quad \left((1-1/\gamma) + \frac{\tilde{G}_{ia}(p_{ia}(c_{ia}^0)/q_{ia}^0(c_{ia}^0))}{\tilde{g}_{ia}^p(p_{ia}(c_{ia}^0)/q_{ia}^0(c_{ia}^0))}\right)p_{ia}(c_{ia}) \end{aligned} \tag{44}$$

The last equality follows from that fact that $\tilde{G}_{ia}$ and $\tilde{g}_{ia}^p$ are both Homogeneous degree of 0 in unobserved auction heterogeneity. Therefore, unobserved auction heterogeneity enters the LHS of (11) only through $p_{ia}(c_{ia})$. Thus, combining with the fact that the right hand side of (11) is log-linear in θ_a^f and independent of θ_a^v, I obtain:

$$\begin{aligned} t_1 &= 1, \qquad t_2 = 1/\gamma \\ s_1 &= 0, \qquad s_2 = -1/\gamma \end{aligned}$$

As the derived strategy profile above $\boldsymbol{\{p(c) = \varphi^p(\theta)\,p(c^0),\ q(c) = \varphi^q(\theta)\,q(c^0)\}}$ does not affect the first-order optimality conditions, $\boldsymbol{\{p(c) = \varphi^p(\theta)\,p(c^0),\ q(c) = \varphi^q(\theta)\,q(c^0)\}}$ is also an equilibrium strategy profile if $\{\boldsymbol{p}(\boldsymbol{c^0}),\ \boldsymbol{q}(\boldsymbol{c^0})\}$ is an equilibrium strategy profile. □

8.7 Proof of Proposition 5: Identification of Covariance Structure

I apply the fact that the sample moment of random variables converges point-wise to the corresponding population moment by the law of large numbers. I also apply Delta method which states that a function of parameters can be estimated consistently by applying the same function to the consistent estimator of the original parameters. Therefore, I establish identification of parameters by solving the parameters of the model in terms of population moments which can be consistently estimated from corresponding sample moments.

η_{p1} and η_{q1} are identified by:

$$\eta_{q1} = E[\tilde{q}_{ria}\tilde{q}_{r'i'a}|\boldsymbol{X}_a]^{1/2} \tag{45}$$

$$\eta_{p1} = \frac{E[\tilde{p}_{ia}\tilde{q}_{ri'a}|\boldsymbol{X}_a]}{E[\tilde{q}_{ria}\tilde{q}_{r'i'a}|\boldsymbol{X}_a]^{1/2}} \tag{46}$$

Now, η_{p2} and η_{q2} are identified from the following moment conditions, such that:

$$\eta_{q2} = (E[\tilde{q}_{ria}\tilde{q}_{r'ia}|\boldsymbol{X}_a] - E[\tilde{q}_{ria}\tilde{q}_{r'i'a}|\boldsymbol{X}_a])^{1/2} \tag{47}$$

$$\eta_{p2} = \frac{E[\tilde{p}_{ia}\tilde{q}_{ria}|\boldsymbol{X}_a] - E[\tilde{p}_{ia}\tilde{q}_{ri'a}|\boldsymbol{X}_a]}{(E[\tilde{q}_{ria}\tilde{q}_{r'ia}|\boldsymbol{X}_a] - E[\tilde{q}_{ria}\tilde{q}_{r'i'a}|\boldsymbol{X}_a])^{1/2}} \tag{48}$$

Also, bidder's private information can be identified from:

$$\sigma_\theta = E[\tilde{p}_{ia}\tilde{p}_{i'a}|\boldsymbol{X}_a] - \frac{E[\tilde{p}_{ia}\tilde{q}_{ri'a}|\boldsymbol{X}_a]^2}{E[\tilde{q}_{ria}\tilde{q}_{r'i'a}|\boldsymbol{X}_a]} \tag{49}$$

$$\sigma_e = E[\tilde{p}_{ia}^2|\boldsymbol{X}_a] - E[\tilde{p}_{ia}\tilde{p}_{i'a}|\boldsymbol{X}_a] - E[\tilde{q}_{ria}\tilde{q}_{r'i'a}|\boldsymbol{X}_a] \tag{50}$$

Lastly, I obtain identification of σ_μ and σ_u from:

$$\sigma_\mu = E[\tilde{q}_{ria}\tilde{q}_{ri'a}|\boldsymbol{X}_a] - E[\tilde{q}_{ria}\tilde{q}_{r'i'a}|\boldsymbol{X}_a] \tag{51}$$

$$\sigma_u = (E[\tilde{q}_{ria}^2|\boldsymbol{X}_a] - E[\tilde{q}_{r'ia}\tilde{q}_{ria}|\boldsymbol{X}_a]) - (E[\tilde{q}_{ria}\tilde{q}_{ri'a}|\boldsymbol{X}_a] - E[\tilde{q}_{ria}\tilde{q}_{r'i'a}|\boldsymbol{X}_a]) \tag{52}$$

which completes the proof.

8.8 Reduced-Form Factor Model: Estimation Procedure

Let $\boldsymbol{Y}_a$ denote a vector of prices and reviewers' evaluations stacked on top of each other for auction a. Also, let $\boldsymbol{\Sigma_a}$ be the variance covariance matrix of $\boldsymbol{Y}_a$ conditional on a vector of auction-bidder characteristics $\boldsymbol{X}_a$. I denote the number of bidders and reviewers in auction a by N_a and R_a, respectively. Note that $\boldsymbol{Y}_a$ has a length of $N_a(R_a + 1)$ as every bidder submits a price bid and a design proposal which is reviewed by R_a reviewers. Thus, $\boldsymbol{\Sigma_a}$ is a $N_a(R_a + 1) \times N_a(R_a + 1)$ matrix with elements consisting of variance covariances specified in (26). Then, integrated log-likelihood function, $l(\Theta)$, can be written as:

$$l(\Theta) \quad \equiv \quad ln \left(\prod_a \frac{1}{\sqrt{2\pi}^{N_a(R_a+1)} |\boldsymbol{\Sigma_a}|} \exp\{-\frac{1}{2}(\boldsymbol{Y}_a - \boldsymbol{X}_a\boldsymbol{\beta})^T \boldsymbol{\Sigma_a}^{-1}(\boldsymbol{Y}_a - \boldsymbol{X}_a\boldsymbol{\beta})\} \right)$$

where $\boldsymbol{\beta}$ is a vector of coefficient parameters. A direct maximization of $l(\Theta)$ is difficult as the log-likelihood function is not concave in variance components in general. Therefore, depending on initial guess of the parameters, a gradient based algorithm may converge to different estimates or diverge. In order to deal with this problem, I estimate the variance components in three steps. First, I estimate $\boldsymbol{\beta}$ consistently by OLS of $\tilde{p}_{ia}$ and $\tilde{q}_{ria}$ on $\boldsymbol{X}_a$. Then, I obtain the estimates of residuals from the OLS, which is denoted by $\hat{p}_{ia}$ and $\hat{q}_{ria}$.[39] Second, initial guess for the variance components are obtained using the sample variances and covariances of $\hat{p}_{ia}$ and $\hat{q}_{ria}$, such that I solve the following 8 equations in 8 unknowns.

$$\begin{array}{cccccc}
\frac{\sum_a \sum_i \hat{p}_{ia}^2}{AN_a} & = & \eta_{p1}'^2 + \eta_{p2}'^2 + \sigma_\theta' + \sigma_e' & \frac{\sum_a \sum_{i>i'} \hat{p}_{ia}\hat{p}_{i'a}}{AN_a(N_a-1)/2} & = & \eta_{p1}'^2 + \sigma_e' \\
\frac{\sum_a \sum_{i>i'} \hat{p}_{ia}\hat{q}_{ria}}{AN_a R_a} & = & \eta_{p1}'\,\eta_{q1}' + \eta_{p2}'\,\eta_{q2}' & \frac{\sum_a \sum_r \sum_{i>i'} \hat{p}_{ia}\hat{p}_{i'a}}{AR_a N_a(N_a-1)/2} & = & \eta_{p1}'\,\eta_{q1}' \\
\frac{\sum_a \sum_{r>r'} \sum_i \hat{q}_{ria}\hat{q}_{r'ia}}{AN_a R_a(R_a-1)/2} & = & \eta_{q1}'^2 + \eta_{q2}'^2 & \frac{\sum_a \sum_r \sum_i \hat{q}_{ria}^2}{AN_a R_a} & = & \eta_{q1}'^2 + \eta_{q2}'^2 + \sigma_\mu' + \sigma_u' \\
\frac{\sum_a \sum_r \sum_{i>i'} \hat{q}_{ria}\hat{q}_{ri'a}}{AR_a N_a(N_a-1)/2} & = & \eta_{q1}'^2 + \sigma_\mu' & \frac{\sum_a \sum_{r>r'} \sum_{i>i'} \hat{q}_{ria}\hat{q}_{r'i'a}}{AN_a R_a(R_a-1)(N_a-1)/2} & = & \eta_{q1}'^2
\end{array}$$

Lastly, using the initial guess of parameter values (denoted above by $'$), I apply Quasi-Newton algorithm to maximize the log-likelihood function. The maximum likelihood estimates of Θ is denoted by $\hat{\Theta}$.

Table 12: Test of Heteroskedasticity

Model Specification Observables	(1), (2) $ln(\hat{p}^2_{ia})$	(1), (3) $ln(\hat{q}^2_{ria})$	(3), (4) $ln(\hat{p}^2_{ia})$	(2), (4) $ln(\hat{q}^2_{ria})$
Project Size	−.00284	.00539	−.00502	.00456
	(.00682)	(.00289)*	(.00669)	(.00293)
ln(Distance)	.0737	.00697	−.0134	.0504
	(.0885)	(.0613)	(.0928)	(.0651)
ln(Rival Distance)	−.0489	.0609	.0623	.0298
	(.168)	(.0857)	(.158)	(.0874)
Utilization Rate	.808	.237	−.424	.154
	(.365)**	(.169)	(.394)	(.178)
Rival Utilization Rate	−.146	.155	−.429	.0648
	(.500)	(.248)	(.509)	(.264)
# of Bidders	.121	.0107	−.0956	.0225
	(.339)	(.130)	(.391)	(.131)
Project Fixed Effects	Yes	Yes	Yes	Yes
Obs	338	1296	338	1296
R^2	0.060	0.024	0.050	0.025

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Standard errors in parentheses

8.9 Reduced-Form Factor Model: Test of Heteroskedasticity

8.10 Equilibrium Computation Algorithm

Step 1: Set the level of horizontal reviewer heterogeneity arbitrarily large. Denote the pseudo horizontal reviewer heterogeneity by τ^j where τ^0 is the initial level of pseudo horizontal reviewer heterogeneity. Then, I draw 200 types from $v_{ia} \sim N(0,1)$, $\theta^f_a \sim N(0, \sqrt{\sigma_\theta})$ and $\varepsilon_{ia} \sim \hat{F}_\varepsilon(.|N, R)$.

Step 2: Guess a strategy of each bidder and denote them by $\psi^{j,k}_i(c_i)$ for $k = 0$. Given the strategy of bidders, compute the equilibrium strategy by applying the following Quasi-Newton map to every bidder of every type simultaneously (Here I suppress the dependency of strategy on $\mathbf{c_i}$ for the sake of visual clarity).

$$\psi^{j,k+1}_i = \psi^{j,k}_i - H^{j,k} J^{j,k} \tag{53}$$

where $J^{j,k}$ and $B^{j,k}$ are the Jacobian and inverse Hessian of π^{int}_i. Since inversion of Hessian matrix is

[39] A test of heteroskedasticity is conducted and presented in Appendix. I find weak evidence of heteroskedasticity.

computationally very expensive, I approximate $H^{j,k}$ (say $\hat{H}^{j,k}$) by BFSG method, such that:

$$\hat{H}^{j,k+1} = \left(I - \frac{(J^{j,k+1} - J^{j,k})(\psi_i^{j,k+1} - \psi_i^{j,k})^T}{(J^{j,k+1} - J^{j,k})^T(\psi_i^{j,k+1} - \psi_i^{j,k})} \right)^T \hat{H}^{j,k} \left(I - \frac{(J^{j,k+1} - J^{j,k})(\psi_i^{j,k+1} - \psi_i^{j,k})^T}{(J^{j,k+1} - J^{j,k})^T(\psi_i^{j,k+1} - \psi_i^{j,k})} \right) + \frac{(\psi_i^{j,k+1} - \psi_i^{j,k})(\psi_i^{j,k+1} - \psi_i^{j,k})^T}{(J^{j,k+1} - J^{j,k})^T(\psi_i^{j,k+1} - \psi_i^{j,k})}$$

A necessary condition for a Bayesian Nash Equilibrium is $\psi_i^{j,l+1} = \psi_i^{j,l}$ for all $l > K$ where K is some arbitrarily large integer.

Step 3: Upon convergence, I reduce horizontal reviewer heterogeneity by $\kappa > 0$, such that $\tau^{j+1} = \tau^j - \kappa$. Then, use equilibrium strategy $\psi_i^{j,k}$ as an initial guess for $\psi_i^{j+1,k}$.

Step 4: Repeat step 2 and 3 till $\tau^j = \hat{\tau}$, such that pseudo horizontal reviewer heterogeneity meets estimated horizontal reviewer heterogeneity in the data.

8.11 Counterfactual Experiment: Simulation Steps

Denote the realization of winning price and objective design quality in a hypothetical DB auction a by p_a^* and q_a^*, respectively. By definition:

$$\begin{aligned} p_a^* &= \left\{ p_{ia} : \frac{p_{ia}^{real}}{q_{ia}^{real}} < \frac{p_{i'a}^{real}}{q_{i'a}^{real}} \ \ \forall\, i' \neq i \right\} \\ q_a^* &= \left\{ q_{ia}^0 : \frac{p_{ia}^{real}}{q_{ia}^{real}} < \frac{p_{i'a}^{real}}{q_{i'a}^{real}} \ \ \forall\, i' \neq i \right\} \end{aligned}$$

where p_{ia}^{real} and q_{ia}^{real} are realized price and quality score of bidder i. q_{ia}^0 is the objective design quality of bidder i's design proposal. That is, p_a^* (q_a^*) is the price bid (objective quality) of the bidder with lowest price per quality score. Now, let p_a^{**} (q_a^{**}) be the winning price (objective quality) in auction a in the absence of horizontal reviewer heterogeneity, such that:

$$\begin{aligned} p_a^{**} &= \left\{ p_{ia}^{real} : \frac{p_{ia}^{real}}{q_{ia}^0} < \frac{p_{i'a}^{real}}{q_{i'a}^0} \ \ \forall\, i' \neq i \right\} \\ &= \lim_{R \to \infty} p_a^* \qquad (54) \\ q_a^{**} &= \left\{ q_{ia}^0 : \frac{p_{ia}^{real}}{q_{ia}^0} < \frac{p_{i'a}^{real}}{q_{i'a}^0} \ \ \forall\, i' \neq i \right\} \\ &= \lim_{R \to \infty} q_a^* \qquad (55) \end{aligned}$$

where (54) and (55) follow from the law of large numbers. The price (quality) cost of horizontal reviewer heterogeneity, denoted by pc_a (qc_a), is therefore defined as:

$$\begin{aligned} pc_a &\equiv \frac{p_a^* - p_a^{**}}{p_a^{**}} \\ qc_a &\equiv \frac{q_a^* - q_a^{**}}{q_a^{**}} \end{aligned}$$

Finally, define $F_p(.|N, R, \tilde{\Theta})$ as the conditional distribution function of the random variable pc_a. Analogously, the distribution of quality cost is denoted by $F_q(.|N, R, \tilde{\Theta})$.

I now describe how to estimate $F_p(.|N, R, \tilde{\Theta})$ and $F_q(.|N, R, \tilde{\Theta})$ under the assumption that a change in horizontal reviewer heterogeneity does not affect bidders' behavior. Although independence of bidders' strategies with respect to horizontal reviewer heterogeneity is a strong assumption, it has the advantage that I can obtain simple estimates of the cost of horizontal reviewer heterogeneity.

First, I obtain the parameter estimates ($\hat{\Theta}$) from the previous section by maximizing the integrated likelihood function. Second, I adjust the parameter values based on the type of experiments, and draw pseudo random variables from respective distributions characterized by N, R, and $\tilde{\Theta}$. Pseudo random variables are denoted by $\tilde{z}_a$, $\tilde{v}_{ia}$, $\tilde{\theta}_a$, $\tilde{e}_{ia}$, $\tilde{\mu}_{ra}$, and $\tilde{u}_{ria}$. Third, compute p_{ia}^{real}, q_{ia}^{real}, and q_{ia}^0 using the factor model as the data generating process, such that:

$$\begin{aligned} p_{ia}^{real} &= h_p^{-1}(\bar{\mathbf{X}}\,\hat{\boldsymbol{\beta}}_{\boldsymbol{p}} + \hat{\eta}_{p1}\,\tilde{z}_a + \hat{\eta}_{p2}\,\tilde{v}_{ia} + \tilde{\theta}_a + \tilde{e}_{ia}) \\ q_{ia}^{real} &= \sum_r h_q^{-1}(\bar{\mathbf{X}}\,\hat{\boldsymbol{\beta}}_{\boldsymbol{q}} + \hat{\eta}_{q1}\,\tilde{z}_a + \hat{\eta}_{q2}\,\tilde{v}_{ia} + \tilde{\mu}_{ra} + \tilde{u}_{ria})/R \\ q_{ia}^0 &= h_q^{-1}(\bar{\mathbf{X}}\,\hat{\boldsymbol{\beta}}_{\boldsymbol{q}} + \hat{\eta}_{q1}\,\tilde{z}_a + \hat{\eta}_{q2}\,\tilde{v}_{ia}) \end{aligned}$$

where $h_p(.)$ and $h_q(.)$ are monotone transformation functions, and thus invertible. I consider log transformation for price (i.e., $h_p(.) = ln(.)$), and identity function for quality (i.e., $h_q(.) = .$) for computational simplicity in this exercise. Lastly, I obtain the estimates of cost distributions by:

$$\begin{aligned} \hat{F}_p(\tilde{pc}|N, R, \tilde{\Theta}) &= \sum_a \frac{\mathbf{1}\{pc_a < \tilde{pc}\}}{\tilde{A}} \\ \hat{F}_q(\tilde{qc}|N, R, \tilde{\Theta}) &= \sum_a \frac{\mathbf{1}\{qc_a < \tilde{qc}\}}{\tilde{A}} \end{aligned}$$

where $\tilde{pc}$ and $\tilde{qc}$ are constants, $\mathbf{1}\{.\}$ is an indicator function, and $\tilde{A}$ is the number of repeated simulation.[40]

Further, $PWS(N,R,\tilde{\Theta})$ is estimated by;

$$P\hat{W}S(N,R,\tilde{\Theta}) \quad = \quad 1-\sum_{a}\frac{\mathbf{1}\{p_{ia}^{real}/q_{ia}^{real}=p_{a}^{*}/q_{a}^{*}\}\mathbf{1}\{p_{ia}^{real}/q_{ia}^{0}=p_{a}^{**}/q_{a}^{**}\}}{\tilde{A}}$$

The vector of parameters that are varied for each experiment are summarized as follows:

$$Experiment\ 1: \qquad R\text{and } N$$

$$Experiment\ 2: \quad \tilde{\sigma}_e,\ R,\text{ and } N$$

[40]I set $\tilde{A}=100000$